Gasdynamics: Fundamentals and Applications
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Lecture 17
Normal Shock - II b

So we are discussing the analysis of normal shocks and going through the details of how to gather all information about flow variables downstream of the normal shock. And the last class we had discussed about getting to the Prandtl's relation for normal shocks that is using the conservation equations.

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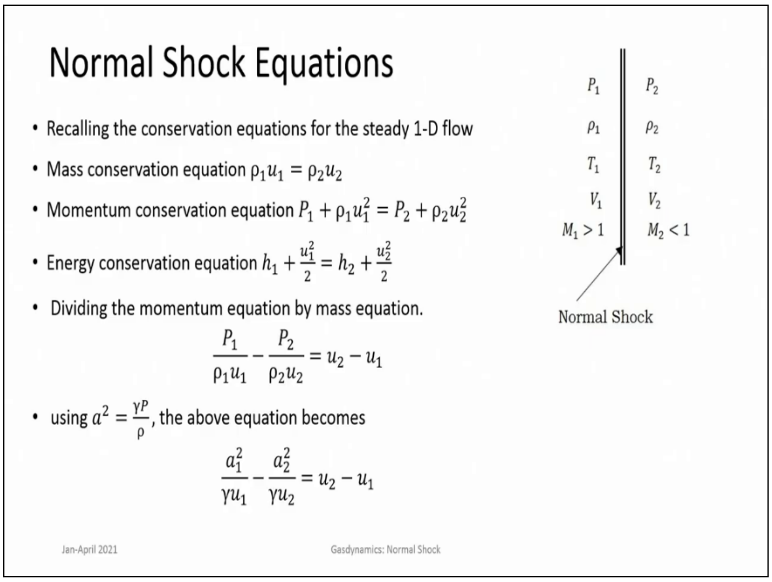

This is something that was done the previous class. So, I will quickly go through this and the conservation equation of mass, momentum, energy for one dimension is used to analyse the normal shock. It is an adiabatic process without any work done. So, you can apply these equations along with the equation of state and relation for the enthalpy.

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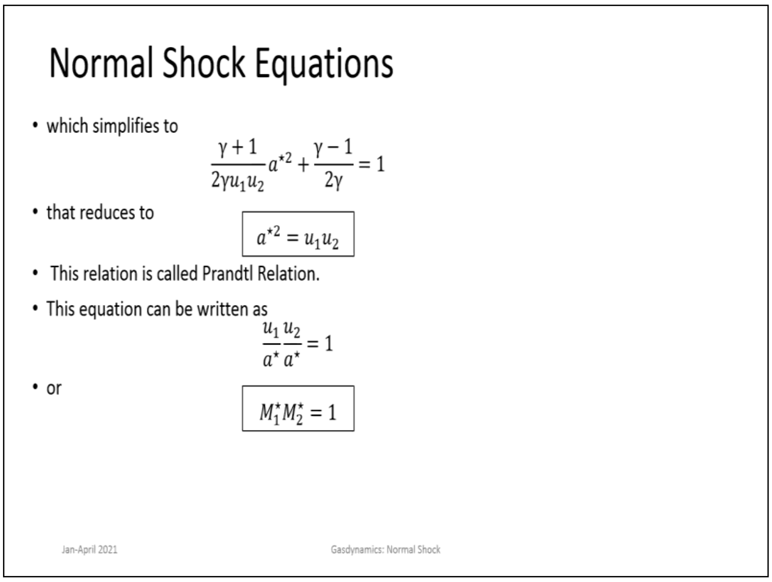

So, by using all three equations together, we come to a relation $M_1^\star M_2^\star = 1$. This is called the Prandtl's relation and using this we can convert the equations.

