Gasdynamics:
Fundamentals and Applications

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Lecture-02
Flow Regimes

So, in this class we look at different flow regimes in when we consider a compressible flow. We had talked about subsonic flow, supersonic flow in the previous class when we introduced compressibility, compressible flows, what is the most important non dimensional number which is the mach number and so on and we looked at several nice images about flows where compressibility effects are important. Usually they are important in gaseous flows at high speeds.

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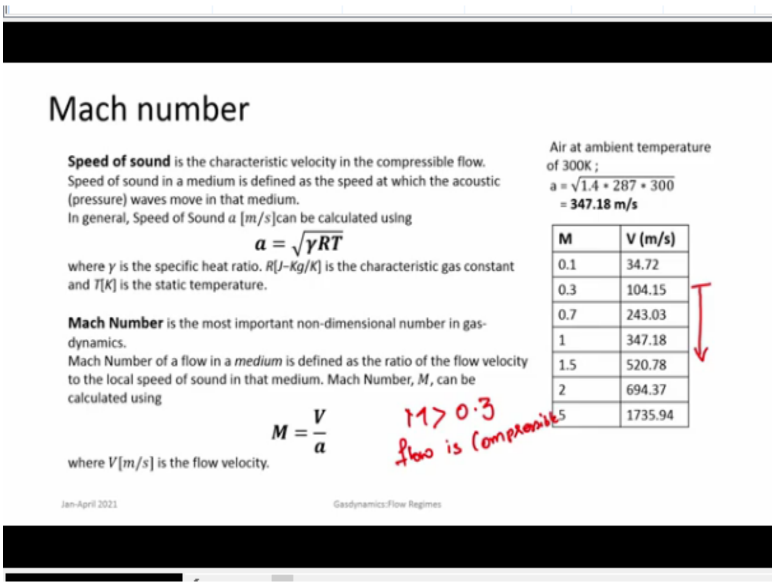

And so if we look at the most important non dimensional number in compressible flows or gas dynamics it is the Mach number which is the ratio of the local flow velocity to the speed of sound at in that particular medium at that point. So, Mach number is really a local quantity and the speed of sound is a very important characteristic velocity in describing compressible flows which is for a perfect gas it is $\sqrt{\gamma RT}$.

And whenever mach numbers become greater than 0.3, then we say compressibility effects are important. So, flow is compressible. So, but it can go really all the way 0.3, 0.7, special case is when it becomes equal to the speed of sound itself which is mach number equal to 1 and flows beyond that when mach number becomes greater than 1. What are the differences in such flows, what is so special and so on? So, let us take a look at this and see how the flow can be classified into different regimes.

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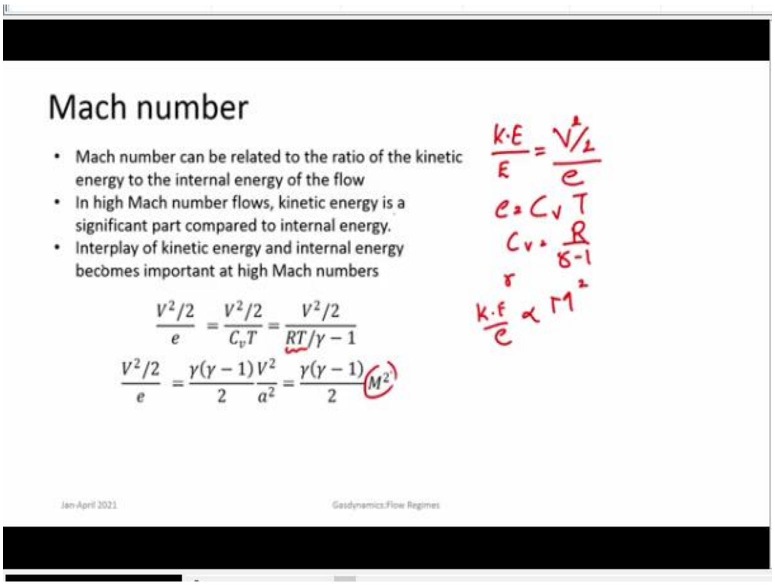

So, even before we go there another important a sort of idea that can be attached to mach number is it can be related to the ratio of kinetic energy to the internal energy of the flow. For example, you take kinetic energy by internal energy so E, kinetic energy is $\frac{v^2}{2}$ and e is specific internal energy per unit mass. And we let us consider a perfect gas for now and we will see that e is nothing but c_v multiplied by T where c_v is specific heat at constant volume.

