Gasdynamics: Fundamentals and Applications
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Lecture 20
Normal Shocks - Numerical

So, we have been looking at normal shocks in great detail. We have looked at the shock relations and also on their applications to Pitot measurements in supersonic flow, the Rayleigh Pitot formula and also the moving shock, how to analyse moving shocks they can be done by taking properties I mean writing the equations relative to the shock wave by jumping on to the shock.

And then the static variables are related by the same shock equations for the stationary shock. So these are the important concepts. Now let us apply these concepts to some numericals in this class.

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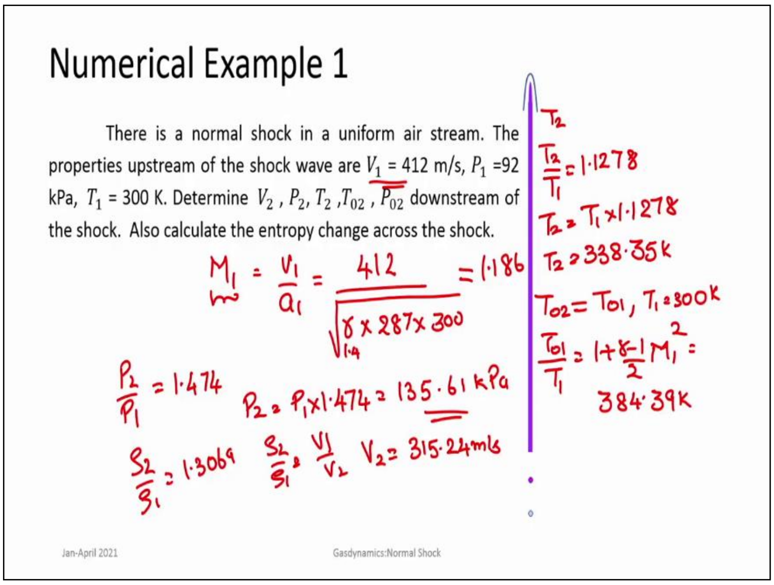

So the first example, there is a normal shock in a uniform air stream. The properties upstream of the shock wave are V1 = 412 m/s , P_1 is 2 kPa, T_1 is 300 K. Determine properties downstream of the shock V2, P2, T2, T02, P02. Also calculate the entropy change across the shock.

So how to go ahead with this. So we know that shock, the equations for shock are all dependent on the upstream Mach number M_1.

So, if you determine $M_1 = \frac{V_1}{a_1}$. You will be able to get to all other values. But also keep in mind that about the conservation equations where they can be applied for some quick results. So here if you look at $M_1 = \frac{V_1}{a_1}$ and this is $M_1 = \frac{412}{\sqrt{1.4 x 287 x 300}}$ = 1.186.

Now you can look at the tables for normal shock. Normal shock tables are exactly similar representation as is for isentropic tables you have a tabulation of Mach numbers, different Mach numbers and pressure ratio, temperature ratio, density ratio, downstream Mach number all of them are given, and also the Rayleigh Pitot formula or Rayleigh Pitot is also given. So that is given in the appendices of textbooks or the other way is to look at some gas dynamic calculators they are now very much available.

Some of this information was shared to you in earlier classes. So, you could use the same things for they have separate modules for normal shocks. So, you could use that from them you can generate the values for the properties. So now what we need to measure is what is P_2 / P_1? So, for pressure after the shock P_2, P_2 / P_1 for Mach number of 1.186 is 1.474. P_2 / P_1 = 1.474. So, P_2 is nothing but P_1 multiplied by 1.474. $P_2 = P_1 \times P_2 / P_1 = P_1 \times 1.474$ = 134.65 kPa.
Now next we need to determine V_2. One way to go about doing this is of course, you have to go and calculate M_2, M_2 is known once M_1 is known. Another quick way of doing that is using $\frac{\rho_2}{\rho_1}$. You know $\frac{\rho_2}{\rho_1}$ =1.3069.
In this case V_1 is given already. So $\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2}$. So, you can use this to get V_2.

So, V_2 = 315.24 m/s. Then what about T_2? So T_2 is do look at $\frac{T_2}{T_1}$ which is $\frac{T_2}{T_1}$ =1.1278 .For this Mach number and corresponding to this you can get T_2 .
It is $T_2 = T_1 \times 1.1278$ and T_2 = 338.35 K.

Now what is T_{02}? That is T_{02} is stagnation temperature. Now is an adiabatic flow so stagnation temperature will remain constant across the shock so $T_{02} = T_{01}$ .a
nd for T_{01}, T_1 is given 300 K.
And you know M_1, so now here you have to use the isentropic table or you have to use the isentropic relations. So be very careful here $\frac{T_{01}}{T_1} = 1 + \frac{(\gamma-1)}{2} M_1^2$. So, you will see that from now on as we go further into gas dynamics all these concepts will be used together. You have

to apply not only normal shock relations you will have to apply isentropic relations and they will come together.

