Gasdynamics: Fundamentals and Applications
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Lecture 25
Unsteady Flows - Numerical

We have in this module we have looked at the unsteady flows particularly focusing on our attention to the shock tube and that is a very good template problem to understand 1D unsteady gas dynamic flows. And we went through in detail solving all parts of flow features that are present in shock tube. Now let us just do few numerical to get these concepts little more clear. So let us do some numerical.

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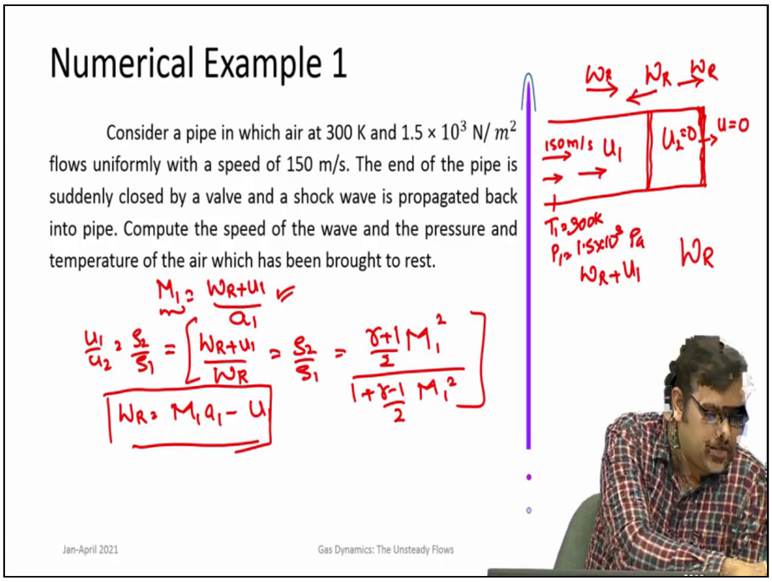

So consider this example numerical example 1. Consider a pipe in which air at 300 K and 1.5x 10^3 N/m^2 Pascal flows uniformly with a speed of 150 m/s. The end of the pipe is suddenly closed by a valve and a shock wave is propagated back into the pipe. Compute the speed of the wave and the pressure and temperature of the air which has been brought to rest.

So, this problem if you look at it, so you know that initial condition, the initial condition is that you have a flow that is going at 150 m/s in originally it is a open tube. So 150 m/s and T_1 = 300 Kelvin and P_1 =1.5x 10^3 Pa. Then suddenly this valve or this portion is closed by while which is sudden. This is important, this has happened suddenly. So instantaneously the velocity at this wall should be going to 0 so you should get u = 0 at the wall.

Now this you should sort of connected to whatever we had discussed in our classes on the unsteady flows and moving shock waves. When is this accomplished? This kind of motion can be accomplished when you have to suddenly bring a flow to rest which is high speed flow 150 meters per second is quite a good speed. This done by a shock this is very similar to the reflected shock problem of the shock tube where you have the incident shock.

Behind the incident shock there is mass motion of the gas. This mass motion is not insignificant, they have relatively good velocities. And on striking the end wall of the shock tube immediately the velocity has to go to 0. If this has to be accomplished then a reflected shock is formed or a shock wave is formed which moves into the gas which moves in. In this case it moves into the pipe with a speed W_R and this is u_1 and we can say this is u_2, $u_2 = 0$ in the fixed laboratory frame of reference.

How do you analyze this problem? It is a moving shock problem and all we have to do is change the frame of reference from laboratory frame you move on to the shock. If you do that you will impose a velocity equivalent velocity W_R here. Now u1 is also in the same direction. So the relative velocity here is $W_R + u_1$ and relative velocity here is W_R and now you can apply stationary equations, stationary gas equation.

So if I say in the stationary frame of reference what is M_1? $M_1 = \frac{W_R + u_1}{a_1}$. Now also consider what is $\frac{u_1}{u_2}$? $\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1}$. $\frac{W_R + u_1}{W_R} = \frac{\rho_2}{\rho_1}$ of the shock which is for the shock with the Mach number M_1 this is well known.

