Flight Dynamics II (Stability)

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Module No. # 12
Aircraft Response to External Inputs
Lecture No. # 39
Wind Profiles, Longitudinal Mode Response to Wind Shear

We are still looking at aircraft response to inputs, and my inputs are atmospheric inputs. In the last class, we looked at the plunging motion of aircraft. We took sharp-edged gust in front of the aircraft. This is called sharp edged gust, it is like a step input to the aircraft and we found response in time as this (Refer Slide Time: 01:47) This tau is the time constant of the motion and it is depending upon the equilibrium speed and the derivative Z alpha, $\tau = -\frac{u_0}{Z_\alpha}$.

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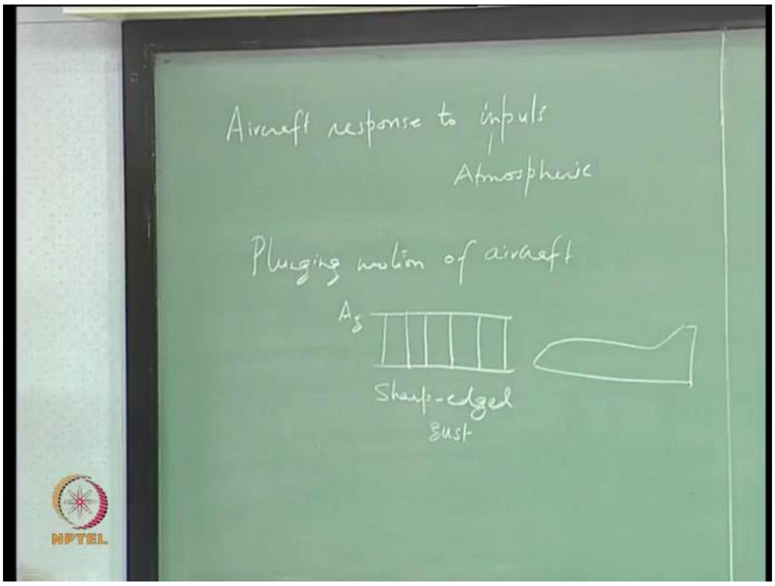

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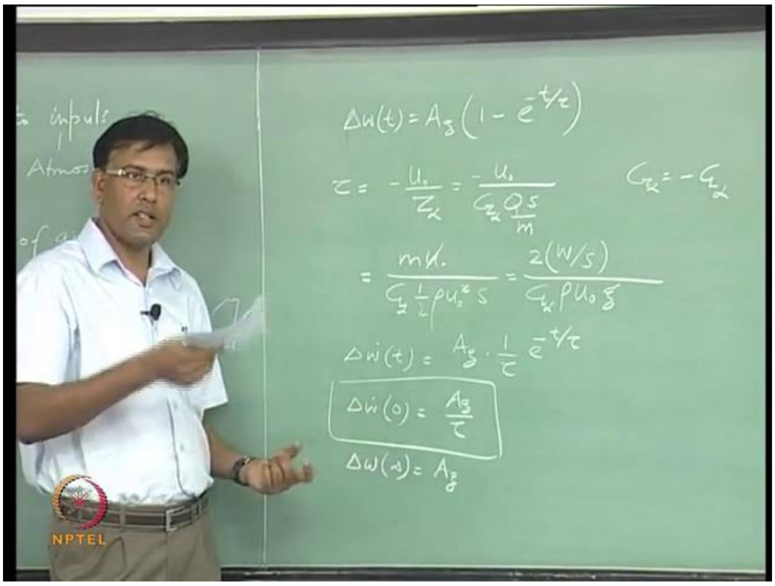

So, now let us try to write this down in terms of aircraft other parameters, $C_{Z\alpha}QS/m$. And this $C_{Z\alpha}$ is nothing but minus of $C_{L\alpha}$. The way we define the axis, Z is in the downward direction and lift is in the upward direction. So, what we get is m u naught over CL alpha half rho u naught squared into S and this is equal to 2 into W over S which is the wing loading over CL alpha rho u naught into g.

$$\tau = \frac{2(W/S)}{C_{L\alpha}\rho u_0 g}; \quad \Delta\dot{w}(t) = A_g . \frac{1}{\tau} e^{-t/\tau}; \quad \Delta\dot{w}(0) = \frac{A_g}{\tau}$$

The response is going to depend upon what is the initial condition, initial condition as an we are trying to find out what is the acceleration at time $t = 0$. We are trying to plot this $\Delta w(t)$ as function of time. We need the slope at time $t = 0$ and that slope is nothing but the acceleration.

