Gasdynamics: Fundamentals and Applications
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Lecture 30
Oblique shock and Expansion waves - Numerical

In the previous classes we have been discussing about Oblique shock waves and Expansion waves, their analyses and their use for finding out pressure over bodies in supersonic flow. As well as what happens when these waves interact with walls or free pressure boundaries. So, constant pressure boundary, so the concepts we have covered, one elaborate numerical we have done. Now let us do a few numerical so that the application of these concepts is clear.

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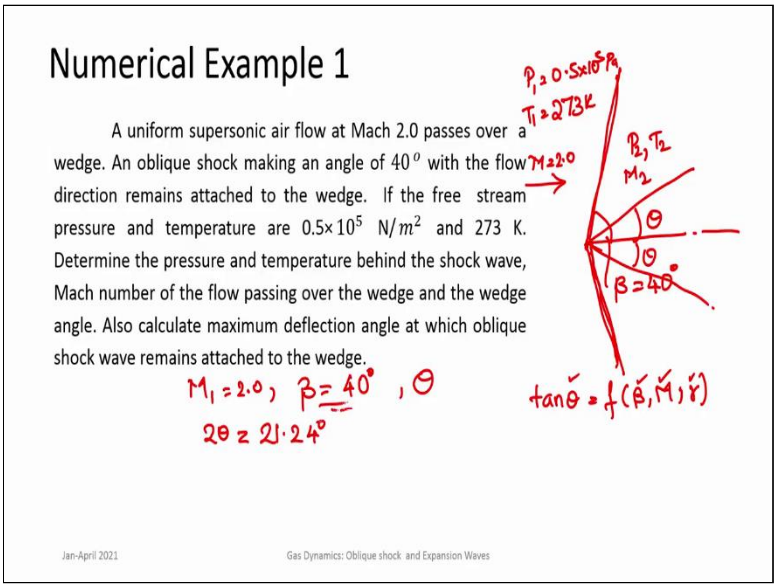

Take numerical example 1, a uniform supersonic air flow Mach 2.0 passes over a wedge. So, you can draw the wedge, so it is a wedge, Mach 2 flow is passing over it 2.0. An Oblique shock making an angle 40° with the flow direction remains attached to the wedge. So, what is given is the Oblique shock angle $\beta,$ is given 40°, remains attached if the free stream pressure and temperature, P_1 is 0.5 bar and T_1 is 273 K. Determine pressure and temperature behind the shock wave Mach number of the flow passing over the wedge and the wedge angle so, what is $\theta,$ also calculate the maximum Deflection angle at which Oblique shock remains attached to the wedge?

So, wedge is usually a symmetric profile. So, you have both θ's and so it is symmetric about this line. So, this is the wave feature that is given. So, Mach number and Oblique shock angle is given 2.0 and beta is given as 40°. So, θ can be calculated directly here. You can use the $M - \theta - \beta$ relations directly, θ you find here will be half angle, the semi half angle.

So, the wedge angle is actually; 2θ and the wedge angle turns out to be 21.24°. So directly by looking at the charts you can find this out. So, since beta is given now we have to look at what is the Pressure, Temperature after the shock.

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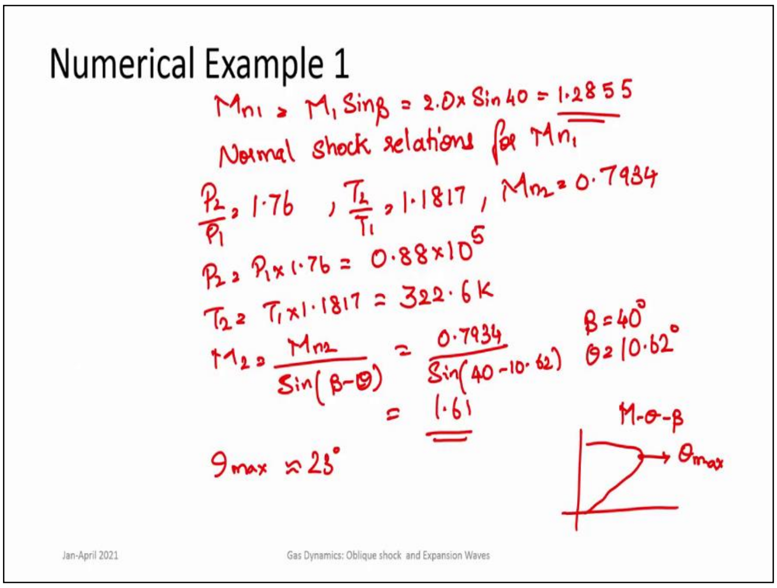

