Gasdynamics: Fundamentals and Applications
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Lecture 19
Normal Shock - IV

So, in this particular lecture we will look at moving shocks. This is a slight difference from stationary shocks which was what we were analysing in depth till now. We derived the expressions for the pressure ratios, temperature ratios of the shock also looked at the Hugoniot which is an expression which relates the thermodynamic variables across the shock and how the normal shock equations are applied for the case of flow measurement in a Pitot in supersonic flow.

In a supersonic flow shock always stands ahead of the Pitot. So, one measures the stagnation pressure after the shock and not the stagnation pressure of the free stream. This is an important concept one has to remember. So that was about stationary shock analysis.

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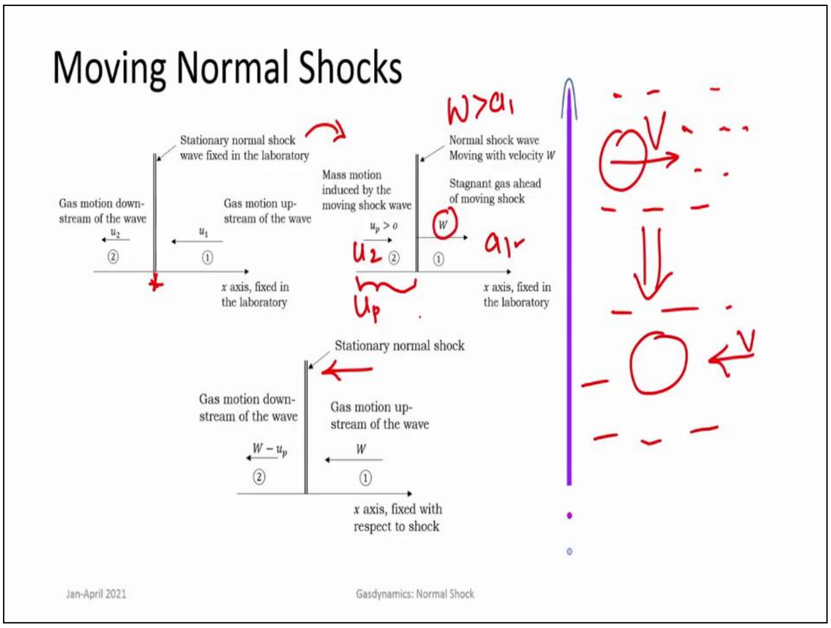

Now let us look there are several cases where one can find shocks moving examples are in the shock tube and which is a device used for many applications. And the other one is during explosive events which generates a blast wave. A blast wave is a shock wave that moves in all directions. So you can look at these schematics what we normally considered till now is that of a stationary shock that is fixed shock.

So in a fixed shock what is happening is you are already having a supersonic free stream. In supersonic is flow is happening and the shock wave stands at a particular location it is fixed and the flow gets processed by the shock wave and you get lower velocities, higher pressure, temperature and densities across the shock. And in contrast if one looks at the case of a moving shock. Here the shock moves with the velocity W and very clearly this shock can move only at supersonic speeds. So, the velocity of shock wave that is W will be greater than the acoustic speed in that medium which is in this it is moving into region one. Region one is the quiescent medium here, it is stationary. So, it is stagnant, so it is moving into the stagnant medium. So the acoustic speed there is a_1 and W is greater than a_1 so this must be very clear to all of you.

Now as this wave moves into the medium it also induces a mass motion. So there will be a movement of gas behind the shock wave though that is very different from any normal wave motion that one describes where we say that particles behind or as a wave passes once it has passed the particle will still remain at the same position. But in a shock wave it grabs the flow the fluid and there is a mass motion induced by the moving shock and you get a speed u_2 sometimes it is also referred to as u_p i.e u(piston), up.

This u_p terminology is coming from the piston analogy that we had discussed very early classes of normal shocks. So other representations are u_2. So how do we analyse this moving shock? So, this follows from the principle which is very often used in flows that if you have a body moving in a flow with a velocity V. So, if this is moving with velocity V in a fluid, an equivalent problem is it this is same as saying that the body is stationary.

And fluid on the other hand is moving with velocity V in the opposite direction. So, it is the same problem, it is a transformation. So, you are doing the transformation by in applying the velocity V in the opposite direction of the motion of the body. So, same way the moving normal shock can be analysed by taking the same velocity W and imposing it on the opposite direction. So, once you impose it in the opposite direction then the moving normal shock gets converted into a stationary normal shock with respect to this stream of gas.

So this is the moving normal shock is in the laboratory fixed frame of reference while stationary normal shock is in moving frame of reference. So, there we have induced the motion or we have applied a motion in the opposite direction. So we have applied a transformation that means whatever was u_p over here now gets converted to $(W - u_p)$.

