Gasdynamics: Fundamentals and Applications
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Lecture 26
Oblique Shock Waves

We move on from One-dimensional flows that we have been doing till now to Two-dimensional flows, increase the dimension and we look at Shock waves, in Two-dimensions they form what are known as Oblique shocks. We will understand Oblique shocks and Expansion waves in this module and look at how to apply them. So, till now we have been doing Normal shocks we did stationary and moving normal shocks.

We understood the unsteady flows in the context of a Shock tube. So, now we move on many of the principles we learn there will be applicable here in Oblique shocks, also something new because you have an added dimension.

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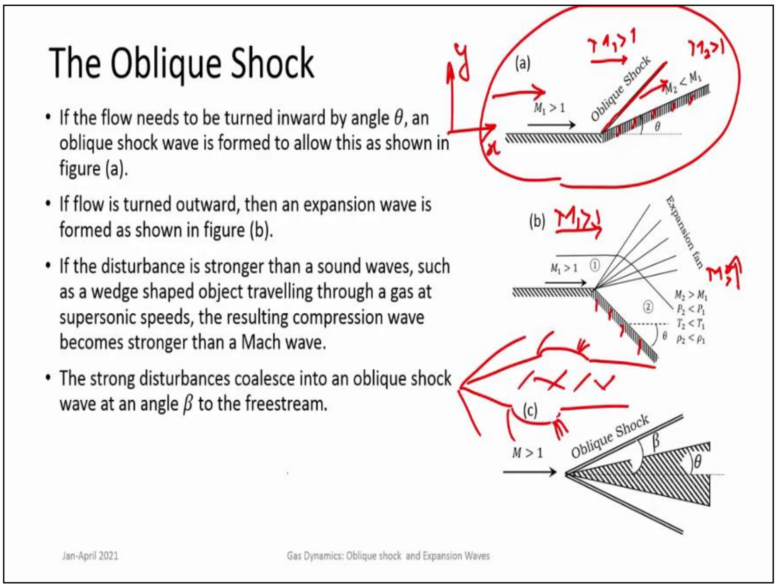

So, when are oblique shocks formed? Whenever the flow must take a turn, for example, as depicted in this schematic over here. You have an oncoming flow which is supersonic, always you should remember the Shock waves are found in supersonic flows. So, a supersonic flow is coming and then it faces a turn. There is a turn which turns the flow towards into itself or towards itself.

So, when the flow turns towards itself then it forms an Oblique shock. The way Oblique shock is formed in principle is the same as we discussed in the case of a Normal shock, you can have several compression waves and all of these compression waves will ultimately join together to form the Oblique shock. Now you can see this Oblique shock is at an angle to the upstream flow.

After the Oblique shock, the flow gets turned towards itself and parallel to the wall that is over here. So that is what happens across an oblique shock, and it is a shock wave, it is a compression wave. So, pressure, temperature, density increases, and Mach number decreases across the Oblique shock, but unlike the Normal shock you will find that in an oblique shock it is possible to have Mach numbers downstream of the shock to be greater than 1.

So, this is 1 case where you have a shock wave, Oblique shock wave, flow before the shock wave is greater than 1, and flow after the shock wave can continue to be greater than 1. It is a possibility in Oblique shocks. So, one kind of a turn that the flow can experience is a turn towards itself and producing a compression in that case it produces an oblique shock. The other case is when the flow turns away from itself which is described over here the flow is coming at supersonic speeds.

Now it must turn away, now you see the wall is turned away by an angle 'θ' and so we have already discussed, something that you should always remember that Shocks are always compressive in nature you do not have an Expansion shock. Similarly, in this case also you will not have an Expansion shock, but instead you have an Expansion fan which is an isentropic process where entropy remains constant and across the fan the angle is turned away from the initial direction.

This expands or rather accelerates the flow, the Mach number after an expansion fan increases. So, Mach number increases, pressure, temperature, and density will decrease. These kinds of shocks are found in any kind of bodies that are there in supersonic flows. If you have a typical structure something of this kind having many different shapes, then you will find Shock waves here.

You can have Shock waves at this corner but when the flow turns away here you can have Expansion fans. These are typical to any aircrafts or spacecrafts or in other cases duct flows that are present. We discuss here, in the context of Two-dimensional flows, that it is in x-y domain. So, they are like flows over flat plates or wedges, then they are the class of flows like flows over cones or axis symmetric bodies their equations are different, we will see towards the end of this lecture. There are slight differences, but basic principles do not change.

So, if you understand Oblique shocks, you can understand Expansion fans, you will understand several aspects of flow features around bodies in supersonic flow.

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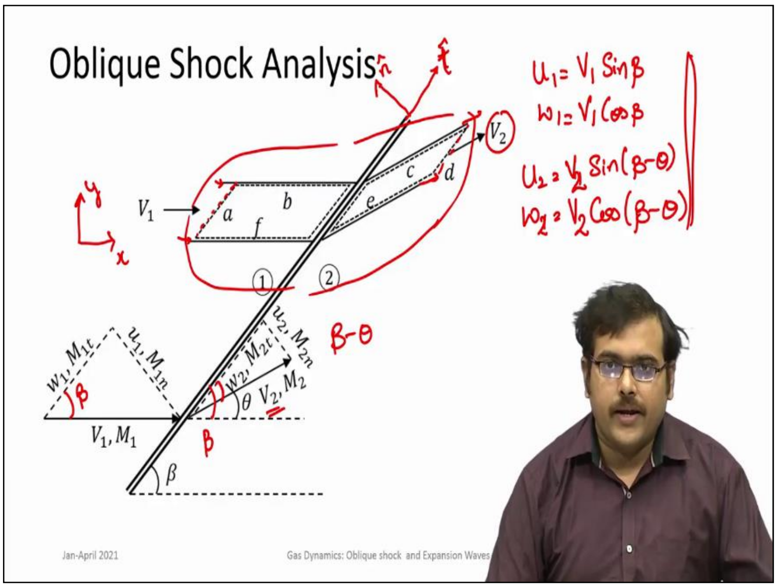

So how do we analyse oblique shock? So oblique shock analysis borrows things from Normal shocks as well as from the idea, that you had when doing moving Normal shock that you jump on to the shock. So even in Oblique shock, it is the same thing, you do this as a stationary flow, so it is a steady flow, it is not an unsteady flow, it is a steady flow. But now you see this has 2 dimensions, the Shock is oblique, it is at an angle to the flow, it is not normal to the flow.