So, the way to go about this is look at the normal component of the shock. So,

$$M_{n1} = M_1 \sin\beta = 2 \times \sin 40^o = 1.2855$$

then use Normal shock relations for the normal component of the shock that is how Oblique shock problems are solved. So, for Mach number of 1.2855, use normal shock relations for M_{n1} you can use a table, or you can use a calculator and get P_2/P_1 as 1.76, T_2/T_1 is 1.1817 and M_{n2} is 0.7934.

So P_2 is P_1 multiplied by 1.76 it turns out to be 0.8 10 bars, while T_2 is T_1 multiplied by 1.1871 this is 22.6 K. Now we need M_2,

$$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)}$$

from the velocity triangles. So, we can get M_2 as 1.61.

So, you can see this is a weak shock in Mach 2 and the flow remains supersonic after the shock. Now the last question is what is the maximum Deflection angle at which flow remains attached to the wedge? So, this again you can look at the $M - \theta - \beta$ charts tables and you can look at the chart the graph of $M - \theta - \beta$ and find out the maximum angle so it would be an angle like; so, this is the maximum angle θ_{max}.

So, this has to be read out for a particular Mach number. So, for the Mach number of 2 this θ_{max} is close to 23^o. So, this is how you can solve this problem it is a direct problem on Oblique shock waves over wedges.

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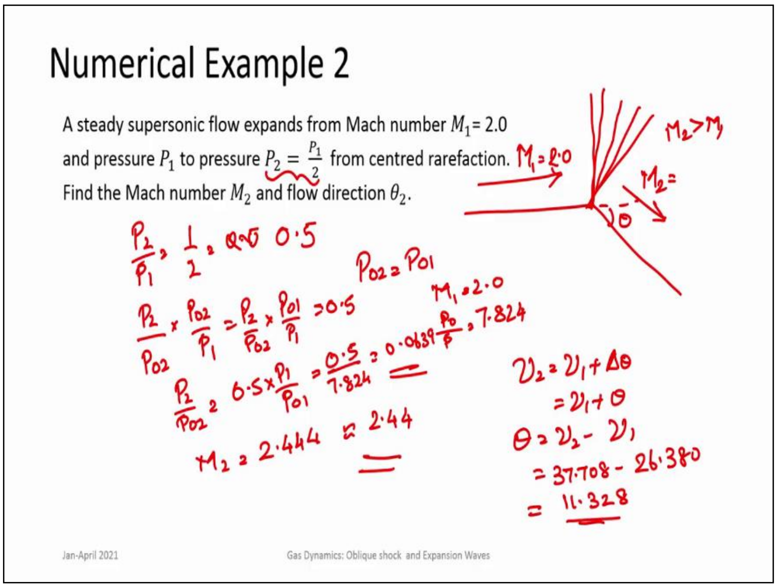

Now let us look at the next numerical example, a steady supersonic flow expands from Mach number M_1 = 2.0 and pressure P_1 to pressure P_2 = P1/2. So, there is it is an Expansion fan and whenever there is an expand flow passes through an Expansion fan pressure decreases. So that is evident here P_2 = P1/2 from centred refraction. So, you have a centred Expansion fan you have all these various expansion fans.

So, it is an isentropic process M_1 is 2.0 after expansion it undertakes an expansion into an angle θ, $M_2 > M_1$. So now what we can find Mach number and flow direction what do we know we know P_2/ P_1. So P_2/ P_1 is 1/ 2 or 0.5. So, we know this value the main idea here is flow through an Expansion plan is isentropic. So, if you know P_2/ P_1 you can calculate the Mach number.

The idea the steps are straight forward here P_2/ P_1 can be written as

$$\frac{P_2}{P_1} = \frac{P_2}{P_{02}} \times \frac{P_{02}}{P_1}$$

across an expansion fan the flow is isentropic therefore $P_{02} = P_{01}$. P_1/P_{01} can be found out for the Mach number equal to 2 isentropic charts, P_{01}/P_1 is 7.824.