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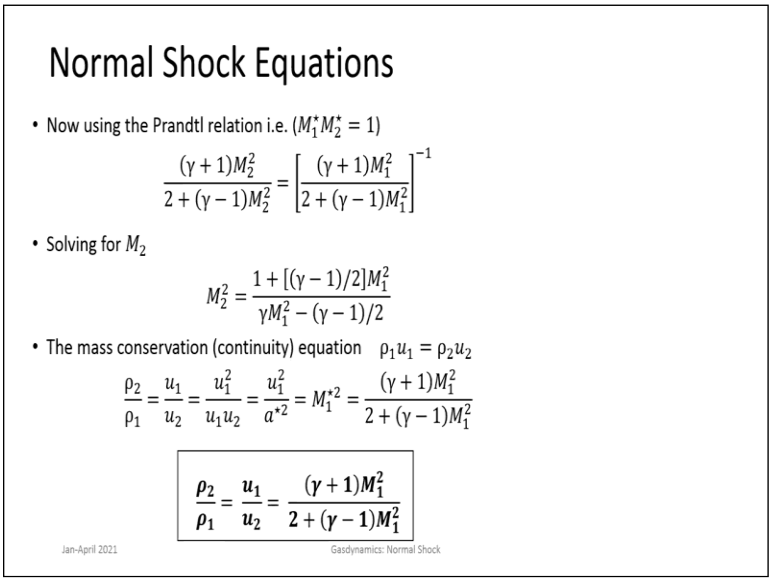

So that we can exclusively get the equation for downstream Mach number M_2^2 in terms of M_1^2 and γ. Similarly using the continuity, once we had done this, we can get the equation for the density ratio $\frac{\rho_2}{\rho_1}$ in terms of only Mach number and γ. So, this is something that will come up again and again

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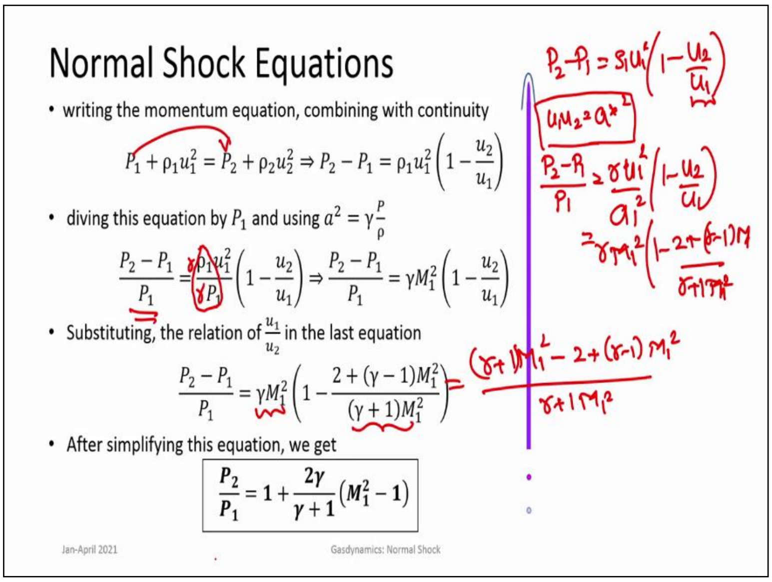

So now we are left with the pressure ratios, temperature ratios. Let us see how to get to them. So here we take the momentum equation again equation $P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$ and do the subtraction the sort of moving the variables from here to the side is $\rho_1 u_1^2$. Now once we have the Prandtl's relation that is relating $u_1 u_2 = a^{*2}$ for conditions across the normal shock.

Our intention in further analysis would be to try to use this form formulation Prandtl's relation in order to get to the other flow variables. So here you get $\frac{u_2}{u_1}$so this quantity is there now you can divide this by P1 which is done over here and you can $\frac{P_2 - P_1}{P_1} = \frac{\rho_1 u_1^2}{P_1}\left(1 - \frac{u_2}{u_1}\right)$. Now immediately some things that again come again and again in gas dynamics is the combination $\frac{P}{\rho}$. If you come across this combination $\frac{P}{\rho}$ and it can be converted to the speed of sound for a perfect gas by multiplying and dividing by γ.

