Gasdynamics: Fundamentals and Applications
Prof. Srisha Rao M V
Aerospace Engineering
Indian Institute of Science – Bangalore

Lecture 14
Numerical

In the previous classes we have looked at important properties of the gas dynamic flow which is the Stagnation properties and the Star properties. To get the concepts clear we will do 3 numericals now and look at various aspects related to stagnation and star properties. So, to just to recap the stagnation conditions can be defined at any point in the flow and they are achieved by an isentropic process going from that velocity, pressure, and temperature conditions to a condition where velocity goes to '0' and this is achieved isentropically.

And using the energy equations and isentropic relations one can relate what happens to temperature and pressure at stagnation conditions those conditions are referred to as stagnation temperature and pressure. Then the counter part of that is the star or the sonic conditions. In this the flow is taken to a state where the velocity of the flow is equal to the acoustic speed at that point. So the Mach number is equal to 1.

So whatever, be the Mach number of the flow at that particular through an isentropic process you take it to a Mach number which is equal to 1 and then the property that you calculate at that point is known as the sonic conditions. In an adiabatic flow both the stagnation and the star properties remain constant and that is something that is going to be widely used while solving problems, so that is the basic concept.

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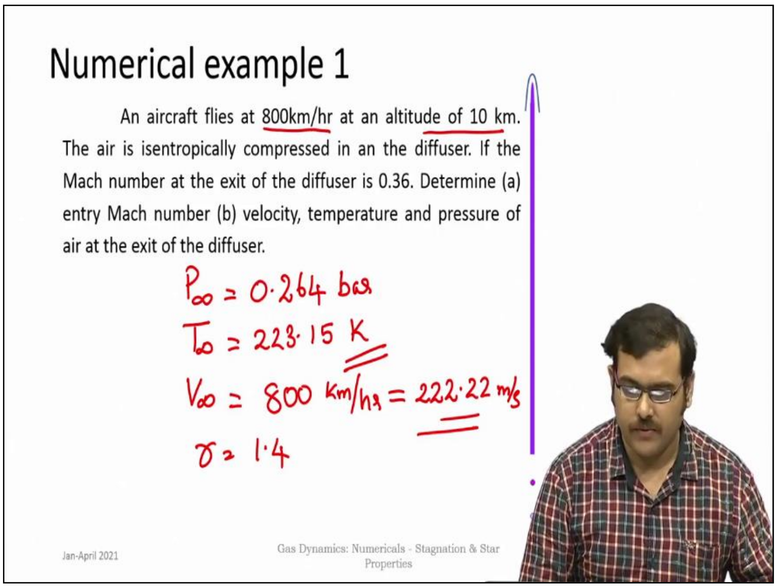

So now let us look at the numericals. The first example here is an aircraft flies at 800 km/hr at an altitude of 10 km. The air is isentropically compressed in a diffuser. if the Mach number at the exit of the diffuser is 0.36. Determine the entry Mach number, and velocity temperature and pressure of the air at the exit of the diffuser. So, see there is an example where an engineering device known as the diffuser.

The diffuser is any device which reduces velocity and consequences of that is that pressure and temperature increase, this we have seen even in our discussions in the isentropic relations that when it is decelerated pressure and temperature will increase. So these are present in many devices and this is a typical example. So here the ways to begin with this you are given the altitude.

So again some concepts that become important here is the flight velocity and altitude is given. So, static conditions and stagnation conditions the distinction between them. So once you know the altitude you can refer to standard atmosphere and get the pressure at those altitudes. The pressure is P_∞ this is 0.264 bar and temperature which is T_∞ is 23.15 K. So, this we can easily find out by using a reference atmosphere.

Then we are given the velocity of the aircraft, so aircraft velocity is given that is V_∞ this is given in 800 km/hr. This can be converted to ‘m/sec’ the SI unit which is normally used. And the conversion is this turn out to be 222.22 m/sec. So, one should notice here that the temperature is quite low, and you can expect the velocity is significant.