So, you get this as 0.5 by 7.824, therefor P_2/P_{02} is 0.0639 and corresponding Mach number for is 2.444. So now that you know Mach number how do you find θ? Theta so $\nu_2 = \nu_1 + \Delta\theta$ or, so $\Delta\theta = \nu_2 - \nu_1$, find the Prandtl-Meyer angles these are again given in charts tables or use a calculator.

So, this is 37.708 minus 26.380 this is 11.328 so this is the deflection angle. So, the flow is deflected away from M_1 by 11.328°. So, this is a problem on Expansion fan. So here the interesting fact was the pressure ratio was given and from there knowing the upstream Mach number we have calculated all other parameters. So again, you need 2 parameters to solve either Oblique shock or expansion fan.

So, they can be a combination of any 2 parameters upstream Mach number pressure ratio or Mach number and the angle.

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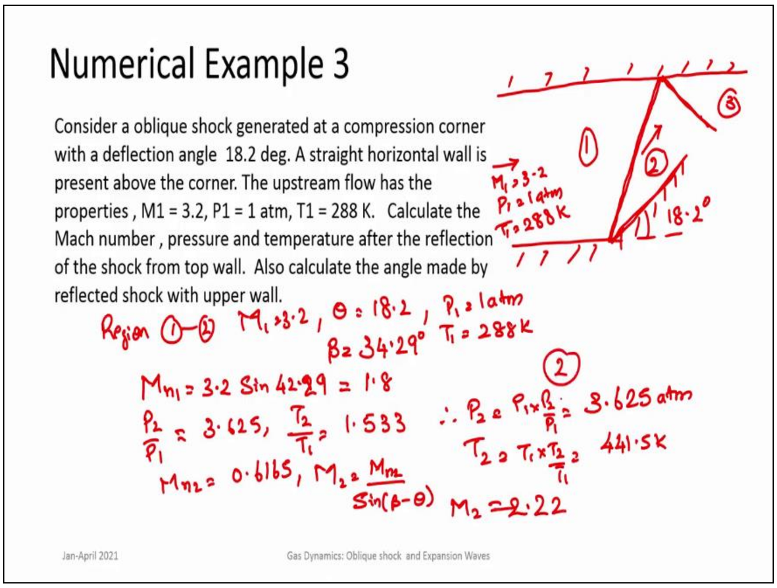

So, these are the various combinations you can always work out different problems with different combinations of these parameters. Now we look at a problem on Oblique shock reflection we had an elaborate discussion on various kinds of reflection there were regular reflection Mach reflection, Mach reflection is a form of irregular reflection but calculating regular reflections are quite involved.

Normally one uses numerical methods to calculate them or graphical means like using a shock polar. But regular reflection can be done on paper. So, we can do a problem on regular reflection. Consider an Oblique shock generated at a compression corner with a deflection angle 18.2^o. So, compression corner is a corner one which the wall has just turned into itself. So, deflection angle is given 18.2^o that is θ and a straight horizontal wall is present above the corner.

So above this corner there is a straight horizontal wall the upstream flow is $M_1 = 3.2$, $P_1 = 1$atm and $T_1 = 288$ K and calculate Mach number, Pressure, Temperature after reflection. So, what happens, because it is a compression corner and there is a turn of the flow into itself a shock wave is formed. This shock wave impinges on the wall which is present above.

After impingement, the flow has to turn in such a way that it is parallel to the top wall. So, after the compression corner the flow would have turned parallel to the compression corner. Now at the wall the flow will turn parallel to the wall. So, the wall behaves like another wedge for the flow in region. So, this is region 1 this is region 2 and this is region 3. So, we are required to calculate properties in region 3.

So, we progress from region 1 to region 2, region 2 to region 3. So, from region1 so region 1 to 2, so what is given M_1 is given 3.2, theta is given 18.2^o, P_1 is 1 atm, T_1 is 288 K knowing these parameters we can calculate region 2. We know M and theta, if M and θ are known β can be found out use a chart or a calculator. So, β turns out to be 34.29^o.

So, once you know beta then way to go about Oblique shocks is calculate M_{n1} normal component of the Oblique shock to the Oblique shock. So, $M_{n1} = 3.2 \sin(34.29)$, this is 1.8. So now P_2/P_1 is 3.625 and T_2/T_1 is 1.533 therefore we get P2 3.625 atm and T2 = 441.5 K.

