Flight Dynamics II (Stability)
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Module No. # 04
Longitudinal Stability and Neutral Point
Lecture No. # 08
Static Margin and CG Limits

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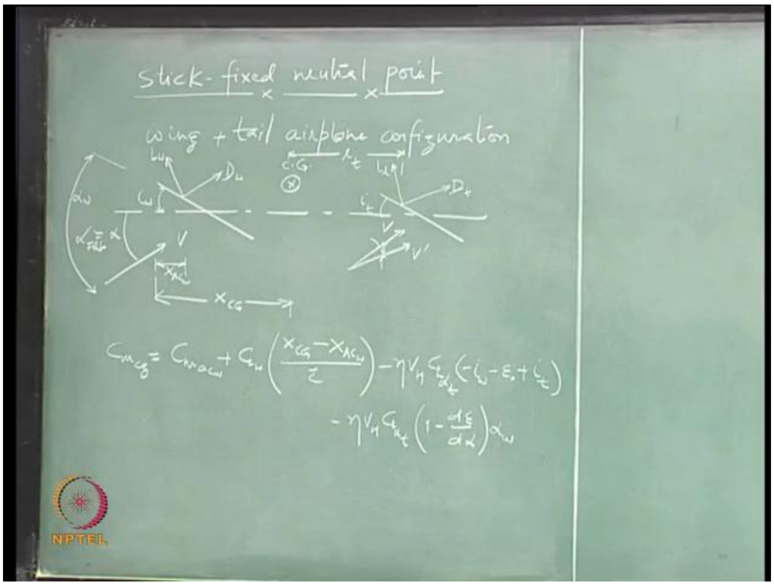

So, we are discussing about…

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Stick-fixed neutral point: we are still talking about wing plus tail airplane configuration.

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So, we have written down the expression for Cmcg using this picture.

(No audio from 01:16 to 03:23)

Expression for Cmcg, after some simplifications Cm CG equal to Cm ac w plus CL w into XCG minus X ac w over c bar minus eta into VH into CL alpha tail into minus i w minus epsilon 0 plus i t minus eta into VH into CL alpha tail into one minus d epsilon over d alpha into alpha w.

$$C_{mCG} = C_{mac,w} + C_{Lw}\left(\frac{X_{CG} - X_{ACw}}{\bar{c}}\right) - \eta V_H C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha_w$$

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Is this right? Now this CLw C_{Lw} has two components; one is changing with alpha and other one is a constant. So, we can write down.

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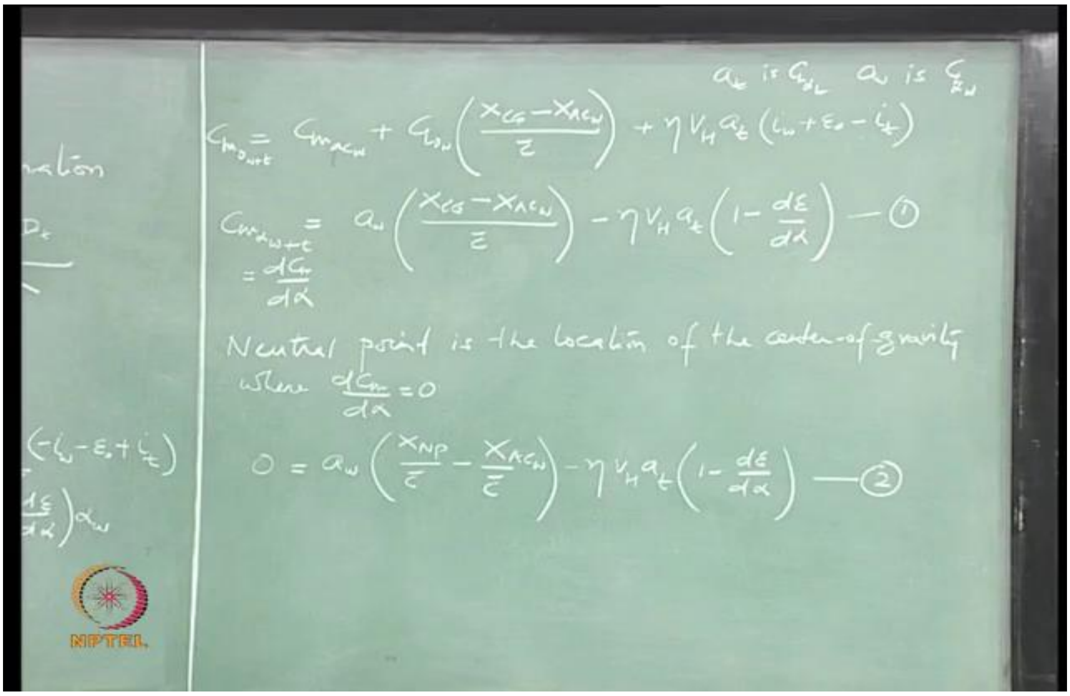