So, there are both components in x and y direction, in both directions. So, x direction as well as y direction and the way to analyse this is jump onto the shock and sit at the shock and then analyse the flow features before and after the shock in the frame of the shock so that is how it is done. So, the analysis is done by taking components parallel and normal, so this is normal this is the tangential components.

So, you take components normal and tangential to the shock. And as we did for normal shock here also, we use the control volume. So, you can see the control volume being drawn here

‘V_1’ is the velocity upstream velocity, and after the shock the velocity is ‘V_2’. The flow has been turned by an angle ‘θ’. Oblique shock forms at an angle ‘β’. θ and β are not the same.

So, β is always higher than θ. So, the important fact here is you take a control volume, and this control volume is drawn enveloping the shock wave and these 2 faces are parallel to the shock wave. So, you are doing the analysis in the frame of the shock Oblique shock. Also remember this velocity diagrams the decomposition of the velocities into different components which is normal and perpendicular to the Oblique shock.

So now this is the Oblique shock, this is the incoming stream here V_1 and the stream forms an angle β, or the shock wave forms an angle β to the upstream incoming flow. So, this angle is going to be β. So, the stream is now decomposed into components which are parallel and perpendicular to the Oblique shock. So, ‘u’ is the perpendicular component so,

$$u_1 = V_1 \sin\beta,$$

while ‘w’ is the tangential or the parallel component

$$w_1 = V_1 \cos\beta$$

The flow after it passes through the Oblique shock, it has a velocity ‘V_2’ and flow has been deflected by an angle θ, this deflection may be due to the presence of a wedge as shown in the previous slide. Now the relevant angle here is to decompose the velocities in the frame of the shock.

$$u_2 = V_2 \sin(\beta - \theta)$$

$$w_2 = V_2 \cos(\beta - \theta)$$

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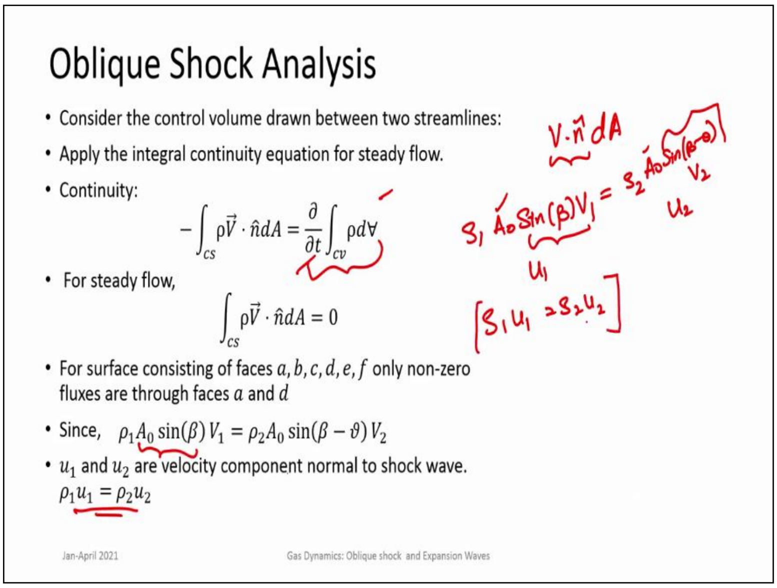

We always start with the Continuity equation, steady flow. Continuity equation in the steady flow, there is no change in the mass of the control volume. So, it is ‘0’ and only we are looking at the fluxes across the control volume. So, it is bound between 2 streamlines so there is no flux of mass across lateral directions. So, there is no flux, so there it is a streamline and only flux that goes in is entering through ‘a’ and leaving through ‘d’.

So that is what we must do. we must calculate what is the mass flux entering through ‘a’ and leaving through ‘d’. Since there is no accumulation of mass in a steady flow then these 2 fluxes must be the same. So, to find what is the flux, so flux is ‘$\vec{V}.\hat{n}\ dA$’. So, where; you have to take the component which is perpendicular to the free stream velocity.

So, the area perpendiculars to the free streams are into the stream so that area is ‘A_0 sin (β)’ where, ‘A_0’ is the face that is across.

$$\rho_1 A_0\ \sin(\beta) V_1 = \rho_2 A_0\ \sin(\beta - \theta) V_2$$

Now ‘V_1 sin β’ is the perpendicular component, perpendicular to the Oblique shock, which is u_1 and ‘V_2 sin $(\beta - \theta)$’ is u_2 perpendicular components to the shock downstream of the shock and A_0 is the same. So, you get

$$\rho_1 u_1 = \rho_2 u_2$$

this is the continuity equation for the Oblique shock, flow through the Oblique shock and ultimately you get the equation that is the same as the continuity for the normal components.

