Gasdynamics: Fundamentals and Applications
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Lecture 10
Speed of Sound- Numerical

In the previous classes we had looked at concepts of Quasi-1D flow and we looked at an application to speed of sound. Now let us apply to some problems to see how these principles can be applied.

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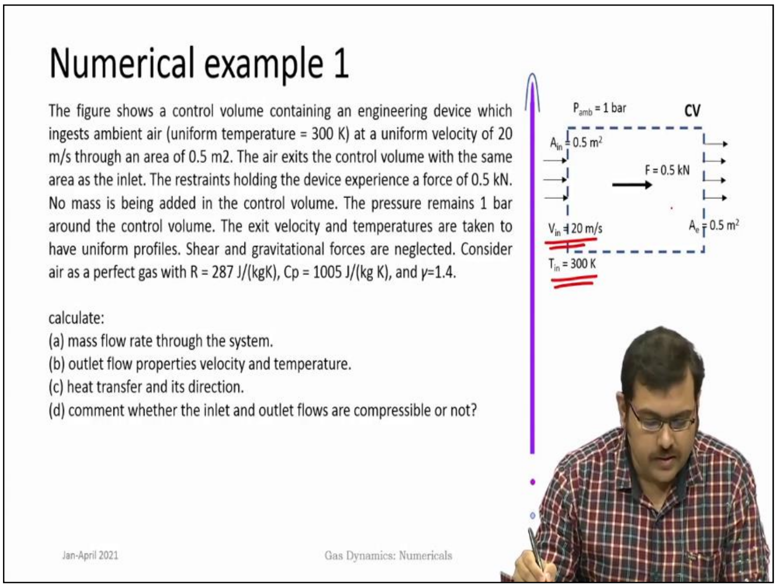

So take this particular numerical example so the figure shows a control volume containing an engineering device which ingests ambient air. So air is ingested into this control volume and at uniform temperature so at $300\ K$. So it is ingesting at $300\ K$ and velocity is $20\ m/s$ and through an area of $0.5\ m$^2. The air exists the control volume and the area of exit is the same as the area of inlet.

So A_{in} and this device is being held the restraints were holding the device experience a force of $0.5\ kN$. So the force experienced by the restraints is $0.5\ kN$. No mass is added into this control volume and pressure remains all over the surface at $1\ bar$ and the exit velocity and temperatures are taken to have uniform profiles. So they are having the same profiles shear and gravitational forces are neglected.

Consider air as a perfect gas with R is 287 $\frac{J}{kg-K}$, c_v is 1005 $\frac{J}{kg-K}$ and γ is 1.4. So first is calculate mass flow rate through the system outlet flow properties that is velocity and temperature heat transfer and its direction. So and comment whether the inlet and outlet flows are compressible or not.

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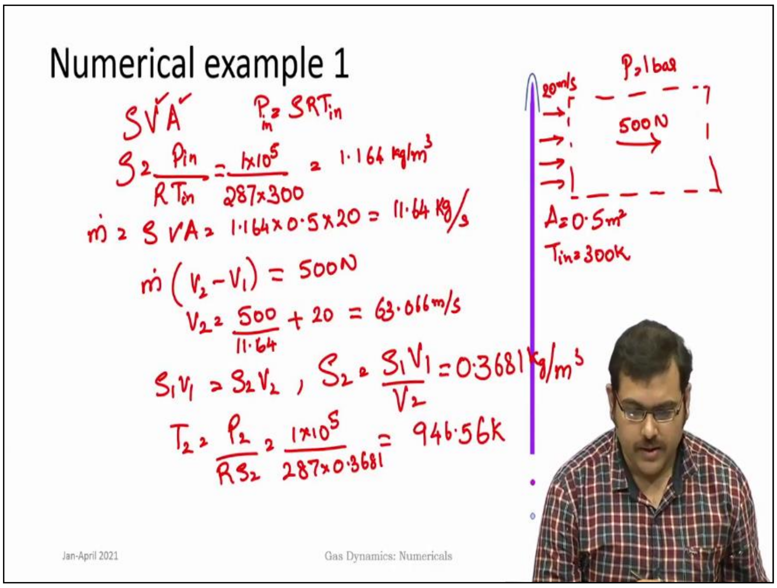

So let us look at this particular example. So from the picture, so this is the control volume and area is given area is $0.5m^2$ and ambient pressure is 1 bar everywhere it is 1 bar and flow is coming in $20\frac{m}{s}$ and temperature at this end is given as $300\ K$. And it is given that flow properties are uniform at the other end and there is a force of $500\ N$ that is getting applied over here.

