Gasdynamics: Fundamentals and Applications
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Lecture 13
Sonic Star Properties

In this module we are looking into important quantities that can be defined within any gaseous flow. Those are stagnation properties and the other corresponding one is the sonic or star properties. We have looked at stagnation properties and looked at their applications to the compressible Pitot in the previous class. So now let us look at these sonic or they are also sometimes referred to as the star properties.

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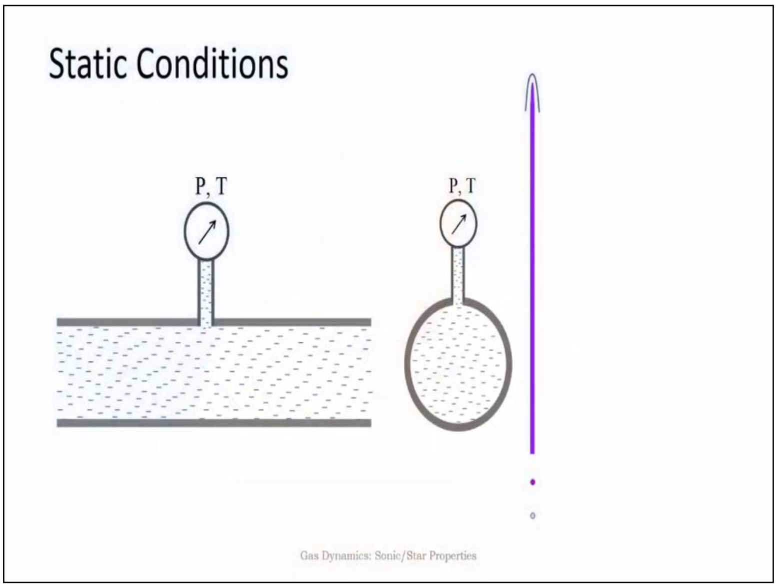

Now what does again common between all of them is the static condition in a flow which is normal pressure and temperature that is measured at any particular point in the flow. So now this is known and it is defined and the flow is moving with the velocity ‘V’.

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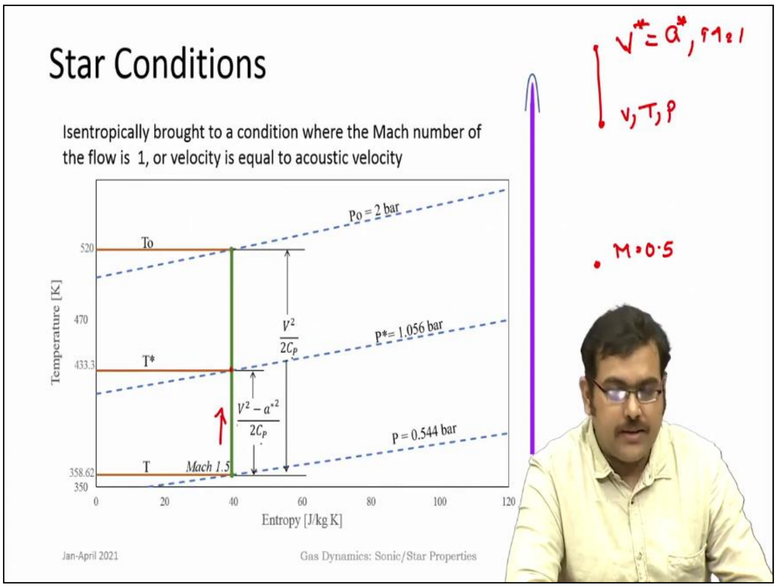

If you consider the flow to undergo a process, similar in conditions to the stagnation process which is reversible adiabatic without any work. So that means it is an isentropic process with no work being done. It is moved from any particular velocity or at that particular point to a point where the velocity becomes exactly the same as the acoustic velocity or the speed of sound.

That means the Mach number from any particular point in the flow if it was having a certain velocity, temperature, and pressure. The flow will be moved to a point where this particular point is referred to as the sonic point or the start point 'V*' will be equal to the 'a*' at that particular point. So now you should understand now it is an isentropic process. So as the process is going on, we have seen in the previous classes on stagnation temperature also.