So, we can write it in that form and therefore you get c_v is 1 by $\gamma - 1$ into R, R by $\gamma - 1$ where R is the specific gas constant and if you substitute that in the equation you get RT by $\gamma - 1$. Therefore you get if you multiply and divide by a γ you get $\frac{\frac{v^2}{2}}{e}$ as γ some constant which is related to the ratio of specific heats γ by a square or it is mach number square.

So, you see that ratio of kinetic energy to internal energy is proportional to mach number square (M^2). So, if mach number increases significantly what we find is the kinetic energy becomes a very significant fraction of internal energy and if there are changes in kinetic energy which can happen in the flow changes in mach number velocity and so on .

Then large part of that energy will start interacting with internal energy. So, there is a close interaction between kinetic energy and internal energy in these kinds of flows and compressible flows you cannot just neglect the energy equation you have to consider not only mass momentum conservation but also energy conservation because of this important aspect. So, density is a variable not only is that you have good interaction between various forms of energy.

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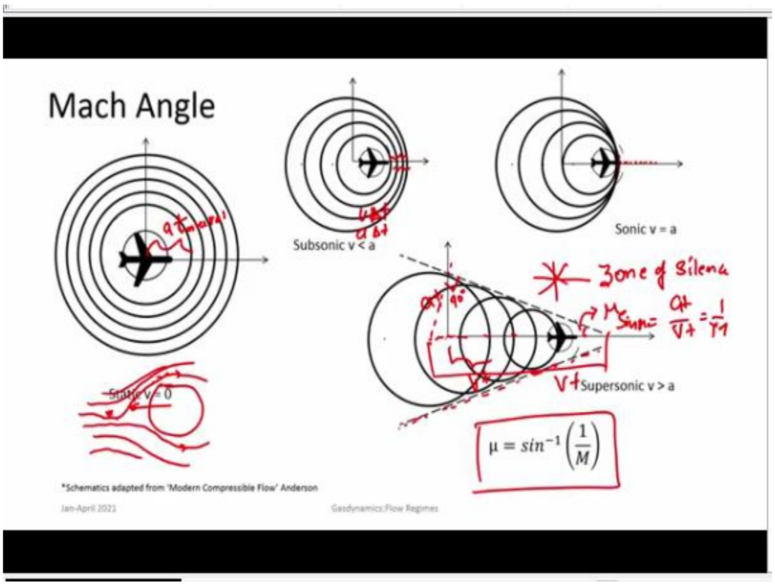

So, now you have understood mach number in different aspects, one is the ratio of velocity to acoustic speed, the other its importance in the context of kinetic energy. Now let us look at it in the context of how it these waves and how is the speed of sound so important in gas dynamics. So, one when we take a look at this at the outset one should ask a question that if a body is moving in a medium like it is moving in air say car is moving or you are walking in air.

Then we know that if there is a body some arbitrary body and its moving in this direction then we know that air flows past it some of these figures you would have come to know in your classes in fluid dynamics then you should pause and ask a question if you look at the way these stream lines behave you see that march earlier to the point when the body comes to a particular point upstream.

So, this is the upstream of the body which is moving into the gas or into air over here a much upstream the flow already knows that it has to turn over the body. So, in some sense the flow has a knowledge or is already knowing that it has to take a turn around the body. How does it know? How does it move? What is the method by or the mechanism that is happening over here? And it moves so smoothly around the body. This is by means of information propagation by means of pressure waves as a body moves it produces these pressure waves.