So, you can expect compressible effects here. First thing to do here as we know in our relations of stagnation conditions and static conditions that everything is dependent on gamma and Mach number, this is air, so gamma is 1.4. So, the way to do this is to connect the given quantities to these stagnation quantities and then from there come to the exit of the diffuser. Since the flow is isentropic the stagnation pressures and stagnation temperature do not change within the flow.

So once this is sort of understood then it is easy to go ahead with the solution of this problem. First point is, we have to get the Mach number for this flow.

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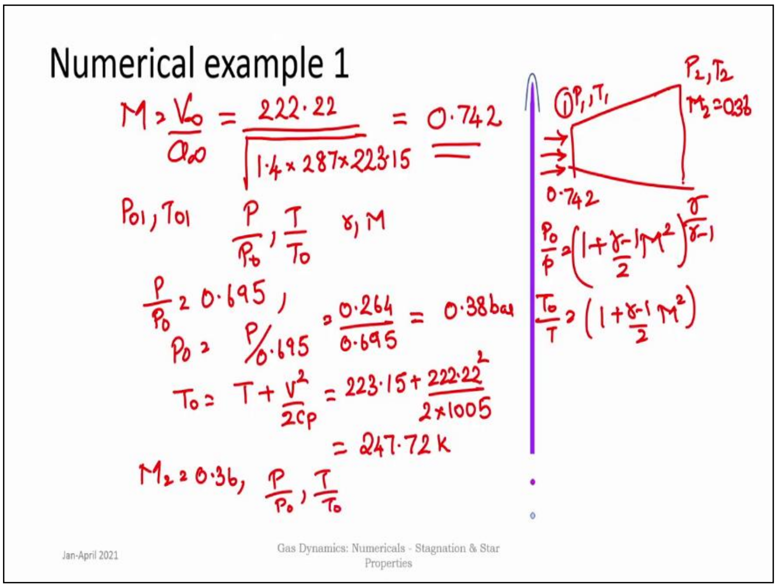

For Mach number is 'V_∞/a_∞' that is the Mach number of the flight this is given 2.2 divided by $\sqrt{\gamma RT}$, which is 1.4287 and temperature is the static temperature at that condition so it is 223.15. This Mach number if you do the calculation, it will come out to be 0.742 is the Mach number of this flight. So, this is known now we must get the P_{01} and T_{01}.

The idea is at the exit of the diffuser. So, diffuser is for this is a subsonic flow. Y ou have a diffuser something of this kind which so this is the entry where it is coming in and conditions are known here Mach number is 0.742. P_1, T_1 is known then P_2, T_2 should be found, when M_2 is known to be 0.36. So, this is known to you so and the flow is isentropic, so P_{01} and T_{01} remain constant.

We must find what is P_{01} and T_{01} the way to do this, as I had discussed last time is that once you know the Mach number and gamma you can calculate this from the relations

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

you can use these relations calculate this over computer or using a calculator this is quite simple. You can do it over a calculator, or you have tables and charts for this which can be used.

Or you can also use an online calculator where you can feed in the gamma and Mach number and you can get results for what is usually the results will be in terms of P/P_0 and T/T_0 for a given gamma and Mach number. So, this can be easily found out by various means you are free to choose the means and what you can do then is get P/P_0 for this given Mach number in this case for 0.742, P/P_0 turns out to be 0.695.

So that means P is known and so P_0 will be P/0.695 and P is already given at 0.264 so this is '0.264/0.695', which is 0.38 bar. Now for stagnation temperature also you can go ahead with similar formulation you can find out T_0/ T, the other way once you know directly you know the velocity the other way to get T0 is using the equation,

$$T_0 = T + \frac{v^2}{2c_p}$$

for a calorically perfect gas you can use this equation also. Here all the quantities are known to you, so this is '223.15 + 222.22^2/2 *Cp' and T_0 turns out to be 247.72 K. So now you know P_0 and T_0 now getting to the exit is straight forward these quantities remain the same P_0 and T_0. So now Mach number has changed to M_2 is 0.36 now again for this value of M2 one has to find out what is P/P_0 and T/T_0. If we find this then you can multiply the equation.