Soon we will look at flows through nozzles and diffusers there it is a variable area duct problem. So you will have a problem associated with variable area ducts but in them shocks can be present. So do not consider each module separately as we go through these modules all of them are connected to each other as we go more and more into gas dynamics. All that you had learnt earlier will be used ever more.

So T_{01} here will be 384.39 K. So this remains the same for the shock waves across the shock wave.

What about stagnation pressure? Will the stagnation pressure be the same or it is different you already know there is an entropy change across the shock.

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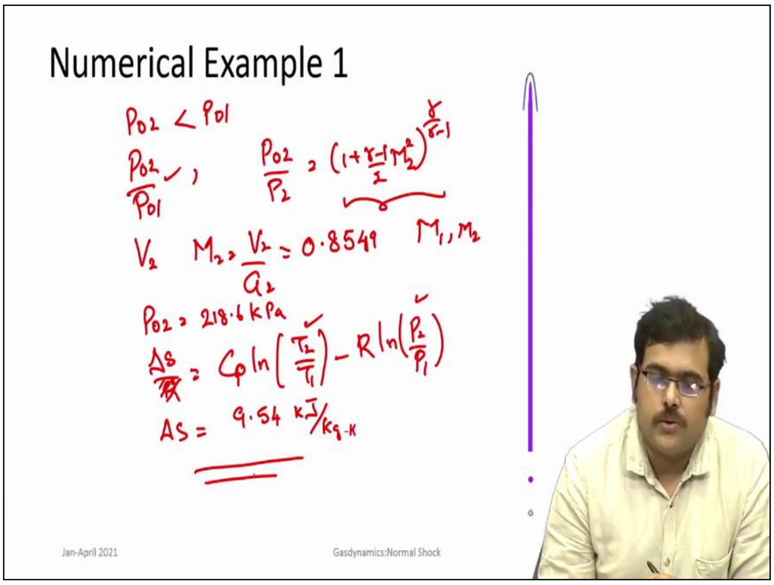

So P_{02} will be less than P_{01} .If you are using tables $\frac{P_{02}}{P_{01}}$ is tabulated you can readily use the tabulation or you can also do this by using $\frac{P_{02}}{P_2} = \left(1 + \frac{(\gamma-1)}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}$. You can use this where you have already calculated V_2 across the shock. So you can express M_2 $M_2 = \frac{V_2}{a_2}$ and $M_2 = 0.8549$. Now this is readily available once you know M_1 you can get M_2.

So all these properties are tabulated or you can get the relations through online calculators directly they will give you $\frac{P_{02}}{P_{01}}$ and you can apply that and get to the value of P_{02} and

P_{02} = 218.6 kPa. So now what is the change in entropy across the shock that is $\frac{\Delta S}{R}$ by this is $\frac{\Delta S}{R} = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$. Both quantities are calculated so you can calculate ΔS , not $\frac{\Delta S}{R}$ but ΔS.

So, ΔS = 9.54 kJ/kg K.

So now we have solved this problem let us go to the second problem.

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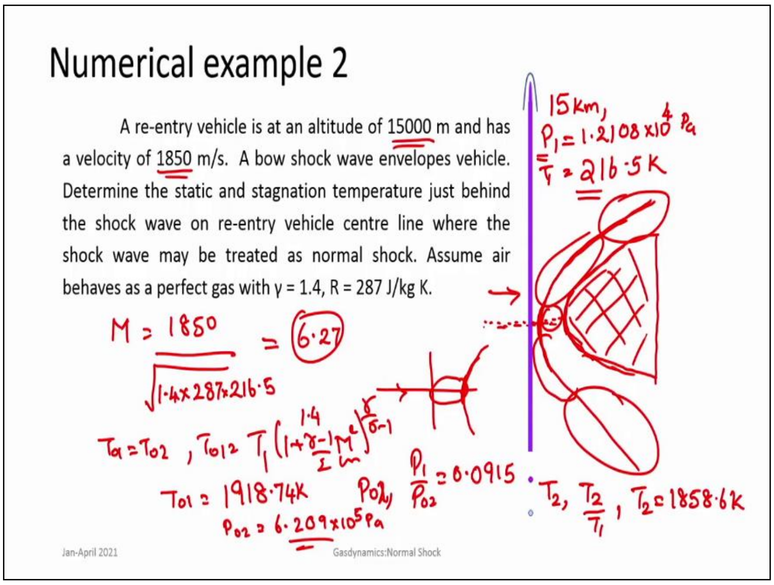

The second problem is that of a re-entry vehicle is at an altitude of 15,000 meters and has a velocity 1850 m/s. A bow shock envelops the vehicle. Determine the static and stagnation temperature just behind the shock wave on re entry vehicle center line where the shock wave may be treated as normal shock. Assume air behaves as a perfect gas with γ = 1.4,
R=287 J/ kg K.