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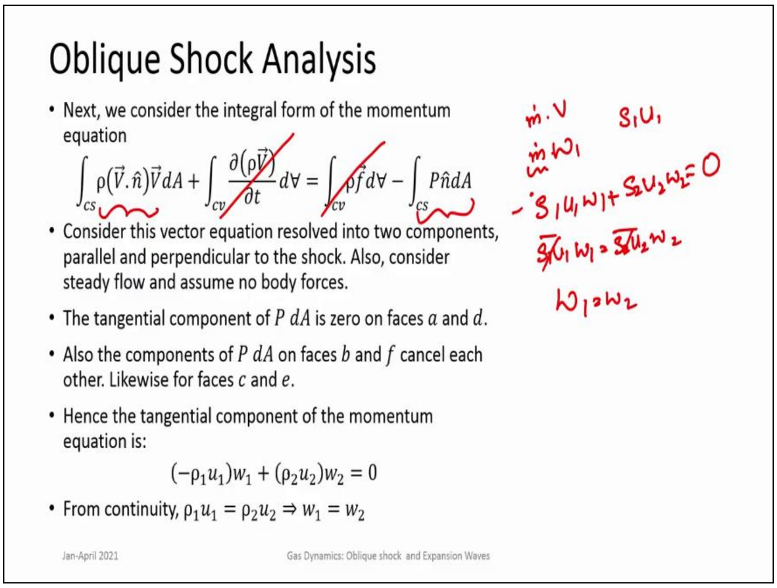

Now from continuity the next part is always the momentum. So, we go to the momentum equation, again it is a steady flow. So unsteady terms are not there, and we are not considering any body forces or any viscous forces so even this term considering the body forces are not there. Only we are looking at the flux of momentum and balance it with pressure forces. But now this is a vector equation you have 2 components, 1 is parallel to the Oblique shock or tangential to the Oblique shock, the other one is normal to the Oblique shock.

So first let us consider the parallel component or the tangential component to the Oblique shock. Now we should always remember that pressure is a normal force. So, it always acts normal to the to the area. So, when we are considering the Oblique shock the pressure force that you are considering will be normal to this area that is ‘a’ and you will have no tangential components across this due to pressure of force at ‘a’ and the flow upstream and downstream of the Oblique shock are uniform flows.

So, it is all ‘P_1, V_1, T_1’ here and ‘P_2, V_2, T_2’ here on the downstream side. So now when you look at components across the faces ‘b’ and ‘f’ they get cancelled to each other because its uniform around and ‘c’, and ‘e’ also, similarly, the pressure forces get cancelled to each other, so there are no pressure forces that appear in the equation for the tangential components of the momentum flux.

So, momentum flux is nothing, but ‘$\dot{m}$’ multiplied by the velocity. So, we are looking at tangential component so it will be ‘$\dot{m}w_1$’. So, $\dot{m} = \rho_1 u_1$ this we did just previously in the continuity equation. So ‘$\rho_1 u_1 w_1$’ is the momentum flux in the tangential direction and you get,

$$-\rho_1 u_1 w_1 + \rho_2 u_2 w_2 = 0$$

So, you get,

$$\rho_1 u_1 w_1 = \rho_2 u_2 w_2$$
$$\rho_1 u_1 = \rho_2 u_2, w_1 = w_2$$

So, the tangential velocity, that is velocity in the direction parallel to the Oblique shock is conserved it remains the same before and after the shock, it is a very important result and that will determine how you will analyse Oblique shocks.

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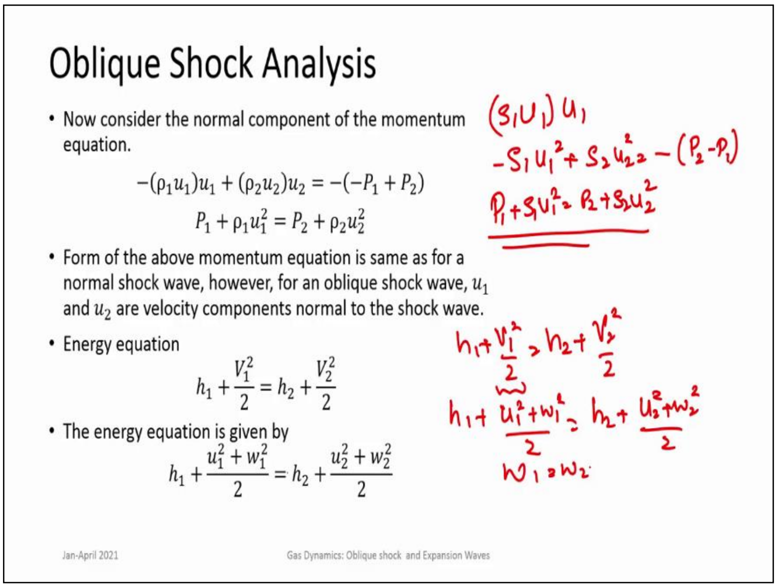

So now let us consider the normal direction. So now normal direction there is a pressure force acting which is ‘P_1’ and ‘P_2’ across the Oblique shock, P_1 before the oblique shock and P_2 after the Oblique shock and the momentum flux is ‘$\rho_1 u_1 \times u_1$’.

$$-\rho_1 u_1^2 + \rho_2 u_2^2 = -(P_2 - P_1)$$

So, you get from here,

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

This is momentum conservation for the normal component across the Oblique shock. Now we come to energy equation, energy equation is an adiabatic flow. So total energy, total enthalpy is conserved

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

Here V_1 and V_2 are the total velocities they are not components, and this is kinetic energy essentially. So,

$$h_1 + \frac{u_1^2 + w_1^2}{2} = h_2 + \frac{u_2^2 + w_2^2}{2}$$

But from our analysis of the momentum equations, we understood that $w_1 = w_2$, the tangential components are the same. So,

