Gasdynamics: Fundamentals and Applications
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Lecture 11
Stagnation properties

So, let us begin this class. Previously we had looked at flow equations for the gas dynamic flows and then came to a very important assumption that is made to understand how such flows behave to forcing such as pressure variations, area variations. To understand these concepts the general assumption that is taken is that across the cross section the flow remains uniform in velocity, pressure, temperature and so on.

That is known as the Quasi-1D assumption. Following this we applied these principles to speed of sound and did a few numericals to understand those concepts. Now we come to very important concepts of what are known as stagnation conditions, and star or sonic condition and why they are important and where are their applications? First, we will begin with the stagnation conditions.

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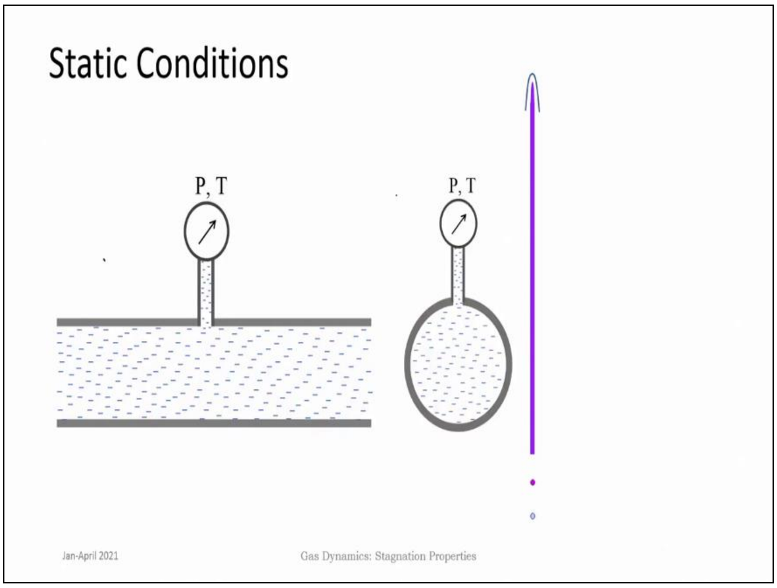

So, the common principle for both of them is the static condition and then you can reach the stagnation conditions or the sonic conditions through a certain hypothetical process. So, static conditions are the normal conditions of pressure, temperature in a fluid flow. For example, you have a fluid flowing through a duct here. At any point if you take, for example at this local particular point, if I measure the pressure and temperature that is this static pressure and temperature.

Now this static pressure and temperature usually the measurement of these conditions will happen in a direction which is parallel to the flow direction. So now if you consider; these static conditions you do not specifically say static pressure and static temperature they are normally written as pressure, and temperature it is understood that it is static conditions that one is talking about.

So, when you see the descriptions in either text textbooks or numericals. When one describes just pressure and temperature it means its static pressure and static temperature at that particular point.

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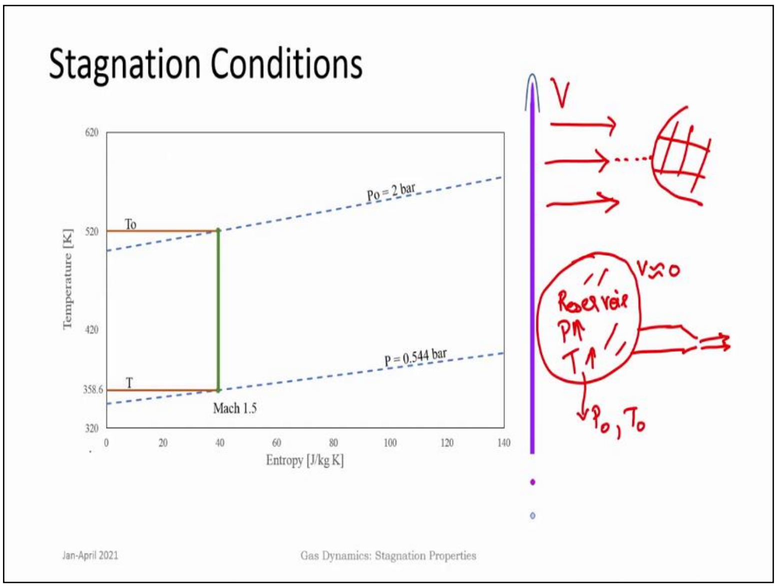

Now let us come to stagnation conditions now at every point in the flow the velocities or Mach numbers can be varying. But at each point one can consider that you can have a hypothetical process that takes the flow from that particular velocity or mach number condition to a condition where the velocity is 0 and this is achieved in a reversible adiabatic fashion without any losses.