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This is the slope of the response at time $t = 0$, which is also the acceleration. So, let us try to get some numbers for two different aircraft.

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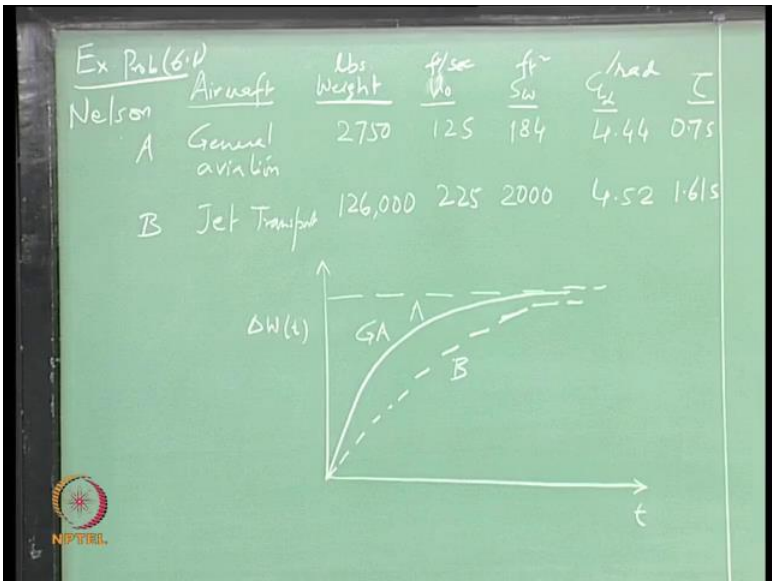

This is a solved example problem in Nelson, Nelson's book. The units are here feet per second, feet squared $C_{L\alpha}$ is per radian, weight W is in pounds and the aircraft are: 2 different aircraft, one is General Aviation - small airplane, and the other one is Jet transport. Weights are 2750 pounds for the General Aviation aircraft and 126000 pounds for the Jet transport. (Operating) Speeds here are different; one case it is 125 the other case it is 225 feet per second. And the wing platform area in the first case is 184 square feet and other case is 2000 square feet. $C_{L\alpha}$ is 4.44 for General Aviation aircraft and 4.52 for Jet transport. What we are interested in finding out? What is the response actually in the 2 cases? So, we need to find out what the time constant is.

Now, everything you know here from the data that is given there. Aircraft A and B. So, you know everything from here. You can find out what the time constant is. In one case it is 0.7 seconds the other one is 1.61 seconds. You also need to find out what is the acceleration at time t = 0; that is the maximum acceleration that you have and it is going to take you to the steady state, when $t \to \infty$, Δw, value of Δw is going to be a constant and that is depending upon the amplitude of the gust.

So, clearly whatever amplitude of the gust may be, the acceleration at time t = 0 is inversely proportional to this time constant. Time constant for the general aviation airplane is lower. So, this slope (Refer Slide Time: 10:15) or the acceleration is larger, is not it? This is for the general aviation airplane A, and for the airplane B this is the response. Now, we will also discuss about other profiles. We can have different wind profiles of this vertical gust and they could be periodic, or arbitrary. ....

