Gasdynamics: Fundamentals and Applications
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Lecture 29
Shock Reflection

So, we are looking at Oblique shock waves and Expansion waves and we have looked at how to analyse Oblique shock waves and Expansion waves. Done an elaborate problem using a Diamond wedge configuration in supersonic flow and saw how we can calculate pressures at each region using the Oblique shock waves and Expansion waves. Now we will move on to a related topic what happens when these waves interact with a wall.

It is often found in engineering applications you can have these Shock waves and then the Shock waves produced by say a body can come and interact with the wall at a downstream location. So, what happens at that interaction. So, these are reflections of Shocks and Expansion waves. So, we will look at reflection.

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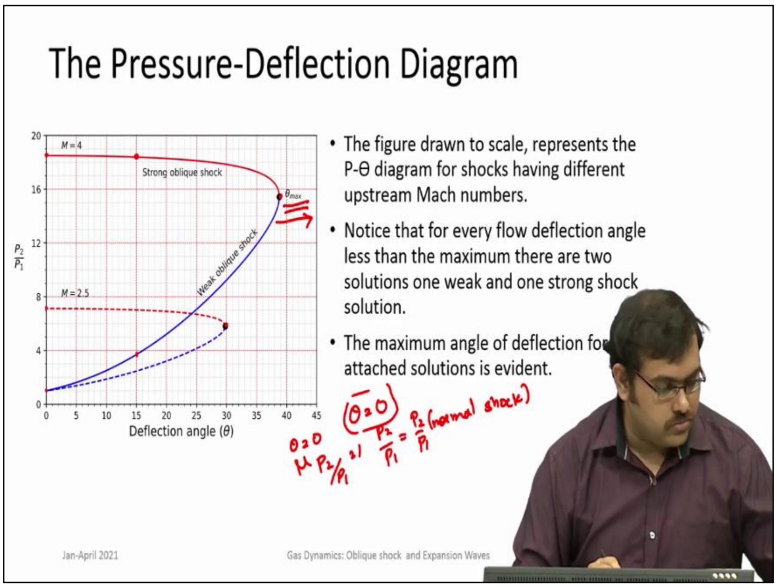

Before we go into reflection itself, it is useful to look at what are known as Pressure-Deflection diagrams. They are a plot of all possible Shock waves; Oblique shock waves also includes the Normal shock, a special case of an Oblique shock. So, if you plot all possible Shock states for a particular Mach number the pressure ratio across the Shock versus the deflection angle for one given.

So here a particular curve is for one given Mach number the top one the solid line is that for Mach number 4 and the dotted line is for Mach number 2.5. These kinds of diagrams are useful when we look at interactions of Shock with say solid surfaces, we will soon come to know what happens there and how these Deflection diagrams, Pressure-Deflection diagrams, can be useful. Now notice that you have 2 sets of solutions for any given deflection.

This we know for an Oblique shock for a given Mach number any given deflection there are 2 solutions. If I take say 15^o there is one weak solution and there is one strong Shock solution. Pressure ratio for strong stock solution is much higher and Mach number downstream of Shock is subsonic.

Indeed, there are attached Shock solutions only until the maximum angle that is angle of deflection so that is why this curve has a maximum that is at θ_{max}. Beyond θ_{max} for this Mach number there is no attached Shock solution. The Shock has to detach. So, you can draw these curves for various Mach numbers and comparison is also given. All of them start with a Mach wave.

So, the weakest possible Oblique shock here is a Mach wave. So, you have that deflection angle there is 0, $\theta = 0$, while the angle of the wave is $'\mu'$ and the pressure ratio is 1. The maximum possible pressure ratio is that for a Normal shock for that given Mach number even there the angle of deflection is 0 and the pressure ratio of the Normal shock.

So, these one should be aware of when one looks at Oblique shocks. So, even for angle $\theta = 0$, you can see there are 2 solutions one is a microwave the weakest possible Oblique shock and the other one is strongest possible Oblique shock that is a Normal shock. And the maximum angle is represented here for another Mach number it will be a different point θ.

So, you should be aware of this Pressure-Deflection diagram, it is often used to represent certain solutions when we look at interaction of Shock waves. So, we will go ahead and look at reflection problem.