So that means if there are no losses, and it is reversible adiabatic and no work is done then it is an isentropic process. Such a process is depicted here in this TS diagram for a typical Mach number which is 1.5 it could be any velocity for that matter. So, the flow is then decelerated to stagnation conditions. Now that, so this particular process is called the stagnation process it is a hypothetical process, and it can be applied at every point in the flow field.

For each point in the flow field one can define a Stagnation temperature and for the same you can also define a Stagnation pressure. Though the by definition we say it is a hypothetical process this can be achieved in several places. Very simple example is that the nose of bodies which are flying or moving through air, so you have a typical body here. If it is moving in air, then air is coming on to it with a particular velocity 'V' and there will always be a specific stagnation point on the body. At the body, the boundary condition is that the flow should have the same velocity relative velocity with the body where relative velocity is zero or the same velocity as the body that means the flow stagnates at the stagnation point.

This process can be considered as completely reversible and there is no work done adiabatic condition. So, this is the stagnation point on a body in a flow. Also, the other kind of application or places where this is often reserved to refer to the reservoir conditions. So, when one is trying to create a flow, by providing a sort of reservoir or a pressure ratio across the device.

For example, here I take a reservoir and flow is taken in a duct and then accelerated in a nozzle and it comes out. Here inside the ducts and in the nozzles the velocity of the flow can be quite high but this is fed from high pressure regions it can also be having high pressure and even high temperature or given temperature. But inside this reservoir the velocities are approximately zero, very small compared to the ones that happen inside the duct.

Then these conditions can be referred to as the stagnation conditions P_0 and T_0 for this particular flow. As a first cut approximation when one analyzes such flows then the losses in the ducts are initially considered negligible. So, that; this can be taken as the stagnation conditions for the flow. In further discussions on many other problems like variable area ducts and other such applications nozzles diffusers. These concepts will occur again and again, so it is important to understand the stagnation conditions, sonic conditions their differences and their applications. So to sort of re-emphasize the stagnation process is achieved at any particular point in the flow. The flow is taken hypothetically from that given velocity, pressure, temperature conditions to a condition where velocity is zero.

And then we look at what happens to pressure and temperature. From energy considerations we can show that both pressure, and temperature increase and they reach the stagnation values.

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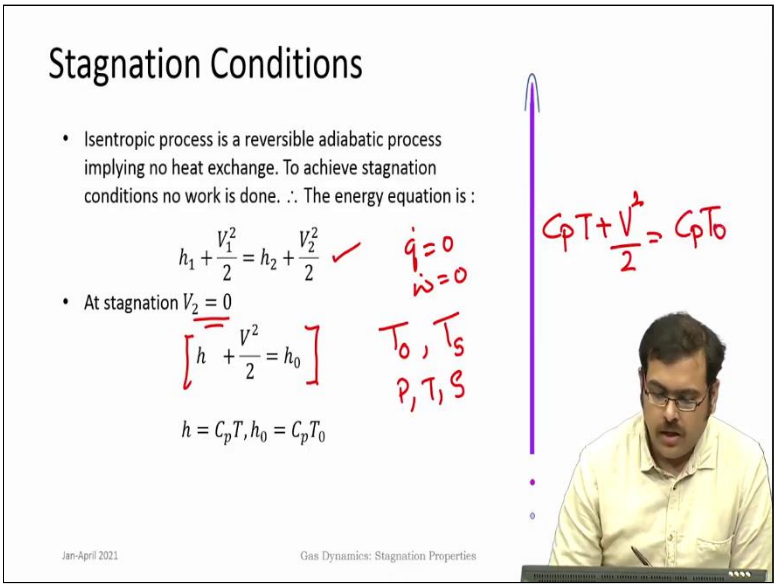

Let us look at the analysis of this stagnation conditions. How do we get them is there any relations between the flow conditions and the eventual stagnation pressures, and temperatures? By definition, the stagnation process is an isentropic process, and no work is done. So, we take the energy equation which is,

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2}$$

There is no heat added so $\dot{q}$ is 0 and $\dot{w}$ is also 0. Now in this equation 'v_1' refers to the static condition 'h_1' is the static enthalpy and when we consider the stagnation condition the velocity goes to 0 or v_2 goes to 0. This equation can be written as

$$h + \frac{v^2}{2} = h_0$$

Usually, the stagnation conditions are represented by a subscript '0'.