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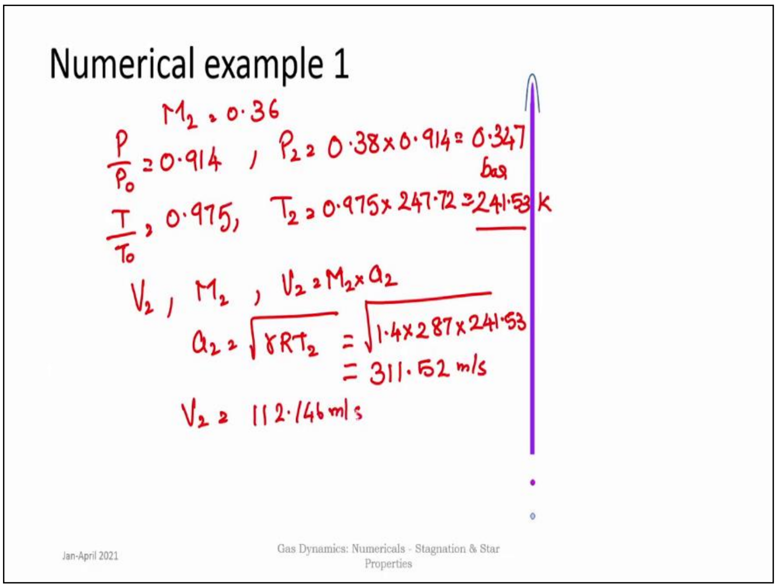

So, for M_2 = 0.36 P/P_0 is 0.914 and so you can get P_2 is P_0 is 0.38 * 0.914 = 0.397 bar. Similarly, T/T_0, T/T_0 is 0.975 so T_2 you can get it multiplied by 247.72 this will be 241.53 K. Now what we need to do is to find the velocity that is V_2 we are given Mach number M_2. So we can get velocity $V_2 = M_2 * a_2$ and the speed of sound at that particular point.

Speed of sound is a local variable, so you have to calculate the speed of sound at 2 this is different from what it was at 1. And a_2 is $\sqrt{\gamma R T_2}$ and T_2 you have found out here in the previous step and so you can substitute that here 287 * 241.53 and multiply this by Mach number. So, this turns out to be 311.52 and V_2 is just multiply 0.36 to this value and you get V2 as 112.146 m/sec.

So, this example sort of showed you how the concepts of stagnation pressure, stagnation temperature can be used to quickly estimates flow properties at different points in the flow path. In this case it was a Diffuser, and it had an isentropic flow through the diffuser. So, for the first estimate this is a very good way to calculate these flow properties.

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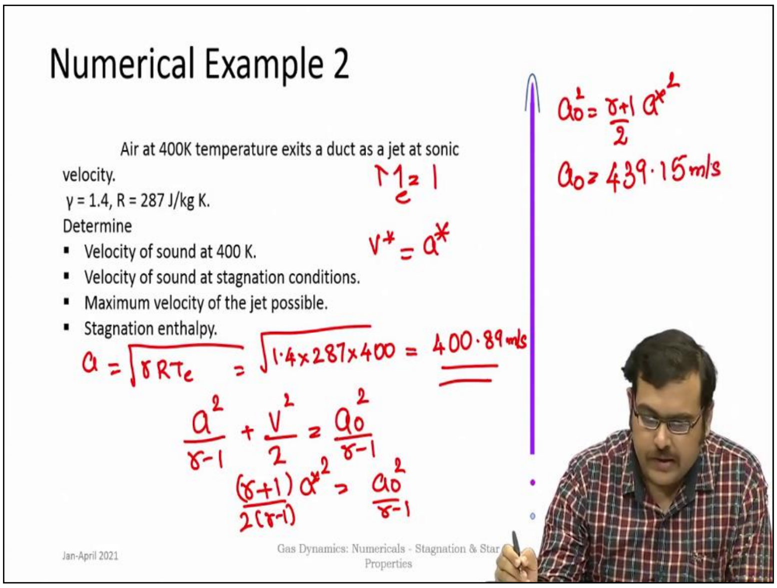

Now let us go to the second example in the second numerical example air at 400 K temperature exits a duct as a jet at sonic velocity. So here it is given that the temperature of air flow is 400K and it is at sonic conditions that mean we know what is sonic condition at the exit of the duct Mach number equal to 1? So, Mach number at exit is equal to 1, gamma and R values are given as 1.4 and 287.