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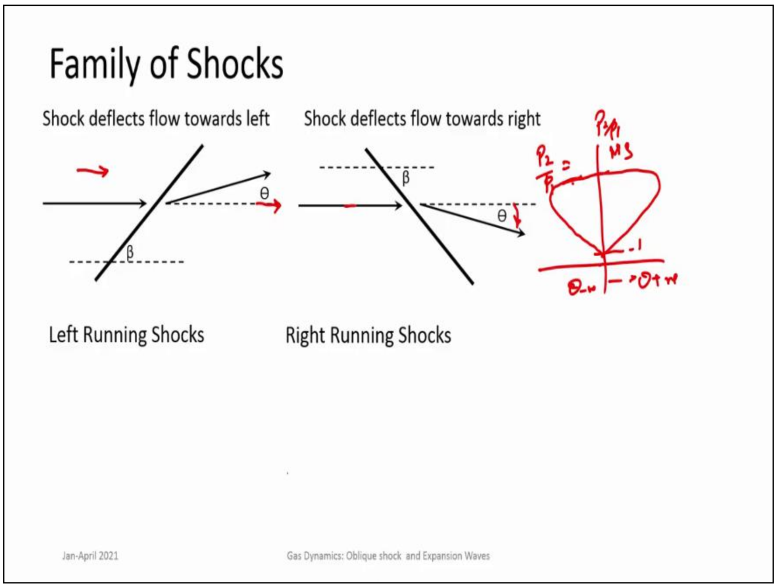

So, one should be aware that the flow can be deflected. So now if you look at the Shock wave alone the flow is deflected towards the flow that is the incoming flow, so this is θ. Now if you consider the incoming flow, it can be deflected either left or it can be deflected towards the right. So, the Shock can do the leftward deflection or rightward deflection. So accordingly, Shocks get classified they are known as families of Shocks.

If it deflects the flow towards the left of the main flow, then the Shock is known as left running Shock. And you can see that the shape of the Shock itself is different from on both sides they have the same angle beta. But the way they produce deflection is different, so this is right running Shock it deflects the flow towards the right. So, each so all left running Shocks form one family all right running Shocks form another family.

But if they have the same angle $'\beta'$ with the free-stream flow direction, the pressure ratios across these Shocks are the same. But the flow deflections if I consider the plot one case say if I take the left running Shocks as positive. So, this is $\theta + ve$ and if; I take right running as $\theta - ve$.

Then Pressure-Deflection diagram would look symmetric across y-axis, which represents ratio of Pressures across the Shock, and it will be symmetric because pressure ratios remain the same. So, this understanding one must have and when looking at Shocks and their interactions you should always check what whether it is left running or right running.

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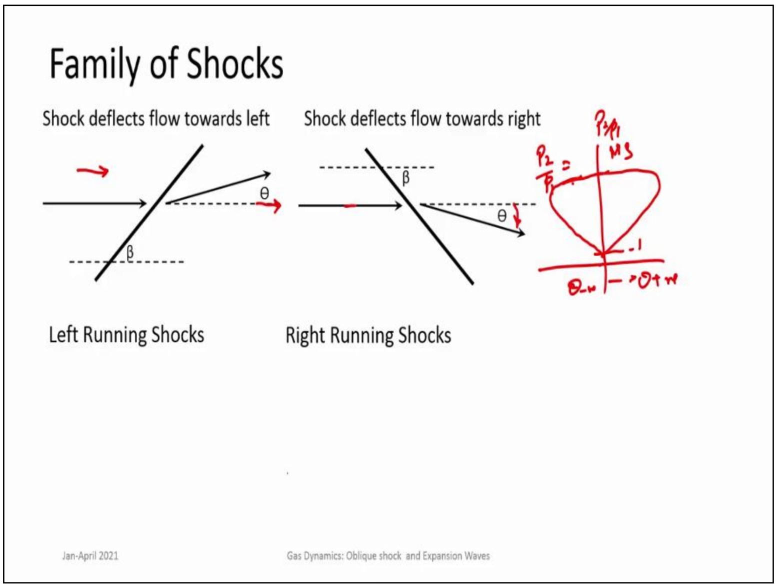

So now we come to reflection of Shock waves. Now this has to be clearly differentiated from reflection of sound or light or such kind of waves. These light waves or sound waves are governed by the linear Hyperbolic equations and their reflections follow the loss of reflection of light, you very well know this if it is a specular reflection. It is a mirror surface then angle of incidence equal to angle of reflection you know this very well from basic physics.