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{1+\frac{\gamma-1}{2}M_1^2}$$

Now can we express, we know that this is the equation for M_1 can we express WR in terms of M_1 ? You can, $W_R = M_1 a_1 - u_1$

So once this is known, so you have this and you have this relation. So we can put them together so we will put it together and write them together.

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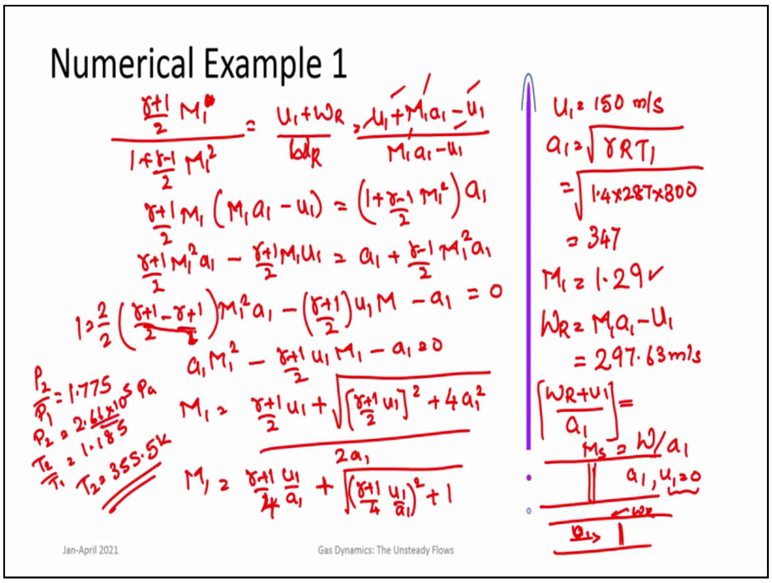

So you have, .

$\frac{(\gamma+1)M_1^2}{1+\frac{\gamma-1}{2}M_1^2} = \frac{W_R+u_1}{W_R} = \frac{u_1+M_1\,a_1-u_1}{M_1\,a_1-u_1}$. So now you get,

$\frac{\gamma+1}{2}M_1\;(M_1\,a_1-u_1) = \;(1+\frac{\gamma-1}{2}M_1^2)\,a_1$

So now we can write,

$\frac{\gamma+1}{2}M_1^2a_1 - \frac{\gamma+1}{2}M_1u_1 = \;a_1+\frac{\gamma-1}{2}M_1^2a_1$

$$M_1^2a_1\,(\frac{\gamma+1}{2}-\frac{\gamma+1}{2}) - \frac{\gamma+1}{2}M_1u_1 - a_1 = 0$$

So now you have we are approaching a quadratic equation, algebraic equation.

$$M_1^2a_1 - \frac{\gamma+1}{2}M_1u_1 - a_1 = 0$$

Roots of this,

$$M_1 = \frac{\frac{\gamma+1}{2}u_1+\sqrt{(\frac{\gamma+1}{2}u_1)^2+4a_1^2}}{2a_1}$$

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$$M_1 = \frac{\gamma+1}{4}\frac{u_1}{a_1} + \sqrt{\left(\frac{\gamma+1}{4}\frac{u_1}{a_1}\right)^2 + 1}$$

So it will have a negative value. So that is not possible. so you take the positive value. Now in this we know, u_1 is known ,150 m/s, . $a_1 = \sqrt{\gamma RT}$, For air $\gamma = 1.4$, $a_1 = \sqrt{1.4 \times 287 \times 300}$. This value is known it is around, it should be $a_1 = 347$ m/s.

You can substitute these values here and find out what is M1?
$M_1 = 1.29$

So what is the wave speed W_R ?
$W_R = M_1\, a_1 - u_1 = 297.63$ m/s

So you can you should understand that Mach number for the reflected way what we consider is actually $\frac{W_R + u_1}{a_1}$. not $\frac{W_R}{a_1}$. So please bear that in mind when you have a primary shock moving into a quiescent medium.

So here this is the primary shock moving into a quiescent medium a_1 , and $u_1 = 0$. So here ,

$$M_s = \frac{W}{a_1}$$

because $u_1 = 0$ here reflected I mean if you transform the coordinates also you will get it. But if you consider a reflected wave going in to a region where there is already a certain velocity u_1 and this is W_R then the speed of the Mach number for that particular wave is $\frac{W_R + u_1}{a_1}$. This has to be borne in mind which is you have to be careful with this.