But more easily it can be seen on this TS diagram that as it goes towards the certain point its temperature is changing that means if its temperature is changing its acoustic speed is also changing. Now we are looking for that particular point where the velocity during the process as it is going is equal to that of the acoustic speed at that point.

That means Mach number will become equal to 1 or this particular condition is known as the sonic condition. So, star condition or sonic conditions are synonymous to each other. Now in this graph that is shown over here for a template or case typical case Mach number is taken as 1.5 that means if the flow has to go towards Mach number equal to 1 it should be decelerated. So it is decelerated and the direction is same as that of the stagnation temperature.

Stagnation temperature also is a deceleration of the flow to ‘0’ velocity while for this sonic process or star process it is moving towards the sonic condition where the Mach number will become equal to 1. But, now if you consider that the flow is subsonic. For example, let us take Mach number is 0.5 then the flow will be accelerated to the sonic condition.

It is quite different from stagnation process in that sense, stagnation process is always the flow will be taken to 0 velocity whether it is subsonic or supersonic. So there will always be an increase in pressure, and temperature. But on the other hand if it is subsonic case and then you are getting to star condition you would see that actually the pressure ratio will decrease similarly, temperature ratio will decrease.

But on the other hand in the supersonic case, it will increase because there it is a deceleration process.

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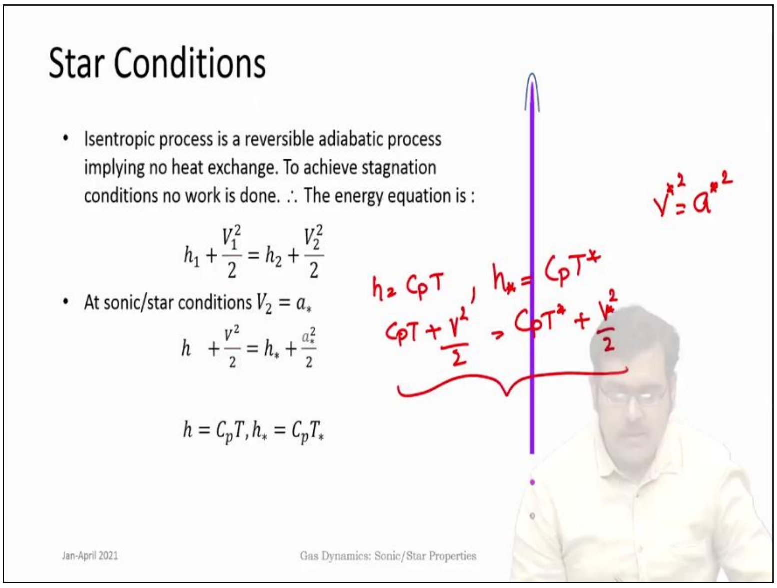

Now again what is the equation that one has to look at? So here the process is again isentropic and no heat exchange or no work done.

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

Now we directly go and apply the perfect gas calorically perfect gas assumption over here and h is c_pT and h_*, that is the enthalpy at the sonic condition is, c_pT_*.

$$c_p\mathrm{T} + \frac{v^2}{2} = c_pT_* + \frac{{v_*}^2}{2} = c_pT_* + \frac{{a_*}^2}{2}$$

a_* which is the definition of ‘a’ by definition of the sonic point or the star point.

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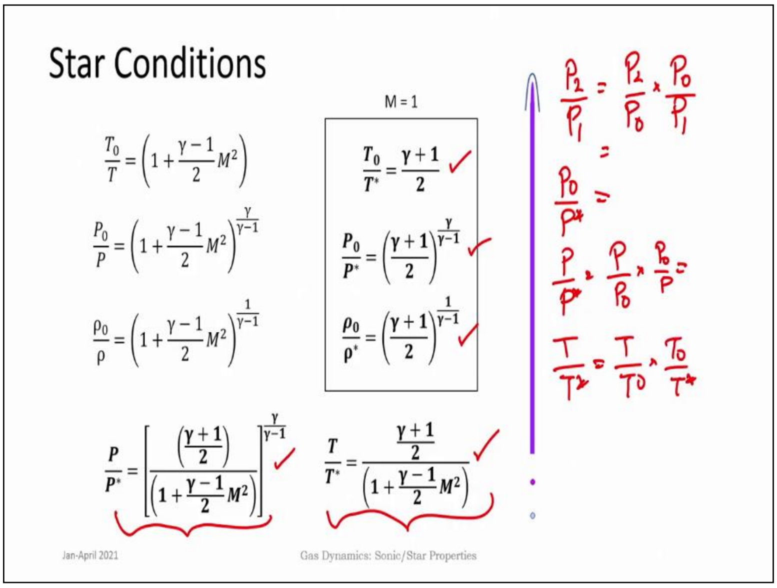