So, if you do that then you get this term which is a group of terms which is $\gamma \frac{P}{\rho}$. So, this term comes out to $\frac{\gamma u_1^2}{a_1^2}\left(1 - \frac{u_2}{u_1}\right)$. Now we will go ahead and see this is M_1^2 .So this is$\gamma M_1^2 \left(1 - \frac{u_2}{u_1}\right)$. Now $\frac{u_2}{u_1}$ from the previous analysis, we found what is $\frac{\boldsymbol{\rho_2}}{\boldsymbol{\rho_1}}$. That is $\frac{u_2}{u_1}$. So $\frac{u_2}{u_1}$is nothing but $\frac{\boldsymbol{\rho_1}}{\boldsymbol{\rho_2}}$ which is $\frac{\mathbf{2+(\gamma-1)M_1^2}}{\mathbf{(\gamma+1)M_1^2}}$ **.**

So, we just have to import that into this formulation and what we get here then is $\frac{P_2 - P_1}{P_1} = \gamma M_1^2 \left(1 - \frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right)$. So that is what you get over here. So, this can be simplified here so you get $\frac{P_2 - P_1}{P_1} = \gamma M_1^2 \left(\frac{(\gamma+1)M_1^2 - 2 - (\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right)$. And if you do the algebraic manipulations on this you get $\frac{\boldsymbol{P_2}}{\boldsymbol{P_1}} = \mathbf{1} + \frac{\mathbf{2}\boldsymbol{\gamma}}{\boldsymbol{\gamma}+\mathbf{1}}(\boldsymbol{M_1^2} - \mathbf{1})$

So this is now straight forward here. So, this is the equation to relate $\frac{\boldsymbol{P_2}}{\boldsymbol{P_1}}$. So, we have used the momentum equation as well as the conditions that we just derived here that is $\frac{\boldsymbol{\rho_2}}{\boldsymbol{\rho_1}}$ is $\frac{u_1}{u_2}$ from the mass conservation. All of them are always they will have their roots towards the Prandtl's relation $u_1 u_2 = a^{\star 2}$

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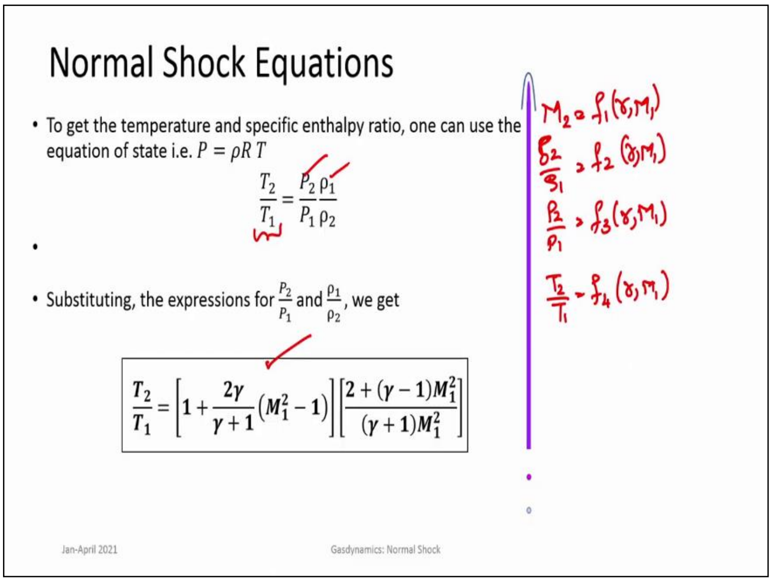

Now once you get the pressure ratio then getting temperature ratio is nothing but using the ideal gas equation of state. So $P = \rho R\, T$. $\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2}$. Now $\frac{P_2}{P_1}$, we have the expression for this, $\frac{\rho_1}{\rho_2}$ we have the expression and so we can multiply the 2 expressions,

$$\frac{\boldsymbol{T_2}}{\boldsymbol{T_1}} = \left[\mathbf{1} + \frac{\mathbf{2}\boldsymbol{\gamma}}{\boldsymbol{\gamma}+\mathbf{1}}(\boldsymbol{M_1^2} - \mathbf{1})\right]\left[\frac{\mathbf{2} + (\boldsymbol{\gamma} - \mathbf{1})\boldsymbol{M_1^2}}{(\boldsymbol{\gamma}+\mathbf{1})\boldsymbol{M_1^2}}\right]$$

So this is the final expression for $\frac{\boldsymbol{T_2}}{\boldsymbol{T_1}}$. Now if you observe you again observe this f_2 is only function of $\boldsymbol{\gamma}$ and Mach number that is Mach number then you got $\frac{\boldsymbol{\rho_2}}{\boldsymbol{\rho_1}}$ density ratio is function of only $\boldsymbol{\gamma}$ and Mach number and pressure ratio is function of only $\boldsymbol{\gamma}$ and Mach number. So this sort of emphasizes that for a normal shock if you know the upstream conditions then you can completely determine the downstream conditions.