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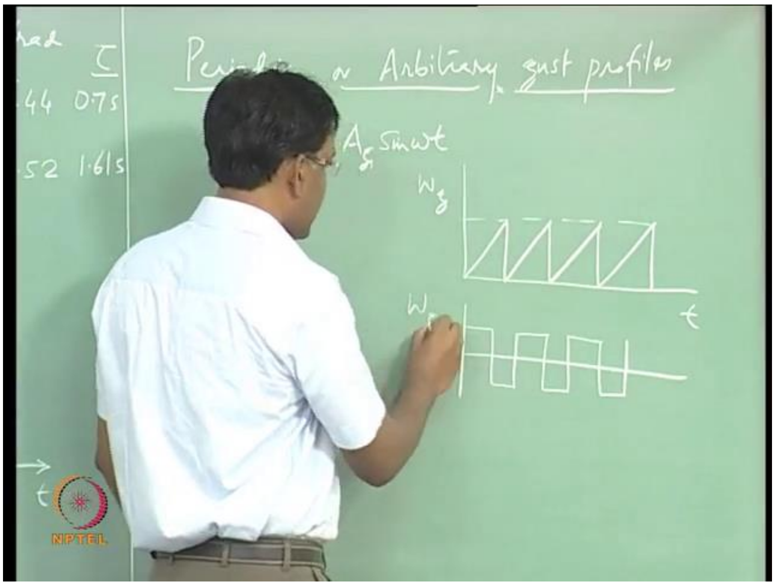

So, we can take w_g to have the form which is this, $A_g \sin \omega t$. Any other periodic function also, you know how to write them as sum of various frequencies using Fourier series. If we have a signal which is looking like this (sawtooth wave or square wave), that also you can write as sum of harmonic, series, or may be .. But of course, you are not going to see such profiles? Wind can have any arbitrary profile and can look completely different from what you are seeing there. So, it could be something like this, (Refer Slide Time: 13:18); but still deterministic?

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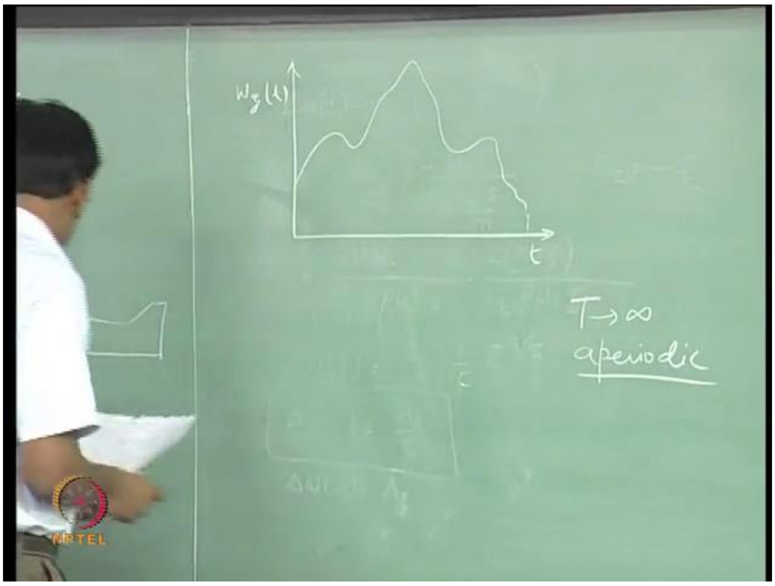

This is a deterministic profile; we know what is the amplitude at any particular time. So, these are non-periodic profiles, what about aperiodic profile? You could have periodic functions when time period is large; large, it is going to infinity. The profile may be repeating itself after a very long time and that can fall into this category, aperiodic profile.

So, we will not talk about these; we know how to find solution for this case. If I know $w(t)$ as a function of time, some function, then, I am still able to find the acceleration, here you have to do something else and that is not what we are going to talk about.