Do Shock waves follow the same thing, no they do not, so Shock wave is not a wave in the sense of a sound wave or a light wave. So, the Shocks do not follow these equations that $\theta_i = \theta_r$, rather the way to analyse the Shock wave reflection is to look at each Shock that is the incident Shock is coming and impinging on the surface and then there is a reflected Shock, reflected wave.

Now a flow is happening, now how would we analyse this problem, the problem has to be analysed by looking at each Shock separately looking at conservation equations across each Shock. So, each of them is an Oblique shock. So, this Shock turns the flow towards the wall so you get a supersonic flow now that is turning towards the wall, but the wall is a boundary it cannot allow flow to go into it.

So, because of that no penetration condition you get a reflected Shock wave which again turns the flow back parallel to the wall. So, this is the concept here, so when we talk about reflection of Shock waves please do not confuse it with already known things about reflection of waves and follow the ideas like angle of reflection is equal to angle of incidence. So that is not true for Shock waves, so for Shock waves you have to do the analysis. So that is important. So, we will see how to go about doing it.

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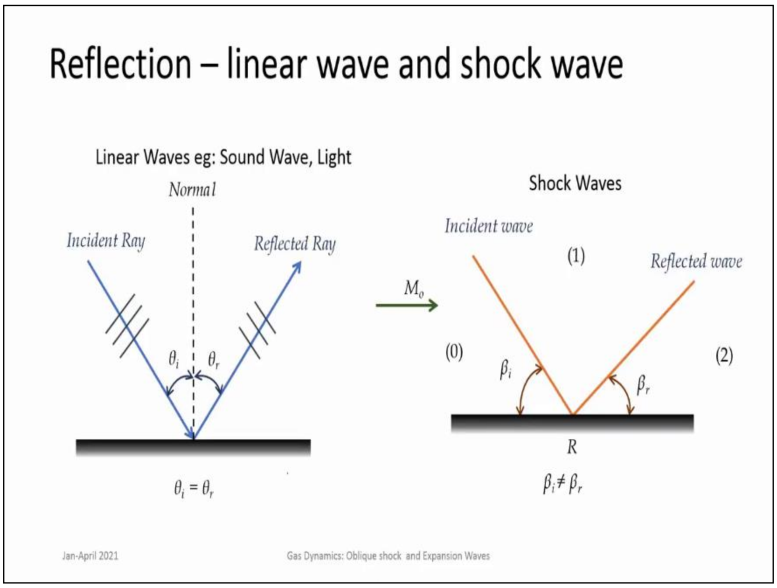

Now if you look at the waveforms formed during reflection. So, Shock wave is do not follow any linear principles they are completely non-linear. So, they form many different reflection patterns, one pattern which was just now we were describing is here it is called a Regular reflection pattern this happens when the incident Shock waves are quite weak for the given Mach number. Then the pattern that formed is you have an incident Shock, to match boundary conditions at the wall a reflected Shock is formed.

We have to do the analysis to get what are these angles β_i, β_r and what are the properties at 0, 1, 2 this analysis has to be carried out. But the formation is that of you have a incident wave and the reflected beam. But there are certain situations you can see when the incident waves are quite strong, and they come and are incident on a wall then situation can so happen that this solution that we have here is no longer possible.

It is no longer possible to satisfy all the boundary conditions at the wall so then consequently you get another kind of waveform this kind of wave form is called an Irregular reflection when we talk about reflection in this course mostly, we will be dealing with Regular reflections. But one should be aware that there are several others, Regular reflection is only a special case there are several other kind of reflection patterns and one kind of a pattern involves these three Shocks that you have an incident Shock you have a reflected Shock, and you have what is known as a Mach stem, at the surface this is normal to the surface.

So, it is at the surface if you have a streamline here it is a Normal shock for this streamline but reaching close to this point where the three Shocks are meeting it is called a Triple point. So, it is T, so you have the same M_0 coming. So now you see that M_0 for this stream the flow passes through 2 different Shocks. So, it passes through 2 Shocks that is incident Shock and reflected wave.