So first is what is mass flow rate mass flow rate is ρVA that is a density, velocity and area so here what do we know at the inlet we know velocity(V) is known area(A) is known but we do not know density(ρ). But density can be found using $\rho = \frac{P}{RT}$, pressure at inlet is known temperature at inlet is known and R is known. So rho is $\rho = \frac{P_{in}}{RT_{in}}$ so this is $\frac{1x10^5}{287x300}$.

So 300 Kelvin so this works out to be $\rho =\ 1.16$. Now you can that is $\frac{kg}{m^3}$ and that can be directly substituted into mass flow rate mass flow rate is then $\rho V\ A$ is $(1.164x0.5x20)$ which is $11.64\frac{kg}{s}$. So this is what is the mass flow rate through the system. So now the question is what is the outlet flow properties velocity and temperature.

Now let us go to the outlet and we know that there is a force getting applied which is $500\ N$ of force. Now mass flow rate through the system remains the same the pressure is also the same. So if you look at the now the area is remaining the same so across from beginning to the end. So if you look at the momentum conservation equation so that is the principle that we have to use here which says $\dot{m}(V_2 - V_1)$ should be equal to the force applied that is the force applied here is $500\ N$.

So we can V_1 is known here which is $20\frac{m}{s}$ and so let this is $\dot{m} = 11.64\frac{kg}{s}$ so V_2 can be directly found turns out to be $63.066\ \frac{m}{s}$ so there has been an increase in the velocity of the flow. Now if you look at this now we need to find what is the temperature? But we do not have any information even on the velocity that again you can use mass conservation because $\rho_1 V_1 = \rho_2 V_2$. So now we know both V_1 and V_2.

So from here we can find out what is the density of the in the case of the second at the section 2. So then you can write ρ_2 is $\frac{\rho_1 V_1}{V_2}$ and if you make the substitutions you can get this density is smaller $0.3681\ \frac{kg}{m^3}$. So now we know the density now the pressure is also known at the section which is one bar.

So temperature we can find out $T_2 = \frac{P_2}{R\rho_2}$ so T_2 comes out to be $946.56\ K$. So you see that the temperature has increased significantly its clearly indicating that there has been an addition of heat. Now that is the; I think next question that how much heat is transferred and its direction. So we can look at the amount of heat transferred so for that we have to go to the principle of conservation of energy.

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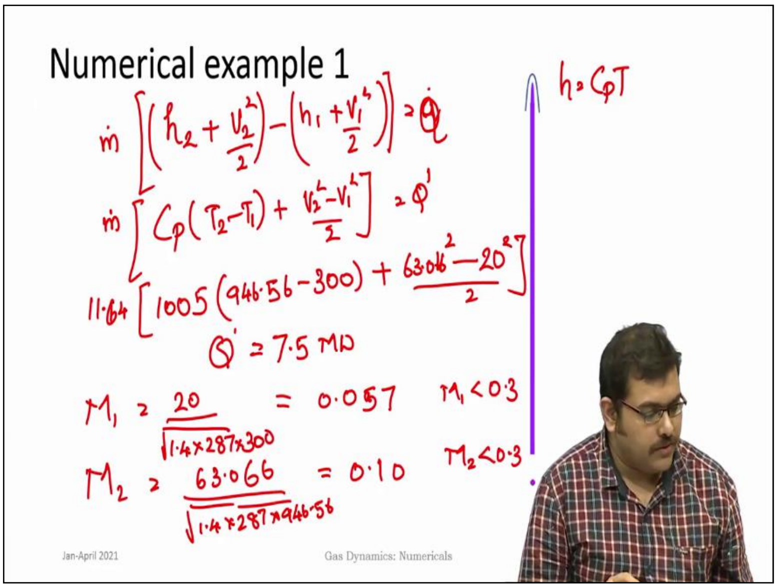

So there it is $\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)$ = to the amount of heat added $\dot{q}$ and so there is a mass flow rate so if you multiply this with $\dot{m}$ with total heat, $\dot{Q}$ added will be given. Now h is actually $c_P T$, through you can write this as $\dot{m}\left[c_P(T_2 - T_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right] = \dot{Q}$. So this is 11.64 and c_P is given 1005 this is T_2 is 946.56 – T_1 is 300 V_2^2 square V_2 is given we have found that out that is 63.066 – V_1 square is 20 by 2.

If you do this you should get $\dot{Q}$ as it is close to 7.5 MW because $W = J/s$. So it will be in watts, MW that is the correct unit to be put here. Now the next question here is comment whether the inlet and outlet flows are compressible or not. So that is something that one should pay attention this is a variable density flow because density is changing from section 1 to section 2.