So, from here these relate the static quantities across the normal shock. How can stagnation quantities be related across the normal shock?

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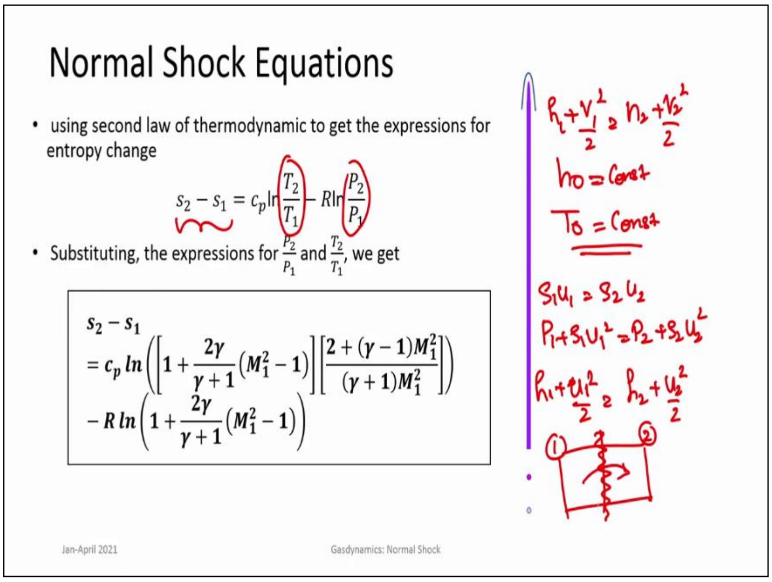

So if we look at the normal shock it is an adiabatic process, so there is no heat transfer or work done that means across the normal shock the total enthalpy $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$,total enthalpy h_0 remains a constant. So that means T_0 for a calorically perfect gas, T_0 does not change, T_0 is a constant. This is a very important idea that T_0 remains constant across normal shock. This is for a standing normal shock. Stationary normal shock in a supersonic flow this is what we are analysing.

Now let us go and look at is there any entropy change across the normal shock. So, we began our analysis by saying that it is an adiabatic process but never said that it is a reversible adiabatic process. And reasons are that shock waves generate entropy just because within the shock wave. Within that small very infinitesimal thickness of shock wave the transport process becomes very, very important.

And they give rise to entropy and the high gradients across the shock give rise to the entropy and so entropy change across the shock is you use the relation.

Now, you know $s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$.

For $\frac{T_2}{T_1}$ and $\frac{P_2}{P_1}$ are determined in the previous analysis. So, we substitute for them in terms of $\frac{T_2}{T_1}$ and $\frac{P_2}{P_1}$. And so, this expression once you know these values you can directly determine the entropy change across the shock wave.

Now, one should notice that the conservation equations if you take a $\rho_1 u_1 = \rho_2 u_2, P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$ and $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$ just relate the balance of mass, momentum and energy across a discontinuity. And we are doing control volume analysis for the states before the shock and after the shock. So this is the shock but by this just by these three equations one cannot determine what should be the conditions across the shock. Can there be conditions that velocity suddenly increases across the shock.
That kind of a shock can be called as an expansion shock because the normal shock whatever we are discussing is a compression shock. It compresses the gas increases the pressure and density. Now we can ask the question that is it possible that there is a shock which will suddenly increase the velocity, reduces pressure and density and temperature can there be such a solution? And if you look at the at only the conservation laws, they do not tell you what the conditions are whether such solutions can exist or they cannot exist.

And for this you need to bring in the second law of thermodynamics and the entropy considerations. And now we have the relations for entropy and now that we know this is an adiabatic process then clearly the change in entropy should always be positive. There are irreversibilities in the system. That means change in entropy will be greater than 0. If you apply this condition to this expression and try to find out what is the condition?