In some places it is also represented by a subscript 's' to refer to stagnation conditions. While if you refer to only pressure, temperature, and density without any such subscripts then this refers to the static conditions. So, this is direct from the energy equation and now this condition is general for any flow. Now we go to the case that we always describe that is of a perfect gas that is specifically calorically perfect gas where 'c_p' is a constant.

So now if we do that then ‘h’ can be written as ‘$c_p T$’. Similarly, stagnation enthalpy can be written as ‘$c_p T_0$’. Making that substitution this becomes

$$c_p \mathrm{T} + \frac{v^2}{2} = c_p T_0$$

We proceed from here and relate this to the Mach number.

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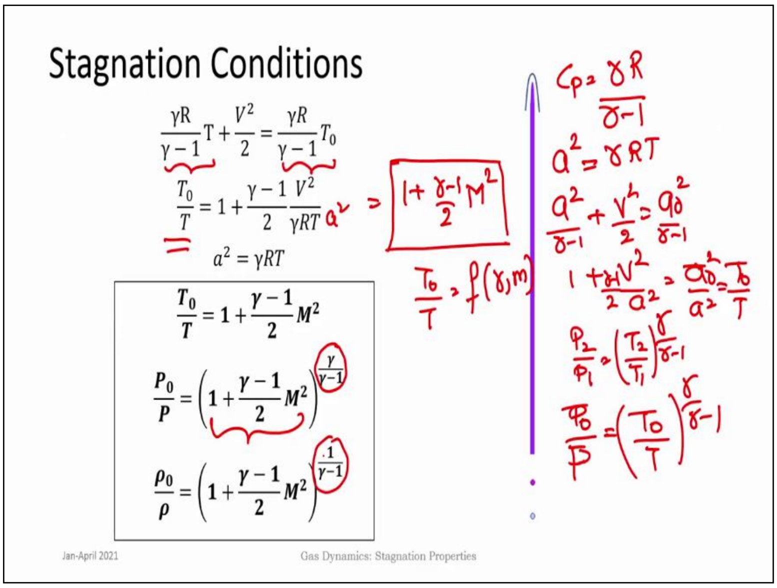

This is using the relation that ‘$c_p = \frac{\gamma R}{\gamma - 1}$’, gamma ($\gamma$) is the ratio of specific heat for the gas and R is a specific gas constant. So once that is substituted you get these 2 equations here. And we also use the fact that $a^2 = \gamma RT$, where ‘a’ is the Speed of sound. This equation then becomes $\frac{a^2}{\gamma - 1}$.

So now dividing this entire equation by $\frac{a^2}{\gamma-1}$, so you get

$$1 + \frac{(\gamma - 1)v^2}{2a^2} = \frac{a_0^2}{a^2} = \frac{T_0}{T}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} \times \frac{v^2}{a^2} = 1 + \frac{\gamma - 1}{2} M^2$$

This is an important relation, which relates stagnation temperature to the static temperature as a function of gamma and Mach number.

Now once you know the stagnation temperature ratios that is ‘T_0/T’, then we can find pressure ratio and density ratio by using the principle of isentropic process. For an isentropic process,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

So here the process is from T to T 0,

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

Now from the relation between T_0/T, γ, and M. You have the relation for 'P_0/P' with an exponential '$\frac{\gamma}{\gamma-1}$'.

$$\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

Similarly, you have the expression for 'ρ_0/ρ'

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}$$

So now these three equations form the set for defining the stagnation conditions in a calorically perfect gas flow and it can be applied at any point.

Depending on the flow there can be variations we will soon discuss about a particular variation for the stagnation pressures. If it is an adiabatic flow without any heat transfer, then a condition that can be applied is that the stagnation temperature does not change, or it remains constant which is directly seen from the energy equation. If there is no heat added or work done h_0 will remain a constant. So that can be very useful for many applications.