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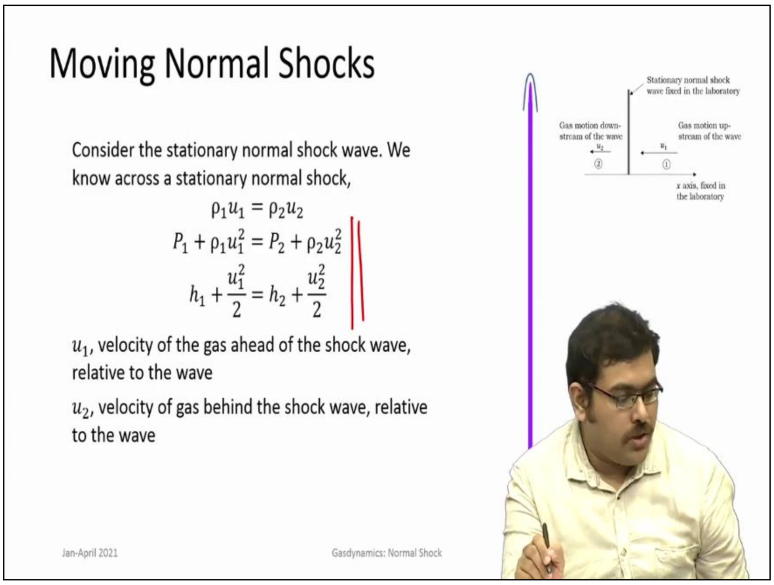

So now how do we apply the equations of shock? So now we have applied the transformation. Now it is a stationary normal shock, so one can apply the equations of conservation. It is a steady shock wave now. So you can apply the same equations of conservation $\rho_1 u_1 = \rho_2 u_2$,

$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$ and $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$.

So you can apply the same equations, u_1 is the relative velocity that is upstream of the shock wave and u_2 is the relative velocity downstream of the shock wave relative to the shock. So that is you change the frame of reference to the shock.

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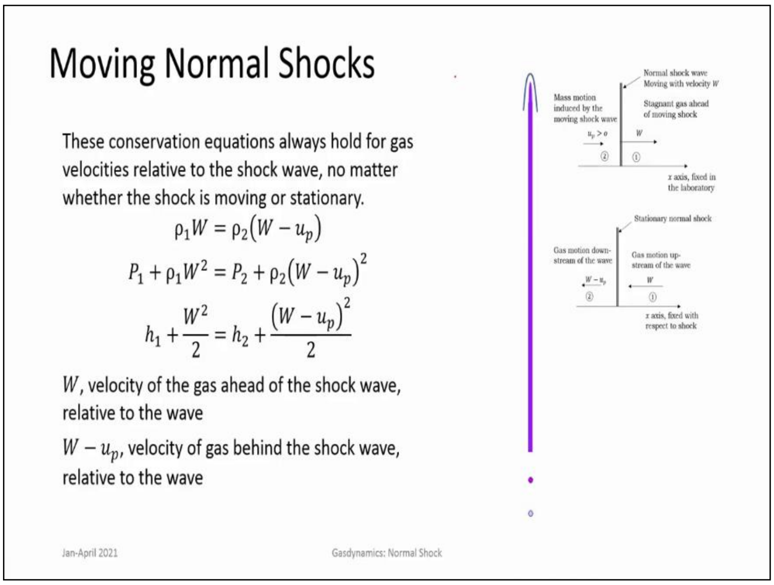

So, now we know upstream of the shock u_1 is equal to W and downstream of the shock the relative velocity with respect to the shock is (W – up). So we can write these equations in terms of W and (W – up),

$\rho_1 W = \rho_2(W - u_p)$ is mass conservation ,

$P_1 + \rho_1 W^2 = P_2 + \rho_2 (W - u_p)^2$ is momentum conservation and

$h_1 + \frac{W^2}{2} = h_2 + \frac{(W-u_p)^2}{2}$ is energy conservation.

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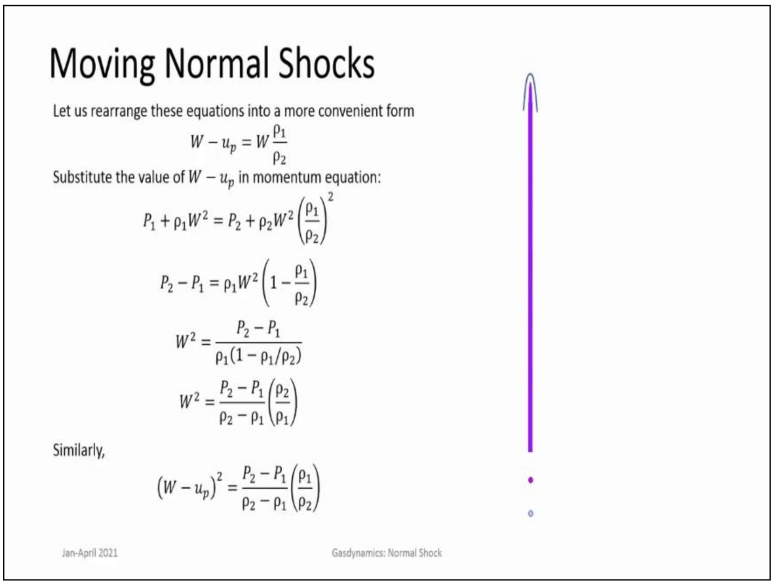

So these equations are the same for a stationary shock wave. We can go ahead and do an analysis very similar to what we had done for the Hugoniot equation because you can then take out the information of velocity and look at what is happening to the thermodynamic quantities across the shock. The development is very similar. So we will just brush through this. So this is the mass conservation where you can express $(W - u_p)$ in terms of W and $\frac{\rho_1}{\rho_2}$.