But an M_0 coming close to the wall, below the Triple point, it passes through a single strong Shock over here. So, then region 3 and region 4 they will have differences we know there is a virtual line here which is the Slip line. Boundary condition here is that across the slip line pressure should be equal, at the place where the slip line leaves the angles of these 2 should be, they are parallel to each other.

So that is the conditions that we have. So, this kind of a structure is known as a Mach reflection. There are many kinds of Irregular reflections. So will not go into too many details of all kinds of Irregular reflection but this flow picture one has to be aware of that there are Regular reflections and there are Irregular reflections.

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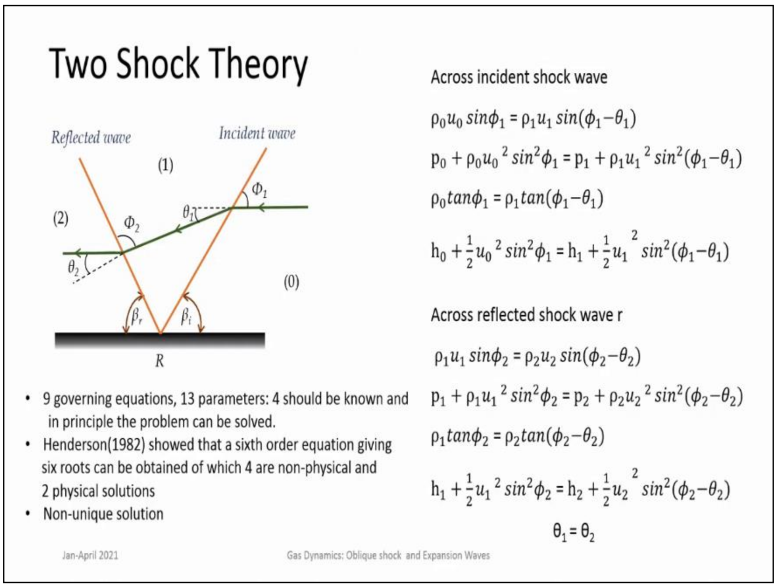

So how do you analyse these reflection patterns. So that is what you have to solve between region 0 and region 1. So, this is the set of equations that is written for the Oblique shock between region 0 and region 1. Similarly, you have to write set of equations for the regions 1 and 2. So these are the regions 1 and 2. So, you write separate set of equations between them.

You have the boundary condition at the wall that the flow remains parallel to the wall after all these interactions again it becomes parallel to the wall. So initially it had a deflection θ_1, the same deflection in the opposite direction must be given. So, this is θ_2 and $\theta_1 = \theta_2$. So, what you have here is series of such equations and you have 9 equations here and 13 parameters and you need to know certain parameters, 4 parameters should be known to solve this problem.

You do not have all same, so you do not have any unique solution. So, there are non-unique solutions and usually 2 physical solutions are possible. So, this is how to do it analytically we will see a better way to look at this is using the Pressure-Deflection diagrams.

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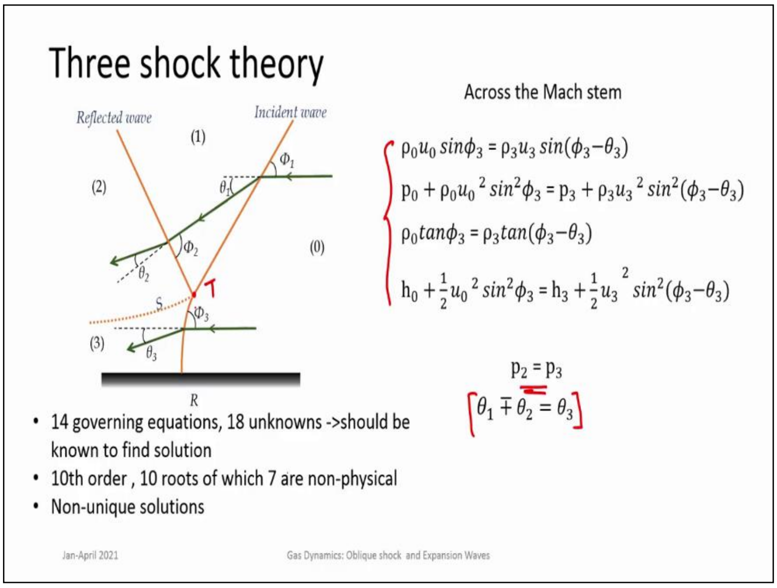

