Flight Dynamics II (Stability)

Prof. Nandan Kumar Sinha
Department of Aerospace Engineering
Indian Institute o f Technology, Madras

Module No. # 03

Longitudinal Static Stick Fixed Stability

Lecture No. # 06

Horizontal Tail Contribution

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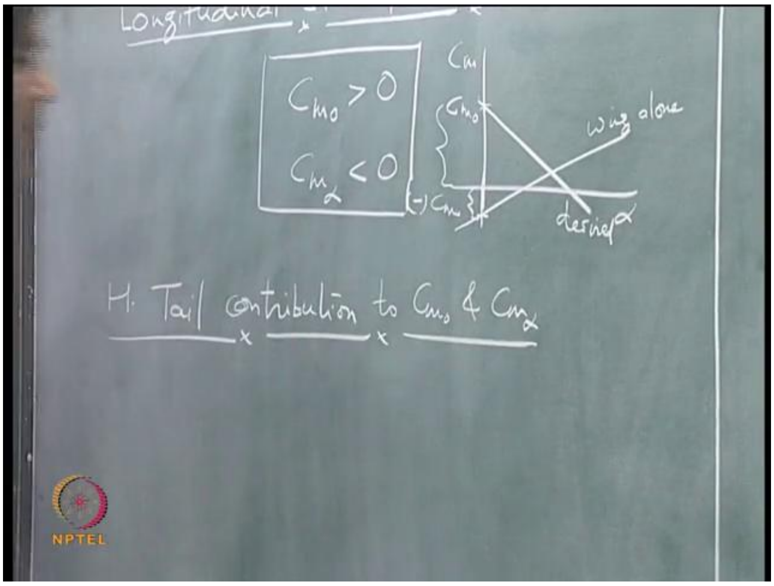

So, in this class, we will continue with our discussions on longitudinal static stick fixed stability of the aircraft. We will discuss an airplane to have stability in pitch and also to be able to fly at positive angles of attack, it should satisfy two condition or rather it should have two properties and they are C_{m0} which has to be positive and $C_{m\alpha}$ must be negative.

We want to put it graphically. This is how the curve for the whole airplane should look like. This quantity is C_{m0}. In the last class, we looked what is the, for a conventional airplane, where wing airfoil is suppose to have positive camber for higher lift and for the CG to be located behind the aerodynamic center of the wing, we saw in the last class that the graph of the wing alone contribution; wing alone C_m versus α curve looks something like this.

So, for positive cambered airfoil for the wing, gives you the C_{m0} which is negative and $C_{m\alpha}$ is having a positive slope. And this is what is desired. This is what you get from the wing alone contribution and this is what is desired (Refer Slide Time: 3:10) for the airplane. Horizontal tail is actually used on the airplane to change this curve to this. So, it is actually used to augment C_{m0} and $C_{m\alpha}$ so that this is preserved.

This should have, the aircraft should be designed in such a fashion so that you should have C_{m0} which is positive and $C_{m\alpha}$ must be negative. In the last class, we saw that actually wing alone is not enough to satisfy both these criteria. So, for this reason, what we do is, what is done is - a tail is added or horizontal tail is added to the aircraft behind the wing so that you can meet these criteria. So, today, we are going to see or discuss tail contribution, and I can also put H here in front of this, so that its horizontal tail.

