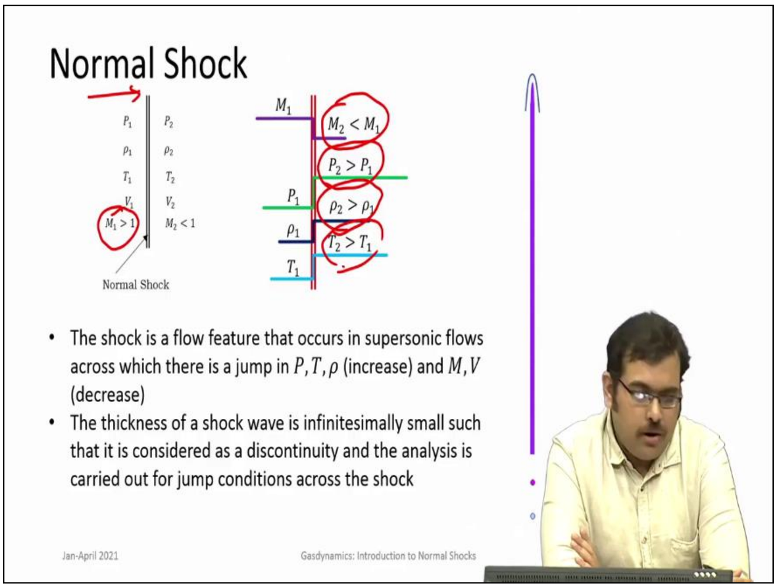
Gasdynamics: Fundamentals and Applications
Prof. Srisha Rao M V
Aerospace Engineering
Indian Institute of Science – Bangalore

Lecture 15
Normal Shock - I

Until now we have been looking at what are known as 1D flow and we have looked at important definitions like the star properties and the stagnation properties. Now let us look at something that is more interesting and very often found in compressible flow situations particularly when the flow goes supersonic that is the shock wave and a particular condition is normal shock.

Shock waves are presented in supersonic flows and now we will analyse and understand about normal shock which is one specific case of the shock waves.

(Refer Slide Time: 01:22)



So, let us look at what is a shock wave? So, shock wave is a flow feature that appears, and you should understand very strictly that it appears only in supersonic flows, that the flow should have a Mach number greater than one. In any flow, shock wave appears only in conditions where the flow is supersonic, Mach number is greater than 1.

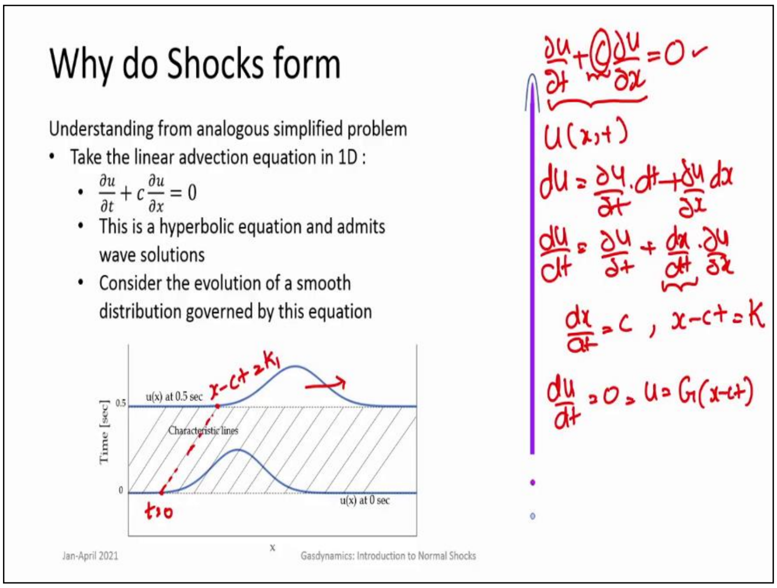
Across the shock wave there is a sudden jump in pressure, temperature, and density they all jump to much higher values while the flow decelerates across the shock as velocity decreases and its Mach number decreases. So, in general the principle of the shock is clearly given by this schematic $M_2 < M_1$, across the shock $P_2 > P_1$, $\rho_2 > \rho_1$, and $T_2 > T_1$.

