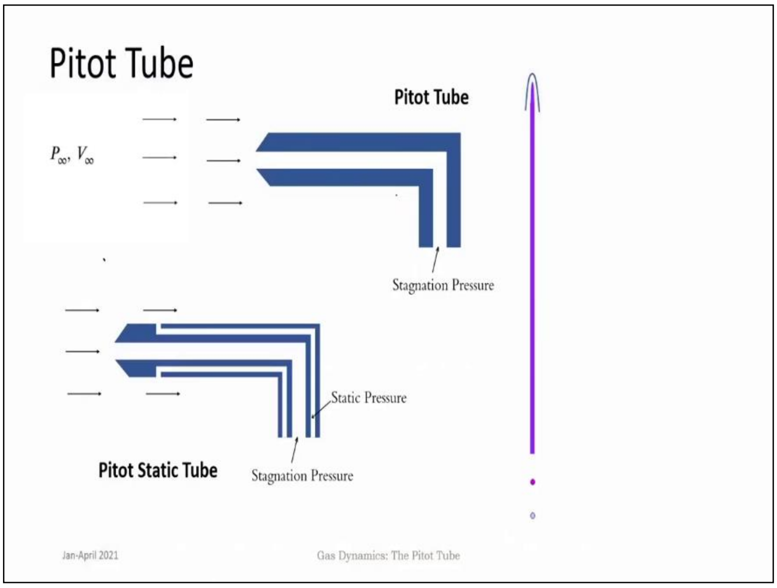
Gasdynamics: Fundamentals and Applications
Prof. Srisha Rao M V
Aerospace Engineering
Indian Institute of Science – Bangalore

Lecture 12
Pitot Tube

In this lecture we look at a particular application of the stagnation flows or stagnation properties that we saw in the previous class which is in flow measurement of compressible flows and that is in particularly to the Pitot tube.

(Refer Slide Time: 00:39)



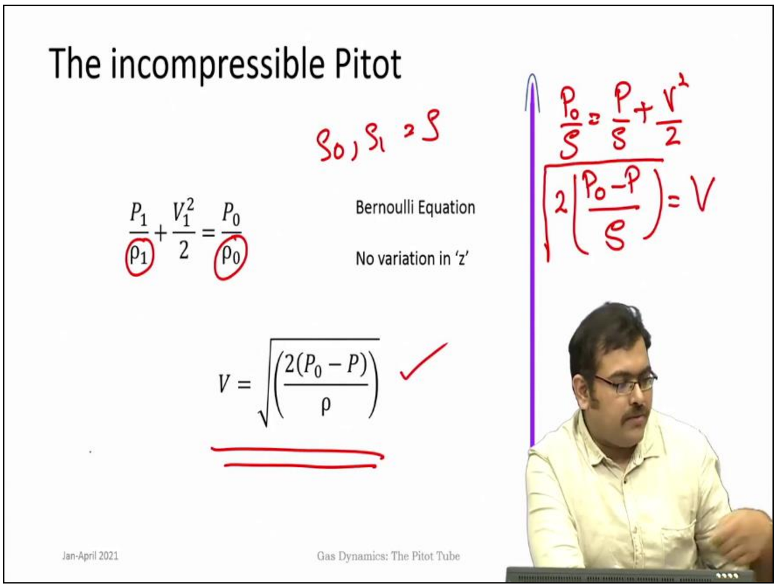
Pitot tube must be familiar to all; it is a flow measurement technique, it is simple in constructions basically a tube. Hollow tube into which the flow coming in, at any particular point if you introduce a Pitot tube then the flow that is coming into the tube goes and stagnates within the tube and the pressure within at that particular point is measured using any pressure measurement system.

The Pitot tube measures the stagnation pressure of the flow. So always it is kept normal to the flow direction. So, if you are measuring the static pressure it is always measured parallel to the flow direction, or if you are inside some flow with a stack the static pressure is always measured parallel. While a Pitot tube measures stagnation pressure, or stagnation pressure is measured normal to the flow. When the flow stagnates within the tube then you measure the stagnation pressure. So effectively Pitot tubes measure stagnation pressure. But just by knowing stagnation pressure we cannot convert that into velocity because that is our final interest to know either velocity or Mach number of the flow at that point. Then you also need information on the static pressure.

Often this is done by a combination known as the Pitot static tube, which has 2 ports you can see that the central port is normal to the flow, which measures the stagnation pressure. Around the periphery you have holes through which pressure which is parallel to the flow is measured, that is static pressure. A Pitot static tube measures both stagnation pressure and static pressure.