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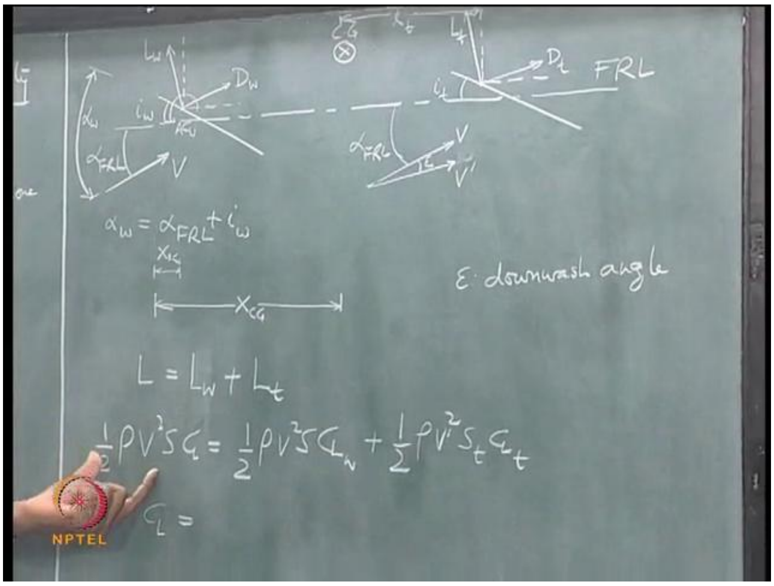

So, let us say this is our fuselage reference line (Refer Slide Time: 5:51). The wing is located here and relative wind is coming onto the aircraft at an angle of attack which is measured with respect to the fuselage reference line, and therefore, is called α_{FRL}. So, this is your V relative wind, and this wing is at some incidence angle i_w with respect to the fuselage reference line. So, total angle of attack at the wing is $\alpha_w + i_w$.

Now, we assume that all the forces, aerodynamic forces are being shifted to the aerodynamic center of this wing with a moment. So, for positive, positively cambered wing, we should have C_{m0} which is negative. This we have seen in the last class. So, I will not go on describing this completely in this class, we will focus on what is happening at the tail.

So, tail will also, tail is also like wing, smaller wing and is inclined at an angle i_t to the fuselage reference line, and CG is located somewhere here, CG of the aircraft. This is aerodynamic center of the wing, this point, and aerodynamic center of the… so, they are both located at their respective quarter chord locations, and remember, we said we are going to measure all the distances from the wing leading edge.

We are going to measure all the distance from this point (wing leading edge). So, this is, right, all right. (Refer Slide Time: 9:50). So, this part actually we have covered in the last lecture. We are going to focus on the tail now. As I said, because of the tail location which is in the wake of the wing, we have to account for what is called the downwash angle and that is because of the tip and the trailing edge vortices at the wing. That is going to cause a change in angle of attack of the fuselage reference line at the tail associated with a change in velocity.

So, this, if I draw a parallel to that velocity is something like this and lets say this is α_{FRL} at the at the wing. And here, we are going to change, see a change in velocity which is smaller than this velocity vector. And this angle is the angle of downwash. So, this is the original relative wind speed and this vector is V prime. So, dynamic pressure at the tail is going to change because of this downwash angle and angle of attack is also changing. (Refer Slide Time: 12:17).

Let us assume that the aerodynamic center of the tail is located at a distance L_t after the center of gravity. And this is the lift created at the tail and the drag D_t. Now, if I want to find out what is the total lift coefficient of this wing plus tail configuration, then you have to add the two lift contributions. L, lift coming from the wing and the lift coming from the tail ($L = L_w + L_t$).

So, we want to look at the lift coefficient of the complete airplane. This is how you write the lift which is half rho V squared S into CL $\frac{1}{2}\rho V^2 S C_L$. This S is a reference area which is taken as equal to the area of the wing. So, this is wing planform area, and this V is the velocity which the wing is seeing, and rho ρ is the density (of air) at sea levels condition or any other condition, this is going to change with height. Now, I am adding a CLw C_{Lw} here.

These quantities are going to be same, but I am adding a CLw C_{Lw} here because I am not using the tail to produce a lift. Tail is being used to change Cm naught C_{m0} and Cm alpha $C_{m\alpha}$. That is only objective why we are using the tail for stability and control purposes, not to produce lift. But of course, this is the lift which is going to change this profile, but only for that reason, not for actually changing the lift of the total aircraft. So, lift of the wing is just sufficient for taking care of the weight.