So, shock is something which compresses the gas and increases its pressure, temperature and density while reducing its velocity. Now if you look at this shock wave these shock waves are extremely thin when they are produced their thickness is typically of the order of the molecular mean free path itself. So now you see the shock wave this structure of a shock wave or what happens inside the shock wave is something which falls under the regime of higher Knudsen numbers.

Being so small when you look at continuum flows then shock wave is actually considered as a discontinuity. So, in all our analysis in gas dynamics we take the shock to be a discontinuity in the flow and we analyse for the conditions across the shock wave, not for what is happening within the shock wave. Inside the Shock wave the processes of viscosity and thermal conductivity they sort of dominate or but it is really we cannot put it under the same class.

So rather for us you need to know it is so thin that you can consider that as a discontinuity and then you need to know what happens to pressure, temperature, and velocity across the shock wave. So that is what we do in gas dynamics we analyse for the jump conditions across the shock. Even before we go there, what we need to understand is? why do and how do these shock waves develop? and they how do they form?

(Refer Slide Time: 05:03)



So, to understand that it is really useful to look at some simple problems which are analogous to the fluid flow equations and see what happens to those equations when nonlinearities become important and then see how that can lead to the formation of shocks. To understand this, we will take a very simple 1D partial differential equation which is Advection equation.

Which is

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

Where, u is any variable that is getting advected. So, this is a hyperbolic equation and it admits wave like solutions it is some quantity ‘u’ is getting advected with a constant speed ‘C’ and this is a linear advection equation in 1D. If you look at its nature and we have discussed the Navier stokes equations in previous classes you would see there is a familiarity of this term which appears in the acceleration terms temporal and convective acceleration terms of the Navier stokes equation.

So let us see how this behaves if there is a smooth initial solution. If there is a smooth initial condition applied, then how does this behave? To look at this since it is a hyperbolic equation, we look at how it propagates in space and time and it can be shown very easily. See that ‘u’ is a function of x, t and if you are really looking at the total derivative ‘du’.

$$du = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx$$

Now if you look at what is its going to change in time this is,

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt}$$

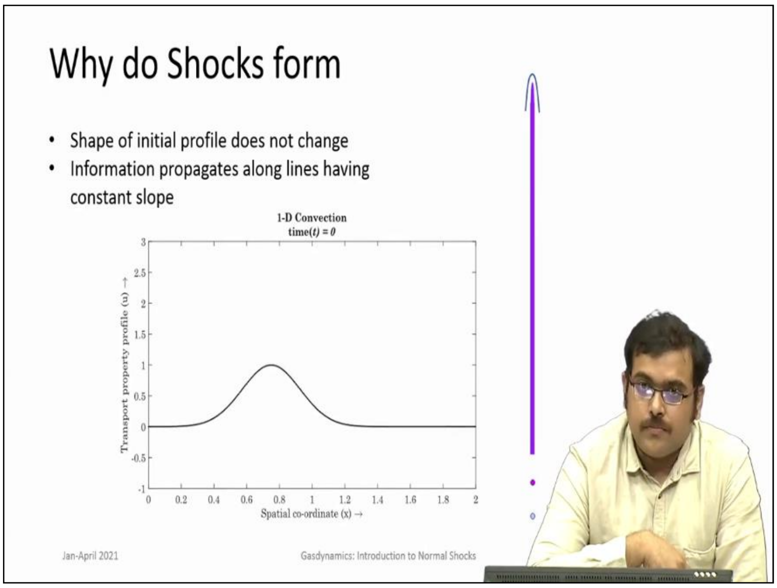
Now you see there is this term dx/dt and if you compare it with the original equation at its place you have the speed of convection C which is taken to be a constant here. So if you write dx/dt = C, then you get these are actually lines ‘x – Ct’ equal to some constant, so some constant k.