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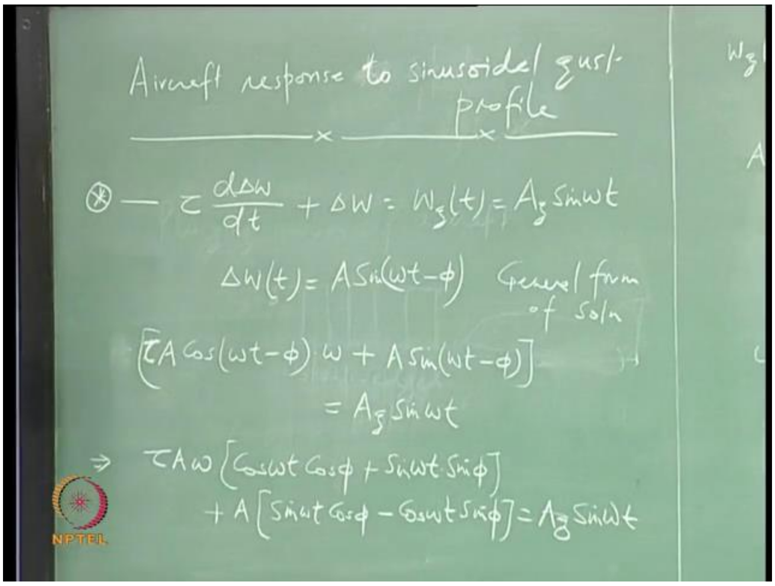

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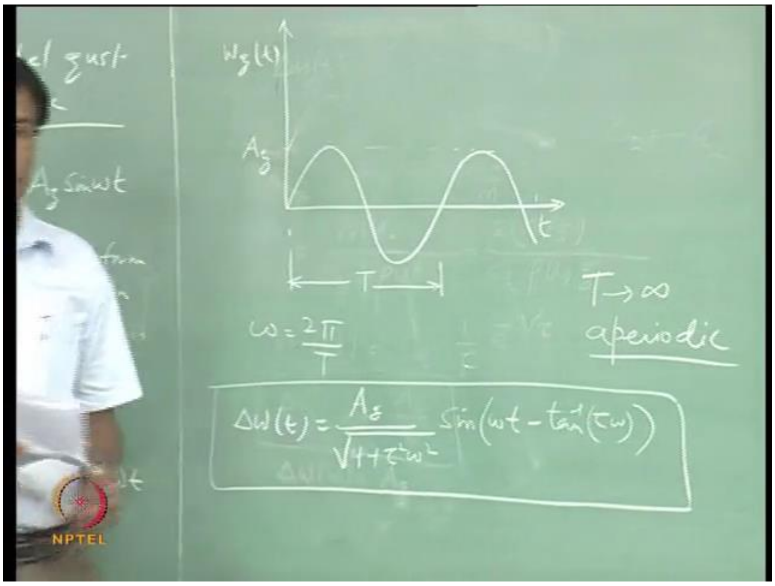

We will look at simpler cases. We will look at first aircraft response to sinusoidal gust; we are still looking at plunging motion. So, equation of motion is not going to change. This $w_g(t)$ is now $A_g \sin \omega t$ (Refer Slide Time: 15:50). So, let us look at how this profile looks like .... . This frequency is 2π over the time period. You can use Laplace technique to find solution for this also, but here it is much simpler; you can also write down the solution directly and that is something like this Delta w(t) equal to Ag over square root of one plus tau square omega square into sine omega t minus tan inverse tau into omega. $\Delta w(t) = \dfrac{A_g}{\sqrt{1+(\tau\omega)^2}} \sin(\omega t - \tan^{-1}(\tau\omega))$