From here and useful expression is u_p. This u_p now is in fixed frame of reference or laboratory frame of reference is nothing but $u_p = W\left(1 - \frac{\rho_1}{\rho_2}\right)$. So, this is a useful expression we will come to this soon. And the combination of the mass conservation into momentum conservation and applying similar principles that we express W square in terms of pressures and $\frac{\rho_2}{\rho_1}$ ratio of difference of pressure and density and $\frac{\rho_1}{\rho_2}$ giving $(W - u_p)^2$.

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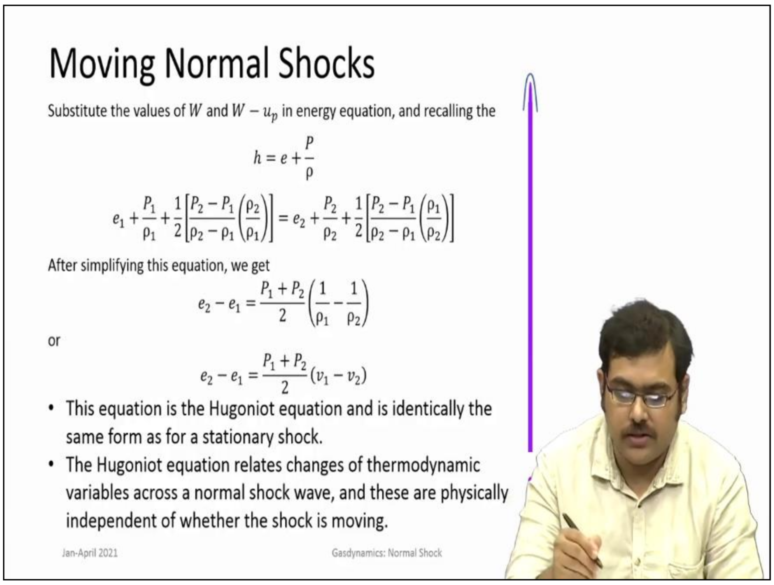

And then putting this into energy equation and coming up with the final expression for Hugoniot. So this is the Hugoniot expression for moving shock written relative to the shock wave and it is the same as the stationary shock. So the shock properties do not change that is the thermodynamic variables pressure ratio, temperature ratio, density ratio do not change across a moving shock when we analyse the shock relative to the shock wave itself.

So this is very important whenever you look at problems on moving shocks always remember that the analysis proceeds that you first change the coordinates to come on sitting on the moving shock. Then the equations are written so that it is relative to the moving shock and that transforms this into a stationary shock analysis. And you can proceed to get all the variables according to the stationary shock. So this is a very important principle.

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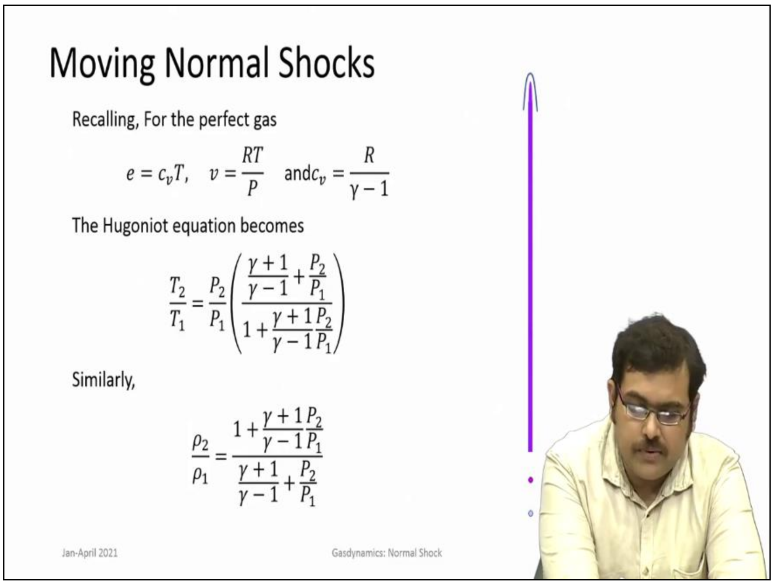

So now let us see how we can express all the different quantities. We are particularly interested in what is this gas motion how much of this movement can be produced because of a shock moving? And to do that we use expressions from the Hugoniot where all the ratios $\frac{T_2}{T_1}$ and $\frac{\rho_2}{\rho_1}$ can be expressed only in terms of $\frac{P_2}{P_1}$ where $\frac{P_2}{P_1}$ is nothing but the strength of the shock.