Now how can this; be converted into the information on velocity in an ideal flow?

(Refer Slide Time: 03:29)

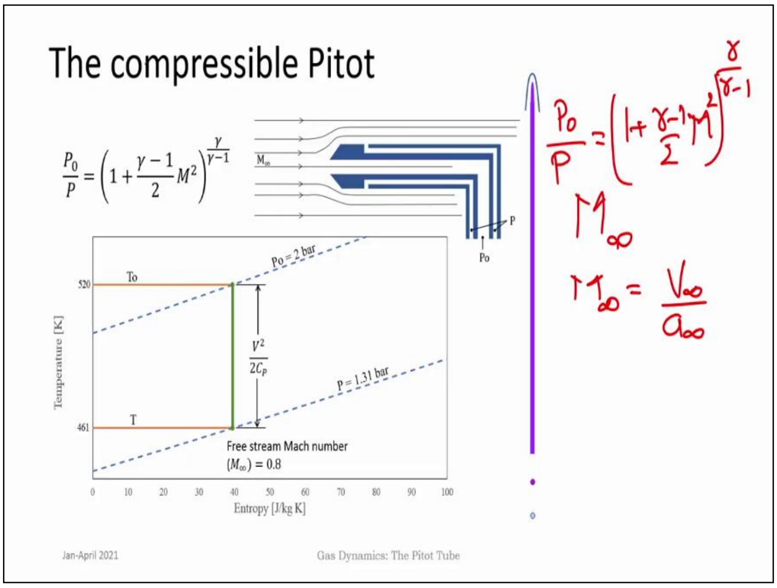


This condition must be very familiar to all, this is the incompressible Pitot equation. In this the principle used is directly the Bernoulli equation there are no losses that is the basic definition of the stagnation process. This is a incompressible flow so density remains a constant. These ρ_0, and ρ_1 all of them are equal to 'ρ' and so you write the Bernoulli equation considering that there is no variation in the potential energy.

Then this is directly $\frac{P_0}{\rho_0} = \frac{P}{\rho} + \frac{v^2}{2}$ and that is, $v = \sqrt{\frac{2(P_0 - P)}{\rho}}$, by this equation. So this is for the case when density can be taken to be a constant in incompressible flows. When applying this in the context of gaseous flows then this is for flows with very low velocity. We have already discussed that we consider flows to be compressible or compressibility effects are important once Mach number starts becoming more than 0.3.

For all flows less than 0.3 this is a valid equation to use but just in the previous class we saw that as Mach number increases if you take the stagnation properties and see they vary very rapidly with Mach number. So, this Bernoulli equation can be no longer applied in the case of a compressible gaseous flow. Then one has to look at the basic definition of the process itself and that is nothing but the Pitot tube.

(Refer Slide Time: 06:14)



The process happening within the Pitot tube is a stagnation process, where you have the flow coming in and it stagnates within the Pitot tube. Once you measure both P_0 and P then this is a compressible flow now you have Mach numbers which are greater than 0.3 then, have to use the stagnation process and under stagnation process the relationship between P_0 and P for calorically perfect gas is,

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}.$$

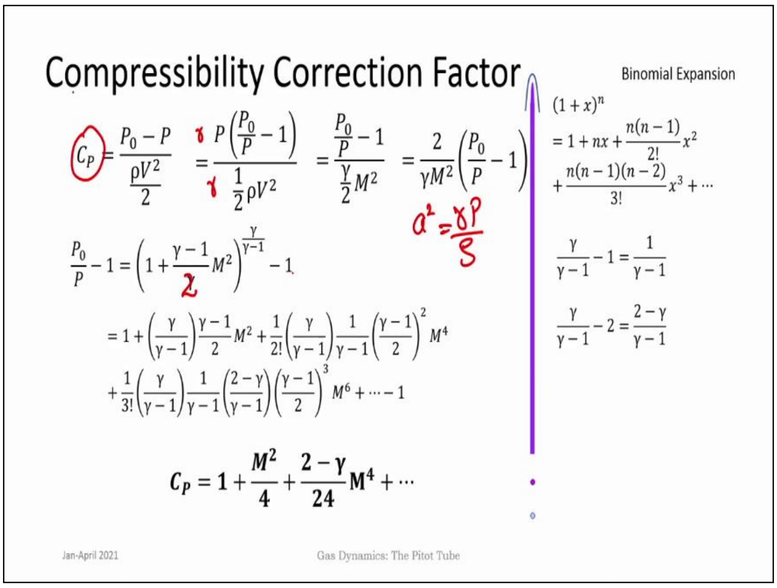
At this point often when we refer to these Pitot measurements or flow measurements, you have to understand various terminologies, Pitot's are usually used to measure the air speed in flights. One is interested there is to know the velocity of air flow and in a consequence to that this often is referred to as the free stream velocity or the free stream Mach number and it is generally denoted 'M_∞' that is for the free stream.