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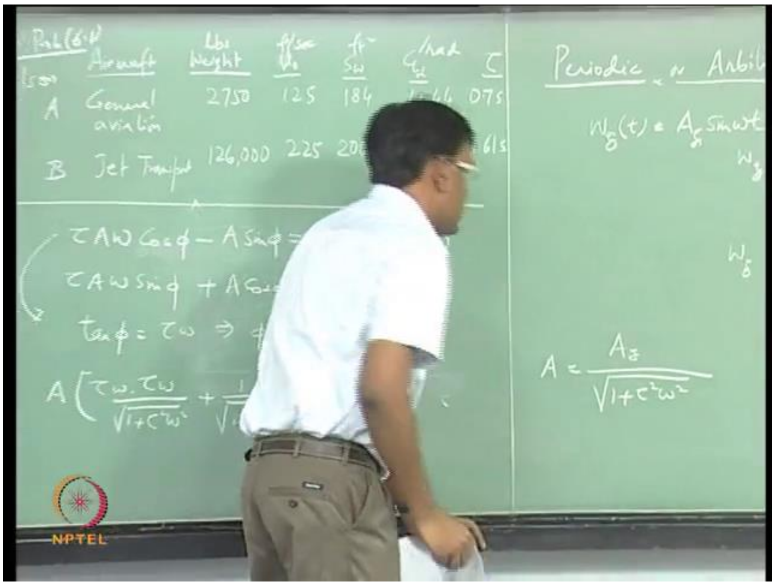

Let us try to plug this general form of solution: $\Delta w(t) = A\sin\omega t + B\cos\omega t$ in this equation. $\tau\frac{d\Delta w}{dt} + \Delta w = w_g = A_g \sin\omega t$ .What you get is (Refer Slide Time:19:00) ….

Now, collect terms common to $\cos\omega t$ and $\sin\omega t$ from both the sides and what you have is: tau A omega into cos phi minus A sin phi, that comes from these 2 terms, and there is no term appearing on the right hand side which is having the factor cos omega t, and corresponding to the sin omega t term …. This gives me the time lag in response which it is coming here (Refer Slide Time: 21:35). Now, A is the amplitude of the response; let us write down A in terms of the parameters that are known right; time constant and omega the frequency of the gust profile - these are the known parameters and I want to look at the response in terms of those parameters.

$$\Delta w(t) = \frac{A_g}{\sqrt{1+(\tau\omega)^2}}\sin(\omega t - \tan^{-1}(\tau\omega))$$

So, what A is? A is Ag over …. , so that your response is …. this (Refer Slide Time: 23:36). So, there is a time lag involved here which depends upon the time constant and the frequency of the gust profile, and even the amplitude depends upon these 2 parameters.

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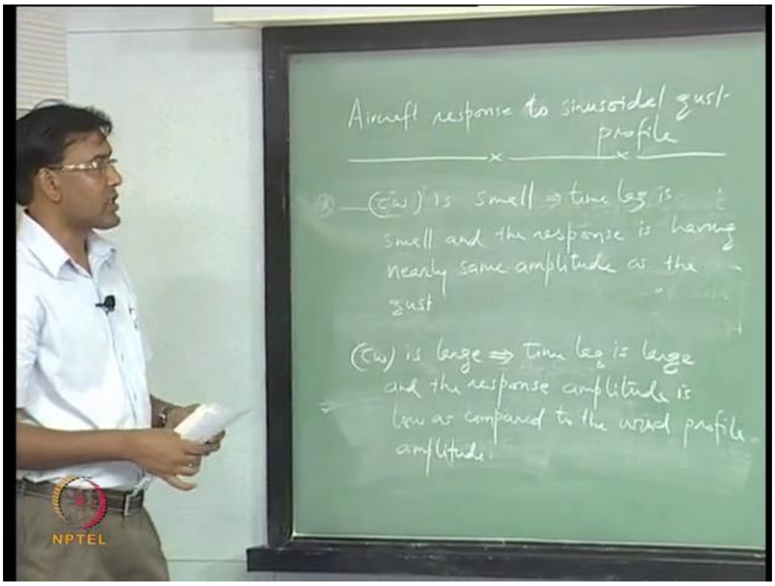

So, when this parameter ($\tau\omega$) is small …. when this parameter is small, what is happening? This is very small, this is also small (Refer Slide Time: 24:18). So, lag is small and the amplitude is almost same as the amplitude of the gust. Its very close .... This is the time lag .... Closely following the amplitude of the gust profile with a small time lag. ...