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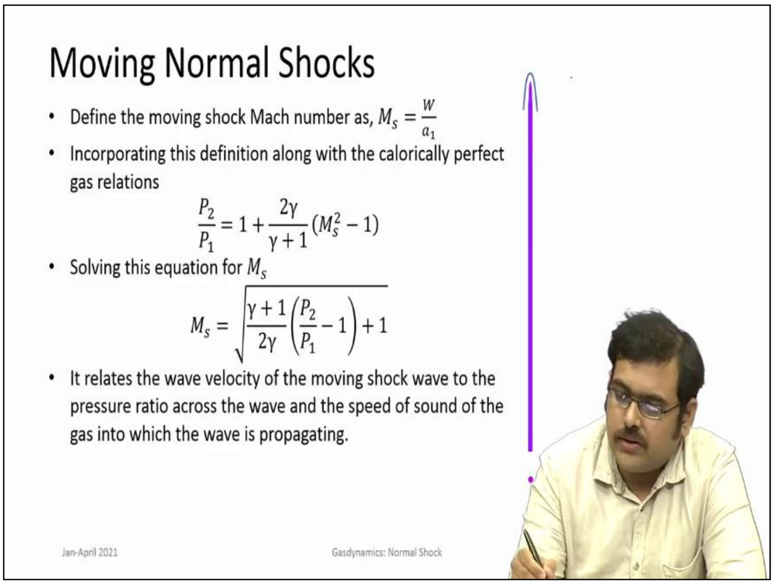

So now if you do that and you express the Mach number of the shock is always so $M_s = \frac{W}{a_1}$ always which is a_1 is the medium into which the shock is moving. So M_s is the Mach number of the shock that number of shock is $M_s = \frac{W}{a_1}$ .This is always greater than 1. And $\frac{P_2}{P_1}$ now is given by this expression which is the same for the stationary shock also but now this is for a moving shock. And from here one can express Ms in terms of the ratio $\frac{P_2}{P_1}$.

$$M_s = \sqrt{\frac{\gamma + 1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right) + 1}$$

So pressure ratio of the shock and the Mach number of the shock wave they are related to each other.

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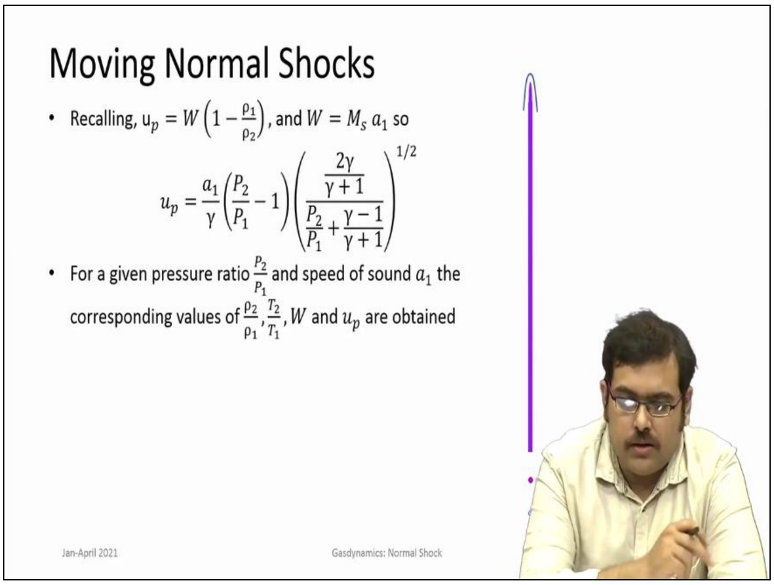

So then we come to how we can express the speed of the gas moving behind the shock. So to do that we use the expression from the continuity equation which was represented there and u_p is , $u_p = W\left(1-\frac{\rho_1}{\rho_2}\right)$ and W is $W = M_s\, a_1$ and you can express this $\frac{\rho_1}{\rho_2}$ in terms of $\frac{P_2}{P_1}$. Then you come to this particular equation for u_p which is expressed in terms of a_1 which is the speed of sound in the medium into which the shock is going and $\frac{P_2}{P_1}$and γ .

$$u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1}-1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1}+\frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

So this is what is obtained. So you can use this expression so $\frac{P_2}{P_1}$ is expressed in terms of shock Mach number.

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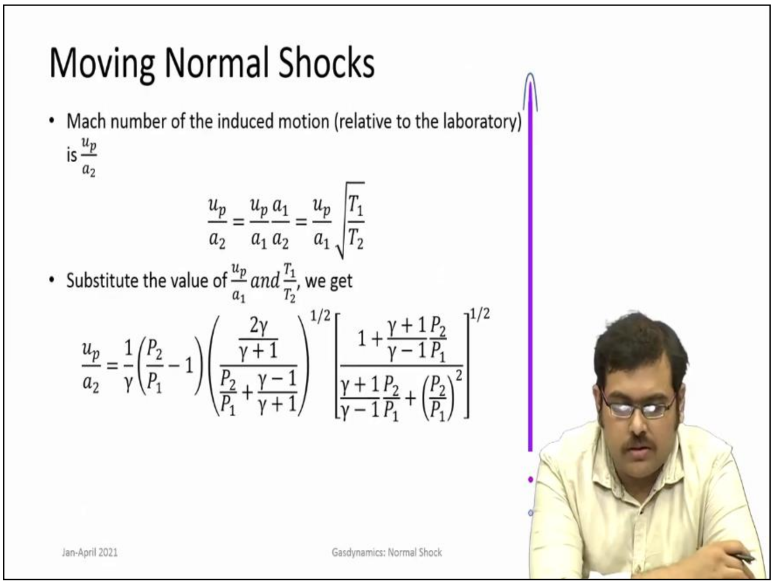