So we still have the values for pressure and temperature behind the shock. So that is once you know M_1 , it is straight forward just have to look at either the tables or look at a calculator and find out what is $\frac{P_2}{P_1}$. We know $M_1 = 1.29$

$\frac{P_2}{P_1}$ = 1.775. So consequence P_2 = 2. 66 x 10^5 Pa.

Similarly, $\frac{T_2}{T_1}$ = 1.185, you can put this value and you will get T_2 = 355.5 K

So you can do this methods straight forward. So now once this is known now let us move to the so this is a good example of shock wave reflection from the end and it is applied to a problem with suddenly the valve is brought to close and the flow is brought to rest. So those kinds of problems also have relations to the shock tube problem.

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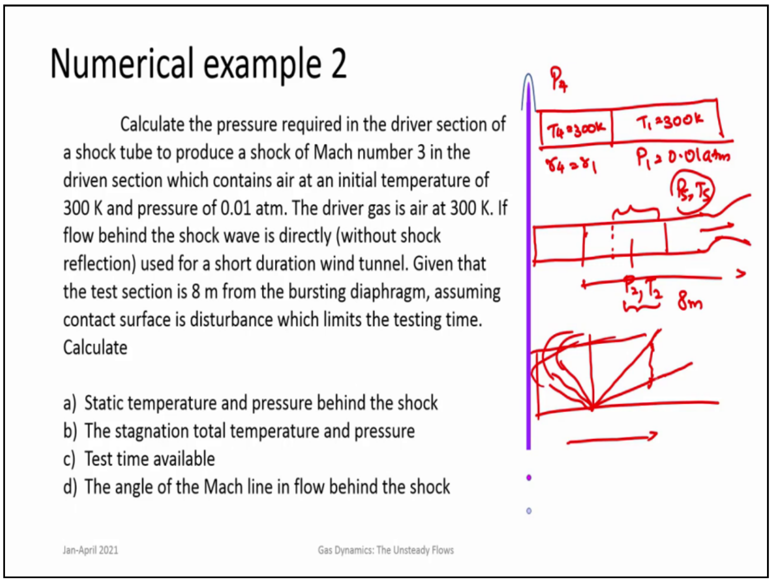

So the second example is, calculate the pressure required in the driver section of a shock tube to produce a shock Mach number 3 in the driven section which contains air at an initial temperature of 300 Kelvin and pressure of 0.01 atmosphere. So its low pressure we can draw the schematic here and here you know T_1 = 300 K and P_1 = 0.01 atmospheres. We do not know what is P_4 . We have to find P_4. Driver gas is air at 300 Kelvin.

So it is the same gas T_4 = 300 K and $\gamma_4 = \gamma_1$ If flow behind the shock wave is directly used without shock reflection for a short duration wind tunnel. So this is a case where you are taking the shock tube and at the end of it you are attaching a wind tunnel. So this combination is known as shock tunnel we will discuss this in coming classes but principle almost all of it you are for the shock tube is already known.

So after the diaphragm ruptures you have a shock wave and you have a contact surface. There is a slug of gas between this having high pressures P_2, T_2 and this slug of gas is then utilized for the wind tunnel. You may use this by having a shock reflection back and increasing the pressures to P_5, T_5 and that forms the reservoir section for the internal or you can use just the P_2, T_2.

And what is said here is there is no shock reflection that means directly we are using P_2, T_2. Given that the test section is 8 meters away. So the distance between the diaphragm station and the test section is 8 meters and contact surface is the disturbance which limits the testing time. So if you draw the xt diagrams which we have been drawing for some time now in the previous classes.

We saw that there is a shock discontinuity, there is an contact surface discontinuity, in there are expansion fans. So essentially this test gas between the shock and the contact surface is the gas that is used for any kind of experiments or aerodynamic testing but you could get other effects like if you allow this to run for longer time the expansion fans can reflect off the end wall and come and make the flow non uniform here.