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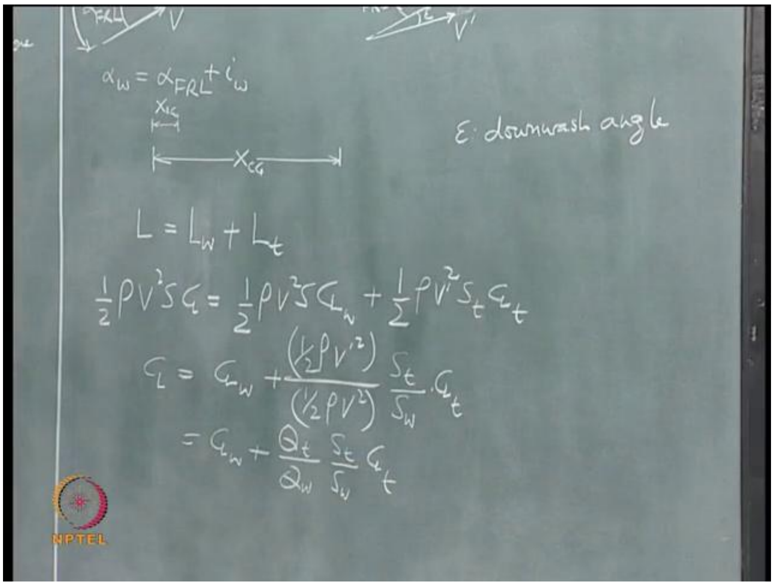

You can divide the right side by this quantity (half rho V squared S $\frac{1}{2}\rho V^2 S$) you get, so, you can also write this as Sw S_w because wing planform area is the one which I am using as reference area. What is this quantity (Q t over Q w Q_t / Q_w)? This is the ratio of two dynamic pressures.

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Ratio of the dynamic pressures Q t over Q w Q_t / Q_w is also known as tail efficiency factor (η). So, this CL CL is now modified to CL equal to CLw plus Q t over Q w into St over Sw into CL t. $C_L = C_{Lw} + \frac{Q_t}{Q_w} \cdot \frac{S_t}{S_w} \cdot C_{Lt}$. The value of this quantity (Q t over Qw Q_t / Q_w) lies between 0.8 and 1.2. So, when do you think it will be 0.8 and when you think it is 1.2? It is 0.8, when the tail is lying behind the wing, in the wake of the wing. If this tail is lying in front of the wing, then this quantity is greater than one.

20:17

So, Q t is less than Q w $Q_t < Q_w$ when the tail is lying aft of the wing. Q t is greater than Q w $Q_t > Q_w$ when the tail, you will not call it a tail, right, in that case is, it is, being called tail because is lying behind the wing. What do you call this surface when it is lying ahead of the wing, it is canard. For canard configuration, you have Q t Q_t which is larger than Q w Q_w. And this is mainly because you are seeing an upwash in front of the wing, ahead of the wing, and a downwash behind the wing.

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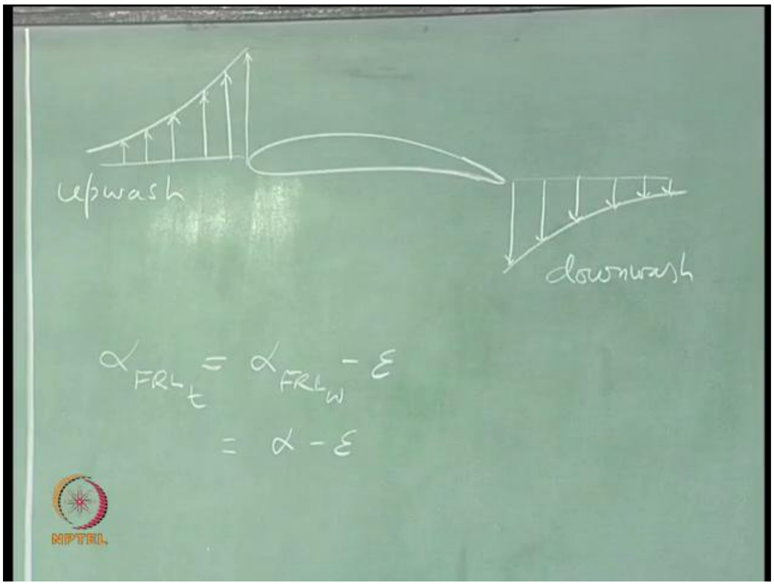