Cm naught C_{m0} for the wing plus tail airplane configuration is Cm acw plus CL0w into XCG minus X AC w over c bar plus eta into VH into a t into i w plus epsilon 0 minus i t.

$$C_{m0w+t} = C_{macw} + C_{L0w}\left(\frac{X_{CG} - X_{ACw}}{\bar{c}}\right) + \eta V_H a_t (i_w + \varepsilon_0 - i_t)$$

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And Cm alpha $C_{m\alpha}$ for wing plus tail configuration is a w into XCG minus XAC w over c bar minus eta into VH into a t into one minus d epsilon over d alpha.

$$C_{m\alpha w+t} = a_w\left(\frac{X_{CG} - X_{ACw}}{\bar{c}}\right) - \eta V_H a_t\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

(No audio from 06:17 to 07:01)

So, this is dCm over d alpha $\frac{dC_m}{d\alpha}$. What is the definition of neutral point? When this quantity becomes 0? So, I can replace this quantity by XNP X_{NP} right. So neutral point is the location of the CG where dCm over d alpha become 0 $\frac{dC_m}{d\alpha} = 0$.

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So, we can set this to 0 and find out what is… What is XNP X_{NP}.?

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Let us call this 1, and the other one is 2, and lets subtract 1 from 2.

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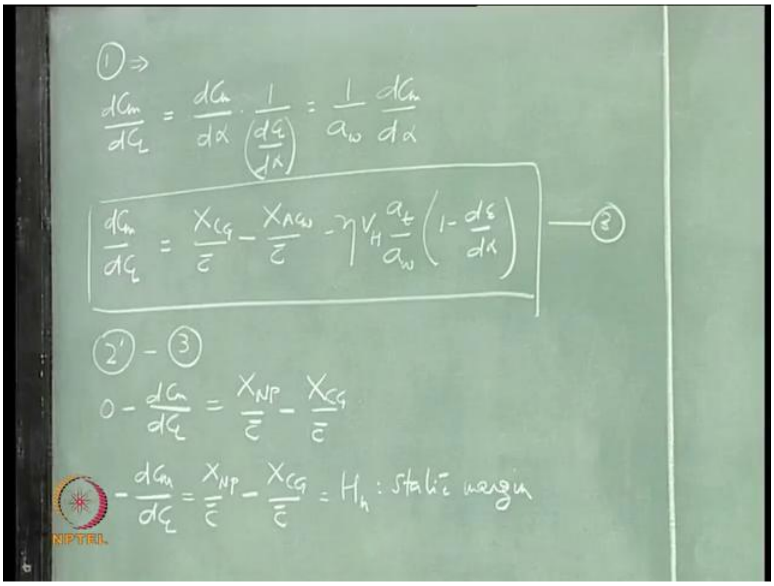

So, let us first also rewrite this equation 1 in some other form which is dCm over dCL. In many books, you will see, they are finding an equivalent expression for this, which is dCm over dCL $\frac{dC_m}{dC_L}$. dCm over dCL equal to dCm over d alpha into one over dCL over d alpha $\frac{dC_m}{dC_L} = \frac{dC_m}{d\alpha} \cdot \frac{d\alpha}{dC_L}$.

(No audio from 09:35 to 09:55)

C_L alpha $C_{L\alpha}$ is going to be positive for any airfoil you choose, negatively cambered, positively cambered or symmetric. So, this is not going to have any effect on the stability, because the positive quantity right. When you do this, you normally take this C_L as the C_L of the wing (airplane total lift coefficient $C_L \approx C_{Lw}$), because that is dominant. The C_L (corresponding lift) produced by the wing is much more than the lift produced by the tail. So, this is of the wing.