But how do you decide whether the flow is compressible or not that has to be done by calculating the Mach number as we had said in our initial classes that compressibility effects become important once mach numbers become > 0.3. So, if we can look at the mach number so at mach number at station 1 is $\frac{20}{\sqrt{\gamma RT}}$.

So this mach number at one it is quite small it is you get it around 0.057 you get at such low numbers. So definitely this is not a compressible flow it is its mach number is less than 0.3. So it is incompressible and considers station 2 M_2 here it will be $\frac{63.066}{\sqrt{\gamma RT_2}}$, temperature is different so Mach number is again a local quantity. So it has to be evaluated at those sections.

And here you can find the Mach number is close to 0.1 but even this mach number < 0.3. So both these flows at section 1 and section 2 are incompressible. So but this is not a constant density flow. So this example considers different concepts it applies the Quasi-1D relations combinations of conservation of mass momentum and energy as well as the thermodynamic principles of ideal gas law.

And we are able to get this complete this solution. So this applies all the different concepts that we had learnt in the following in the previous sections.

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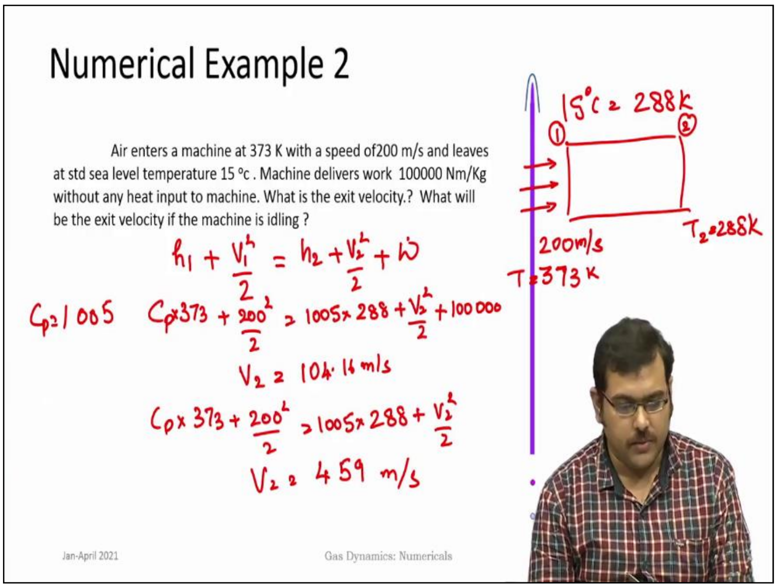

So now let us look at another example which also a combination of control volume analysis and thermodynamics. So air enters a machine at 373 K with the speed of 200 $\frac{m}{s}$ and leaves at standard sea level temperature at of $15\,^{\circ}\mathrm{C}$, $15\,^{\circ}\mathrm{C}$ is $288\,K$. Machine delivers work of $1{,}00{,}000\,\frac{J}{kg}$ without any heat input what is the exit velocity?

What will be the exit velocity if the machine is idling? So this is given here so again we draw a control volume around the machine we know that certain air is input it is coming in at 200 $\frac{m}{s}$ and 373 K is temperature. And it leaves so at this end T_2 so this is section one at section 2 you have 288 K. So now we have to calculate the exit velocity.

So we go this we demands the conservation of energy principles then you can write the equation that $h_1 + \frac{V_1^2}{2} = W + h_2 + \frac{V_2^2}{2}$ so now h is $c_P T$ so $c_P(373)$ here and c_P is taken as 1005. So + V_1 is known $1005(373) + \frac{200^2}{2} = 1005(288) + \frac{V_2^2}{2} + W$; V_2 is not known and this is known the work done.

So now you can find what is the velocity? So V_2 comes out to be $104.16\ \frac{m}{s}$. So that you can see that there has been a reduction in the velocity due to the work that has been taken out of the machine. Now if it is idling when there is no so when the idling no work is taken out the equation again still remains the same which is the conservation of energy. Now that no work is being taken out of the machine and if it still continues to exit at the same temperature then it would have much higher velocities.

And the relevant equation is 288 + $\frac{V_2^2}{2}$ and then the velocity can be higher much higher which is 459 around 459 $\frac{m}{s}$. It will come to around this value. So here again control volume principles and energy conservation equations have been used to solve this. So in gas dynamics once you come in you always have to apply fluid flow equations along with energy equation and the ideal gas laws or the equation of state.

And this is true when we do even with simplification we do Quasi-1D relations there also we have to solve all these system of equations. So with that we will end this class and from next class we will come to some very important concepts known as the stagnation concepts and the star concepts or the sonic concepts conditions. So that would be the next class.