And these pressure waves move at the speed of sound. So, let us take a look at this consider a case when a typical body here this body is represented ok it has a certain shape but actually what we are talking about is an arbitrary body it can be a point particle also and it is currently we are taking it has to be stationary and let us say it produces sound or it produces tiny fluctuations of pressure which moves the speed of sound.

So, if it does that then at a certain instance of time these waves propagate by certain distances which is equal to if you calculate this distance it will be nothing but a multiplied by that specific time interval**.** So a t, ok some specific distance here, the body is not moving. So, you will see you get a set of these circles. So, you just get a set of such concentric circles ok.

Now let us go ahead and let this body start moving and we take a sample of these pressure waves which are sent out from the body at particular intervals of time. Now here the body is also moving. So, this is the initial case at when the body is at this particular location it emits a sound wave and that wave propagates at a certain other instance the body has moved by a certain distance which is v t, that is v multiplied by that small change $v\Delta t$.

While the sound has moved by a distance a Δt with a small time interval Δt, the sound has moved to such a distance but the body has more only by a small distance and we see that the sound wave actually moves much faster than the body. So, you can see these circles are sound waves that are emitted at subsequent intervals of time. These sound waves are always a head of the body.

So, any changes in small changes in pressure immediately gets transferred upstream of the body downstream of the body everywhere and the flow in turn comes to know that a body is approaching its point, therefore it has to turn and it turns. So, this is the case when the velocity of the body is lower than the speed of sound and you find that the body is always moving slower than the speed of sound and the sound is able to move much faster than the body.

So, you get these kinds of circles which indicate such emotion. But now let us take a case, now you see that if you look at this particular picture due to the motion you will see that upstream there is a sort of compression or convergence of these waves of sound. Now but if the body moves exactly with the speed of sound at each instant of time the body moves at exactly the same distance as the speed of sound.

So, if we do just look at it you see at every point it is moving exactly at the speed of sound and all these sound waves at the upstream portions they all coalesce, they all come together because now you see that it becomes difficult for fluid particles which are ahead of this particular body to feel that the body is coming towards them. Because that was the mechanism the pressure waves were the mechanism by which they would have known but now the pressure waves are moving at the same speed as the body is moving.

So, now the fluid particles ahead of the body do not come to know readily that the body is approaching them and suddenly they will come to know that the body is there. So, this is the basis for how shock waves are produced that there is suddenly there is a change which happens in speeds greater than the speed of sound much before that it all upstream locations know that there is some change happening place.

And therefore solutions are smooth but once you go beyond mach number equal to 1 or beyond the acoustic speed now let us consider the case when the object is moving at supersonic speed that is its velocity is greater than the speed of sound. Then what happens is at particular instant if a sound wave or if a pressure perturbation is produced it moves at a speed.

This speed at which it moves is ‘at’ but the object moves much faster than that this speed is ‘vt’. So, every subsequent circle is actually this sound is left behind and the object moves much faster than that. So, you see that the object is moving much, much faster. So, if you just draw a tangent to all these circles it represents a certain cone, it represents a certain angle beyond which at all these locations the flow or the medium never knows that a body is moving at speeds which is much greater than the speed of sound.

So, this we can call is zone of silence, this is the reason why when objects move at speeds greater than speed of sound we do not hear them we first see them and we hear them only once these waves which are known as mach waves which is this the coalescence of several sound waves only when these mach waves go past a particular point only at that time you come to know the presence of the object.

But by that time the object would have really gone past and it would be at some other location. Now what is this particular angle you can construct the simple trigonometric relations the angle is this here is 90^{o} it is a tangent. So, you are looking at this angle which is known as the mach angle μ and this speed is 'at' while the distance it would have covered at the same time is 'vt'. So, sine of this angle is $\frac{at}{vt}$ is $\sin\mu$ and therefore you see this $\frac{a}{v}$ is nothing but 1 by mach number.

So, $\mu = \sin^{-1}\frac{1}{M}$. So, this is something that you would come to understand very well during this course that in supersonic flows there is a particular direction to which information can propagate or pressure waves can propagate or so on. And that directionality is very important in supersonic flows. In subsonic flows this directionality is not there it is absent and so there are consequences on how these different kinds of flows are evaluated or analysed and so on.