So x - Ct = k, some constant. So, these are lines in the field or in this t-x space and if you look at these lines if you take the solution along these lines then according to this equation du/dt =0 or ‘u’ remains constant, another constant along the lines x - Ct. So, these lines ‘x – Ct’ are called the Characteristic lines they propagate the information in the t-x plane.

The solution progresses along these characteristic lines. So now these lines have been drawn in this t-x plane across like this is shown by these black lines. So, if you want to find the solution at certain another time let us say in this case at ‘0.5 sec’ then all you have to do is to drop a characteristic back all the way this is ‘x – Ct’ equal to some constant say k1 it is dropped all the way back to the time when t = 0.

Then at that point whatever the value of ‘u’ is there that value gets at this point at 0.5 sec. So, what you see is that whatever initial smooth form was given that has simply convected by seeing that there is no change in the shape or the form itself and it has moved just in the same way. There is no change in shape. Let us compare this to what happens if there is a simple non-linearity.

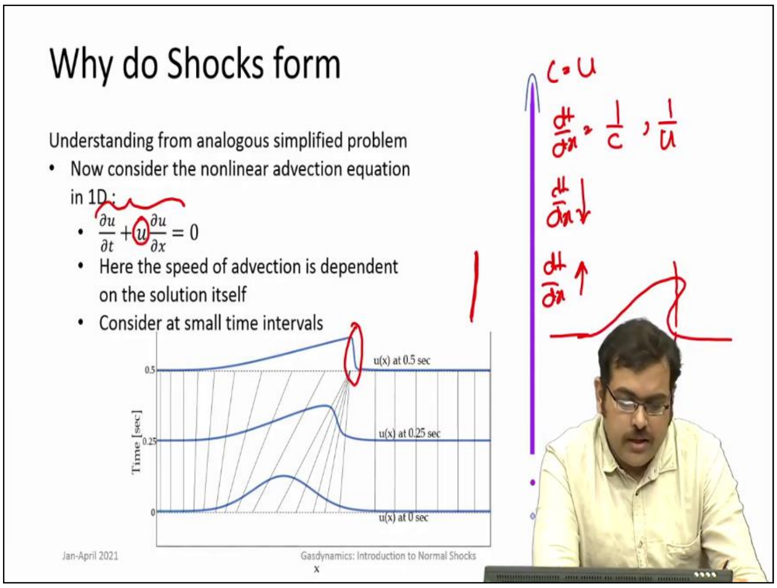
(Refer Slide Time: 10:55)



But before going there just look at the solution an animation of the solution in time which is looped over again and again. So that I will stay at this point for some time so you can understand that starting from the same profile it is simply convecting down without any change in the shape. So, it just propagates. So, this is by solving the same equations whatever has been described in the previous slide in numerically and the solution is being plotted in time.

Now there you saw that the slopes which were actually going all the way in constant slopes they were having a value ‘C’.

(Refer Slide Time: 12:00)



Now let us consider the case problem of a nonlinear advection equation in one dimension. If you consider the nonlinear advection equation, here we introduce a non-linearity that the speed of advection or convection is dependent on the solution itself 'C = u'. So, at every instance depending on the solution the speed of advection will also change.

Now the development of these equations is very similar just like we saw in the previous case that they move along lines. But here the slopes of those lines are not constant, but it depends on the solution. So now let us see that we have a very smooth initial profile, it has a smooth nature you see first it is increasing and then it is decreasing.

Now this is a t-x curve. So, if you look at this essentially the slope is dt/dx which is 1/C or in this case it is 1/u where, 'u' is the solution here. So let us see how this changes now when 'u' is '0' these are straight lines vertical lines. Now as 'u' slowly begins to increase we see that dt/dx starts decreasing that is they start moving towards the slopes start moving towards the x-axis.