But there is also the other way to look at this process or these equations to get to the star properties, that is consider since both these processes are isentropic process they would fall on the same line and that is the starting from the given Mach number. So, according to the TS diagram over here, so it is on the same line. Stagnation temperature and pressure for an isentropic process will remain the same.

So if one has to look at say P_2/P_1 in an isentropic process this can be written as,

$$\frac{P_2}{P_1} = \frac{P_2}{P_0} \times \frac{P_0}{P_1}$$

We are looking for the point where the flow goes to Mach number one. So if we can find out what is P_0/P_*,then this can be used to determine the star conditions for a given flow at that particular Mach number and this is easily done by substituting M=1 in the equations for T_0/T, P_0/P and ρ_0/ρ.

Will get these equations T_0/T_* is $\frac{\gamma+1}{2}$, $P_0/P_*$is $\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$. Similarly, $\rho_0/\rho_*$is $\left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}$. So, these three equations tell you how the stagnation process and the star properties are related to each other. Now let us see for any given point how we can compute the star condition that is straight forward once this is defined because you know this is the relationship,

$$\frac{P}{P_*} = \frac{P}{P_0} \times \frac{P_0}{P_*}$$

Upon substitution,

$$\frac{P}{P^*} = \left[\frac{\left(\frac{\gamma+1}{2}\right)}{\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{\gamma}{\gamma-1}}$$

Similarly,

$$\frac{T}{T^*} = \frac{\frac{\gamma+1}{2}}{\left(1+\frac{\gamma-1}{2}M^2\right)}$$

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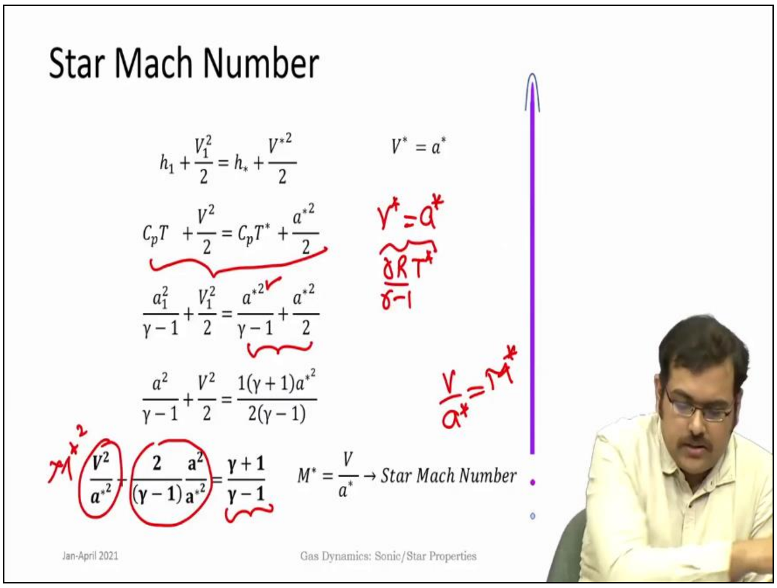

The equation that we had just seen the star energy equation or energy equation where V_* is considered and to be a_*can be taken forward and we can define a new kind of Mach number known as the star Mach number. It has its utilities, and we will see it's significance. Continuing there this equation is written by using the point that at star condition $V_* = a_*$.