Now determine velocity of sound at 400 K, velocity of sound at stagnation conditions, maximum velocity of jet possible stagnation enthalpy. So here now the velocity of sound at 400 K is the same as the velocity of jet because you are having it at critical condition that is V*=a* so this is at sonic speed. So, velocity of sound here at the exit is the same as $\sqrt{\gamma R T_e}$, which is $\sqrt{1.4 \times 287 \times 400}$ = 400.89 m/sec.

Now next, what is the speed of sound at the stagnation conditions? T_0 is the stagnation temperature you can calculate the velocity of sound at stagnation conditions, at stagnation temperature, that is the stagnation velocity of sound or from here you can go in very different methods you are possible all are equivalent one is to use the charts to you know the speed for this its Mach number is one.

The relationship between the P/P_0, T/T_0 at Mach number equal to 1 is known and then you can find out what is T_0 and from there get a_0. But another sort of straight forward method is directly using the alternate form of the energy equation which is

$$\frac{a^2}{\gamma - 1} + \frac{v^2}{2} = \frac{a_0^2}{\gamma - 1}$$

this is directly from the alternate form of energy equation.

Here for stagnation conditions the velocity is 0. Here both V^2 and a^2 are known because they are the same. So, this turns out to be

$$\frac{(\gamma+1)}{2(\gamma-1)}a^{*^2} = \frac{a_0^2}{\gamma-1}$$

This gives you directly the relation for a_0 because a* is known you have already found it out. So a_0 turns out to be 439.15 m/sec.

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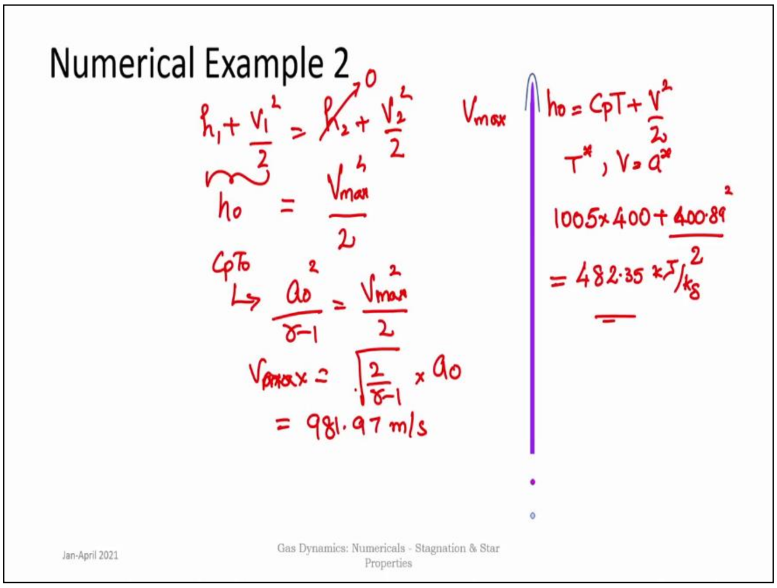

Then the next question is what is the maximum velocity of the jet possible? Now here also we can use the energy equation to get to this number, the idea here is that if you look at the energy equation for an adiabatic flow. You can see that any change in velocity involves interaction between the enthalpy and energy kinetic energy. So it is a balance between the enthalpy and kinetic energy.

So what if we ask the question what is the maximum velocity that is possible that the answer is simply that if all the enthalpy and internal energy gets converted only to kinetic energy then that yields the maximum velocity that can be extracted from the flow that is V_{max}. So that is the concept here.