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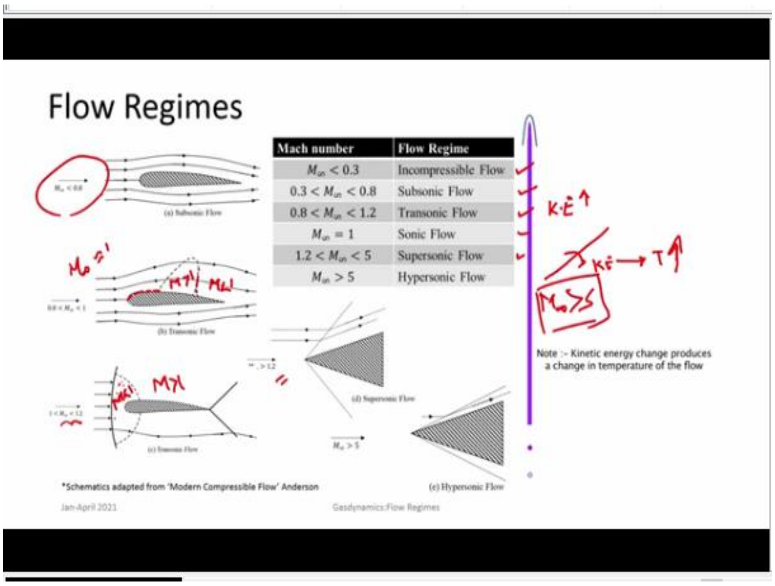

So, if you look at the so different flow regimes you really have such different regimes for mach numbers which are less than 0.3 you essentially do not consider the compressibility effects and you say that is an incompressible flow, for mach numbers greater than 0.3 you have to consider compressible flows but these flows initially they are having velocities which is less than the speed of sound they are more smooth.

These flows are smooth and you call them subsonic flows, when you approach the speed of sound which is mach number equal to 1 and you have a body for example let us consider this kind of an airfoil in a uniform flow, in this case what we are talking about is the mach number of this uniform flow. Then if you consider such a body which is where the mach number of the uniform flow M_∞ approaches 1 from the subsonic side?

So, it is approaching 1 and it is in still in the subsonic region but when you see the flow happening over a body then if you take say for an airfoil, initially flow accelerates. So, in some local area or the local region in the airfoil the flow can become supersonic, but the conditions on the airfoil are such that or in the flow are such that it cannot support a full supersonic flow.

So, later on at some certain point you get shock waves and flow becomes again subsonic. So, you have a pocket of supersonic flow and then a pocket of subsonic flow you have a mixed region of flow. So, this happens to flow over bodies or such kind of flows when mach number approaches really close to 1. This can also happen in the supersonic side also, it is very close to 1 but not very high also.

So, it just say mach number 1 to 1.2, you again for such flows as we had discussed the flow would not know that a body is there. So, a shock wave develops but across the shock wave since it is very close to 1, the region over here is mach number is less than 1 but the flow again accelerates becomes greater than 1 and so on. So, you have pockets of supersonic and subsonic flows existing when mach numbers are very close to 1 and that kind of flow is known as transonic flow okay. So, that is the transonic flow.

Subsonic flow, in subsonic flow the flow is always subsonic throughout, but then once you get to mach numbers which are really higher, say mach numbers greater than 1.2 and so on then the shock waves become more oblique and you get dominantly supersonic flow all over and that kind of flow is known as supersonic flow. When you have very high mach number flows then certain important phenomena occur like what we were discussing the kinetic energy becomes very high.

So, when you have a shock wave and the velocity decreases across the shock wave this kinetic energy change gets dumped into internal energy or it goes to increasing temperature of the body significantly and you get certain other things happening due to such high temperature effects and that changes lot of flow physics in those bodies, such things that happen at very high mach numbers is known as hypersonic flow.

And that typically a ballpark number is M_∞ greater than 5, it is a rough estimate and a brief introduction to hypersonic flow will be provided towards the end of the course.