Now the Mach number for the induced motion will be with respect to the medium behind the shock wave that is when the shock is moving here into this medium. Now this is a_1 as the shock moves into it there it produces a motion u_p and across the shock temperature will increase. So T_2 will be greater than T_1 as a consequence a_2 will be greater than a_1. So what is M_p that is or M_2 behind the shock is actually $M_2 = \frac{u_p}{a_2}$.

So $\frac{u_p}{a_2}$ that is this is with respect to fixed frame of reference that is with respect to the laboratory.

And it is expressed as $\frac{u_p}{a_2} = \frac{u_p}{a_1}\frac{a_1}{a_2}$ and a = $\sqrt{\gamma RT}$. so this turns out to be $\frac{u_p}{a_2} = \frac{u_p}{a_1}\frac{a_1}{a_2} = \frac{u_p}{a_1}\sqrt{\frac{T_1}{T_2}}$.

So if you substitute that you get this expression solely in terms of the strength of the $\frac{P_2}{P_1}$.

$$\frac{u_p}{a_2} = \frac{1}{\gamma}\left(\frac{P_2}{P_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}\left[\frac{1 + \frac{\gamma+1}{\gamma-1}\frac{P_2}{P_1}}{\frac{\gamma+1}{\gamma-1}\frac{P_2}{P_1} + \left(\frac{P_2}{P_1}\right)^2}\right]^{1/2}$$

This is the motion of the gas.

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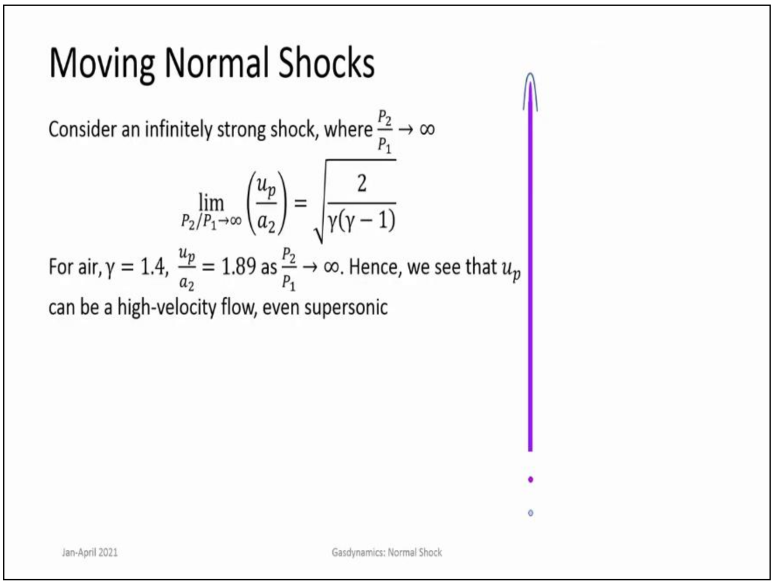

So now let us take the limit as we had done previously for very large shock waves. So, let us consider a very strong shock wave, is infinitely strong shock wave then $\frac{P_2}{P_1} \to \infty$ that we had seen that as Mach number goes to infinity $\frac{P_2}{P_1} \to \infty$. But we also saw that M_2 that is Mach number downstream of the shock goes to finite values as $M_1 \to \infty$ .That means the gas motion behind the shock will also reach finite value.

So, let us see what is that number if you put the appropriate limit in this expression $\frac{P_2}{P_1} \to \infty$. Then you get $\lim_{P_2/P_1 \to \infty} \left(\frac{u_p}{a_2}\right) = \sqrt{\frac{2}{\gamma(\gamma-1)}}$, so this is the expression. And now if you substitute γ is 1.4 you get up is 1.89. So that means for a very, very strong shock wave the Mach number of the gas flow behind the shock wave is nearly 2. 1.89 its supersonic . Its very high speed.
And so it is not a very small velocity. It is very high velocity flow behind a strong shock and it can carry huge amount of momentum along with it and that is what affects bodies once shock waves besides the jump across the shock wave.