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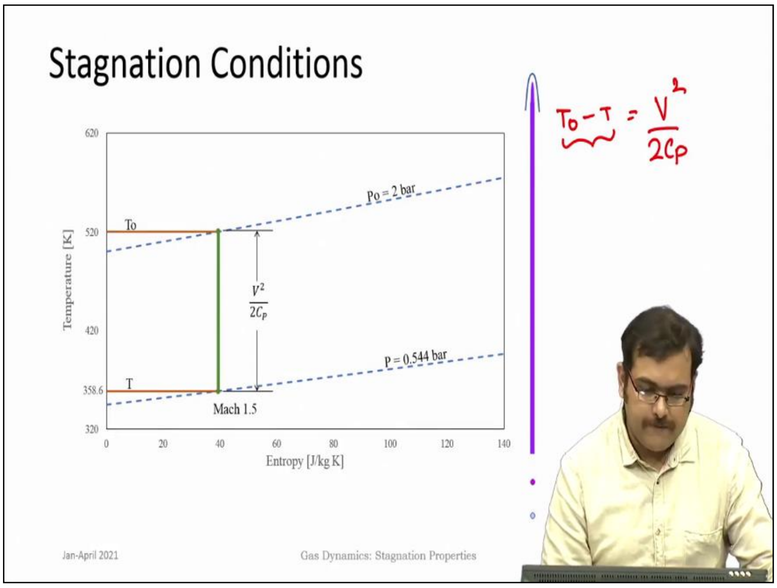

So now if you look at the equation then you can easily make out from this condition that,

$$T_0 - T = \frac{v^2}{2c_p}$$

So, in the plot that was shown in the TS diagram for a calorically perfect gas. We can really see that this ordinate as it changes is $\frac{v^2}{2c_p}$. This height is also an indication of the velocity of the flow when plotted to proper scales.

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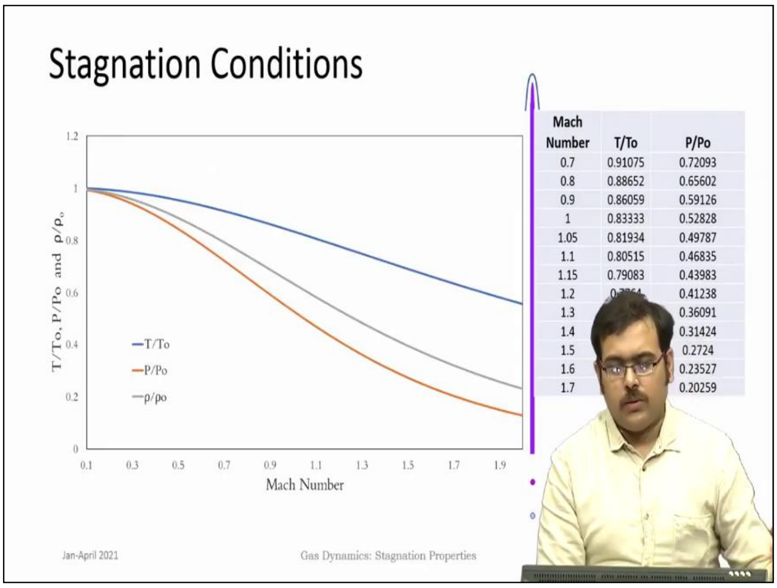

You can immediately notice that this formulation '$1 + \frac{\gamma-1}{2}M^2$' for temperature ratio is fairly straight forward. But for pressure ratios and density ratios you have exponentials. While calculating, the way these principles are used is usually that you will know the velocities or Mach numbers and have to estimate the stagnation conditions.

These relations can be applied in different places during the numeric computations or numericals or you know the stagnation values and you have to get back the Mach number these are the 2 kinds of operations mathematical operations that need to be performed. While going the forward way, that you know the Mach number and getting the conditions is quite easy, but the reverse is not that easy for the pressure ratios and density ratios.

Of course, with current day calculators and computers these processes are very easy, but the classical way is using tables, charts, and graphs. They also provide a visual means to see what happens with increasing mach number to these stagnation properties. Usually, they are given as T/T_0 or P/P_0 in many places. They can usually be found at the end and the appendices of all good textbooks of gas dynamics.