So here we know the altitude, so it is 15 km, you can use a reference atmospheric property so P_1 static properties will be the same as $P_1 = 1.2108 \times 10^4$ Pa and T_1 = 216.5 K. So this property is known to you. Now a vehicle, re-entry kind of vehicle you would have come across such shapes they are usually of a blunted kind and this bluntness is because of the extreme temperatures that are seen at the nose of such vehicles when they re-enter.

So the speed is known it is nearly it is 1.85 km/s that means in one second it travels 1.85 kilometers is extremely fast. And when such fast flow impinges on the body then all the kinetic energy of the gas at the stagnation point can be converted into heat, stagnation temperature and you can get very high temperature. So, the shapes of these bodies are meant to cater to such.

So, it is an aerodynamic shaping in order to minimize the effects of these very high temperatures and it is a supersonic flow. So, a shock wave develops about this body and usually over such bodies the shape of the shock is like a bow, it will be of this kind. So that is why it is called a bow shock. We will soon come to the discussions of shocks which are at an angle to the flow like over here and over here.

These are known as oblique shocks they are oblique all the way right from all these regions are oblique shocks but right at the center line. At the center line where this bow has this curve at that particular point if you zoom in then it is just a normal shock then this is the body zoomed Type equation here.up version and here you have a normal shock. So at stagnation point or in the region near the stagnation point properties can be calculated by using the normal shock relations flow is subsonic here in this region.

So how do we get this now? So you know temperature, so we can calculate the Mach number of the flow it is $M_1 = \frac{1850}{\sqrt{1.4x287x216}}$ and the Mach number comes out to be 6.27, $M_1 = 6.27$. quite a good high Mach number and if you want to calculate the stagnation properties you know the static stagnation temperature, can be calculated directly because T01 is equal to T02 you just have to calculate $T_{01}= T_1 x\left(1+\frac{(\gamma-1)}{2} M_1^2\right)$

And you can calculate this T_0 , you have to put M_1= 6.27 here , γ = 1.4 and will get T_{01} as $216x\left(1+\frac{(1.4-1)}{2} 6.27^2\right)$ = 1918.74 K. So that is quite a high temperature considering that the static temperature is only 216 K. Now as we are doing stagnation temperature, let us also calculate the stagnation pressure P_{01}. P_{01} you can calculate in this case P_1 is given to you so you can calculate P_{01} using the Rayleigh Pitot formula which is which relates P0 sorry we have to calculate P_{02} that is after the shock.

So P_{02} and P_1 is what we have to relate and that is related by Rayleigh Pitot formula $\frac{P_1}{P_{02}}$ and this is $\frac{P_1}{P_{02}}$ = 0.0915 for this particular Mach number.

So you can get $P_{02} = 6.209 \times 10^5$ Pascal's. So this is what you get. And in order to calculate temperature T_2, static temperature after the shock you just need T_2 / T_1 for Mach number of 6.27. So you can use that and get to T2. So T_2 = 1858.6 Kelvin.

So, this involved looking at the stagnation region of a body where the shock can be treated as a normal shock.

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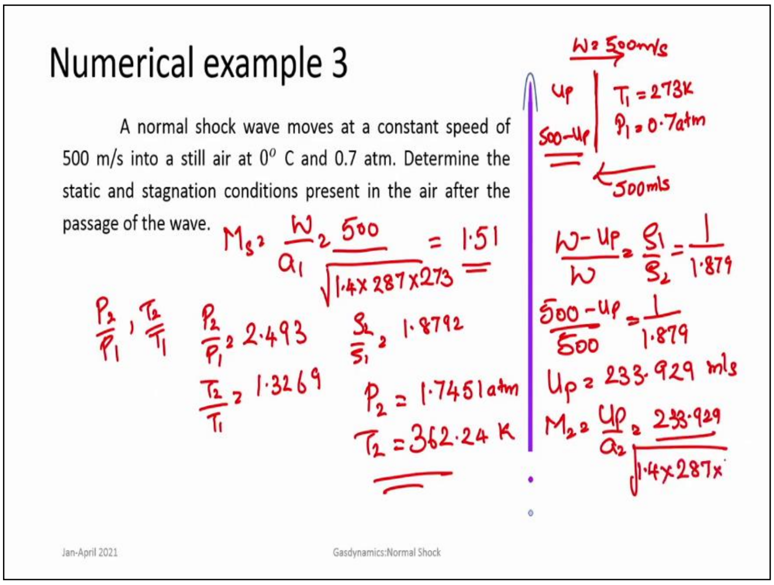

Now let us do a problem on moving shock wave. So, a normal shock moves at a constant speed of 500 m/s into a still air at at 0^o C and 0.7 atm. Determine static and stagnation conditions present in the air after the passage of the wave.