So, the flow field around the wing will look something like this. This is downwash, this is upwash. How is angle of attack going to change at the tail now because of this downwash? So, alpha FRL α_{FRL} at the tail is going to be α_{FRL} at the wing minus this angle (ε). And I have said that we are going to treat this angle of attack as the angle of attack of the aircraft. So, we are going to drop this FRL and write this as alpha α. So, here also this is nothing but alpha plus i w $\alpha + i_w$.

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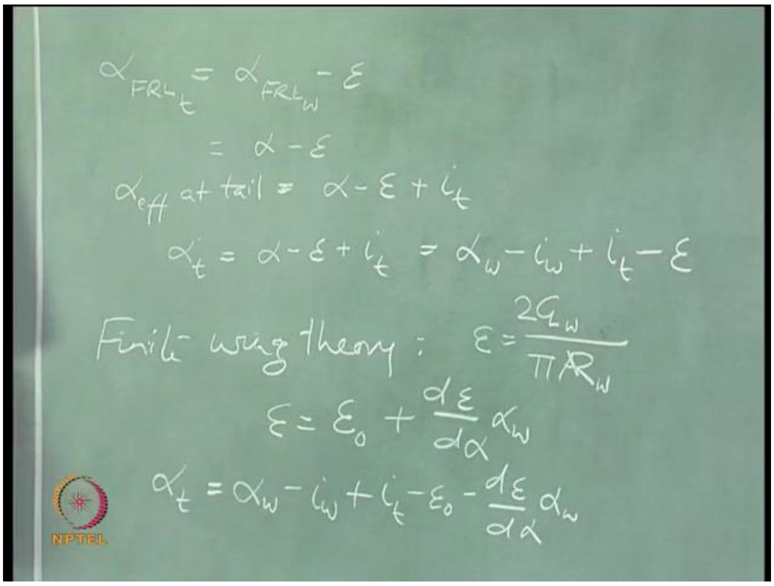

So, what is the effective angle of attack at the tail? What is that?

alpha minus epsilon plus i t. $\alpha - \varepsilon + i_t$.

So, I am going to write this as alpha t α_t at the tail, and, that is, it can be also written as, so, this alpha α is alpha w minus i w $\alpha_w - i_w$. Now, the question is how do you quantify this angle? Of course, for everything in this course, we will assume some empirical relation. But, so you have to measure the quantity and just plot it. So, that is the empirical relation.

But there have been some work on this to quantify this epsilon ε analytically also. So, there are, the finite wing theory gives you an expression for this epsilon ε which is epsilon equal to C Lw lift coefficient of the wing over pi into aspect ratio of the wing $\varepsilon = \frac{2C_{Lw}}{\pi AR_w}$.

Yeah

(( ))

yes this is 2 into C Lw over

Sir it will depend up on the (( ))

Yeah it depends on lot of factors like, where this tail is located behind the wing. How much above or how much below, how close, all that is going to affect this angle. But this course we will assume that this relation is correct. That is something you have to go back and find out from aerodynamics book. I will also in this course itself, after a couple of lectures, I will give you empirical relation. So, I will give you this plot of epsilon ε against some parameter of the aircraft, and then, it will be much clear.

So, you will know what this epsilon ε is, when a wing is lying at a particular distance from the wing. So, that kind of plot is available, but for now, we will keep this. This epsilon ε is also a function of angle of attack. You can write this as epsilon ε equal to epsilon naught at 0 angle of attack plus 0 angle of attack of the wing $\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha_w$. So, you can further expand this. There is going to be another term here which is this. Any questions so far?