Since now this expression is only function of γ and Mach number. So that is upstream Mach number you will get that M_1 has to be greater than 1. So even from entropy considerations you will find that always shocks exist in supersonic flows and second thing is that the expansion shock is not permitted because it reduces entropy it is not possible that you can have a sudden increase in Mach number.

So you cannot increase Mach number by a discontinuous jump so because of these conditions these entropy considerations. So expansion shock is not possible only compression shock is possible and normal shock is a compression shock.

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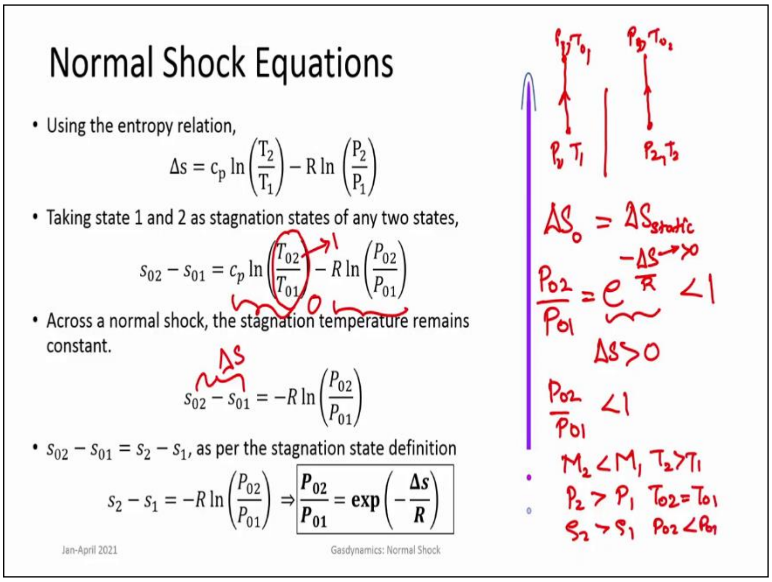

Now we know that stagnation temperature across the shock is constant by the virtue of the process being adiabatic but what about stagnation pressure? So stagnation pressure across the shock how can we relate this? For this again we go to the entropy relations. So the entropy relations for across the shock we just discussed this$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$. Now we come to a specific application of the stagnation conditions.

Now we know that every point in the flow. So if I take this is normal shock there is a point before and after the normal shock and P_1 ,T_1 represent the static quantities and P_2 ,T_2 represents static quantities downstream of the shock. Then there is an equivalent point where the entropy remains through an isentropic process. You can take this to P_0 and T_0. These are equivalent conditions P_0 and T_0. So, their entropy at P_1, T_1 and P_0 ,T_0 are the same.

Similarly, entropy at P_2 ,T_2 and P_0 ,T_0 are the same P_{01} and T_{01}. So the same equation that is so change in entropy can be written in terms of changes to the stagnation quantities. And we know that across the shock wave this stagnation temperature does not change stagnation enthalpy is constant. So $\frac{T_{02}}{T_{01}}$ is 1. So this term, $\ln\left(\frac{T_{02}}{T_{01}}\right)$, goes to 0. So you can get the relation telling the change of stagnation pressure.

So stagnation pressure ratio $\frac{P_{02}}{P_{01}}$ is directly given by this is ΔS. Now ΔS at the stagnation conditions is the same as ΔS at static conditions because they are related by an isentropic, each state is related by an isentropic process. So what we find here is that $\frac{P_{02}}{P_{01}}$ you can take the inverse logarithm of this process $\frac{P_{02}}{P_{01}} = \exp\left(-\frac{\Delta s}{R}\right)$. Now just now we had discussed that the entropy always increases across the normal shock.

So ΔS is always greater than 0 that means this term is greater than 0.That means that this term will be less than 1. So$\frac{P_{02}}{P_{01}}$ is less than 1. That means stagnation pressure decreases across the shock. So this is a important sort of expression that stagnation pressure decreases across shock. So if you now you have determined for both the static and the stagnation conditions across the shock.

And you know the relationships between all of them that is the back number M_2 decreases across the shock, pressure increases across the shock, density increases across the shock, temperature increases across the shock. But T_{02} that is total temperature remains the same, P_{02} decreases downstream of the shock. So this gives us all the relations. So now let us put them in. We know how these expressions are and let us plot them and see how they vary over Mach number.