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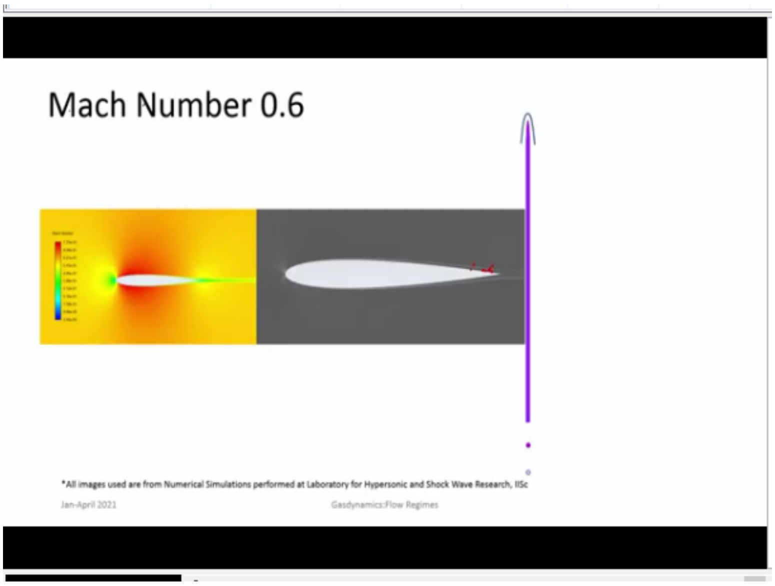

So, let us look at these different aspects that we discussed if we just do a simple numerical computation over some of the standard CFD softwares, that will provide us some idea about what is happening, here we are representing 2 images, these numerical simulations have been performed by our team at IISc and here we are representing 2 images for a particular case a typical airfoil in a flow where the flow mach number here is in the subsonic region of 0.6 it is a compressible flow.

And here what we are seeing is a picture of the mach number variation, mach number contours and the on the right hand side you have variations of density which is similar to a schlieren, here the flow is extremely smooth significant changes you see close to the wall which where viscous effects become important around the boundary layer. But otherwise outside the flow is very smooth.

First there is an acceleration and then the mach number increases and then at the rear part there is a deceleration. So, smooth flows around the airfoil at mach 0.6.

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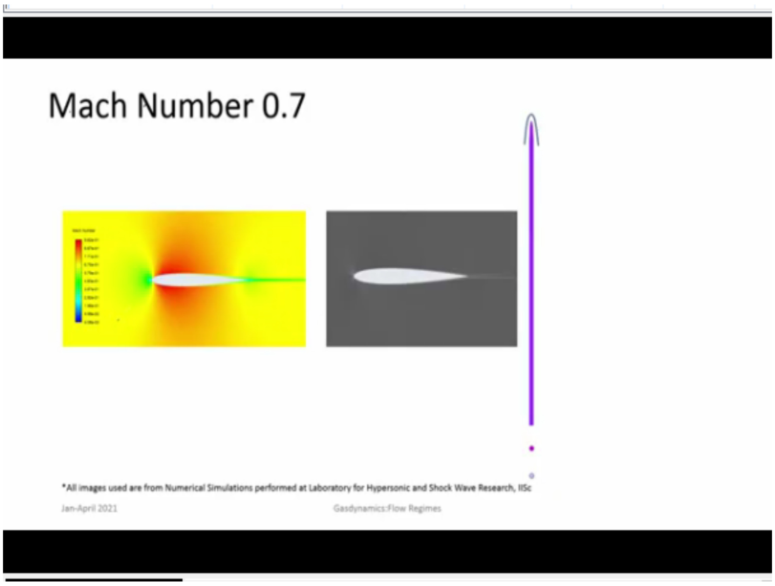

Now if you go to higher and higher mach numbers you find that it is still subsonic, these flows are still smooth initial there is an acceleration, but the mach number does not go to such an extent that it becomes supersonic or reaches mach number 1, highest mach numbers are still around the body are still subsonic.