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

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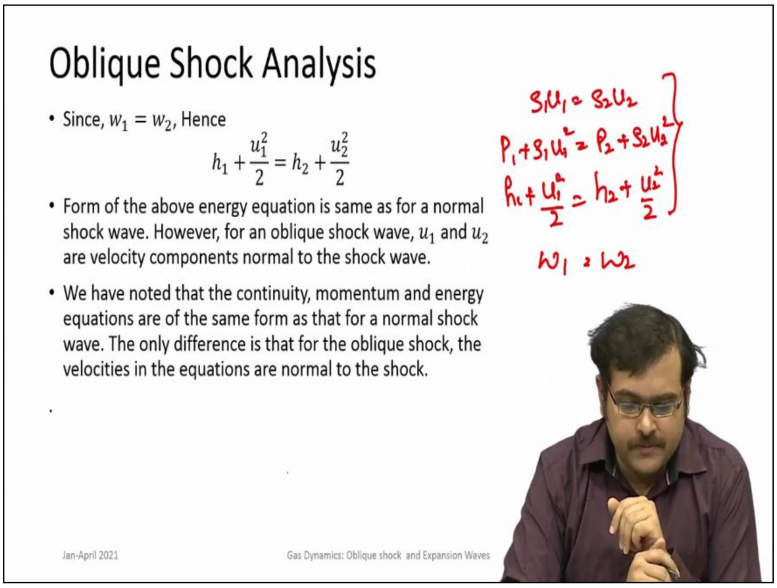

Now if you consider the set of equations for the normal component of velocity,

$$\rho_1 u_1 = \rho_2 u_2$$

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

The moment you see the set of equations you must be reminded of our analysis all this while about Normal shocks. So that is the key idea here, that to analyse Oblique shocks the simple idea is just decompose the velocities into normal and tangential components to the Oblique shock.

Once you do that the tangential velocity remains the same and normal components the relationship is exactly that of a Normal shock wave. So, this is just the equations for Normal shock. So, normal components of velocity you apply Normal shock equations and then apply the condition that $w_1 = w_2$, and you are done with solving Oblique shock equations. So, you see that oblique shocks are present in 2 dimensional flows unlike Normal shocks.

But once you decompose them in the frame of the Oblique shock, you come back to the equations of Normal shocks for the normal component and tangential velocity is conserved. So, this is the key idea once you understand this idea understanding analysis of Oblique shocks is fairly straight forward.

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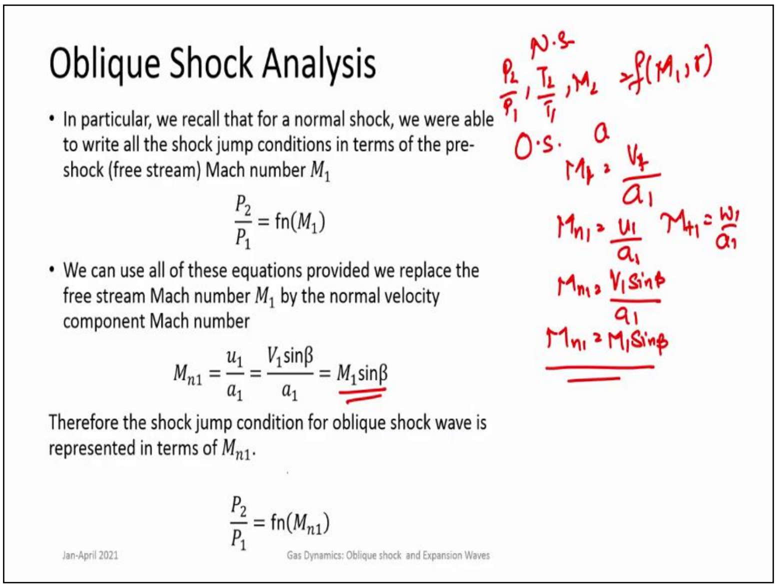

So, we know that the Normal shock relations all relations P_2/P_1, T_2/T_1, M_2 all of them are just functions of the upstream Mach number M_1 and γ, we know this one for the Normal shock. Now for an Oblique shock, we just now has found out that if you take the normal component it is going to be the same as that of an Normal shock.

Now what about speed of sound, speed of sound is a local quantity 'a', so it does not have any direction. So, when we talk about the Mach number so M_2 is V_2/a_2 or $M_1 = V_1/a_1$ and they can have individual components that the M_n, M_{n1} that is normal component of the shock is u_1/a_1 and M_{t1} tangential component is w_1/a_1.

So,

$$M_{n1} = \frac{V_1 \sin\beta}{a_1} = M_1 \sin\beta$$

So, the normal component is related to the Mach number in the same sense as the velocity is related. So you know this the jump conditions across the shock, Oblique shock is given for the normal component of the velocity or Mach number. So, you substitute M_{n1} in the normal shock relations.

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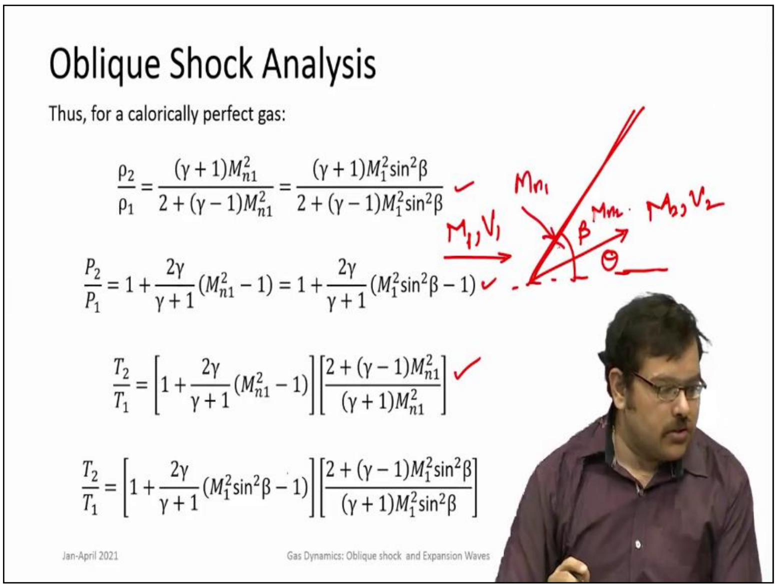

You can get all the relations for pressure ratio, density ratio, and temperature ratio. So, all of that analysis you had done for Normal shocks, you can carry on again over here and do it for the normal component of the Oblique shock. So, you have an Oblique shock here, it is at an angle β creating a flow deflection of θ for an incoming flow of 'M_1', Mach number M_1 velocity is V_1 and results in M_2, V_2.