Now we need M_2 so M_{n1} is 1.8, M_{n2} is 0.6165. So $M_2 = M_{n2} \sin(\beta - \theta)$ substitute the values and you get M_2 is 2.22. So, we know the properties at region 2 so these are all properties at region 2. Now we go from region 2 to region 3.

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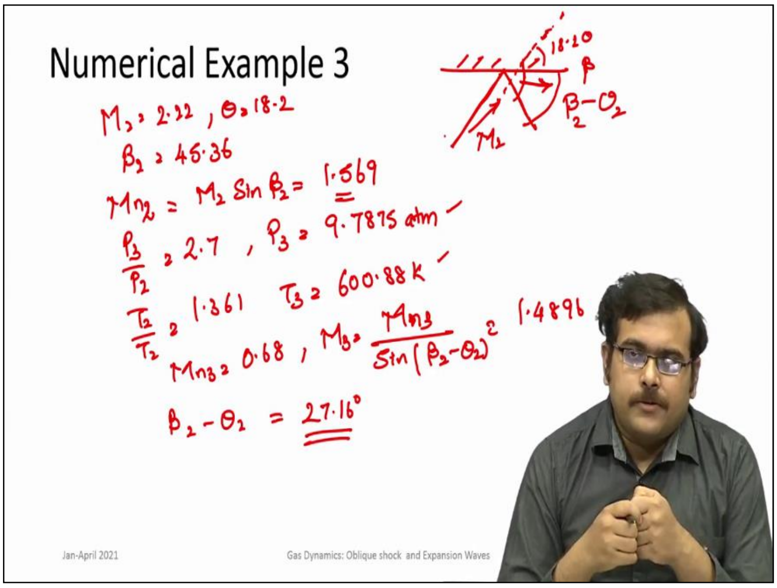

From region 2 to region 3 when we go so, we have let us remember diagram. So, the shock is coming here, and the flow is deflected, and it meets this wall the deflected flow meets the wall and another shock is formed such that after the shock the flow turns parallel to the wall. So, the flow angle that it turns by is the same as it got deflected that is 18.2^o. So, this is the reference line now this is theta while this angle is beta.

So, but Mach number now is M_2, M_2 is 2.22, $\theta = 18.2$. So, for this β is 45.36 calculate M_{n1} or rather it should be M_{n2} because in the region 2, M_{n2} = M_2 sine 1.569, for this you get P_3/ P_2 = 2.7. So P_3 is 9.7875 atm while T_3/ T_2 is 1.361 which means T_3 is 600.88 K and M_{n3} is the downstream Mach number after the normal shock for 1.569 it is 0.68.

So M_3 = M_{n3} sine $(\beta_2 - \theta_2)$ which turns out to be 1.4896. So, you see that Mach number keeps reducing pressure and temperature keep increasing after each shock after the flow passes through each shock and angle that the reflected shock makes with the upper wall what is that angle it is this angle which is $'\beta_2 - \theta_2'$. So, $\beta_2 - \theta_2$ is 27.16^o.

So here we solved a numerical example for an Oblique shock reflection. So, you should understand that solving shock reflections involves solving Oblique shocks for each region and it is not as simple as is this for say reflection of light where you just do $\theta_1 = \theta_2$ angle of reflection is angle of equal to angle of incidence. So, it is not that do not get confused. For shock wave reflection you must calculate the Oblique shock relations for each region as the Oblique shock reflects.

After the reflection the flow will become parallel to the wall. So, in this case the wall was horizontal you could have walls at angles. So, there are various combinations of parameters here also and different such applications can arise. So, with this we have covered extensively on Oblique shock waves and Expansion waves. So, all through the discussion for a while now we have been concentrating on specific flow features, they may be normal shock waves or Oblique shock waves or Expansion waves.

So, they are all flow features that can be found in a flow. Now we move on from this point towards applications of gas dynamic principles to certain devices like nozzles diffusers. They are a class of flows where there is change in area. So, the area changes as the flow happens within a duct. How do we solve such problems or understand in simple terms how these devices behave?

What does area change do to a compressible flow that is the theme of the next module we will begin starting the next module with varying area ducts.