(No audio from 10:37 to 10:46)

So, I can use this one and find out; what dCm over dCL $\frac{dC_m}{dC_L}$ is and that is: dCm over dCL equal to XCG over c bar minus XACw over c bar minus eta into VH into a t over a w into one minus d epsilon over d alpha.

$$\frac{dC_m}{dC_L} = \frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}} - \eta V_H \frac{a_t}{a_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (3)$$

(No audio from 11:29 to 11:47)

We can also rewrite this: zero equal to XNP over c bar minus XACw over c bar minus eta into VH into a t over a w into one minus d epsilon over d alpha.

$$0 = \frac{X_{NP}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}} - \eta V_H \frac{a_t}{a_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (2')$$

(No audio from 11:50 to 12:23)

Is it correct? Now, subtract 3 from 2 prime.

(No audio from 12:36 to 12:46)

What do we get? This part is same in both the equations. So, what we get is: 0 minus dCm over dCL equal to XNP over c bar minus XCG over c bar

$$0-\frac{dC_m}{dC_L}=\frac{X_{NP}}{\bar{c}}-\frac{X_{CG}}{\bar{c}}=H_n: \text{Static Margin}$$

(No audio from 12:56 to 13:27)

Right, this quantity is... So, this dCm over dCL negative $-\frac{dC_m}{dC_L}$ is XNP over c bar minus XCG over c bar $\frac{X_{NP}}{\bar{c}}-\frac{X_{CG}}{\bar{c}}$. This quantity is known as static margin.

(No audio from 13:53 to 14:07)

From pilots experience of flying conventional airplanes, it has been found that for good handling qualities of aircraft, this H_n should be 5 percent of, it should be 5 percent. So, X_{NP} minus X_{CG} should be 5 percent of the mean aerodynamic chord.

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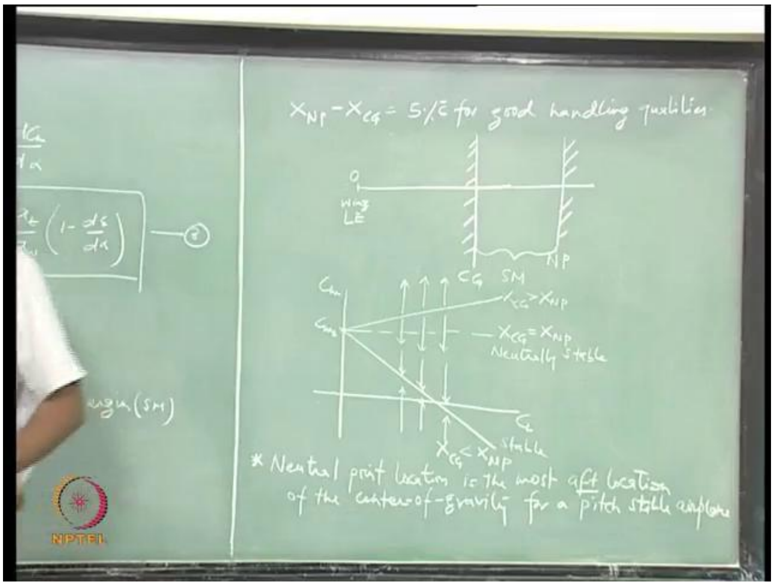

We will talk about this a little later.

So, clearly this H_n has to be positive, that means, X_{CG} has to lie ahead of X_{NP} (for stability in pitch).

(No audio from 15:32 to 15:53)

Zero is at the wing leading edge. So, this is what is called static margin or SM? So, let us try to look at what is happening graphically, draw C_m versus C_L plot.

(No audio from 16:27 to 16:39)

So, dCm over dCL $\frac{dC_m}{dC_L}$ is negative for this curve, and that means, CG is lying ahead of the neutral point.

You move CG backward and then there is a point where X_{CG} is lying right on the X_{NP}, and then you have neutral stability. So, this is for stable case, this line is for…Neutrally stable airplane.

So, clearly if we want to maintain stability, ensure stability for all CG locations - the CG should not go beyond the neutral point, that is what it means. So, I can say that neutral point location is the most aft location of the centre of gravity. If you want to ensure stability. Let me write that statement somewhere or may be there itself.

(No audio from 18:55 to 19:58)

Remember, we have now found the aft most location of CG, you have to ensure that whatever is happening to your aircraft, the CG should not go beyond the neutral point. So, people can be properly seated and not all of them should move simultaneously backward, so, that you know CG has shifted beyond this neutral point location. So, that is the reason why crew will ask you not to move around much (inside an airplane in motion).

(No audio from 20:36 to 20:52)

(( )).

(( )) initial CG of...