What is our objective? We want to find out, yeah.

(( )) .

Actual angle of attack that we are talking about its all in the pre-stall region. So, lets say up to 14 15 degrees.

Should be delta (( ))…

It should be delta.

(( )) if epsilon ε is 0 (( ))

When alpha α is 0, epsilon ε is epsilon naught ε_0. What is our objective? Our objective is to find out the moment of the forces at the tail about the CG of the aircraft.

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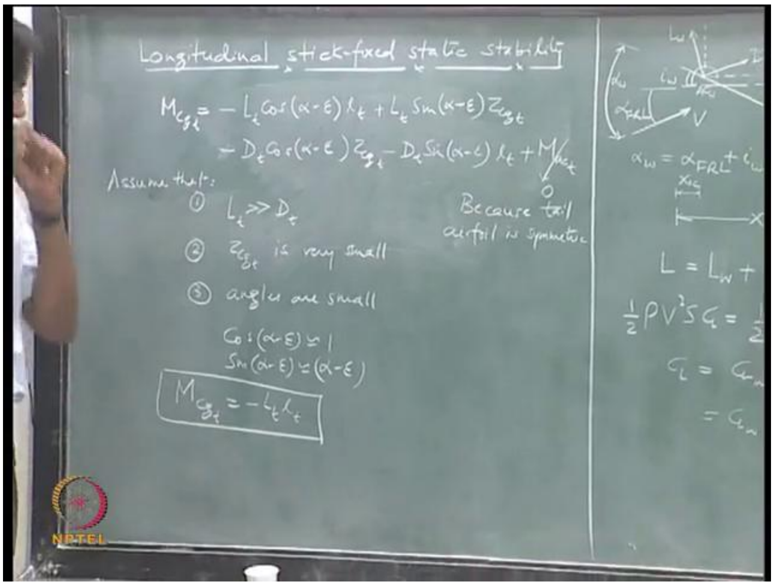

We want to find out an expression for Mcg of the tail. What is that going to be? What is this angle? alpha minus epsilon $(\alpha - \varepsilon)$ and this alpha is alpha FRL at the wing. So that is what I have taken as angle of attack. So, I have dropped this FRL and then you are taking this angle as alpha α. Now, let us try to write this expression. What is it? The cos component of this Lt multiplied by this distance and the direction of the moment is negative.

So, minus Lt cos alpha minus epsilon into lt plus Lt sin alpha minus epsilon and this is going to give a positive moment. That is why I have written this plus in front of Lt (for the second term). So, I have to define a distance now. So, let us say this distance is… the height of Cg above the aerodynamic center of the tail, that is Zcgt. This is ac (aerodynamic center) tail. What about other components? plus or minus? Minus D t cos alpha minus epsilon ($D_t \cos(\alpha - \varepsilon)$).. plus this residual moment.

($M_{cgt} = -L_t \cos(\alpha - \varepsilon) l_t + L_t \sin(\alpha - \varepsilon) Z_{cgt} - D_t \cos(\alpha - \varepsilon) Z_{cgt} - D_t \sin(\alpha - \varepsilon) l_t + M_{act}$)

What is the value of this moment (Mact M_{act}) for the tail? It will depend, what kind of airfoil we have? Shape of airfoil at the tail which is normally symmetric. So, this quantity (Mact M_{act}) is actually 0 because tail airfoil is symmetric.

Now, let us try to simplify this. So, we can assume that the lift at the tail is much greater than the drag (Lt >> Dt $L_t >> D_t$). That is one assumption, and this is how actually you have to design your tail. You cannot have an aerodynamic surface which is giving you more drag than the lift. Let us say this height Zcgt Z_{cgt} of the center of gravity from the aerodynamic center of the tail is very small, and angles are small. So, one assumption is this; second is this, and the third one. (Refer Slide Time: 36:47)

So, this cos alpha minus epsilon ($\cos(\alpha - \varepsilon)$) is 1. Now, this quantity is small because I said the angles are small and this is small. So, both together, product of the two small terms will result in even smaller quantity. So, we will neglect the smaller terms only because they are much smaller as compared to the quantity which is large, so, large and small. We have to look at the magnitude of that and that is how we can drop out terms.