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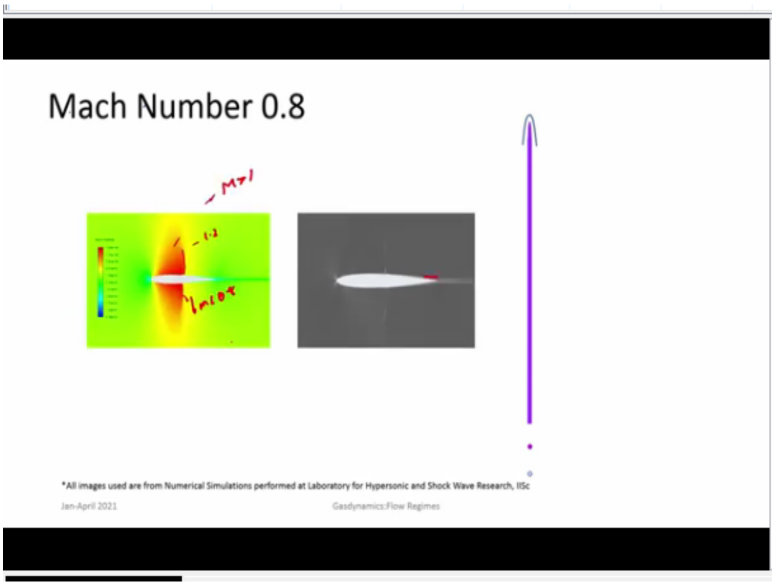

But once you reach mach number 0.8 you find that the acceleration has happened to such an extent that in a local region the mach number increases to beyond mach number 1. So, mach number is greater than 1. But the conditions around the airfoil are not sufficient to support mach number 1 all through. Therefore there is a sharp front here which is a shock wave and this shock wave suddenly changes the mach number to lower values.

So, it gets to about mach numbers close to 1.2 in this region and after the shock wave mach numbers drop down to 0.8 and so on and this has consequences this sudden change will disrupt the boundary layer it can cause separation and so on and this can cause a large increase in drag which is what is known as the critical mach number and you need really good amount of energy to go past this high drag.

So, much earlier they thought that is very difficult to go beyond mach number equal to 1 due to this sudden increase in drag near the critical mach number. But later it was found that once you go past this difficulty it is again possible to have higher velocity. So, now supersonic flight is more common now than earlier.

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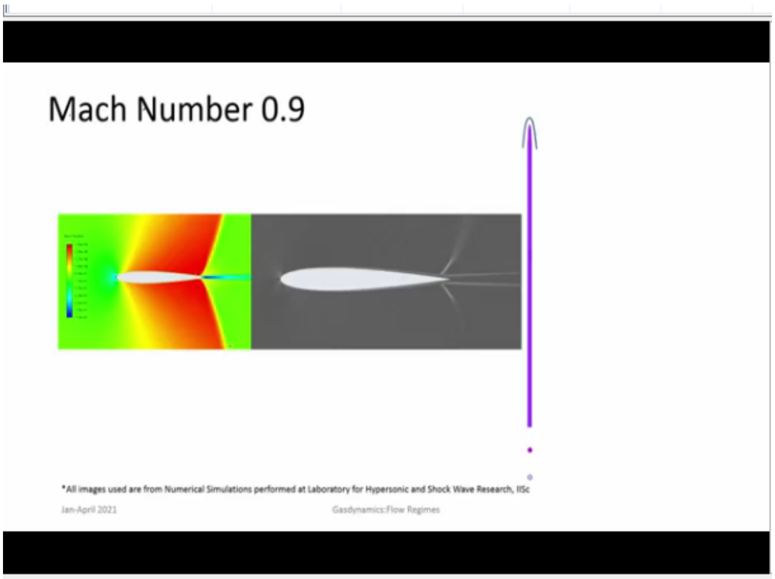

So, you see now that flow structures start changing at mach 0.9 this shock wave is pushed very close to the trailing edge of the airfoil. These regions lie in the transonic regime where you have pockets of supersonic and subsonic flow.