Now the key idea is just decomposing this normal to the Oblique shock, it is M_{n1} and here you will get M_{n2}.

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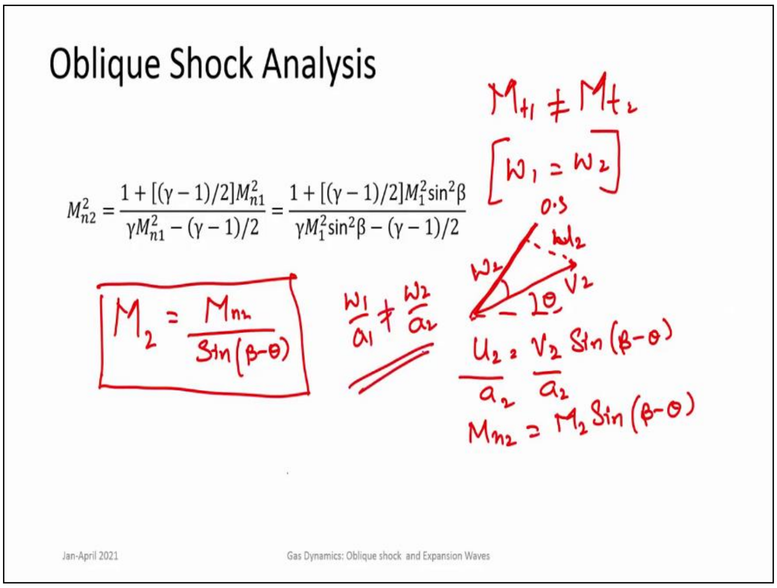

So, you can find M_{n2} which is the normal shock, the flow downstream of the normal shock but M_{n2} is only the normal component of the downstream velocity. There is a tangential component also. So, the tangential component w_1 is same as w_2. Now be very careful here because the velocity triangle or the velocity diagram for this is the Oblique shock; we are looking at the downstream flow it has been turned by an angle θ.

So, this is $u_2 = V_2 \sin(\beta - \theta)$, we can divide both sides by a_2, so we get this is

$$M_{n2} = M_2 \sin(\beta - \theta)$$

From here we can get what is the Mach number downstream of the Oblique shock, it is '$M_{n2}/\sin(\beta - \theta)$'. So, after doing all conversions we should not just leave it at normal component we should convert back to the total velocity which is V_2.

Understand the conservation is for tangential component is for velocity M_{t1} is not equal to M_{t2}. You should understand that because $M_{t1} = w_1/a_1$ while $M_{t2} = w_2/a_2$. Now a_1 is not equal to a_2 so these 2 are not equal but the velocities are equal.

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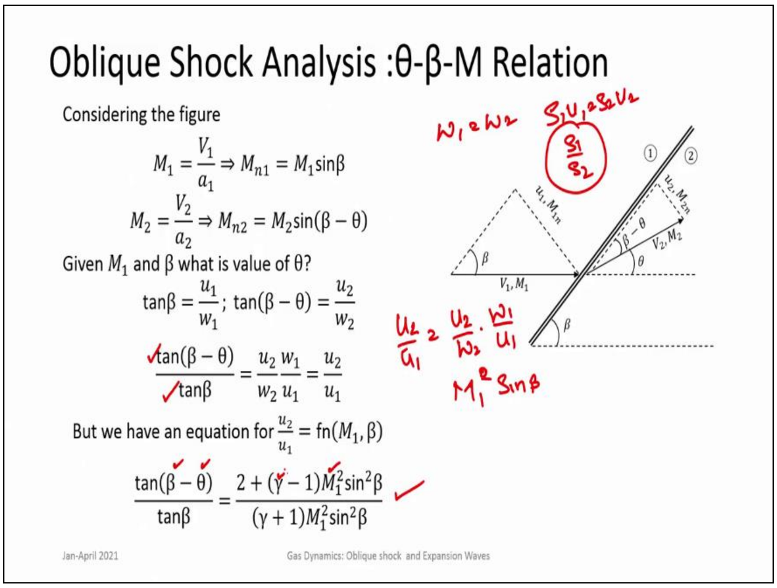

So now we have expressed all quantities flow variables in terms of the Mach number, β, θ. But the relationship between these 3 we do not know what is the relationship between Mach number, beta, and theta? Because it is a 2-dimensional problem you must have 2 variables which is usually you will be knowing Mach number and flow deflection, or you will know Mach number and the shock wave angle.

So, unless you know these 2 variables you cannot solve the Oblique shock problem. If you know only θ and you do not know anything else, then it is not possible. It is unlike the normal shock where you needed the information only about upstream Mach number you know everything about normal shock. But in an oblique shock besides M_1 you need to know either the shock wave angle or θ.

So, what is the relation between M, $\beta, and\ \theta$. How can we find that out is, same that tangential component is conserved. Tangential component is conserved w_1 is equal to w_2 and what about u_2 / u_1? This is a Normal shock relation. So $\rho_1 u_1 = \rho_2 u_2$

$$\frac{u_2}{u_1} = \frac{\rho_1}{\rho_2}$$

Can also write as u2 w2 multiplied by u1 w1 by u1 and w1 equal to w2 that is known and

$$\frac{u_2}{w_2} = \tan(\beta - \theta), \frac{u_1}{w_1} = \tan(\beta)$$

$$\frac{u_1}{u_2} = \frac{\tan(\beta - \theta)}{\tan\beta} = \frac{\rho_1}{\rho_2}$$

How do we get rho1 by rho2 rho1 by rho2 is you get by the normal component of the velocity M1 square sorry M1 sin beta and you use the expression for rho1 by rho2 which is given here these 2 are the same. So here you get a relationship,

$$\frac{\tan(\beta-\theta)}{\tan\beta}=\frac{2+(\gamma-1)M_1^2\,sin^2\,\beta}{(\gamma+1)M_1^2\,sin^2\,\beta}$$

which contains $\beta, \theta,$ and Mach number as well as gamma. So, this is the relationship which relates $\beta, \theta,$ and Mach number.