After, we have, sort of properly placed the loads, and now the aircraft is going to take-off. There are two situations - one is fuel will be consumed and that will have an effect on the CG; the other situation is people can move on the aircraft, but that they will take care of. So, I would assume that the CG when we take-off and even before take-off you will have some run on the runway. So, only major consideration would be how you are going to place your loads on the aircraft, and after that, it is the fuel. Anyhow this margin is going to take care of some other aspects, like how loads are getting re-distributed over the airplane. So, now I can draw some arrows. So, on this line…

(No audio from 22:11 to 22:21)

You know, instability means going away from something; and stability means converging.

(No audio from 22:30 to 22:39)

So, at this point clearly we do not know and you are neutrally stable. I cannot draw any direction whether something will converge to it or diverge, coming and going this arrow through this neutral point line as (( )). Is this alright?

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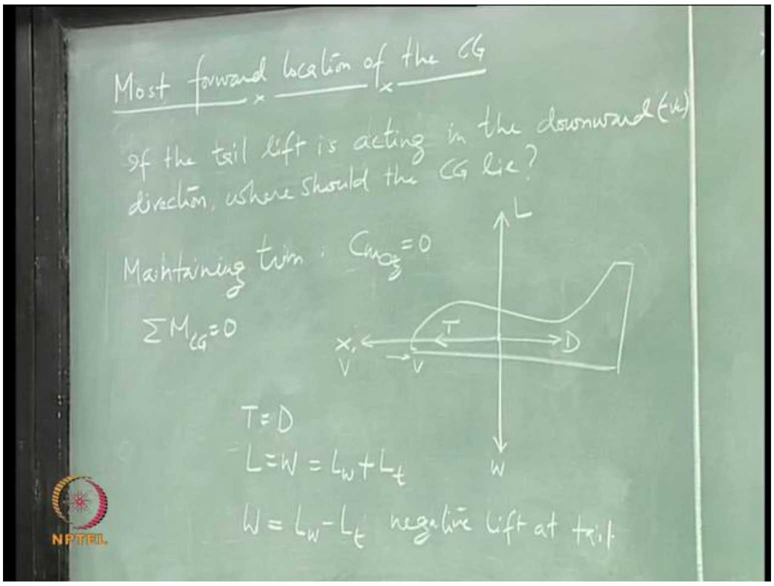

Now, let us discuss about the most forward location of the CG. This we talked about the most aft location of the CG, what is the most forward location of the CG?

(No audio from 23:25 to 23:46)

Yeah

Aerodynamic center.

Aerodynamic center.

(( )) of the wing.

That is in the case, when…

(( )) if it is a flying.

Yeah.

It is a flying wing.

That is only when the lift produced on the tail is acting in the upward direction, is it not? I can always have the lift produced on the tail in the negative direction. So, the effect on stability from the tail is destabilizing, if I have lift on the tail acting in the downward direction then what happens?

(( ))

Is going to give a...

Additional moment to that.

Yeah, positive moment. So, both of them will become actually destabilizing when the lift produced on the tail is negative, then the contribution from the wing, which is already destabilizing. And tail will also start having destabilizing effect on the pitch stability, is it not? If I say lift is this (downward) moment about CG is going to be in this (clockwise) direction. Now where should the CG lie.

(No audio from 25:43 to 25:49)

I can have such a situation right.

(No audio from 25:51 to 26:53)

So, you can clearly see that aerodynamic center of the wing is not the most forward location of the CG. How would you define that? What all you have to ensure, remember, we talked about maintaining trim, and for trim, Cmcg must be 0 $C_{mCG} = 0$ (sum of all moments about CG must be zero at equilibrium or trim condition).

What is the other condition?

Now, let us talk about the trim first. So, we will talk about level flight condition; that is what this is $C_{mCG} = 0$.

Remember we talked about this.

(No audio from 28:16 to 28:45)

Aircraft is going in the forward direction. Relative velocity is V along the X axis, this for the case when alpha is 0, angle of attack is 0 $\alpha = 0$, and then, to find a trim you not only have to balance out the forces, but also the moment. The expression for the moment, I have written here Cmcg should be 0 $C_{mCG} = 0$ or you can write it like this sigma M cg equal to zero $\sum M_{CG} = 0$. So, this we get from here. So, what is the other condition?

(No audio from 29:39 to 29:47)

Thrust should be equal to the drag, let us say we have ensured that, and lift should be equal to the weight. And this lift is the sum of the lift produced by the wing and the tail. This may be small as compared to this, but still we have to account for that when we are talking about producing negative lift. If I am producing negative lift that means, W is Lw minus Lt $W = L_w - L_t$.