Essentially that ends the test time or the other case is that the contact surface passes through the test section. Once that is done again you get all non uniform flows beyond the contact surface. These are the 2 cases so here it says is the contact surface that is the limiting the test time. So the test time is limited by this difference between the shock and the contact surface. So what do we need to calculate static temperature and pressure behind the shock, stagnation total temperature and pressure, test time available. The angles of the Mach line in the flow behind the shock. So first point is to calculate the pressure required in the driver section. So what is P_4 ?

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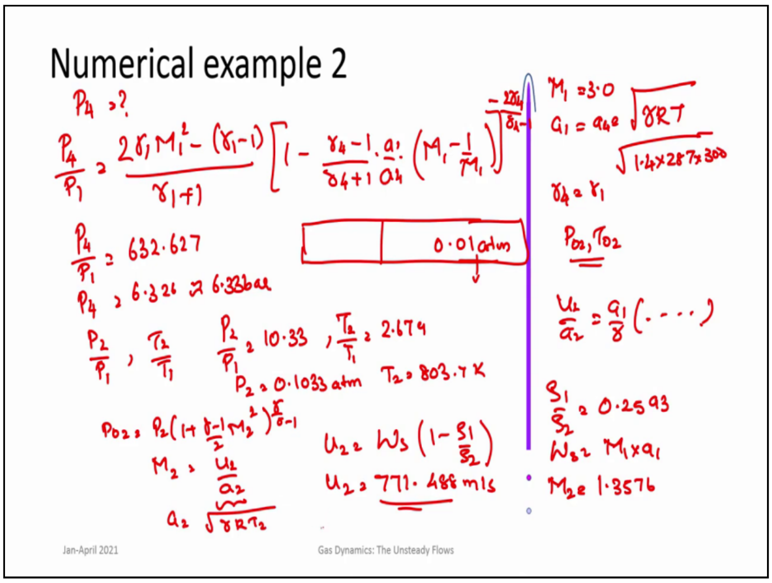

So directly you can apply the equations $\frac{P_4}{P_1}$, So this is direct from you can use the text books

$$\frac{P_4}{P_1} = \frac{2\gamma_1 M_1^2 - (\gamma_1 - 1)}{(\gamma_1 + 1)} \left[1 - \frac{(\gamma_4 - 1)}{(\gamma_1 + 1)} \frac{a_1}{a_4} \left(M_1 - \frac{1}{M_1} \right) \right]^{\left(\frac{-2\gamma_4}{\gamma_4 - 1} \right)}$$

or you can use the notes and so this formula if you put we know M_1 , M_1 is Mach 3 Mach number is 3 so $M_1 = 3$ and $\frac{a_1}{a_4}$ both are $a_1 = a_4 = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 300}$

So, $\gamma_4 = \gamma_1$. We know this and if you substitute this you can get $\frac{P_4}{P_1}$ directly it is $\frac{P_4}{P_1}$ = 632.627. So you can see to produce a Mach 3 flow of Mach 3 shock wave in a shock tube you need 600 times the pressure in the driver section. So that is why you generally keep high pressure as well as low pressure. So you evacuate the section which is driven section is kept at low pressures. In this case 0.01 atmospheres. Otherwise if it was at one atmosphere you can imagine you need to give 632 atmospheres here but you can manage a fairly strong shock by using low pressures here and then the pressure at P_4 is only 6.326 which is 6.33 bar. So that gives you the pressure that pressure ratio across the shock tube that needs to be provided so that you get the shock. And once the shock is formed what is the stagnation pressure and the stagnation temperature so for the flow. So that is P_{02} and T_{02} these are the conditions.

So for that we need first $\frac{P_2}{P_1}$ and $\frac{T_2}{T_1}$

This you can get given that it is a Mach 3 flow,

$\frac{P_2}{P_1} = 10.33$ and $\frac{T_2}{T_1} = 2.679$. So $P_2 = 0.1033$ atmospheres while $T_2 = 803.7$ Kelvin. Now stagnation pressure if you have to calculate for the region 2 you have to do the calculation for P_{02}, $P_{02} = P_2\left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$

what is M_2? $M_2 = \frac{u_2}{a_1}$.