For solving problems one can use those tables to get to these values instead of calculating them by hand every time. But notice this plot of T/T_0, P/P_0, and ρ/ρ_0 verses Mach number, these ratios decrease all the way from very small mach numbers to very high Mach numbers. This just shows that the temperature ratio is the one which has a very the sort of increase or in 'T/T_0' its decrease is minimum both 'ρ/ρ_0' is next which decreases rapidly and 'P/P_0' decreases even more rapidly.

Or if you put it in the other way, that is if you look at the ratio of stagnation pressure to the static pressure it increases enormously with increase in Mach number. The other one is if you get a feel for numbers it is very sort of useful. As indicated earlier one of the important applications is to look at the stagnation point in a flow and at that particular point you would get stagnation conditions.

Let us see what happens if suppose we have the temperature is 300 K, static temperature, and suppose you have Mach 10 flows. If you do the calculation for this, you will find that temperature can go quite high it will be of the range of around 6300 K for Mach number 10. This is higher than surface temperatures of sun itself. These kinds of Mach numbers are usually encountered when bodies re-enter the Earth's surface from space or nowadays even people are talking about hypersonic flight and these mach numbers are common.

They can be even greater than this, definitely at these high temperatures the high temperature effects become important and as we had discussed in the thermodynamics classes c_p will remain no longer constant. But this is indicative of what will happen, and very high temperatures can be seen on the nose of such bodies and engineering them is a challenge and one has to look for good high temperature materials and heat shields and so on.

So how do we solve such problems is one way is to use the tables where at the textbooks and the appendices you will have these numbers written. And you can read directly for Mach number say 1.5, T/T_0 is 0.68966. In case you get a mach number which is in between say, 1.52 or 1.55 then you can do a linear interpolation between these 2 values. It is how you do normally read a table.

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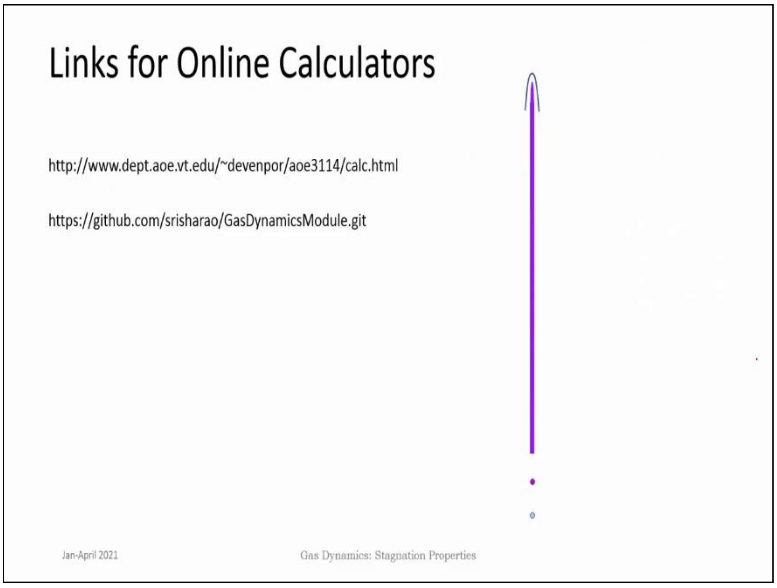

Or there are several online calculators available and one of the common ones that is quite useful is given by this link where they give once you enter the mach number and gamma you can get all the relations not only for isentropic tables but for many other cases also that we will come up soon in the following classes. There is a small gas dynamics module that our group has developed, and it is available in GitHub and it is written in python. So, one can use many of these online calculators also for looking at numericals.