For the Irregular reflection also, you can do the same kind of analysis. So now you have these relations across the incident Shock across the reflected Shock and across the max stem. So, you can write they are all Oblique shocks they are doing the analysis at the Triple point. $P_2 = P_3$ that is known. So, then you can apply the boundary condition here that the flow should be parallel, it depends on how the angles are deflected you can have $\theta_1 \pm \theta_2 = \theta_3$.

So, these conditions can be given again the number of equations is large. You have 14 equations 18 unknowns and again this is a case of non-unique solutions. This is how to do it analytically it is quite involved.

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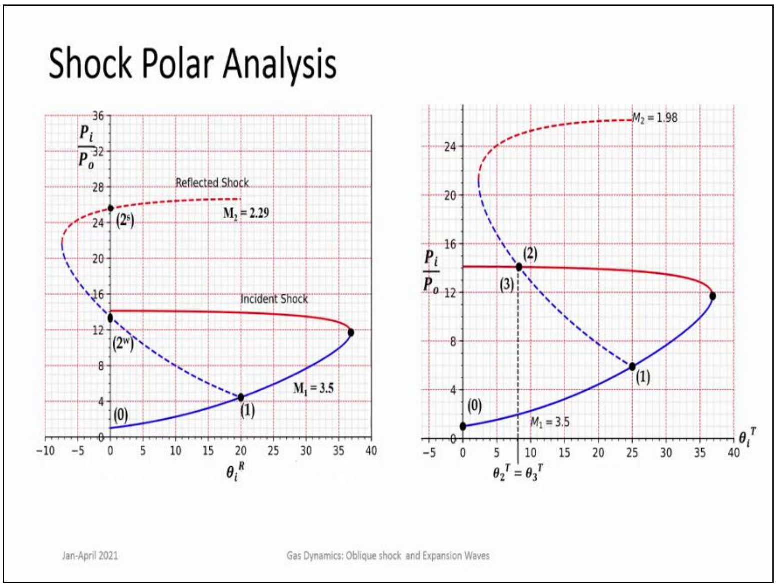

So largely what people do is look at Shock polar, now this Shock polar can be drawn for each region. So, because the Shock polar is a plot of all possible Shock waves. You can look at the Shock polar and try to find the solutions. So, the incident Shock so given a particular incident Mach number and an incident Shock you have all possible Shock waves for that Mach number is given here.

You know the incident Shock value. So, you know what deflection it has given. So, you this is the point that is region 2 for the incident. If I take region 0 and region 1. So, this is the incident Shock, and this is the reflected Shock. It has a certain flow deflection this is known to us. Now at region 1 we know the Mach number from the analysis of the first Shock what is M_1.

Since we know M_1 we can plot all possible Shocks of M_1 and now you can see that these are the families of Shock. So, this was one family of Shock but now the reflection is the other family of the Shock and all possible Mach numbers, all possible Shocks can be plotted it is another Pressure-Deflection diagram. We know the boundary condition at the end that theta should become 0, again initially it was 0 and then finally it is 0.

So, we seek a solution where $\theta = 0$, and we find there are 2 such possible solutions, one is a weak solution and the other is a strong solution. So, this is how we can effectively use the Pressure-Deflection diagrams to look at such interactions. Similar solution can be done for even the Mach reflection. So, in Mach reflection you find that the second Shock polar that is drawn does not intersect with the $\theta = 0$ curve.

But rather at a certain location here on the main on the M_0 curve. So, M_0 curve gives the condition $\theta_2 = \theta_3$. So, this is the solution for a Mach wave regular reflection kind of a problem. So, Shock polar can be drawn for such kind of problems and they give a graphical and visual means to look at these problems.