(No audio from 30:34 to 30:51)

So, when you are producing negative lift at the tail, still you have to ensure that this condition is satisfied. Is it not? This has to be satisfied to maintain the equilibrium. Let us say we can somehow manage to satisfy this. We have enough lift produced from the wing which can take care of this small negative lift produced by the tail. And how do you produce negative lift by the tail, this is going to depend on the camber.

To start with this we said that the tail is always going to be have, be having symmetric airfoil. How we are going to change the camber of the tail?

Elevator. So, this L_t, negative L_t max is going to depend upon, how much maximum elevator deflection you can have in the upward direction. Is it alright? So, you have to think about that situation, corresponding to delta e maximum upward deflection - elevator maximum upward deflection, this L_t max negative, can be compensated by the lift produced on the wing. So, that this quantity $(L_w - L_t)$ is able to maintain (balance) the weight. So with this, now go back to this equation. The question is, if the tail lift is acting in the downward or negative direction, where should the CG lie?

Ahead of the aerodynamic center of the wing…

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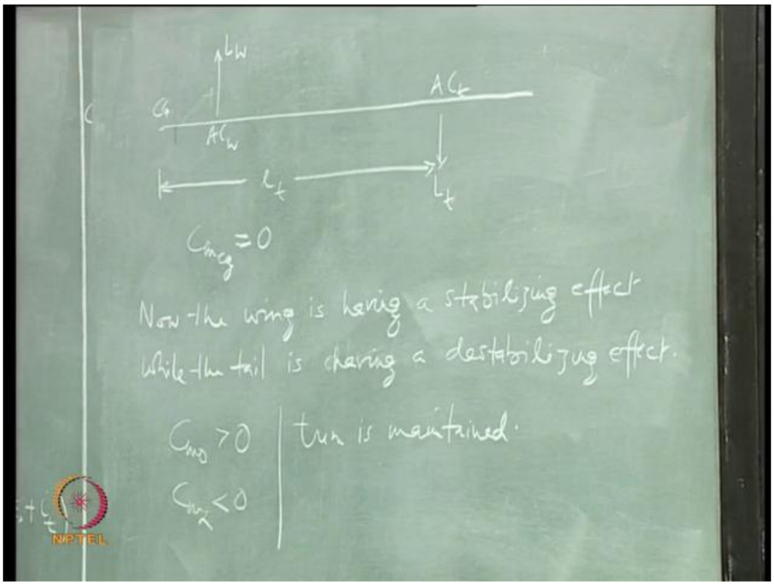

So, in this case, the case when the (tail) lift is acting in the negative direction.

CG can lie ahead of the aerodynamic center of the wing.

Now, question is whether you want that or not. So, C_{mcg} you know can be maintained 0, because this destabilizing effect can be compensated for by the stabilizing effect of the wing, when CG is lying ahead of the AC then you are going to get a stabilizing effect coming from the wing.

(No audio from 34:48 to 35:41)

Would you want to go into such a situation? Would you want, I mean now, clearly we have we can maintain the equilibrium, we can also maintain stability, what will be Cm naught C_{m0}? Cm naught C_{m0} is still going to be positive. So, I am satisfying all the criteria, I am satisfying Cm naught greater than 0 $C_{m0} > 0$ and you have Cm alpha less than 0 $C_{m\alpha} < 0$, and trim is maintained $C_{mCG} = 0$.

Would you want such an airplane, where you can have your CG moving ahead of the aerodynamic center, but clearly we can have it. Mathematically we can prove that CG can go beyond this aerodynamic center of the wing ahead of this point, if the lift can be maintain negative at the tail.

Question is, would you want such an airplane, why?

(( )) the sudden increase in lift will keep on increasing the angle of attack, I mean sir we have to (( )) it depends that is a (( )) it depends suppose there is an increase in lift then alpha will change.

Yes

It is what is (( )) work (( )) to lift the same…

So, performance of the aircraft will go down, is it not? Because lift - negative lift means this wing will take this lift has to be has to compensate for so, the wing has to work more as we said.