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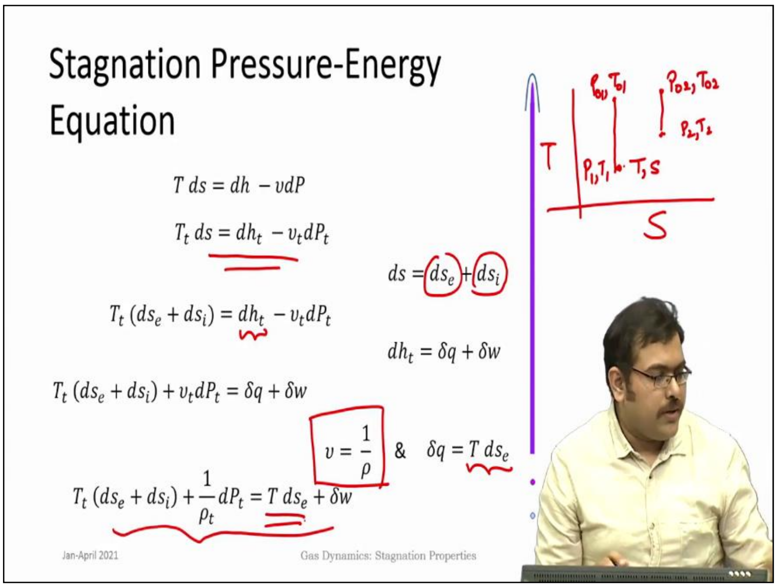

So, coming into the specific case of the stagnation pressure, now if you look at the TS diagram; you can immediately make out that for every point in a flow peak T, s or this is any particular pressure, and temperature. In a TS diagram you can always associate a stagnation condition where the velocity can be taken to be 0. So that is the stagnation condition. Similarly if there is another process another point in the flow which is having P_2, T_2.

You can always find out what happens to its pressure and P_{02} and T_{02}. Can find out it is stagnation pressure, and stagnation temperature. The stagnation points are thermodynamically equivalent points with velocity equal to 0. If you look at this equation, this is nothing but first law of thermodynamics it is written for a 'Tds = dh – Vdp' but this is written for static conditions equivalently you can write for the Stagnation conditions that is where 't' is to total conditions.

So, T_t and 'dh_t' that is total enthalpy and 'P_t' which is total pressure. Where 's' is the specific entropy, so this equation is written for the total conditions or the stagnation conditions for any particular point in the flow. Now the change in entropy can be resultant of energy exchange processes and irreversibilities. So, there can be irreversibilities in the process and there can be energy exchange.

So that is how it is written as the total entropy change is written as a sum of these 2 and also dh_t is equal to 'dq + dw' or heat added, and work done on the system this is first law of thermodynamics. We can substitute for the change in total enthalpy by this heat added and work done. Now heat added is nothing but the energy exchange process which by the definition of entropy is Tds_e.

If you introduce that term into this equation and also use the fact that density and specific volume are related to each other by this equation, then we get such a formulation.

$$T_t\,(ds_e + ds_i) + \frac{1}{\rho_t} dP_t = T\,ds_e + \delta w$$

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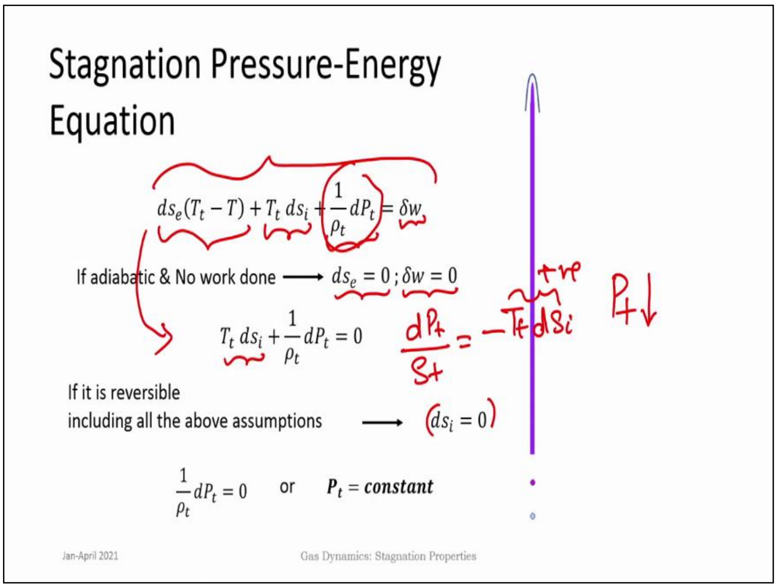

Now this is algebraically rearranged to terms containing only energy exchange process terms containing only irreversibilities, work done and the total pressure. So this equation in total is known as the stagnation pressure and energy equation. So there is stagnation pressure here and the rest of the terms relate energy and work done. Now specifically if you look at this equation and take that it is an adiabatic process. In an adiabatic process there is no heat exchange happening.