To re-emphasize that it is at that particular point this acoustic speed is the same as the velocity. The acoustic speed and the velocity both change during the process. So when velocity changes acoustic velocity also changes there will, occur a particular point where both can become the same that particular point is known as the sonic point.

$$C_pT + \frac{V^2}{2} = C_pT_* + \frac{a_*^2}{2}$$

$$\frac{a_1^2}{\gamma-1} + \frac{V_1^2}{2} = \frac{a^{*2}}{\gamma-1} + \frac{a^{*2}}{2}$$

After few algebraic manipulations,

$$\frac{a^2}{\gamma - 1} + \frac{V^2}{2} = \frac{1(\gamma + 1)a_*^2}{2(\gamma - 1)}$$

Now then we can define a new Mach number that is Star Mach number $M_* = \frac{V}{a_*}$.

$$M^{*^2} = \frac{\gamma + 1}{\gamma - 1} - \frac{2}{(\gamma - 1)} \frac{a^2}{a^{*^2}}$$

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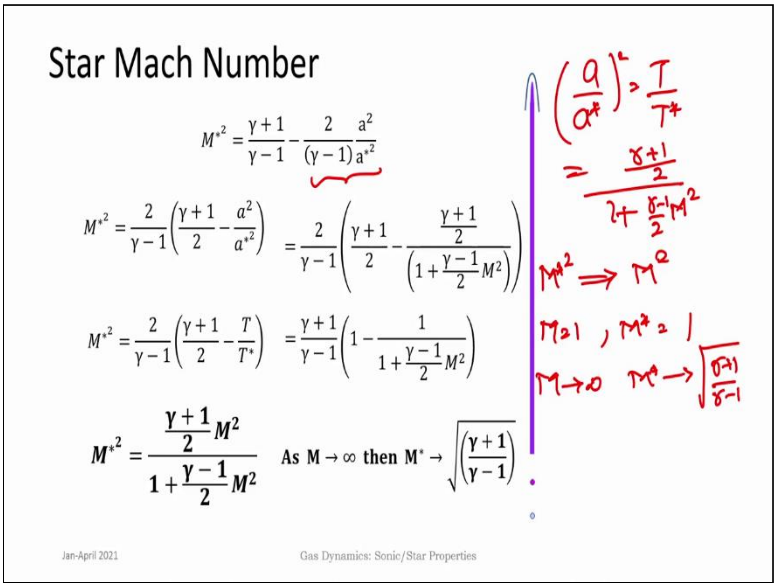

Here in this equation $\left(\frac{a}{a_*}\right)^2$ is nothing but $\frac{T}{T_*}$

$$\frac{T}{T_*} = \frac{\frac{\gamma + 1}{2}}{1 + \frac{\gamma - 1}{2} M^2}$$

So that can be this can be substituted here into this equation and you can do the algebraic manipulations in this equation to come to a relation that is given over here between M* and M.

$$M_*^2 = \frac{2}{\gamma - 1}\left(\frac{\gamma + 1}{2} - \frac{\frac{\gamma + 1}{2}}{\left(1 + \frac{\gamma - 1}{2} M^2\right)}\right)$$

Now the advantage with this Mach number is that as Mach number goes to infinity. So, when Mach number is equal to 1 for this case you see that when M is equal to 1, M* is also equal to 1, but when M tends to infinity then M star tends to $\sqrt{\frac{\gamma+1}{\gamma-1}}$. So, it goes to a finite number and very soon when we discuss shock waves this star Mach number is useful to derive relations in the shock equations. It goes to finite values even when Mach number goes to infinite numbers.

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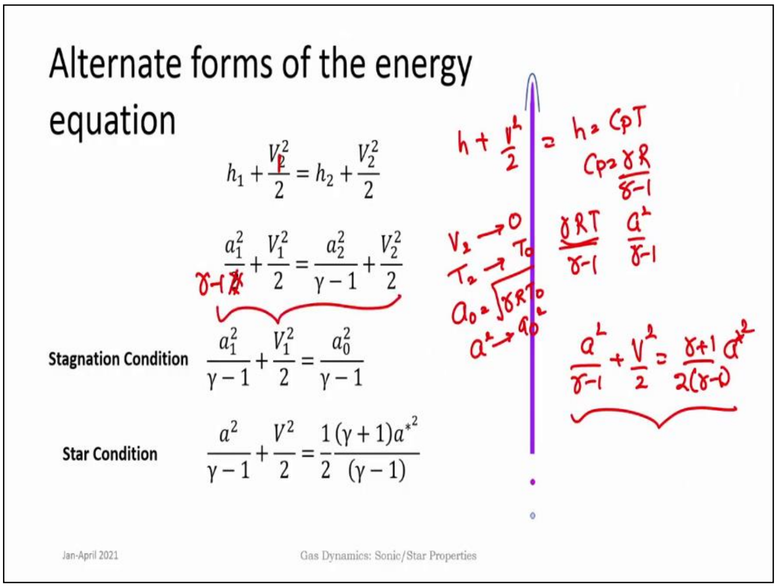