So wave passes it is moving at a speed and W is equal to 500 meter per second. This is the speed and T_1 is given it is 273 Kelvin and P_1 is given is 0.7 atmospheres.

So, how to do this problem? It is a moving shock problem so in order to do a moving shock problem you have to do the transformation. So that you sit on the shock or move relative with the shock and then it becomes stationary and so 500 meter per second is imposed in the opposite direction and here you have a mass motion so you will get (500 - u_P) as the speed behind the gas behind the shock.

So now let us see what the Mach number of this shock is, $M_S = \frac{W}{a_1}$ which is $M_S = \frac{500}{\sqrt{1.4x287x2273}} = 1.51$. So, this is the Mach number of the shock. Once you know Mach number you can find out $\frac{P_2}{P_1}$ and $\frac{T_2}{T_1}$. $\frac{P_2}{P_1} = 2.493$ and $\frac{T_2}{T_1} = 1.3269$. So $\frac{\rho_2}{\rho_1} = 1.8792$. So now once you know these ratios is straightforward to get $P_2 = 1.7451$ atmospheres and $T_2 = 362.24$ Kelvin.

So now what is the velocity of the wave behind the gas. We need the velocity of the gas behind the shock? We need to know this in order to calculate the stagnation conditions. As we had discussed before stagnation conditions should be calculated separately. So we can do that by using this condition $\frac{(W - up)}{W} = \frac{\rho_1}{\rho_2} = \frac{1}{1.897}$

$$\frac{(W - up)}{W} = \frac{(500\ - up)}{500} = = \frac{1}{1.897}$$

u_p = 233.929 m/s

So u_p turns out to be 233.929 m/s. Now you know T_2, T_2 is known so you can calculate what is M_2. M_2 after the shock passes over is $M_2 = \frac{up}{a_2} \frac{233.929}{\sqrt{1.4x287x362.24}}$ = 0.613.

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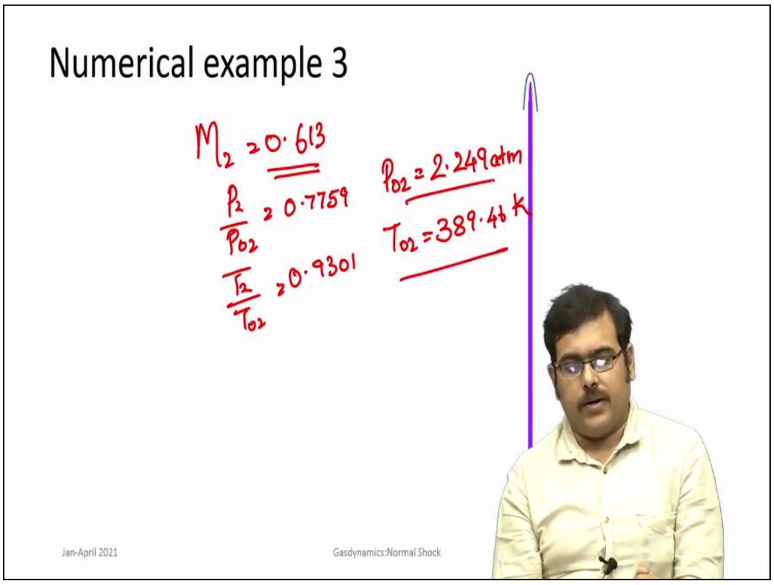

And this Mach number turns out to be M2 is 0.613 and from the isentropic charts for 0.613 or nearest values to 0.61 you can calculate what is $\frac{P_2}{P_{02}}$. So, $\frac{P_2}{P_{02}}$ = 0.7759 and $\frac{T_2}{T_{02}}$ = 0.9301. So you get P_{02} = 2.249 atm and T_{02} = 389.46 Kelvin. So you get the stagnation properties across the moving shock. So moving shock problem when one is doing, one has to be really careful that you use the appropriate transformations.

Look at the frames of references to get the correct answers. These static property relations are the same as that of a normal shock. But stagnation properties have to be calculated in the fixed frame of reference. So we have done 3 problems as examples for normal shocks and moving shocks. From the next class onwards we will look at unsteady flow problems normal the moving shock is a part of it but there is much more to it. So an example of that is the shock tube itself. So we will look at that in the next class.