If you take the flow to 0 velocity, then all kinetic energy is getting converted to flow enthalpy. So that is the stagnation enthalpy. So we can relate that so this is a stagnation enthalpy h_0 and this is equal to in the second case this goes to '0' so this goes to $V_{max}^2/2$, this is a maximum enthalpy that can be taken in so and since this h_0 can be expressed as Cp T_0 and we know this can be expressed as

$$\frac{a_0^2}{\gamma - 1} = \frac{v_{max}^2}{2}$$

Then already this, a_0 is determined in the previous calculation. So, Vmax is

$$v_{max} = a_0 \times \sqrt{\frac{2}{\gamma - 1}}$$

and this turns out to be 981.97 m/sec. So, this is the concept of maximum velocity and finally we have to determine the stagnation enthalpy this is h_0, h_0 is a stagnation enthalpy it is Cp T plus $V^2/2$. So here temperature is T*, velocity is a*, this is already known and it is calculated.

This comes out to be 1005 * 400 K, V* is known as 400.89 /2 and this results in 482.35 kJ/kg. So, we have determined the barrier. So here a new concept that came about in this numerical is that of maximum velocity through an isentropic process and this is determined when all the enthalpy total enthalpy gets converted only to kinetic energy.

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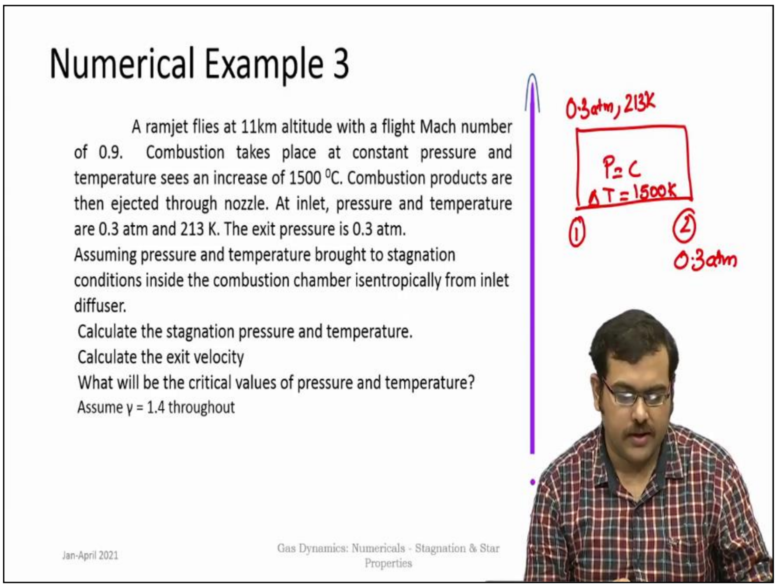

So another example similarly using for a particular device is 3, a ramjet flies at 11 km altitude with a flight Mach number of 0.9 combustion takes place at constant pressure and the temperature sees an increase of 1500 ^{0}C. So, temperature increases by 1500 ^{0}C but during combustion pressure remains constant combustion products are then ejected through the nozzle.

At inlet pressure and temperature are 0.3 atm and 213 K. So, inlet pressure and temperature is given. So let us take it as a box and what is given at the inlet pressure is given 0.3 atm and temperature is given 213 K and at the outlet and the outlet you are given exit pressure is 0.3 atm this is given.

Here you are given pressure is constant while temperature increases by 1500 ^{0}C, it is the same as Kelvin so 1500 K. calculate stagnation pressure and temperature and calculate the exit velocity what will be the critical values of pressure and temperature? So here assumption is that pressure and temperature brought to stagnation conditions inside the combustion chamber isentropically from the diffuser.

So, the diffuser again is an isentropic diffuser and it is taking so flight Mach number is given 0.9 and is taking that and converting it isentropically to the combustion chamber conditions and their pressure is constant.