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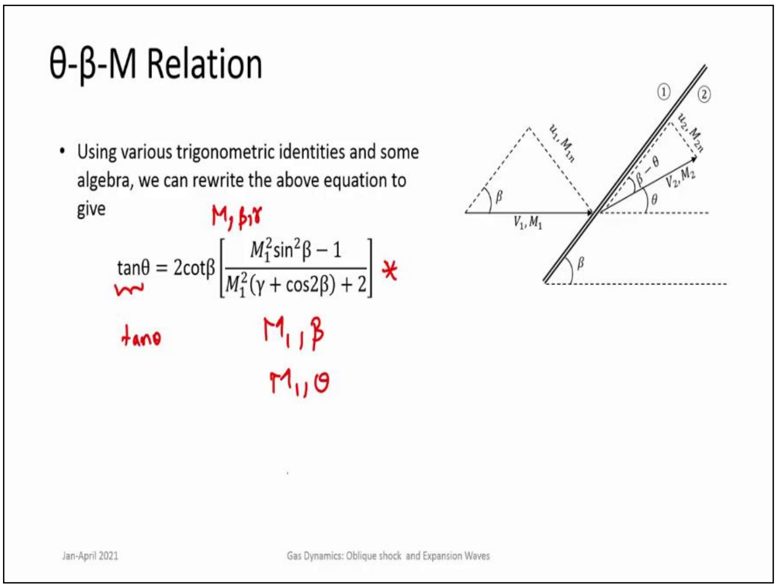

Now this can be simplified through certain trigonometric identities and algebraic manipulations and here it is written explicitly for tan θ, 'θ' is the deflection angle, tan θ is 2 cot β. So, all terms of Mach number β and gamma are on the right-hand side

$$\tan\theta=2\cot\beta\left[\frac{M_1^2\,sin^2\,\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}\right]$$

So, you can explain expect a certain non-linear behaviour here and second thing is if you know M and beta you can easily find out by supplying into this equation substituting M 1 and beta we can find tan theta. But if you know M1 and theta which is often the case that you know the deflection angle you know the shape of the body but and the Mach number where it goes then you need to find the shock wave angle.

Because all the analysis is with respect to the shock wave angle then this cannot be easily solved through some analytical means this is trying to invert this relationship. Inverse for this relationship is not easy you have to use some numerical method to get to the inverse.

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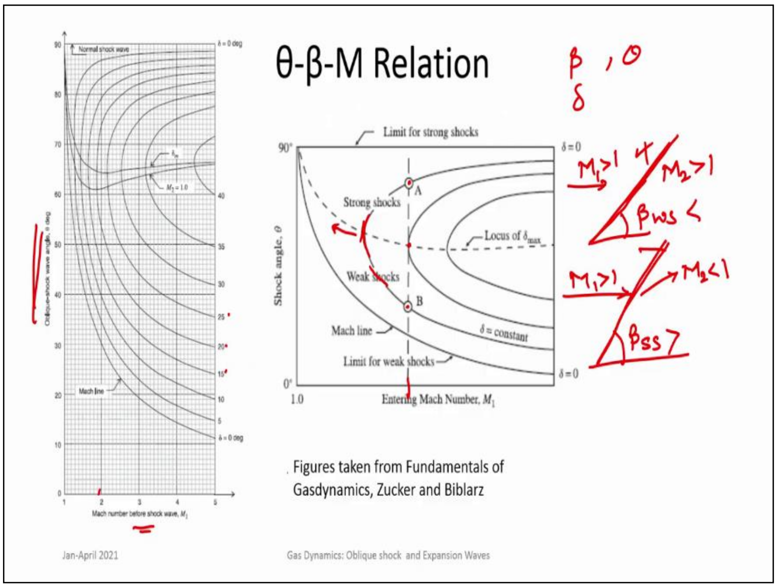

Or the other way around for this is that these are plotted you can find these plots in any textbooks. For example, these plots I have taken from fundamentals of gas dynamics from in Zucker and Biblarz and similarly any gas dynamic textbook will give you the plots of M which is Mach number here and shock wave angle here it is given theta what we have referred to is as beta.

So please look at the textbook that you are using or the graph that you are using and be careful with respect to the conventions they follow. So, different textbooks have different conventions. We in our descriptions we have used beta as the shock wave angle, but this textbook uses θ as the shock wave angle and uses δ as the deflection angle or we use θ as deflection angle.

This is plotted you can look at the plots you can see a non-linear behaviour here. You can also see that for a given Mach number if you take a particular Mach number then there are 2 points at which a vertical line from the Mach number cuts the curve that is having the same delta that is same deflection angle. So, if you have a deflection angle vary Mach number then the plot of the shock wave angle is given here.

This is the plot and if you take a particular Mach number then there are 2 solutions for this problem. So, oblique shocks have 2 solutions for a given Mach number and deflection angle.

So there are 2 solutions that is one solution is known as weak shock, the other solution is the strong shock in weak shocks the angle of the shock wave angle is small is lower. And the Mach number so if incoming Mach number is greater than 1 and then M2 is also greater than 1.

So this is a weak shock, β for the weak shock is small is less is actually small. While for the same case there is another shock there is a possibility of another shock where β is larger this is the strong shock beta is large. Here if the incoming Mach number is greater than 1 the Mach number that goes is less than 1. So strong shocks are shocks in which the downstream flow is subsonic.