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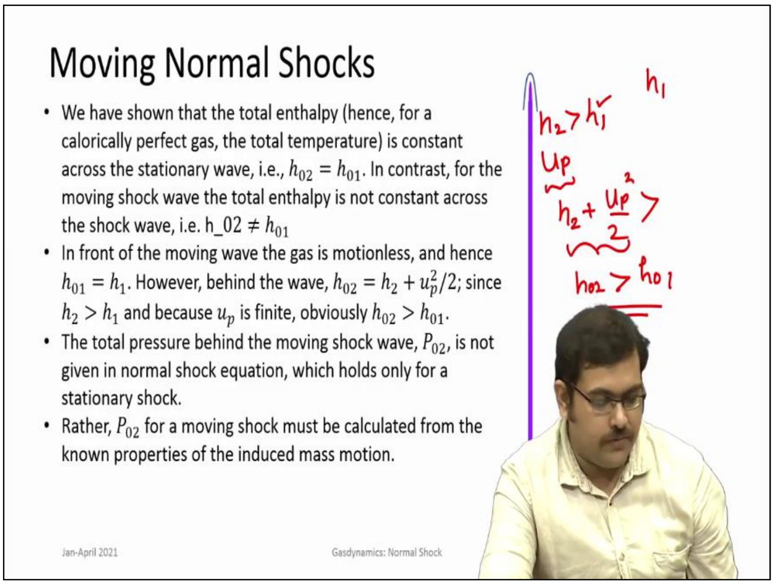

Now whatever we have discussed till now it was on pressure ratios, temperature ratios across the shock wave of a moving shock and to analyse this, the simple technique was to jump on to the shock wave, a moving shock wave and write the equations relative to the shock wave. And that renders the shock stationary and you get all the relations for various quantities but what about the stagnation properties.

So, we know from our understanding of the stationary shocks that it is it is a case of constant stagnation enthalpy because it is an adiabatic flow. But what about moving shock waves. Now we have done a change of reference but the stagnation temperatures and stagnation pressures they are all to be calculated in reference to the fixed frame that is the laboratory frame. So if you do that first thing that you notice is that h_2 that is the enthalpy behind the moving shock is greater than h_1 is what you would get. This is from the shock analysis itself.

Not only this you also find that u_p is induced there is a gas motion induced after the shock passes through the medium. Now the shock is moving to quiscent medium that means before the shock moves the velocity is 0. But once the shock passes over a particular point , gas is moving over that point. At that point with the velocity u_p, so the total enthalpy now is

$$h_{02} = h_2 + u_p^2/2.$$

This is definitely greater than what you had started with initially. So you find that h_{02}. So h_1 is what you started initially because there was no gas motion. You find that h_{02} is greater than h_{01}. So, this is very important so the stagnation properties for a flow behind the moving shock wave should not be calculated using normal shock relations for a stationary gas, stationary shock wave. But they have to be calculated separately once you have calculated the motion of the gas behind the shock wave.

Now is there any contradiction ? We should ask this question if there is any contradiction or not.

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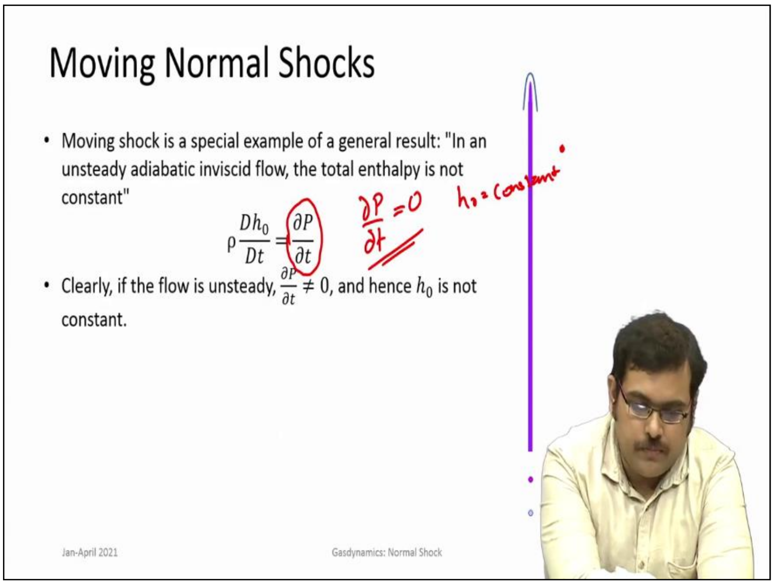

It is not a contradiction because we are actually looking at a case of unsteady flow. If you look at a particular point in that flow, it is not a steady flow anymore because the pressure, temperature changes immediately as a shock passes over that particular point. So it is not a steady flow problem. So when it is not a steady flow problem, if you look at the energy equation then you find that on the right hand side, of course neglecting any heat addition and other terms. On the right hand side you have the term related to unsteady pressure $\frac{\partial P}{\partial t}$.

In a steady flow $\frac{\partial P}{\partial t}$ =0 because it is a steady flow and as a consequence you get in adiabatic flow h_0 is constant. But in moving shock wave is an example of an unsteady flow $\frac{\partial P}{\partial t} \neq 0$ as a consequence h_0 is not constant. So that one should bear in mind, so static properties you across a moving shock is calculated using stationary normal shock relations by doing an appropriate transformation by jumping onto the shock and moving with the shock.