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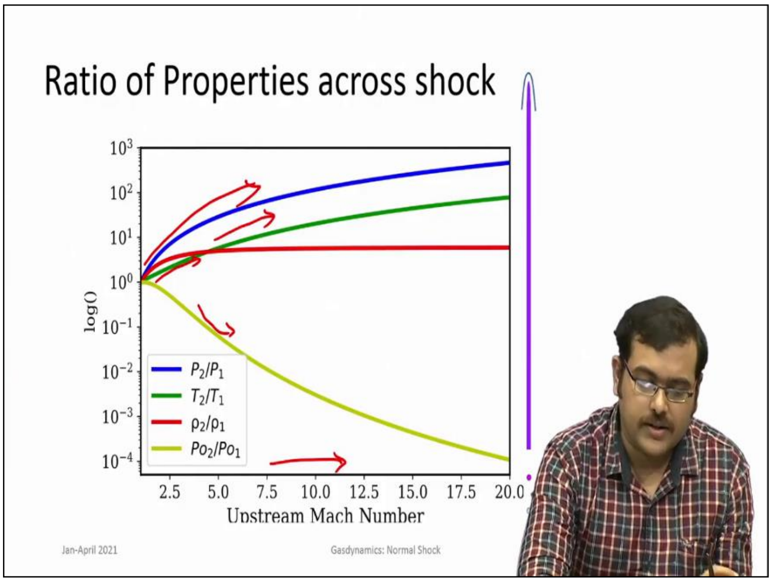

You can see that they vary quite rapidly and because they vary so rapidly, we cannot plot them on a linear scale. And increase in pressure due to increase of shock as the shock Mach number increases is enormous there is a significant increase in pressure. And this increase in pressure is accompanied by an increase in temperature also. So as Mach number changes both pressure and temperature increase significantly.

While if you look at what happens to density it is interesting initially it increases but once you reach higher, higher Mach numbers it sort of saturates. So it does not increase as much as temperature and pressure do. Stagnation temperature remains constant stagnation pressure on the other hand decreases significantly. So what are the consequences of this because there is a significant generation of entropy presence of shock waves in devices is leads to lot of losses.

So it makes it less efficient. So if you consider process of compression you have isentropic compression you have shock compression. So shock compression generates entropy. So lot of the energy is converted to unusable heat energy. So you see that if you consider 2 states one achieved by an isentropic process, for the same pressure ratio another for using a shock process then the temperature achieved in the shock process will be much higher than the one you get by an isentropic process.

So shock processes generate a much higher entropy. So looking at how to use these in designs or how to put them in devices then always we should look at trying to reduce entropy losses. So significant effort is put in so that the strength of these shocks can be reduced. Of course, when one looks at supersonic flows shocks are unavoidable if not in the design conditions but when the device works even slightly at an off design conditions you generate lot of shocks.

So the idea is to minimize the strength of shock. Strength of shock is mainly determined as we know by its Mach number and sometimes it is also referred to by the pressure ratio they are synonymous because once you know the Mach number you have a fixed pressure ratio. And so strength of shock you can say is by pressure ratio or the Mach number. So as weak shock as possible is what we would like.

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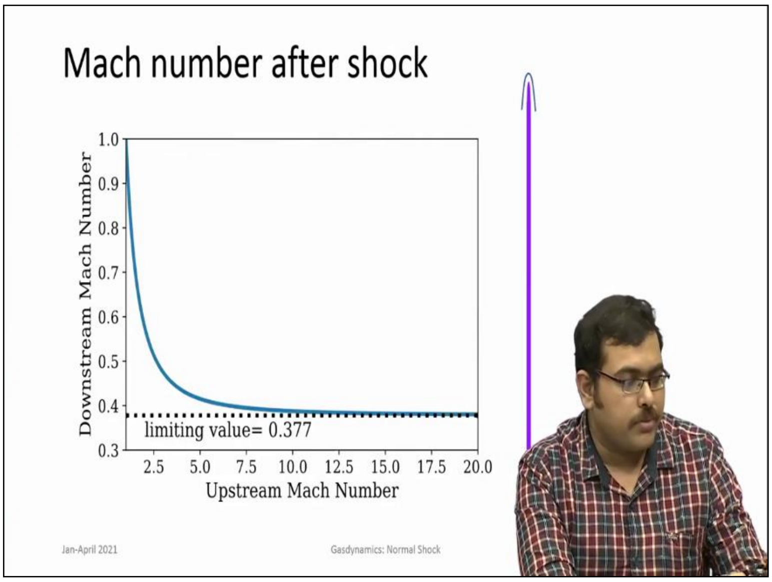