The change in entropy due to heat exchange is 0. Then work done is also 0. So this particular equation then comes down to the change in stagnation pressure is related to the irreversibilities in the system and irreversibility is always give rise to an increase in entropy.

$$\frac{dP_t}{\rho_t} = -T_t ds_i$$

or this term is always positive that means stagnation pressure, the stagnation pressure in a real process with irreversibilities always goes down or it decreases. If the process is reversible then even all the irreversibilities are ‘0’ then stagnation pressure can be considered as a constant. In first order analysis of many cases like the nozzles, diffusers consider that the flows through such devices are isentropic that means there are no irreversibilities in the system and it is adiabatic.

In those analyses the stagnation pressure will remain constant. Also, the stagnation temperature will remain constant. But in case there are any irreversibilities it always leads to a decrease in stagnation pressure.

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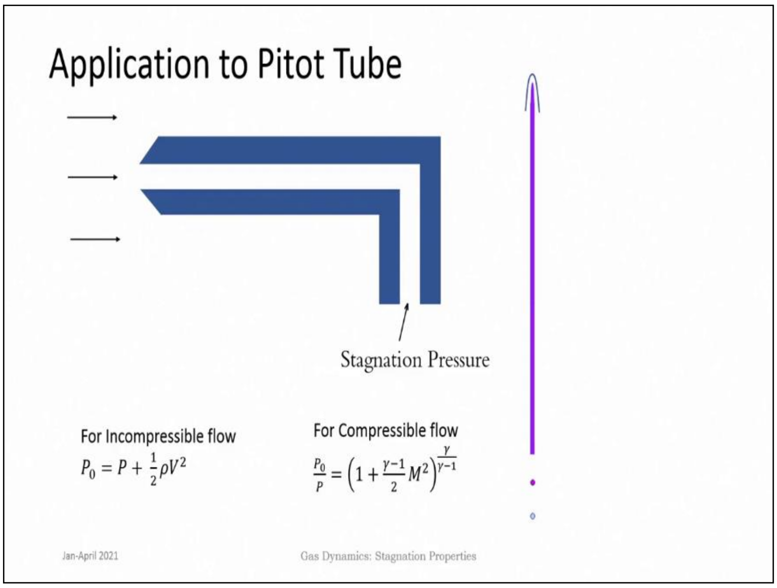

This understanding of the stagnation properties is important and this is applied directly to flow measurement through the use of Pitot tube which will be discussed in detail in the next class.

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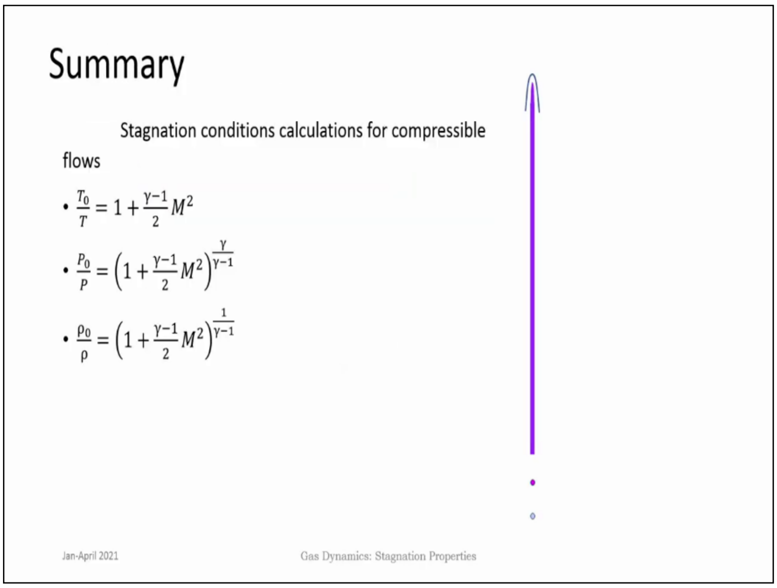

So this class we have completed discussions on stagnation conditions for compressible flows.