But stagnation quantities have to be calculated separately by knowing the properties velocity, temperature, pressure in the fixed frame of reference.

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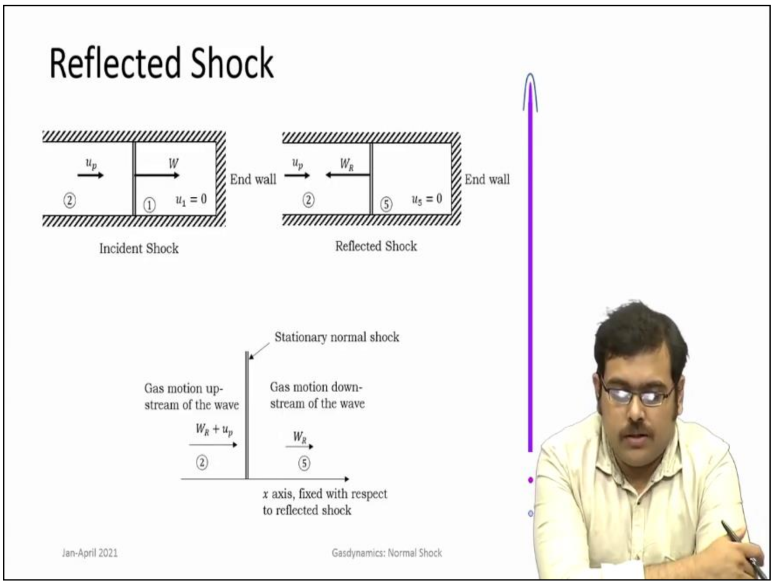

Now let us see often what happens if you have the shock wave moving and it moves and there is a gas motion behind it and it comes and hits on a solid surface. So it can be in the case of a shock tube or it can be otherwise. Also you have rigid bodies which do not allow any motion through them. Therefore the only way this can go about is that this shock goes and hits the end wall and reflects back.

So you have a reflected shock. Now the reflected shock is also a moving shock. So the analysis of reflected shock follows similar lines as the analysis of the moving shock itself. But now you see the boundary conditions or the conditions that you have to apply have changed. First thing is that the boundary condition at the wall is that the velocity should be 0 that means at these conditions, at these places ,the velocity is 0 .V equal to 0.

This term u_5 is used here. It is coming in from shock tube terminology. We will come to it soon but velocity here is 0. Now this is a relative motion which is happening into a flow which is coming opposite to u_p. So the relative velocity here is actually $W_R + u_p$. So you would impose a velocity W_R in order to be on the shock and that means the total velocity coming on to this shock to make it stationary is $W_R + u_p$. While V is 0 behind the gas, the velocity is W_R so this is the problem that has to be analysed.

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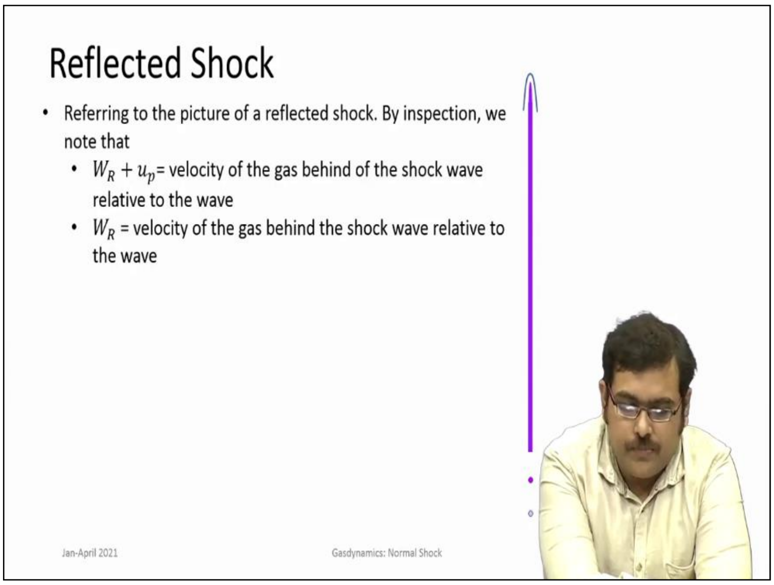

And so relative to the reflected shock upstream velocity is $(W_R + u_p)$ and downstream it is W_R, same set of equations.

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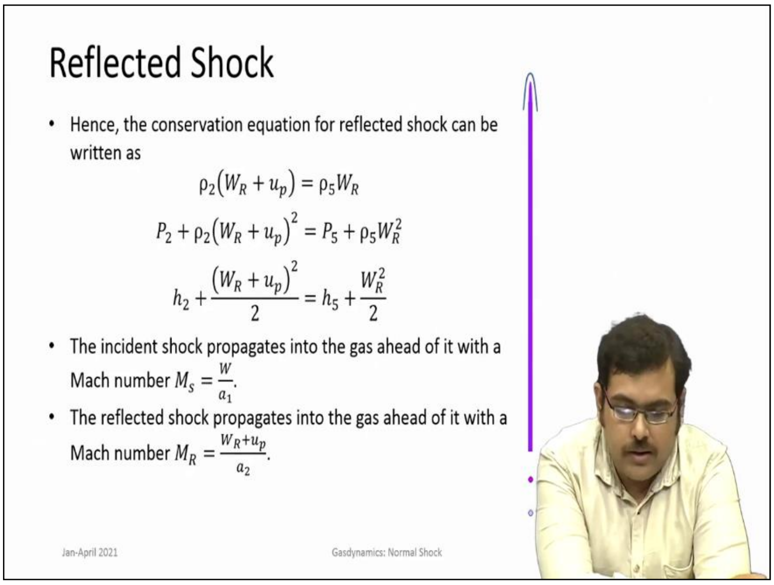