Look at what happens when this ($\tau\omega$) is large (Refer Slide Time: 26:00). When this ($\tau\omega$) is large, the amplitude is going to depend upon the magnitude of this ($\tau\omega$) and it is going to be small. So, almost flattened and having large lag in time. Let us say, this omega is fixed and we have control over this time constant. Time constant is what we have control over; time constant is related directly to the wing loading, is not it? If we go by that argument then this quantity (τ) is going to be large for airplanes having large wing loading, and the amplitude of response is going to be flattened, and less amplitude as compared to the wind profile. ...

This is a very simplistic view, this is not going to be an actual case. In actual case you can see all kind of winds; you can also have what is called wind shear. Wind shear - so, I am plotting here one profile, shear is you know is having a gradient with respect to the distance.

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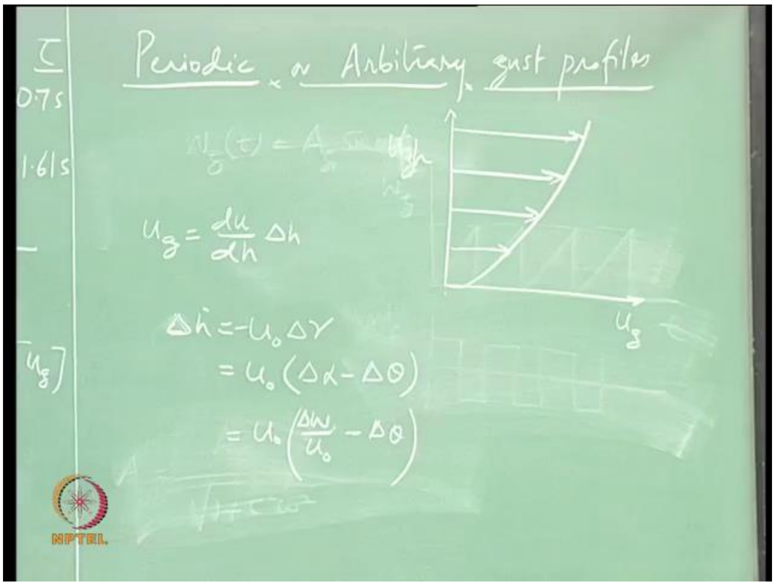

So, if you want to look at one such profile; let us say you are looking at u_g and it is with respect to let us say height. So, this is how ... the horizontal wind is changing with height. Now, how will you deal with such cases? So, u_g is somewhat like this function $u_g = \frac{du}{dh}\Delta h$ (Refer Slide Time: 29:55) where h is being measured from the ground. Now, how do you include this part in the aircraft equations of motion if you want to see the effect of this on the aircraft behavior.

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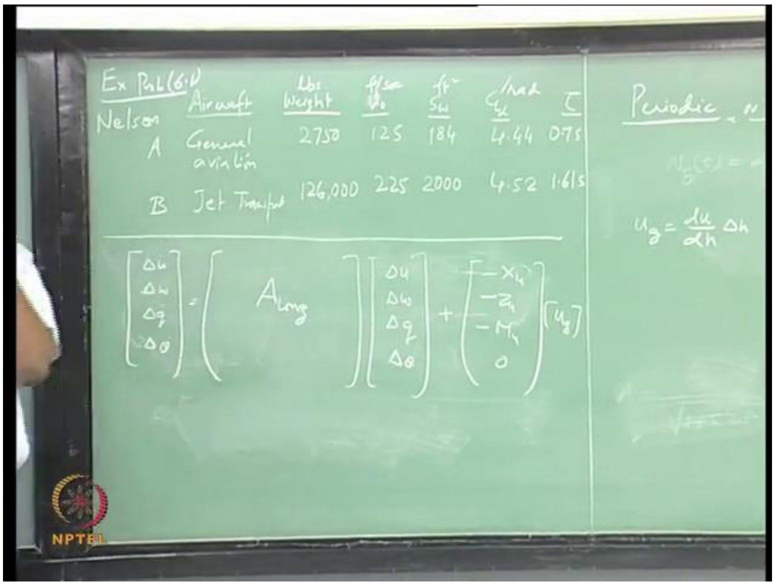