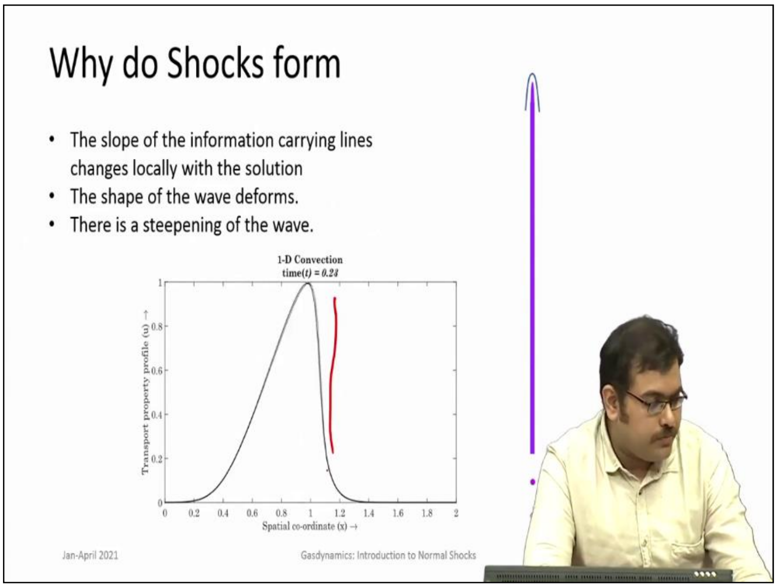
Then as it passes the hump and starts decreasing again, we see that dt / dx, it starts increasing again. Now let us follow the way these slopes move, first they open out expand like a fan they just expand out and then once they start coming back in all of them close together. So, what happens to the shape of the profile when these characteristics sort of expand they stretch the profile but when they come together they bring it compress the profile and make it sharp.

This can continue to happen and thereby defining a very sharp profile. So if you see how the process happens here you see that the wave here what was initially a smooth profile as in succeeding intervals of time it has deformed and continued to deform and has come to a very sharp profile. If allowed to go, on its own this can generate solutions like this where you can have sort of multiple these are in real processes these do not happen.

In real processes you have viscosity and thermal conductivity these play a dominant role when such large gradients are produced. So, when gradients become high those processes kick in and finally what you get is essentially a very sharp front which is the shock front. So, this equation whatever we are seeing over here is the non-linear advection equation.

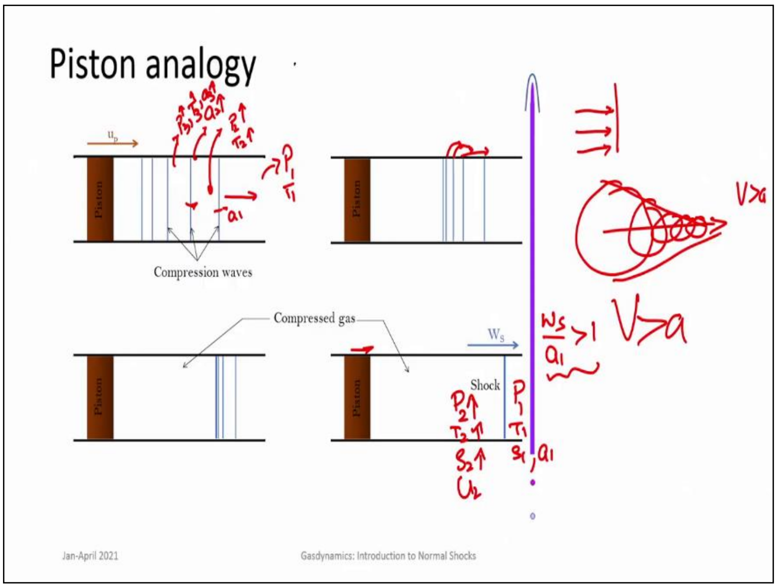
This non-linear part if you take the one-dimensional system of flow equations in the differential form this forms the left-hand side of the equations. So, the non-linearities present in gas dynamic equations lead to the formation of shocks from initially smooth profiles. So, this sort of indicates how discontinuities can be produced in such flows.