This is also often used, similarly flight Mach number is the Mach number of the speed of the flight or speed of the object moving in air divided by the ambient speed of sound, $M_\infty = \frac{V_\infty}{a_\infty}$.

But one should also remember that Mach number is a local quantity and if it is getting measured at different points on a body then that those Mach numbers can be different.

When one reads certain articles or numericals then you have to be careful with these technical words and their distinctions. So now directly we can measure the Mach number by inverting this equation we know P_0/P which is measured by using the Pitot, then getting Mach number is just the inversion of that relationship.

(Refer Slide Time: 09:38)



But for low Mach numbers we can use a small correction factor. This again refers to the points when computational power was quite small but also this gives us a good idea to see when really these compressibility effects are becoming significant. That is the 'C_p' or the definition of the non-dimensional factor, $\frac{P_0 - P}{\left(\frac{\rho v^2}{2}\right)}$.

If you take an incompressible flow this should actually equal to 1. Now as the flow becomes compressible, we see what happens to this coefficient of pressure C_p. Some algebraic manipulations can be done over here if you take P outside

$$C_p = \frac{P\left(\frac{P_0}{P} - 1\right)}{\frac{\rho v^2}{2}} = \frac{\left(\frac{P_0}{P} - 1\right)}{\frac{\gamma M^2}{2}}$$

Now P_0/P has an isentropic relation with 'M', so you can replace that with the isentropic equation. Let us look at small Mach numbers. So Mach numbers are not very large we are looking at points where compressibility becomes important that means the speed is low but it is becoming high.

So Mach numbers are small, when Mach numbers are small then this quantity is quite small is really small. So this expression then is $(1+x)^n$ where, n in this case '$\frac{\gamma}{\gamma-1}$' which can be expanded using the binomial expansion to $1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$.

After some algebra and substitutions in the binomial expansion form will can get the expansion in terms of Mach numbers M^2, M^4, M^6 terms.

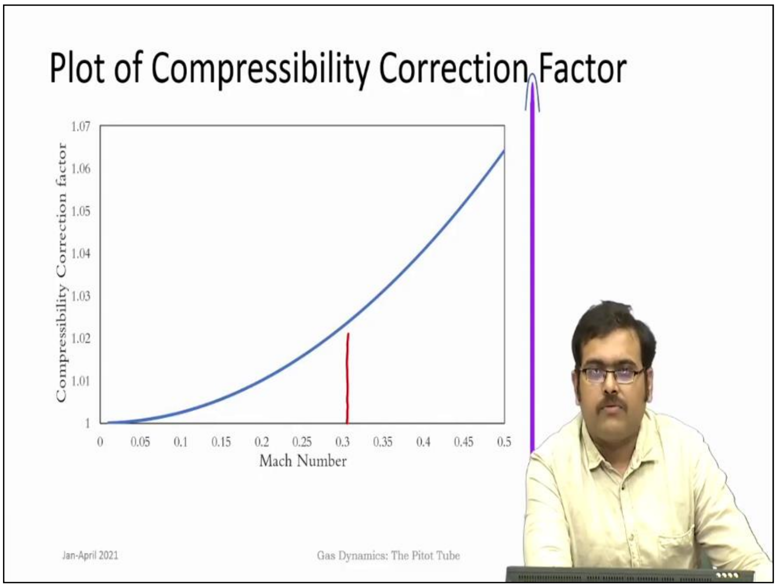
$$\frac{P_0}{P} = 1 + \left(\frac{\gamma}{\gamma-1}\right)\frac{\gamma-1}{2}M^2 + \frac{1}{2!}\left(\frac{\gamma}{\gamma-1}\right)\frac{1}{\gamma-1}\left(\frac{\gamma-1}{2}\right)^2 M^4 + \frac{1}{3!}\left(\frac{\gamma}{\gamma-1}\right)\frac{1}{\gamma-1}\left(\frac{2-\gamma}{\gamma-1}\right)\left(\frac{\gamma-1}{2}\right)^3 M^6 + \cdots - 1$$

Upon further algebraic manipulation and substitution in C_p, expression results in

$$C_P = 1 + \frac{M^2}{4} + \frac{2-\gamma}{24}\mathrm{M}^4 + \cdots$$

Remember that Mach number is small in this particular expansion therefore further all the terms are not considered. And so this will give us a approximate method to calculate, but quite good to calculate the coefficient of pressure at small Mach number. Here as Mach number changes you see that there is a compressibility effect coming into play and Cp changes.