So another interesting thing about the downstream number of shock you see that always you have the Mach number downstream of the normal shock to be subsonic a normal shock exists only in supersonic flow. So upstream is supersonic downstream is subsonic the way M_2 behaves is that it decreases but then similar to the way density sort of saturates by increasing to a value.

The Mach number decreases to a value and saturates and sort of tends to be a constant. So these are how these values occur. There is another interesting thing about shock waves and a comparison with the isentropic compression that even those shock waves produce lot of entropy. In some cases in some applications particularly for aerodynamic testing when one looks at extremely high velocities that need to be achieved this can the stagnation pressures and temperatures required to achieve those high velocity conditions.

Typical to re-entry or very high Mach number flow will become very large. And this is clear to you from the stagnation conditions that we had already discussed also in the perspective of shock waves. So if you look at it as shock numbers increases the pressure ratios are actually higher and go higher and temperature ratios also go higher. So in order to achieve such high pressures and temperatures it is not very easy to do it in an isentropic compression because isentropic compression cannot increase temperature.

When temperature becomes important, enthalpy becomes important, isentropic compression does not give that high enthalpies. So shock compression is often used and they give rise to what are known as shock tube based tunnels.

Shock tunnels where shock compression is used to achieve high pressure and temperature. So, in some cases you need the shock waves and you want to use it for specific purposes.

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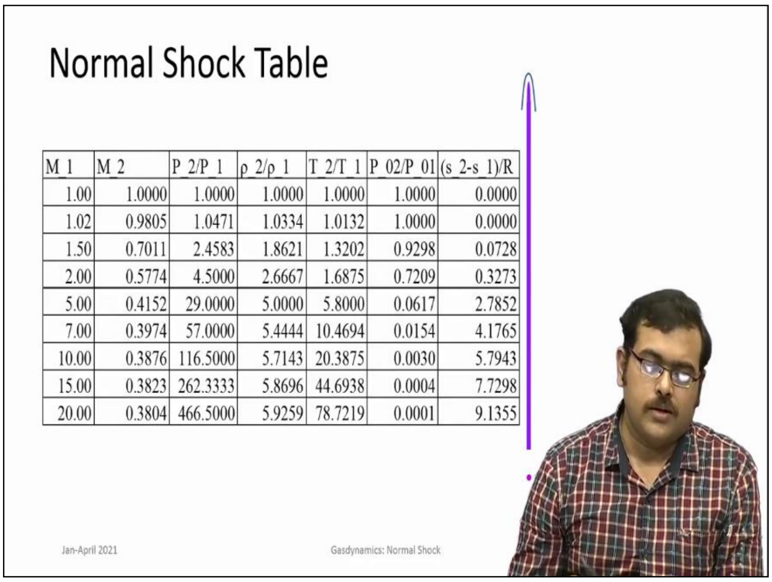

Similar to the isentropic charts normal shocks also you get charts and the way to use the charts and tables is the same whether you use it for isentropic relations or normal shocks. And they are given at the end of text books or in the appendices. So when we look at solving problems then we will use either these tables or you can use online calculators. So with that we come to a close on the analysis of getting to downstream conditions of shock given the upstream conditions.

And we saw we are able to relate all the different flow parameters. Now in the next class what we would see is that we come to what are strong shocks and weak shocks are there any limiting conditions? We have some indications through the graphs but let us look at it. And there is another thermodynamic way of representing the shock known as the Hugoniot relation which does not actually include the information on velocity just the relation between the 2 thermodynamic states that is useful in several cases.

And further on we look at where this can be applied in supersonic flows particularly for the Pitot. So that would be in the next class, thank you.