Now written in terms of $(W_R + u_p)$ and W_R and reflected shock Mach number is $M_R = \frac{W_R+u_p}{a_2}$.

So, in order to make it stationary or we need to bring it into the stationary frame of reference.

So $M_R = \frac{W_R + u_p}{a_2}$ and this can be related to the incoming shock itself that is Ws. It the W_R and Ws can be related to each other.

$$\frac{M_R}{M_R^2 - 1} = \frac{M_s}{M_s^2 - 1}\sqrt{1 + \left(\frac{2(\gamma - 1)}{(\gamma + 1)^2}\ (M_s^2 - 1)\left((\gamma + 1)\frac{1}{M_s^2}\right)\right)}$$

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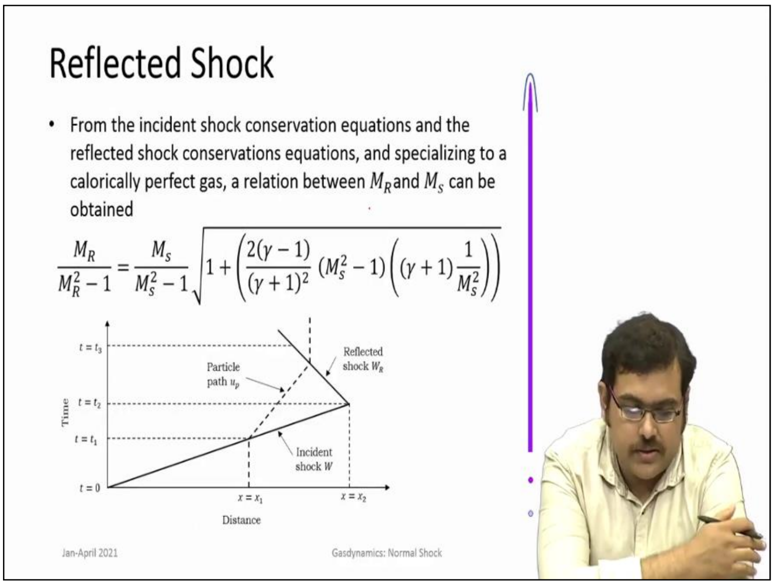

And one can express the reflected shock completely in terms of the incident shock Mach number itself. So this is a single relation which relates reflected shock wave with that of the incident shock wave. So if one is to look at what is happening for a particle? So, as the very close to the end wall, so as the shock wave coming moves into the medium it takes all the gas and moves it along with it with a speed u_p.

And then after that the primary shock comes reflects at the end wall and then goes back. As it goes back it converts all this to a velocity equal to 0. So it again becomes the velocity of particle again becomes 0. But if you draw the motion of the particle in a x-t diagram that is how it is moving in x, I mean in time and its location in x then you see before the shock arrives it is not moving. And the moment the shock arrives due to the induced gas motion it starts moving with a speed u_p and the moment the shock reflects, and it comes back and meets the reflected shock after that it becomes stationary again. But see that there is a fixed motion of the particle due to the combination of shock wave and reflected shock wave. So, the key ideas here that have introduced in this analysis of moving shock wave is that to analyse moving shock waves. All one has to do is sit on the shock and look at the motion of air relative to the shock wave. That makes the system of equations same as that of the stationary shock.

So, the analysis of flow properties static properties across the shock moving shock is the same as that of the stationary shock. But always make sure that you do the correct transformation. Again, when one is looking at the gas motion behind such moving shocks again makes sure that the transformations that you apply are correct. So, if you analyse using the normal shocks stationary normal shock the Mach number after the shock that you get is relative to the shock wave.

So, we should always seem to get the velocity with respect to fixed laboratory frame, we again use the transformation back. Then, these shock waves, moving shock waves, can go and reflect from end walls. And the correct boundary condition there is that once it reflects , in such a way that the flow after the reflected shock passes over is 0 because flow cannot penetrate inside the wall.

And analysis of the reflected shock is again same as a moving shock. But now the coordinate transformation or the transformation is $(W_R + u_p)$ which is what you have to take care of. So moving shocks are important in shock tubes which are devices used in several laboratories and having other applications as well. It is a case of an unsteady flow. So we will discuss, we are now discussing 1D flows.

So, we will discuss unsteady flows in the coming classes. Before we do that let us solve some numericals in the next class to get the concepts of normal shocks more clearly.