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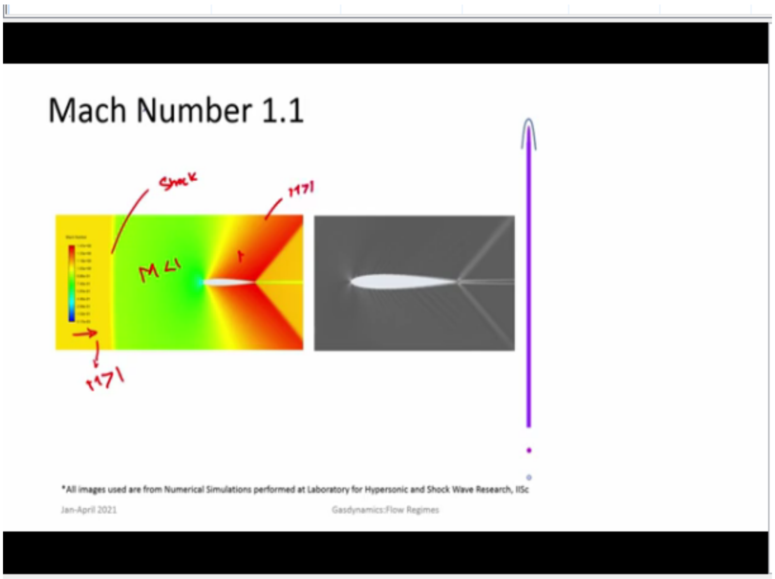

Now you see as mach number increases further you have a region again, this is again in transonic flow, here the mach number incoming is higher, it is mach number is greater than 1, but due to the presence of the body the flow should know that it should happen therefore a shock forms here. This is the shock and then you get mach number less than 1, then flow again accelerates to mach numbers greater than 1. So, you see this is again in the transonic regime.

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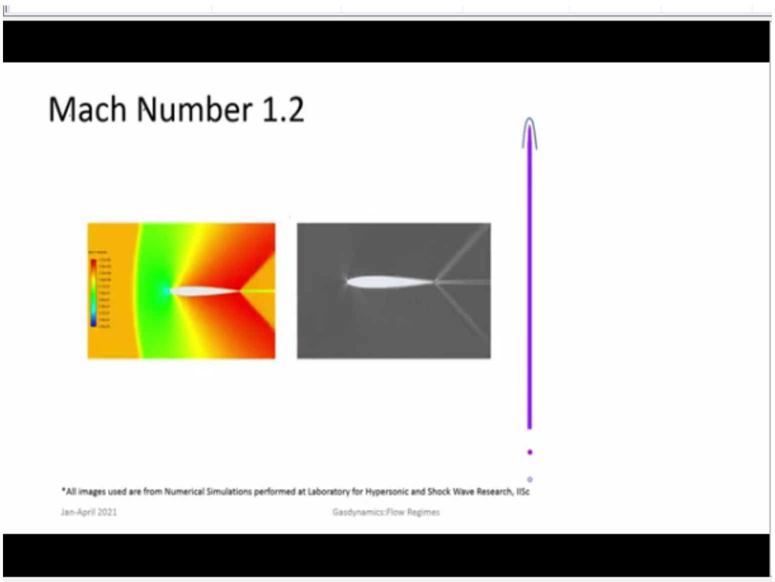

So, 1.2 falls in transonic regime.

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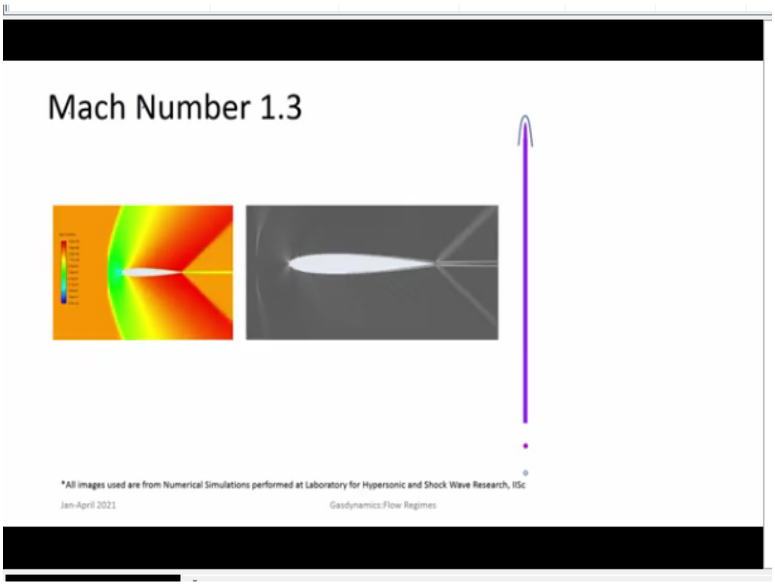