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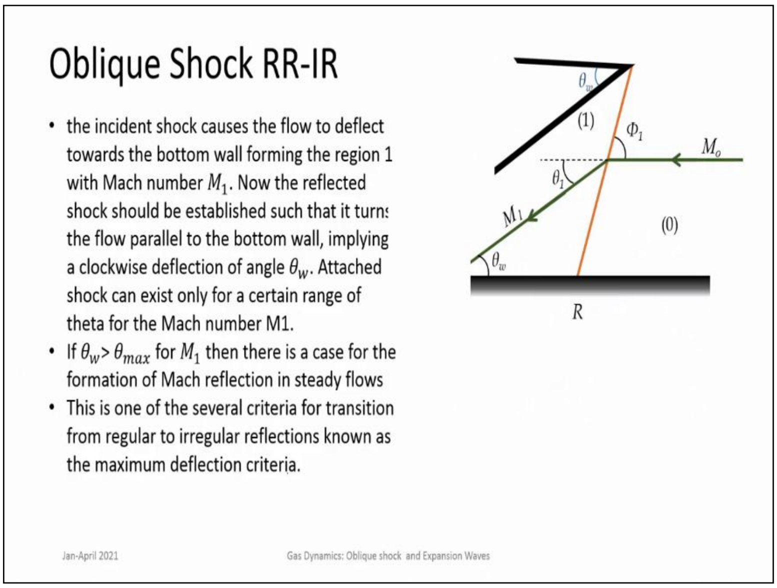

Now just a note that we were talking about Regular reflection and Irregular reflection. So why this kind of differences occurs there are several possible reasons why these occur one of them is that we know that if you look at any Oblique shock problem then there is always a maximum angle beyond which you cannot have attached solutions. The Regular reflection is like an attached Shock problem that the Shock is attached to the wedge.

But what happens when I continue to increase the angle of deflection. So that for the Mach number in region 1, M_1 and beyond the maximum angle which is the case here in the Irregular reflection. See this angle is much greater than the one for attached solution. You needed attached solution at $\theta = 0$ but there is no such solution possible. Therefore, now the flow must take another course and it takes the Irregular reflection.

So, this criterion is known as Maximum Deflection criteria. But in literature one can find many, many different criteria. So, this is just a note and for people to sort of think about that there are such problems in Shock reflections, and it is interesting to look at this non-linear behaviour of Shock reflections.

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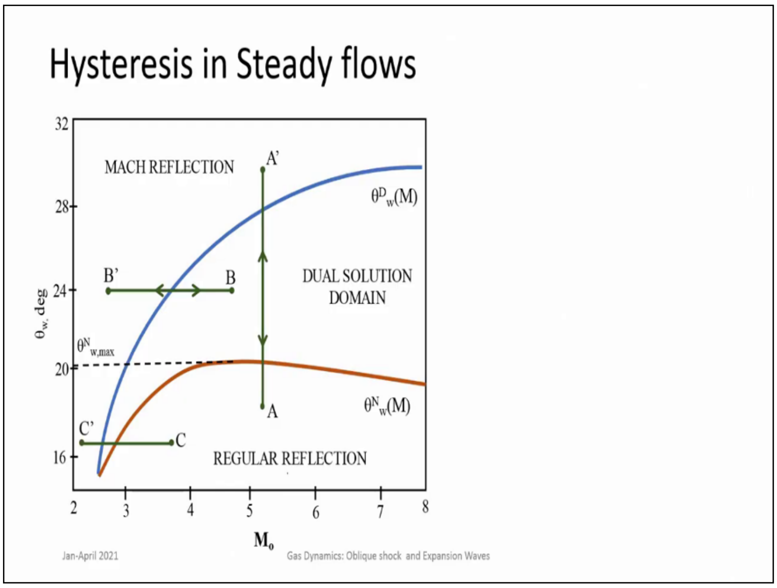

Another important feature of non-linear behaviour is Hysteresis that is the possibility that you can have multiple solutions. In certain situations, you will find one kind of solutions in certain other situation you can find another kind of solution. As you change the parameters it is not necessary that the way point, where the switch over between one kind and the other kind happens remains the same.

For example, this is a sort of a phase plot you are having Mach number and the angle of the wall θ or the angle of the wall that θ_w and you are looking at the reflection. So, there are 2 kinds of reflection shown here Regular reflection and Mach reflection. What lies between these 2 curves this is the region known as Dual solution domain, where you can observe either Mach or Regular reflection.