(Refer Slide Time: 17:22)



Let us see how these transforms. So, what you can see here is the solution of this non-linear form of the equations done numerically and plotted in time. You can notice that an initially smooth profile is changing its shape continuously and becoming sharper and sharper. So, this kind of process is called wave steepening. So, this wave stiffening process ultimately in the end leads to the formation of shock waves in the flow.

(Refer Slide Time: 18:19)



Now let us come to Gas dynamics or similar so from the simple analogous equations or mathematical forms that we were looking at and see physically what is happening in the flow. So similar one-dimensional consideration let us take a long tube in which you have a piston and let us say the piston is getting accelerated. So now as the piston increases its velocity what you should know that piston in a tube as you move it, it will produce pressure waves, compression waves.

So, the moment it begins moving and it starts accelerating at each instant of time it produces compression waves. These compression waves travel through the gaseous fluid inside the tube. So that is what is being depicted over here you have a series of compression waves that are getting generated. Now let us see what happens when you have the first compression wave going into the tube.

Now when the first compression wave goes into the tube it very slightly compresses the flow, the fluid here and here at this point you would have the pressure if this were at P_1 this is just after the compression wave it would be slightly higher than P_1 and similarly temperature would be slightly higher than T_1. Now you look at what happens to the acoustic speed or speed of sound in the region behind the first compression wave.

Because the temperature has slightly increased the speed of sound in this region would be slightly higher. So a_2 will be slightly higher than a_1. Now into this slightly compressed gas another compression wave is then sent forth by the piston. So, the second compression wave that comes in now it will produce another increase in pressure P_3, T_3, and a_3 these are all higher than P_2, T_2 and a_2.

But most importantly the speed with which this compression second compression wave travels will be higher than the speed with which the first compression wave travelled because the temperature has slightly increased behind the first compression beam. Similarly, if you look at all the different subsequent compression waves each compression wave will have speeds which are higher than the previous compression wave that went first.

That means if you allow them to be travelling inside the tube they will soon, that is the second compression wave will soon catch up with the first compression wave. Similarly, third one will catch up and similarly the fourth one will catch up. So, all these compression wave come closer and closer together and then the gradients and the rise of pressure and temperature will become so strong that ultimately you get the formation of a shock wave.

This shock wave will then travel inside the tube. So, if you look at this then this shock wave travels inside the tube, shock wave compresses the gas. So immediately behind the shock wave you will have high pressures this is P_1, P_2, P_2 is high, T_2 is high and ρ_2 is high. Now from this piston motion you should be able to understand that this motion of the piston is important, or it is quite the shock wave is quite unlike the sound wave in that sense that it when shock wave moves it carries a certain amount of gas with it.