Now you have to calculate u_2 . So you can you know M_s , you know $\frac{P_2}{P_1}$, you can directly use the relations of what is $\frac{u_2}{a_1}$. You can also go slightly from the first principles that, $u_2 = W_S(1 - \frac{\rho_1}{\rho_2})$. and if you know $\frac{P_2}{P_1}$ and $\frac{T_2}{T_1}$ you can calculate $\frac{\rho_1}{\rho_2}$ and $\frac{\rho_1}{\rho_2} = 0.2593$ and W_S is nothing but $W_S = M_1 a_1$

So from this you can get u_2, $u_2 = 771.488$ m/s. So it is not small, its very good velocity but a_1 is also large therefore you get $M_2 = 1.3576$. So this is M_2, so correspondingly now once you know M_2, $M_2 = \frac{u_2}{a_1}$, $a_1 = \sqrt{\gamma R T_2}$

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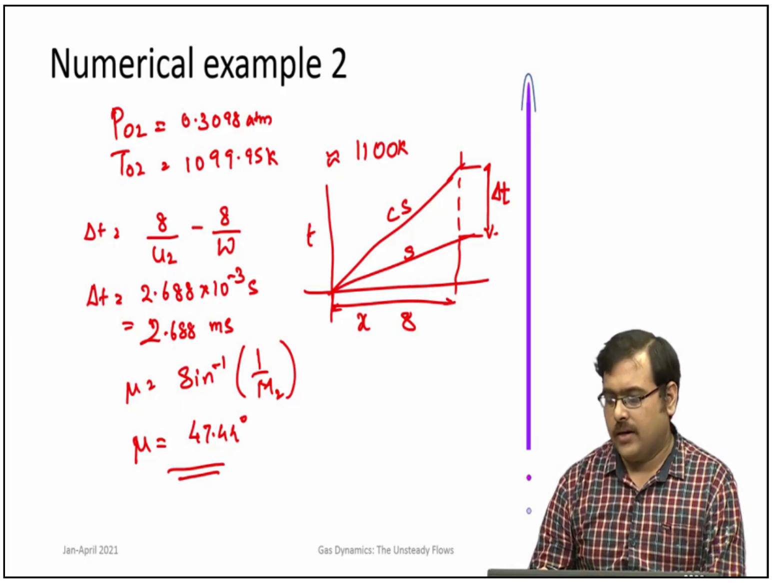

So now P_{02} and T_{02} can be found out. So $P_{02} = 0.3098$ atmospheres and $T_{02} = 1099.95$ Kelvin. So you see its quite high temperature approaching 1100 Kelvin.

Now what is given is that the contact surface is the one which limits the test time and if you draw the xt diagram, the shock wave passes right way and contact surface is here and test section is 8 meters. So this difference in time is the test time basically. So this is the difference in time or delta t which is the test time.

So this is control contact surface ,this is shock. So how do you calculate this the same distance so what is delta t for the same distance? The contact surface takes the time,
$\nabla t = \frac{8}{u_2} - \frac{8}{W}$, $\nabla t = 2.688\ e^{-3}$ s .

So you can see this is quite small 2.688 milliseconds. But these facilities are often used for high temperature flows and high hypersonic flows high enthalpy flows.

And test times there are quite short but the instrumentation is also made similarly it is made advanced and people get a lot of useful results from such facilities. What is the angle of Mach line in M_2 flow. $\mu = \frac{1}{M_2}$. This turns out to be , μ = 47.44 degrees. So that solves this problem.

So there are 2 problems we saw one the first one was the application of the unsteady principles that we had done towards a problem where there is a high speed flow in a pipe and suddenly a valve is closed then what happens. And how do we calculate quantities there.
The second one was a template shock tube problem where we know what is the shock Mach number required? And we calculate various quantities like what is the pressure ratio that needs to be provided so that we can achieve that shock Mach number. And what is the properties of the slug of gas which is usually used for any applications including in wind tunnels.

So with this we come to close on unsteady flows and the next module we will be looking at moving from 1D equation. We were all in the 1D framework to 2D framework and look at shock waves and expansions in 2D framework. So that is when you go to 2D framework the shock waves become at an angle to the flow this is known as oblique waves. So what is normal shock is actually a special form of oblique shock but the angle is normal to the flow.

But at any other angle you get an oblique shock and we will look at oblique shocks and the counterpart which will expand the flow shock waves compress the flow they are expansion waves and Prandtl Myer expansion waves. So we will see at that in the next module.