In most of the case we observe weak shocks very frequently strong shocks are observed in special cases this because the pressure ratio across the strong shock. Now that you get such a large change in Mach number will be extremely high. So that is something that you must bear in mind. So, always when you look at oblique shock look at this problem whether it is a weak shock or a strong shock.

Now you see again how this curve goes on you achieve maximum at a certain point and beyond that for a given curve beyond that there are no solutions. So, for a given Mach number suppose we are considering this Mach number. So, this particular Mach number you see that there is a maximum angle for that particular; so, this is the locus of all the maximum angles.

If the angle of deflection is greater than the maximum angle, then there can be no attached solutions. So, the way it goes is 5, 10, 15, 20, 25 and if I take for example a particular Mach number which is considering the angle 20 degrees. So, it is about 1.9. So, it is very close to 1.9 for 20 degrees yeah for 20 degrees is close to 1.9. So, for a Mach number of 1.9 if the angle of deflection is increased beyond 20 degrees, then there is no attached solution.

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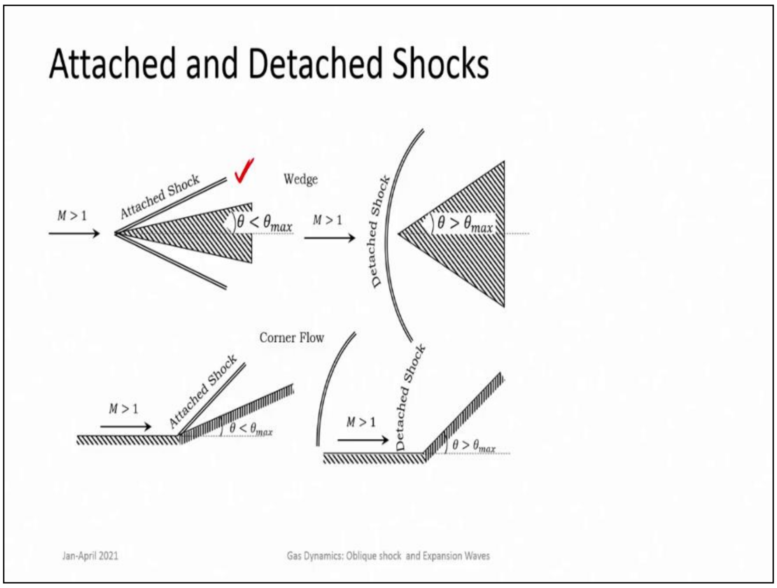

So, this kind of solution where the shock wave is attached to the wedge is called an attached solution. Here the angle of the wedge or angle of deflection is lesser than the maximum angle while if the angle is greater than maximum angle then you call it a detached shock. Then you form detached shocks like this. So, in almost all our analysis we are concerned with attached shocks but we should understand this when you do your analysis or apply things to problems.

Then always see whether the oblique shock is attached or detached, and you should observe how things change. For example, if you take the same deflection angle and you increase Mach number. So as Mach number is increased you are increasing the Mach number this side for the same angle delta being constant you can see that the shock wave angle decreases, or it comes closer and closer to the body so that is important.

Then for the same Mach number if you have the same Mach number and you increase the deflection angle, your oblique shock angle increases until the maximum deflection angle beyond that there is no solution attached solution. And for each Mach number there are 2 solutions where there is an attached shock, one is the weak shock the other one is the strong shock. This is the essence of M theta beta relationships.

So, what is attached shock and what is detached shock when are they formed this is something you have to bear in mind.

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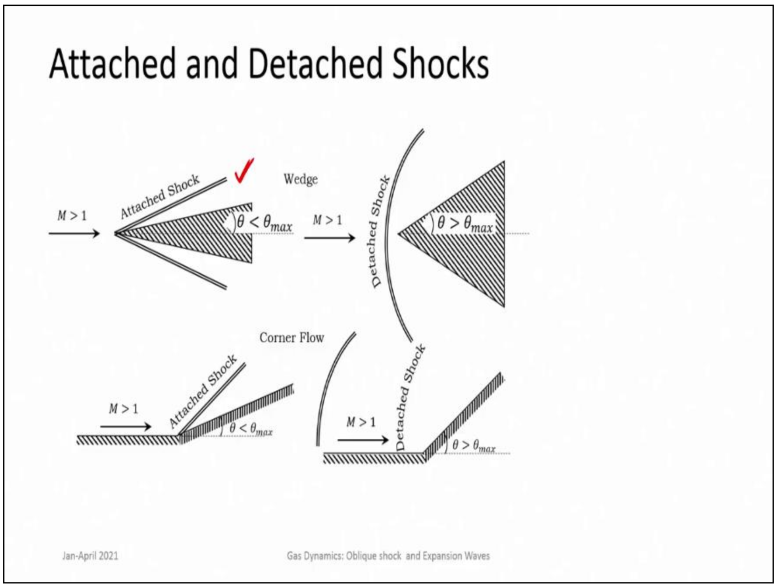

So, ending of these discussions on oblique shocks as I told initially the oblique shocks are the analysis that we did is for a 2-dimensional flow typical are plates at angles wedges and so on. The same kind of analysis can be done for cones axisymmetric bodies but there is a difference, the difference is that in the oblique shocks it is 2-dimensional. So, if you look at the streamlines, the streamlines just go across the shock and then they deflect according to the angle of the wedge across.

So, if you consider the direction which is perpendicular to this sheet of paper across that it will all deflect uniformly to theta. But that is not the case in case of a conical shock because now this is axisymmetric. So, there is another direction where things are also are different, they are not the same because of that in conical shocks the streamlines are not as parallel as in oblique shocks.

They undergo a convergence towards the oblique shock to the conical surface. So, this is quite different from the oblique shock. Of course, an oblique shock is also formed for a conical body. So, this is a cone shock or called as conical shock but the relationships between theta beta theta and the strength of the shock are quite different because of this change or convergence of streamlines towards the surface.