Below this curve you always get Regular reflection above this curve you always get Mach reflection but in between these 2 curves it is possible to see both. For example, while if you start off from a point 'A' and you are going for a given Mach number you keep increasing the angle θ_w. You keep increasing the angle then you might start getting Mach reflection from this point.

So, you will observe Regular reflection all the way until this point. But now if you start reversing the direction and start decreasing the angle you can keep observing Mach reflection all the way until this point and then only it turns to Regular reflection. So, you see that this behaviour is like Hysteresis. It is a hysteresis that you do have regions where both these solutions can be observed, and it depends on the way you go into the history of the flow whether you observe a Mach reflection or a Regular reflection.

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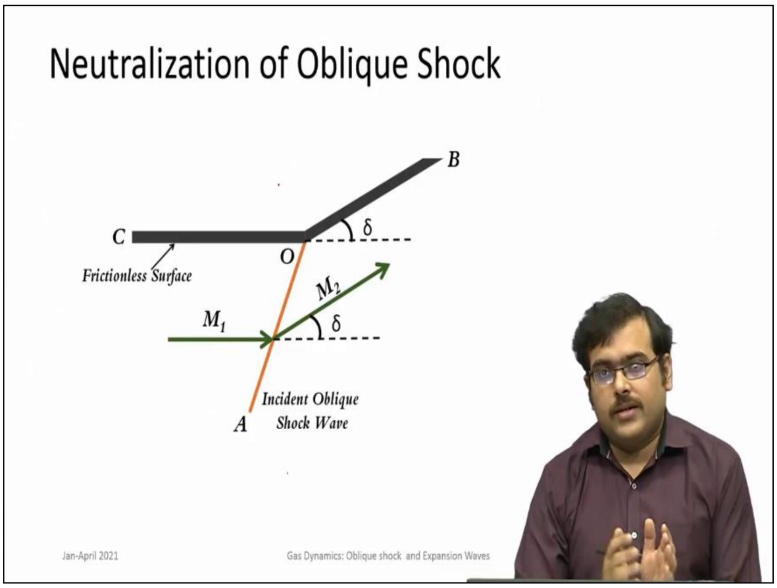

So, this is some interesting non-linear behaviour. Also, so Oblique shocks are quite different from the other waves if you can give to the wall a certain angle such that it matches the angle that the Oblique shock turns the flow with. So, it turns the flow by angle delta and at this point of intersection with the wall. The wall is also turned by angle delta. Now there is no requirement for the flow to do any further turn.

Therefore, you will not get any more waves. So, this is a method by which you can neutralize the Oblique shocks. So, no more waves will be formed here.

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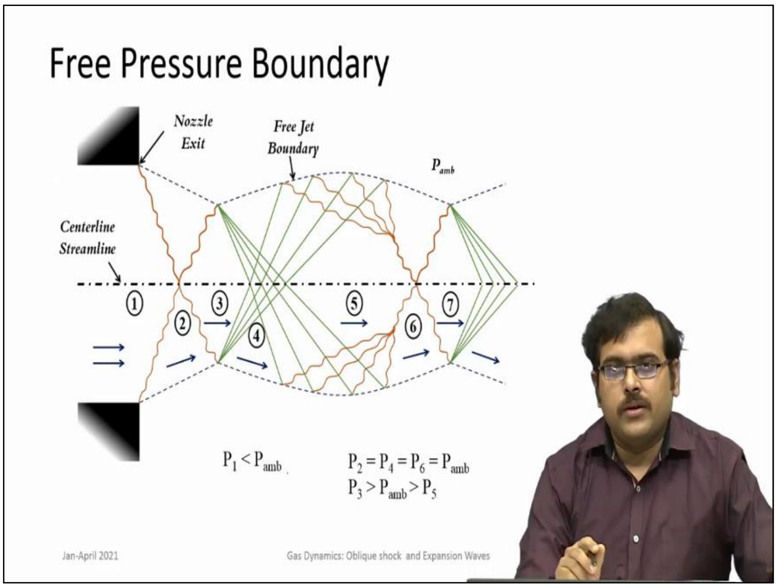

So also, often in flows you find these virtual boundaries in the flow which have constant pressure. So, there you already saw a few of them such as the slip line where you had P_2 and P_3 they are the same, but velocities can be different, or temperatures can be different, densities can be different. But Pressure remains constant across these boundaries in the flow virtual boundaries in the flow.