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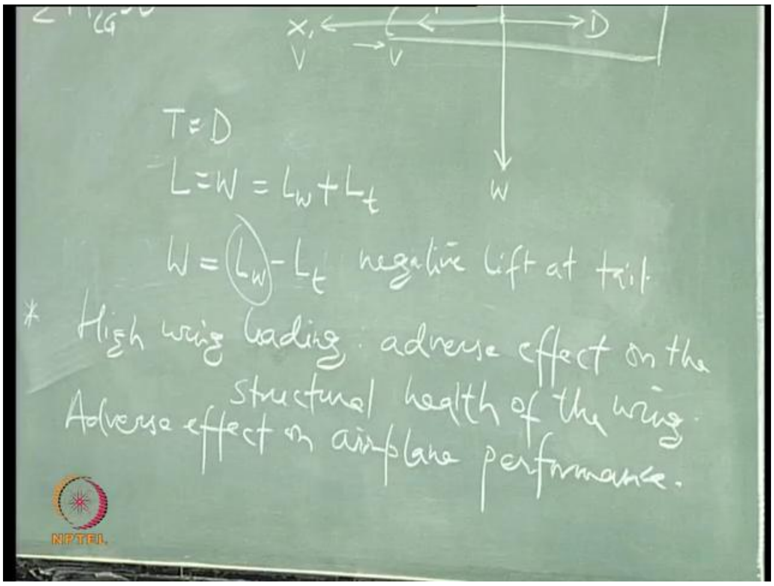

And what is the problem that is going to come the wing because of that? High wing loading.

And that is going to have bad effect on the health of the wing.

(No audio from 38:57 to 39:22)

By the way people do worry about the structural health of now mechanical systems, there is a whole lot of research on this topic.

Performance of aircraft will also suffer, because if this lift has to go high that angle of attack has to be high, and that is going to cause or create more drag; more lift is also giving you more drag. So, definitely this is going to have an effect.

(No audio from 40:08 to 40:38)

But now you know, I mean where you can have the most forward location of the CG. I would not say it should be at the aerodynamic center of the wing. Because if it is going to that side, then we have to again satisfy all the criteria, and also look for the best performance of the aircraft. So, performance and stability are actually connected.

(No audio from 41:09 to 41:48)

Alright. But there are different kinds of airplanes, we do have airplanes with canard in front, the tail in front of the wing. Is it not? So, again we have to repeat the stuff, we have to see, we can maintain the trim. We can see if we have satisfied the stability criteria; we have to see if we can fly it at positive angle of attack. So, all of these have to be worked out for any airplane you talk about, is it not? Any question. But we have not added other effects, the other parts also which are going to contribute to the location of the neutral point; now, we will look at the effect coming from the power plant and also from the fuselage.

Fuselage actually has a coupled effect, it is going to influence the wing - wing is also going to influence the fuselage. So, that is interference effect. Anything else? Which is other body which is looking like a fuselage mounted on an airplane?

Yeah, that is for combat aircraft.

But for normal transport aircraft...

So, power plant - plant is also the encased within what is called engine nacelle? and that is also looking like fuselage, body of revolution.

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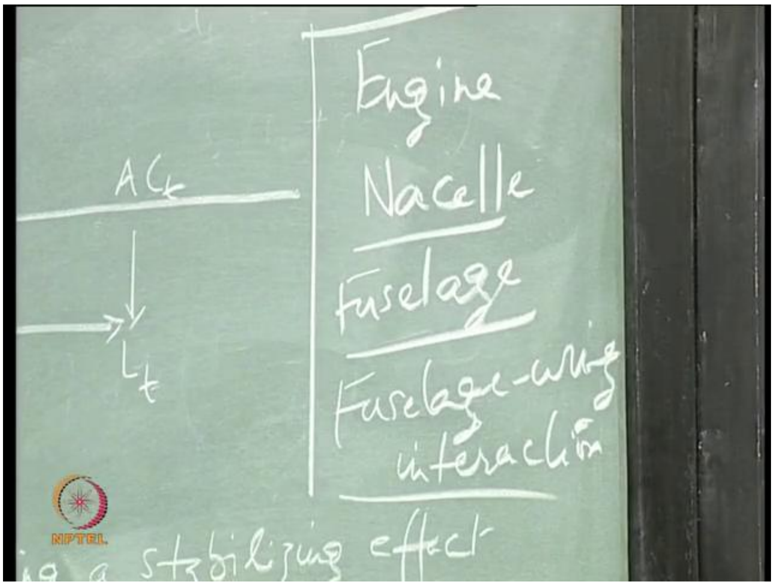

and its going to contribute to Cm alpha $C_{m\alpha}$.

And fuselage wing interaction.

So, all these three effects; now, we are going to add to our equations, anything else?

Effect of the rotating propellers. Jet engines. So, we are going to talk about all that in the next class. So, they will all come together under one subtitle which will be power effects. So, in the next class we will talk about the effects, coming from these 3 components, and then we look at the power effects. So, we can stop for today.