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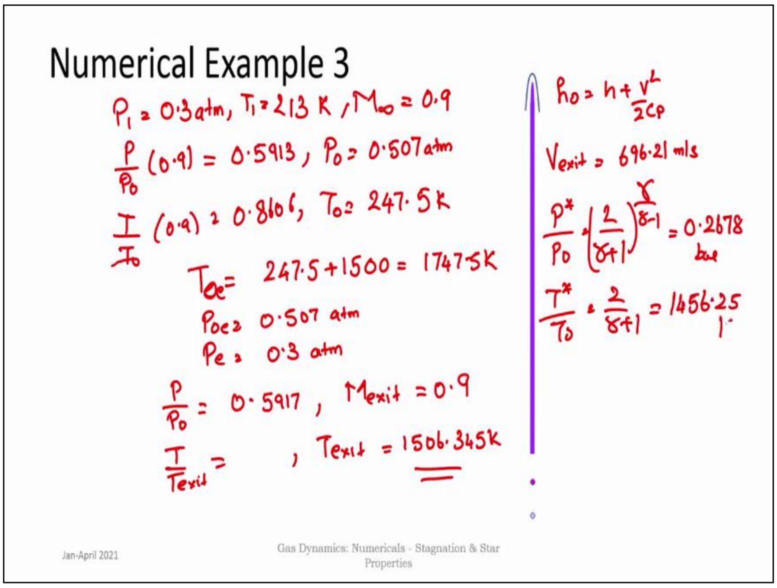

So let us solve this you are given P_1 that is inlet pressure is 0.3 atm is given and T_1 is also given 213 K. So, and Mach number is given, Mach number of the flight is 0.9. Now directly use the relations P/P_0 for Mach number of 0.9, this value is 0.5913. So, from this P_0 is calculated it is 0.507 atm, next is what about temperature T/T_0?

So here T/T_0 at 0.9 is given 0.8606, so T_0 goes to 247.5 K. Given that the increase in temperature is 1500 K. Now diffuser decreases the velocities to very low values so one can unless otherwise stated since there is it is not really stated what is assumed here is that the diffuser brings down the velocities to low values inside the combustor combustion chamber.

If it is low values then its pressure and temperature is almost same as that of the stagnation pressure and temperature this is something we refer to as the reservoir conditions when velocity is very, very low then its pressures and temperatures are very close to that of the stagnation pressure and temperature. We can use that assumption here and find out that what is the final temperature? T_{final} that is T_{exit} or T_0 exit because it is the total temperature so you are adding 247.5 + 1500 = 1747.5 K.

Now the final temperature is known, and pressure remains constant here so that is given it remains constant at 0.507 atm. Now at the exit the pressure is given as 0.3 atm so this is very special case of very ideal kind of analysis. So, in real analysis there will be other irreversibilities that come into picture. We know the pressure ratio here that is P/P_0 is know.

So, this turns out to be 0.5917 and is a consequence is similar to what we started with this so M exit is 0.9 that what is the velocity at exit so now you see that the temperature has increased as a consequence of this the speed of sound has increased the T/T_{exit} that is T/T_0, 4.9 we already know this and from there you can calculate what is T exit.

So, you can calculate Texit which turns out to be 1506.345 K and from here you can calculate the V_{exit} either you can use the conditions that you relate T, T_0 that is

$$h_0 = h + \frac{v^2}{2c_p}$$

you can use this or the other way to do it is find out the speed of sound for this particular temperature static temperature and then find out velocity.

So, then Texit will become 696.21 m/sec. Now to find out critical conditions what are the critical conditions? This you can directly use the relation between P* and P_0

$$\frac{P_*}{P_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$$

Similarly, $\frac{T^*}{T_0} = \frac{2}{\gamma+1}$. So, P* turns out to be 0.2678 bar and here T* turns out to be. 1456.25 K.

So, I think with these 3 numerical examples we can sort of look at we have a clarity on the understanding of stagnation pressure and stagnation properties and star properties. So with this understanding we will move on to normal shocks where these concepts will be used extensively to analyse the normal shock itself. So, that we will; start from the next class.