So, this can be dropped, because there are two terms which are small. The product of those two terms is going to be further small. Here, drag is small, this is small. You can also drop out this term and same case here. So, if we use these assumptions, then I can come down to a single term here which is this Mcgt = - Lt into lt $M_{cgt} = -L_t l_t$. Now, let us try to expand this because all I am interested in is, the expressions for Cm naught C_{m0} and Cm alpha $C_{m\alpha}$.

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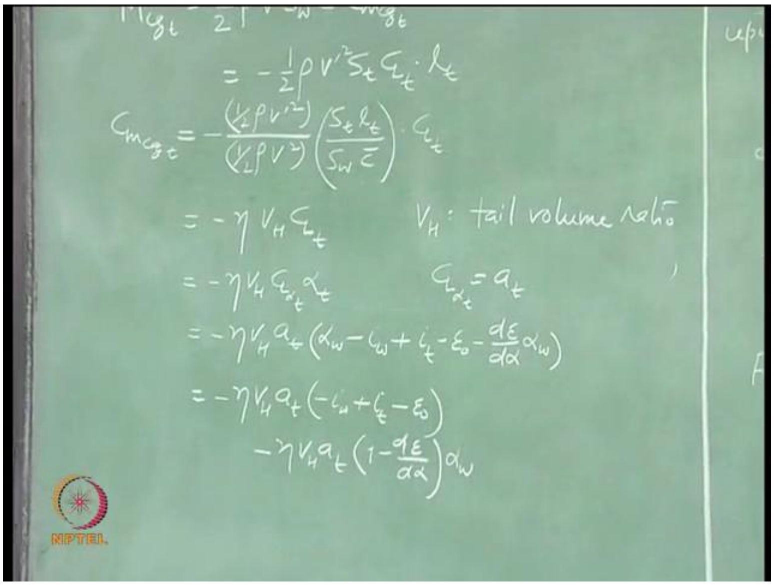

I am going to non-dimensionalize the quantities. So, remember, this is the moment about center of gravity due to the forces on the tail. But if you talk about the coefficients, non dimensionalized coefficients, then the reference is that we have to take these quantities at the wing. So, half rho V squared S w c bar ($\frac{1}{2}\rho V^2 S_w \bar{c}$) which (c bar) is the mean aerodynamic chord of the wing and Cm c g C_{mcgt} tail and that is equal to minus half rho V prime squared ($\frac{1}{2}\rho V'^2$), lift at the tail, we have written the expression for lift at the tail S t S_t into the arm length. So, Cm c g due to the tail C_{mcgt} is minus half rho V prime squared over half rho V squared into St lt over Sw c bar into CLt.

$$C_{mcgt} = -\frac{\frac{1}{2}\rho V'^2}{\frac{1}{2}\rho V^2} \cdot \frac{S_t l_t}{S_w \bar{c}} \cdot C_{Lt}$$

(( )).

S w is S. So, you can S is the reference area which I am taking as the wing plan form (( ))

For the lift also we should use the same (( )).

For the lift also.

(( )).

No no, you are talking about this quantity?

Yes sir

So, how do you calculate lift over any surface? You have to take the area of the that particular surface.

Sir why we are using S w (( ))

This I am using only for non dimensionalizing the quantities, why not?

(( )).

If you write the complete expression for the whole airplane, this will only come as a component. So, when I am writing Mcg for the whole airplane and I am trying to find out Cm for the complete airplane, there I am making this assumption that the reference area is the wing planform area.