So, the streamlines are not parallel they in fact undergo a small change. Across the shock it is a discontinuity after that there are no other sources of entropy so you can consider the flow to be isentropic and you can write the equations.

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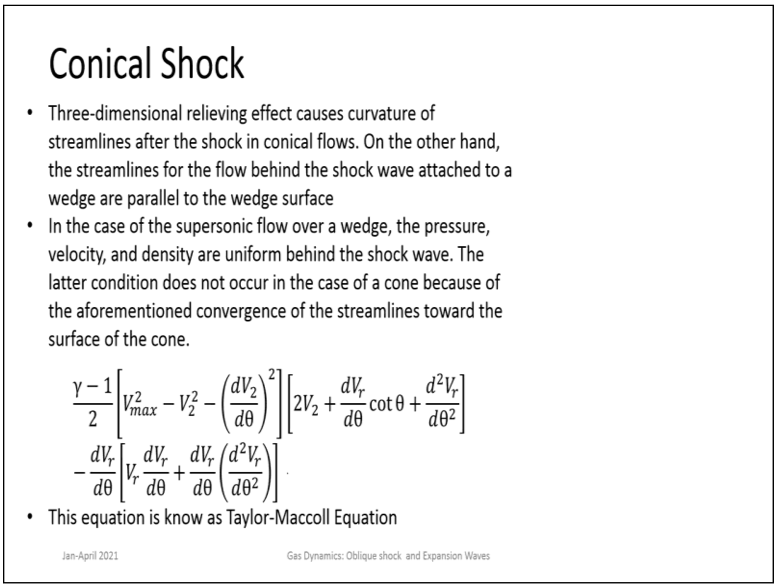

Of course, we will not go into details of these equations these are known as Taylor-Maccoll equation.

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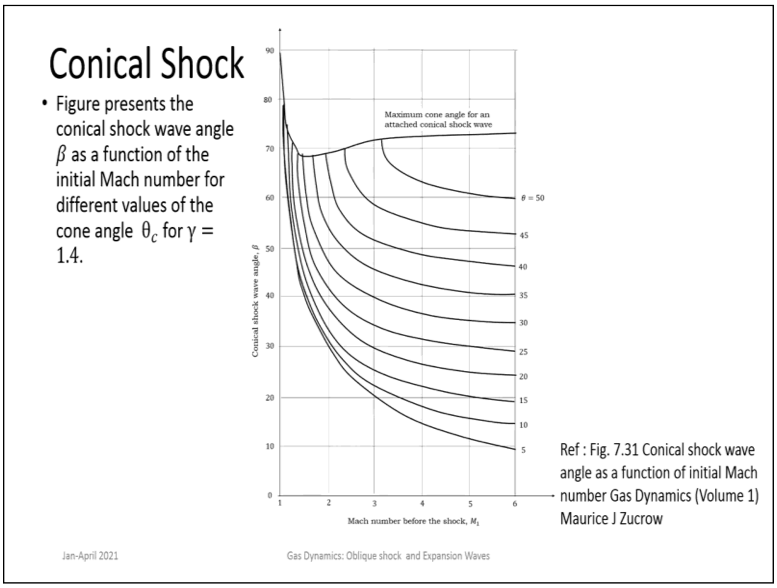

The key idea you must understand is when you are considering a conical body and shock it is the even though the shock appears oblique in nature you cannot apply the oblique shock relations there. You must use the conical shock relations these are plotted you can get these plots. One main feature you must understand is that angle of conical shock is always greater than that for an oblique shock for the same deflection angle.

And what we are interested in is the pressure over the conical surface because you have a change of convergence of streamlines the velocity keeps changing across the rays of these cone rays across these rays. So, the pressure is not uniform in this region it is not a uniform flow. So, we are interested in the pressure on the conical surface on the body and these are plotted.

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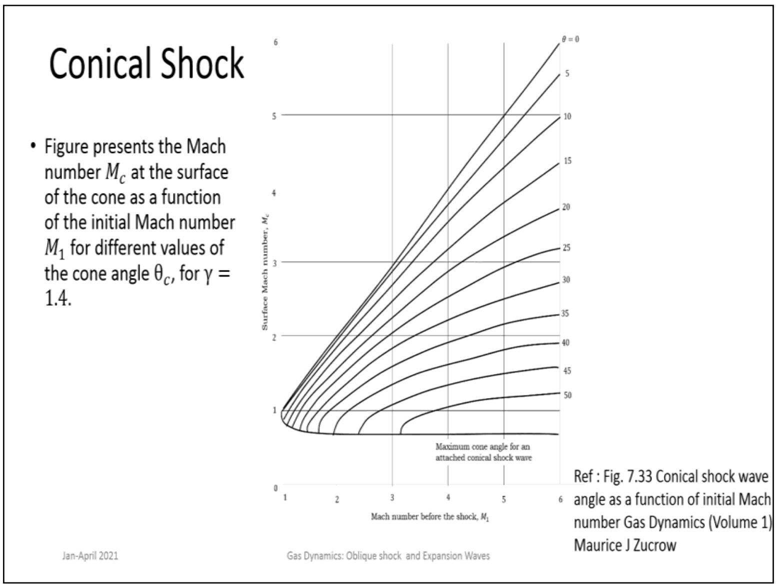

These things can be plotted, so main idea here is though you get Oblique shocks in cones also which is known as conical shock they are not the same as Oblique shocks for wedges there is a slight, there are differences. But the basic principle remains the same that if the flow must take a turn in a supersonic condition and the turn is towards itself it is a compression kind of nature then Oblique shocks are formed.

We have done the analysis for Oblique shocks. So next we will look if the flow must turn away from its initial direction, what to do? So that is the Expansion waves.