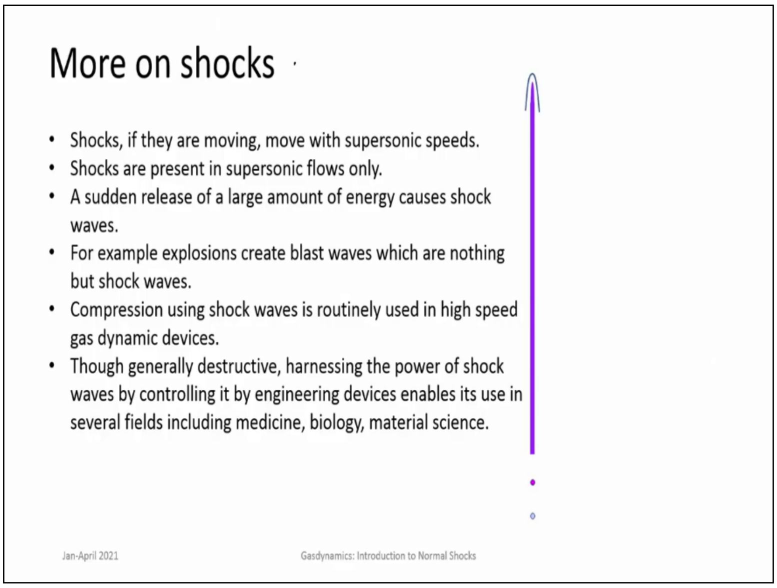
It produces a motion, mass motion of the gas with a certain velocity u_2. So, this is this W_s is always, so here you have T_1, ρ_1 and correspondingly you have W_s/a_1, this is Mach number of this shock will always be greater than 1. So, shock waves always move at supersonic speeds they are they are always present in supersonic speeds. We are discussing special case when we are considering only one-dimensional flow.

In a one-dimensional flow this shock wave will be normal to the flow direction and hence we call it the Normal shock. We saw that the production of this shock is by a coalescence of several compression waves and this happens due to non-linear processes which produce waves deepening. As these compression waves travel within the medium, they change the medium ever so slightly.

The subsequent compression waves can travel faster than the first ones that went and they catch up together and coalesce together to form shock waves. This must also have been seen when we had discussed about flow regimes and we saw that information propagation by objects moving faster than speed of sound does not happen in all directions it happens only along particular directions.

We saw that the acoustic waves produced by an object moving with velocity greater than speed of sound or velocity greater than period of sound will not catch up with it but rather they will all coalesce together along the Mach line. So, this property of wave steepening and subsequently forming a sharp front is sort of important in gas dynamics and it leads to the formation of shocks.

(Refer Slide Time: 26:02)



So important things are is that shock waves are present only in supersonic flows. They can be stationary, or they can be moving, a typical example of a moving shock is when shock wave is produced by sudden release of a large amount of energy typical in explosions or blasts. Then huge amount of chemical energy is suddenly converted to heat energy and mechanical energy.

And because of that the gas ahead starts moving extremely fast and you have a locally you can consider the problem to be like what is known as the piston problem or the piston analogy which we just discussed. Now and in very soon it forms a shock wave, and this shock wave is this first precursor wave that actually or is the first wave that passes over objects when a blast happens and behind it there is a sudden increase of pressure, temperature, and things.

So, shocks also produce enormous compression and along with compression simultaneously they can produce increase in temperature. This is very useful for many applications particularly in aerospace applications where you need to compress objects. So, this used very effectively for objects flying at speeds greater than speed of sound. Then an effect where they can you can use the kinetic energy of the gas itself of the motion of the gas and get it converted to the energy, internal energy, of the gas is thereby increasing its pressure that is known as shock compression. You can use that effectively in objects moving faster than speed of sound and that is the basis for several types of air breathing engines which do not have any rotating or moving components like ram jets and the scramjets. So, if you are able to appropriately control these shock waves then you can use them to compress gas.

Besides a lot of work is happening on applications of shock waves in various fields and including as we showed in the initial slides on the Gas dynamic course it is used effectively in medical devices like the lithotripsy. So, from next class what we would look at is how to analyse these shock waves from basic Gas dynamic principle. Relate the pressure ratios across the shock waves temperature ratios across the shock wave and see what happens to entropy between the flow before and after the shock. What is the Mach number of the flow after it passes over a shock and that can be the basis of several applications of this principle? And all supersonic flows are shock dominated flows shocks are present all the time and most of the analysis of supersonic flows sort of surrounds around how to analyse different kind of shocks, their interactions from which we can know about the pressure distributions and temperature distributions over objects. With that we will close this class and next class we will look more into analysis of shock waves.