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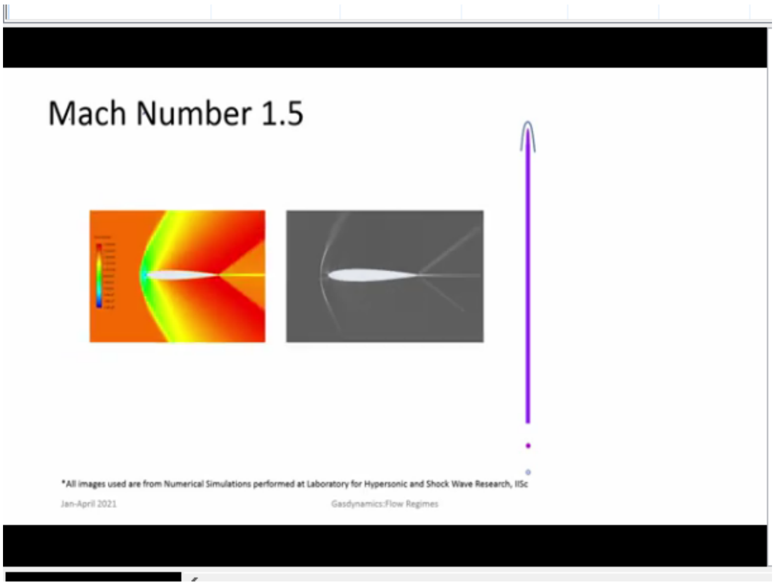

But once you get to higher and higher mach numbers this shock comes very close to the body and now predominantly the body is in supersonic flow.

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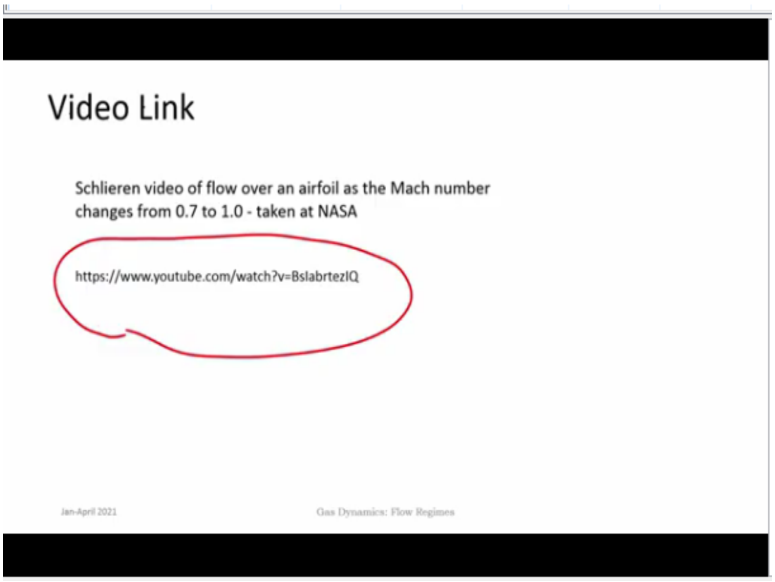

So, this particular numerical simulations that we just showed now, you can see the exactly similar videos over airfoils placed in wind tunnels as the mach number changes from 0.7 to 1 and these were taken by NASA and they are available in youtube and you can watch them the link is given over there and that will give you a better appreciation of what we did here in the class. So, with this we close here and move on to initial parts where we look at thermodynamics in the next class, thank you.