Also, at this point this energy equation can be written and manipulated algebraically to different condition which is useful in many gas dynamic analyses. Particularly we will see as we go on when we look at shock waves and their analysis the alternate forms of energy equations are really useful and that is to convert the enthalpy using the definition of speed of sound. Of course, this is valid for mainly perfect gases.

$$h_1 + \frac{V_2^2}{2} = h_2 + \frac{V_2^2}{2}$$

Substituting 'h' with '$c_p T$' and then 'c_p' with '$\frac{\gamma R}{\gamma - 1}$'. So this Cp is gamma R by gamma minus 1 so you get gamma R by so this should be gamma minus 1 this is R2 gamma R T by gamma minus 1 gamma T is a square.

$$\frac{a_1^2}{\gamma - 1} + \frac{V_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{V_2^2}{2}$$

Now when you go to stagnation conditions, energy equation in terms of speed of sound is

$$\frac{a_1^2}{\gamma - 1} + \frac{V_1^2}{2} = \frac{a_0^2}{\gamma - 1}$$

So this is the other form of equation. You can also use this form of the equation which relates any 2 states and their speed of sounds and velocities. This also is utilized in certain gas dynamic analysis.

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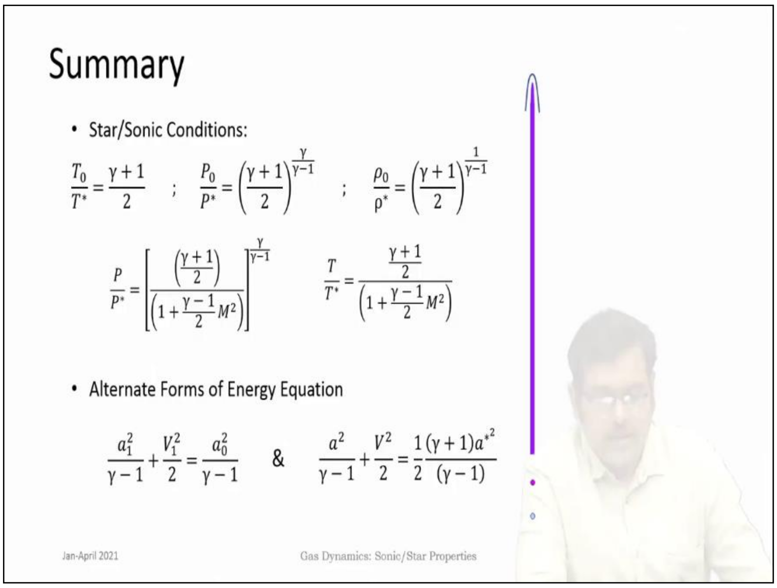

To look at the entire thing we have looked at stagnation processes which are isentropic processes that move that is an hypothetical process at every point you can define a stagnation point where the velocity is 0 or goes to rest. And the pressure and temperature at that particular point is the stagnation pressure and temperature. The other case is also important is the critical point is also referred to as critical point.

Because Mach number at that point where you take flow through a certain process you make it go to Mach number equal to 1, in this case it is an isentropic process. So that condition is known as the star condition sonic condition. Soon when we look at the application of these principles in duct flows you will see that this point of Mach number equal to 1 is associated with changes to certain flow phenomena itself.

There is a switch from one kind of behaviour in subsonic flow regime to another kind of behaviour in supersonic flow regime. So often this Mach number equal to 1 is also referred to as the critical point was that is a switch over point between these different kinds of regimes. So the equations we have derived it from basic principles for a perfect gas and also looked at how the energy equation can be written in terms of the speed of sound and velocity and that is also useful in certain analysis that will come in the classes to come.

So, in the next class we look at certain numericals the where we can apply these principles and understand them better.