This VH V_H (equal to St lt over Sw c bar $\frac{S_t l_t}{S_w \bar{c}}$) is called tail volume ratio. What do you want to find? You want to find out Cm naught C_{m0} due to tail and Cm alpha $C_{m\alpha}$ due to tail. Finally, we have arrived at an expression which is the Cmcg due to tail and that is equal to this. Now, this VH is something you can play with when you are designing your aircraft so that you can have more stability, more Cm naught C_{m0} and that you can find out by expanding this.

So, this VH is the ratio of S t into l t over Sw into c bar . This l t I can play with, S t I can play with after you have fixed here wing. And you can also play with some more parameters which are coming from here. Remember, everything we are doing here or throughout this course, we are doing it only in the pre-stall region. So, I can write this C L t as C L alpha tail into alpha tail $C_{Lt} = C_{L\alpha t}\alpha_t$. So, lift curve slope at the tail can also be written as this a t (a_t) into alpha tail α_t which is alpha w - i w + i t - epsilon 0 - d epsilon over d alpha into alpha w ($\alpha_w - i_w + i_t - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha_w$).

Now, I am going to separate it out in two parts - one part is going to give me Cm naught C_{m0}, other part is going to give me Cm alpha $C_{m\alpha}$. So, all these quantities here are fixed. You can fix your wing incidence with respect to the fuselage reference line, i t i_t tail incidence, and this epsilon naught ε_0. This part is the function of angle of attack.

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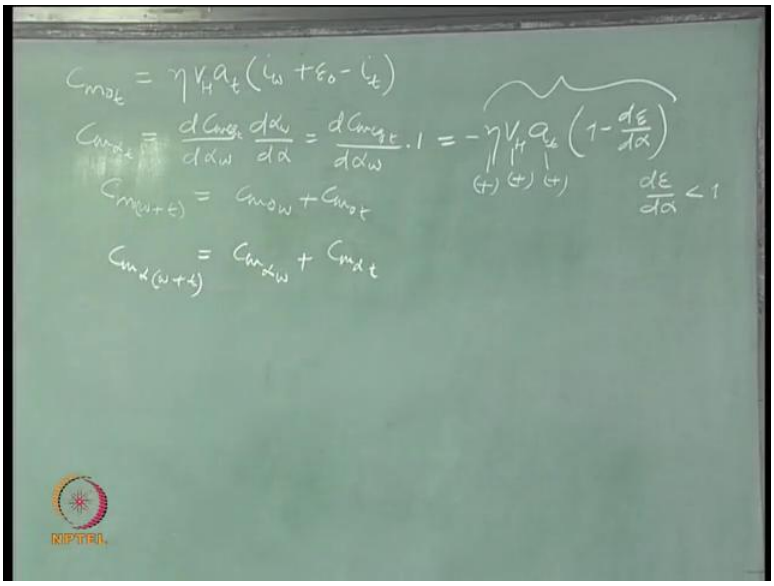

So, Cm naught C_{m0} due to tail is, what is that? minus eta into V H into a t $-\eta V_H a_t (i_w + \varepsilon_0 - i_t)$. The Cm naught C_{m0} has to be positive and Cm alpha $C_{m\alpha}$ has to be negative. Look at these terms here, this is going to be positive, this is also positive, lift curve slope of the tail is also positive. Yeah, any questions?

Yeah the second line d the next term sir.

d Cm cg tail.

d epsilon over d alpha $\frac{d\varepsilon}{d\alpha}$ is actually less than 1, much less than 1. So, overall you are going to achieve what you started with? If you wanted to add a tail, so that you can get Cm alpha curve for the airplane to become negative.

For the wing alone configuration, it was coming out to be positive, the slope. And Cm naught C_{m0} is something you can adjust. You can choose these angles in such a fashion that you get this Cm naught C_{m0} positive. Any question? So, total Cm naught C_{m0} for the wing plus tail configuration is Cm naught C_{m0} due to wing plus Cm naught C_{m0} due to tail, and Cm alpha $C_{m\alpha}$ wing plus tail is the sum of these two quantities. We can continue from here in the next lecture.