Slip, they are usually referred to as Slip lines. Now if a Shock interacts with the slip line example, if one we will soon come across these in further classes when a jet exits into a jet of high-speed flow supersonic flow exists into ambient into the general ambient. Then ambient has a certain pressure ambient $P_{ambient}$. But within the jet the pressure need not be the same as $P_{ambient}$.

So, for example here you have Shock waves, Shock waves are compressive in nature they can increase pressure so P_2 can be quite; so, P_1 here in this case can be quite low compared to ambient pressure. Consequently, a Shock wave is formed so that at the boundary the pressure is maintained constant. So, in P_2 the pressure is equal to ambient, but Shock interactions can happen again with this boundary.

So, there is another Shock here P_3 so if this was P ambient then at 3 pressure has increased again. So, P_3 is greater than P ambient now this Shock comes and interacts with the free pressure boundary. So, when Shock waves interact with the free pressure boundary what should happen in the succeeding region actually this region the pressure should remain constant. But across the Shock pressure will increase.

So, this was P ambient earlier. So, across the Shock pressure will increase but at the boundary pressure should remain constant that cannot happen. So consequently, it generates Expansion waves. So, Shock wave when it interacts with a free pressure boundary it generates Expansion waves.

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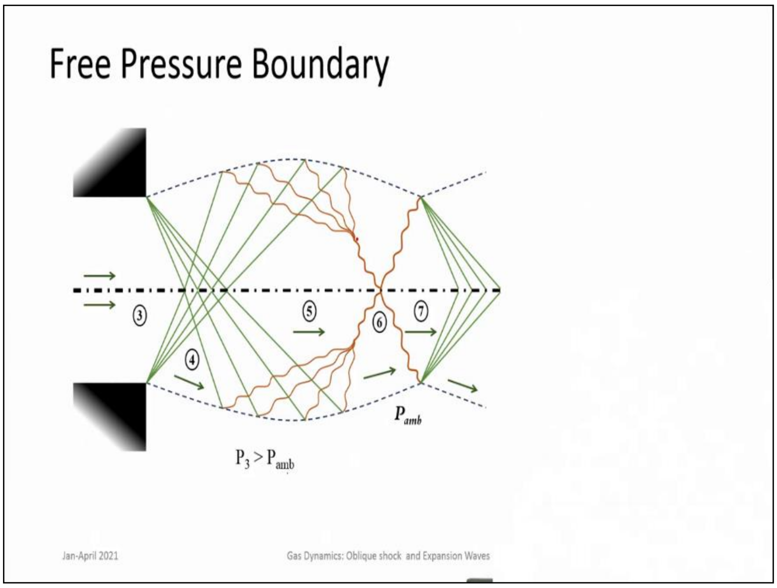

The vice versa holds good Expansion waves when they interact with the free pressure boundary, they lead to formation of compression waves which can coalesce together to form a Shock. So, like waves when like waves interact a Shock interacts with solid wall it reflects as a Shock wave. When it interacts with the free pressure boundary it reflects as an expansion wave it changes its character.

Similarly, Expansion waves when they interact with the solid wall they reflect as Expansion waves but when they interact with a free pressure boundary they reflect as compression waves and which further coalesce form Shock waves. So, this is important so distinguish between reflections from solid wall and free pressure boundaries like slip lines. So, this understanding is important.

So here we have completed our discussions regarding the reflection of Shocks we understood that it is not like sound waves or light waves it is quite different. You have to analyse each Shock apply boundary conditions at the wall and only then you can solve this problem. And it is possible to have different kinds of waveforms at reflection one kind is a Regular reflection which has an incident Shock and a reflected Shock.

The other kind a typical example is a Mach reflection which has an incident Shocks a max stem and an incident and a reflected Shock. So that is a Mach reflection, and several nonlinear phenomena can be present there like hysteresis or you can even neutralize the Shock if you provide an angle of the wall same as the deflection angle for the Oblique shock. And if the Shock waves interact with free pressure boundary they reflect as Expansion waves.

If Expansion waves hit the free pressure boundary they will reflect as Compression waves which coalesce to form Shocks. So let us look at some few numerical in next class to understand this Oblique shocks and Expansion fans.