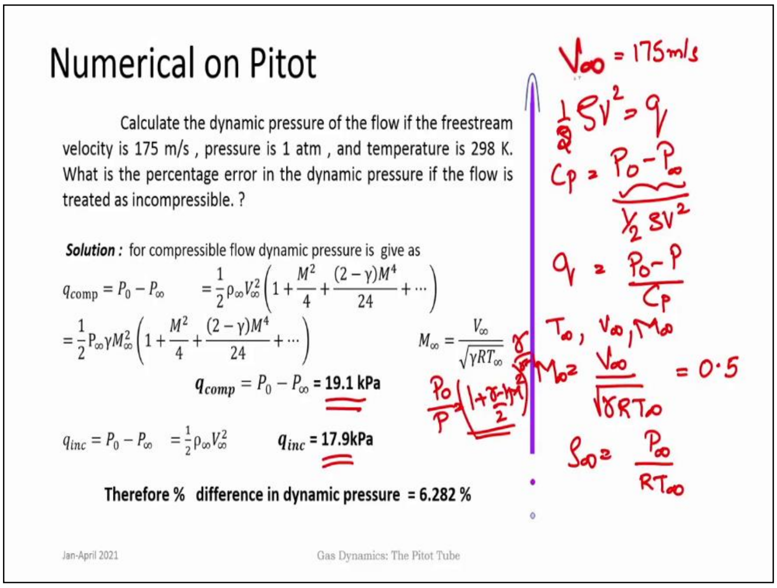
(Refer Slide Time: 15:06)



Cp the correction for compressibility there it is increasing and typically is if you like at Mach number 0.3 that is over here. It is slightly greater than in an increase of greater than 2 percent. So this gives an indication. So incompressible flows when the flow velocity increases beyond Mach number of 0.3 then one can no longer consider the incompressible Bernoulli's equation.

One has to consider the compressible ways of estimating the velocity which is using the stagnation pressure equation in terms of Mach number or if the Mach numbers are low then one can go for a compressibility correction of the kind that has been discussed over here. Then estimate the Mach number from the measured stagnation pressure and static pressure.

(Refer Slide Time: 16:25)



Now just let us look at briefly how these things compare so just by using a simple numerical. Calculate the dynamic pressure of the flow if the free stream velocity. As here it has been written as free stream velocity which is 175 m/sec. Pressure is 1 atm and temperature is 298 K. what is the percentage error in dynamic pressure if the flow is treated as incompressible?

Dynamic pressure is $q = \frac{1}{2}\rho v^2$, and from the definition of Cp you can see that this is nothing but 'P_0-P', so (P_0-P_∞)/$\frac{1}{2}\rho v^2$.

$$q_{\text{comp}} = P_0 - P_\infty = \frac{1}{2}\rho_\infty V_\infty^2\left(1+\frac{M^2}{4}+\frac{(2-\gamma)M^4}{24}+\cdots\right)$$

So if you know temperature and velocity.

$$M_\infty = \frac{V_\infty}{\sqrt{\gamma R T_\infty}}$$

So, gamma is 1.4, R is 287 J/kg-K, T_∞ is 298 K, and V_∞= 175 m/sec. M_∞ close to 0.5 and that can be put into this equation for the compressible correction factor. While here we have truncated terms greater than M^6 and so on. So, you get q compressible for these 2 conditions as 19.1 kPa.

So both are given so you can calculate this and V is known this is 17.9 kPa. Directly you can see that there is about 6.3% difference due to compressibility. So once the flow becomes compressible one has to use compressible flow equations stagnation processes to calculate the velocity or Mach number from the measurement of stagnation pressure and static pressure and not by using the incompressible Bernoulli's equation.

So I think that point is made clear here. So in the next class what we will look at is the star condition. So stagnation conditions and star conditions or sonic conditions are important critical conditions for a gaseous flow. So next class we look at star conditions.