So, let us write down the equation of perturbed motions in longitudinal case. .... So, this is our original (Refer Slide Time: 30:47) *A* matrix in the longitudinal motion ... and we have seen how we can include the gust. So, I am dropping here the control inputs; looking directly at the gust input. Somehow, there this Δh has to appear, otherwise, you are not going to be able to include that part in this *A* matrix right? Anything which goes into the *A* matrix is going to change the dynamic behavior. So, let us try to …

So, we are looking at problem when aircraft is on an approach to land and then this gust is encountered, this wind shear profile is encountered. So, it is coming to land, the altitude is going to go down. So, we can write the linearized change in altitude as $\Delta\dot{h} = u_0 \Delta\gamma$ from $\dot{h} = V \sin\gamma$ .... this (Refer Slide Time: 32:40). .... So you include this equation $\Delta\dot{h} = u_0 \Delta\gamma$ also in this model, because now it is this (linearized) set of equation (with *A* matrix) coupled with this equation. Why is that? Because, we have these 2 terms ($\Delta\dot{h} = u_0 \Delta\gamma = u_0(\Delta\theta - \Delta\alpha)$) appearing here (Refer Slide Time: 33:15) now, $\Delta\alpha$ and $\Delta\theta$. So you can either write this $\Delta\dot{w}$ in terms of $\Delta\dot{\alpha}$ ... or you can do this.

So, let us try to include this gust; the wind shear through this equation, try to take that in into the *A* matrix and then we can look at the eigenvalues.

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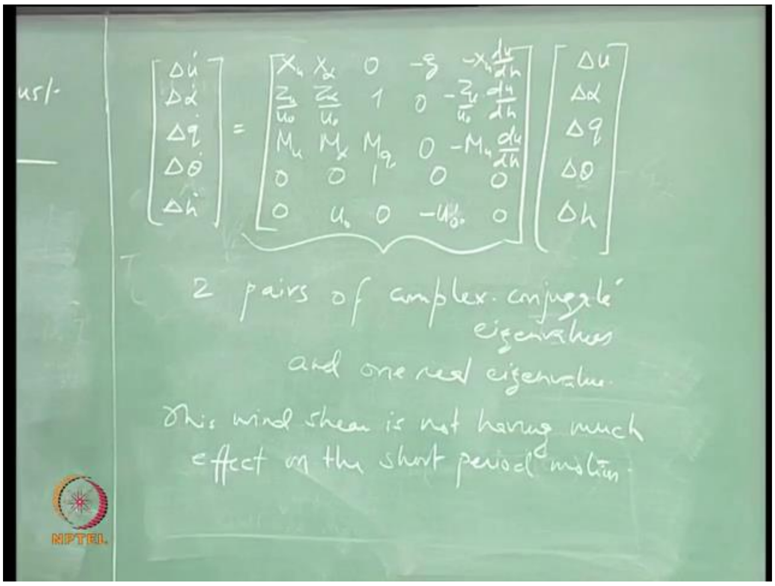

So, u_g is, (du/dh) this Δh. That is how we have defined this wind shear. .... What is it going to change? Now, you have one more equation. So, we would be expecting one more eigenvalue; earlier we had 2 pairs of eigenvalues for this A longitudinal and both pair, a pair of complex conjugate eigenvalues corresponding to the short period and phugoid mode, is not it?

Now, you are going to see another eigenvalue which is non-oscillatory, it is going to be a real eigenvalue. Now eigenvalues of this matrix are 2 pairs of complex conjugate eigenvalues and one real; real means non-oscillatory. So, we have sort of understood that change in u is not going to affect the short period motion and that can be seen here also. When you solve this set of equations it is not going to have much of effect on the short period motion, the phugoid is different.

This wind shear is not having or having negligible effect on the short period mode, ... but it will have an effect on the phugoid motion and if you want to plot how phugoid mode eigenvalues are moving in the complex plane when you change this gradient (du/dh), then what you get is, ...

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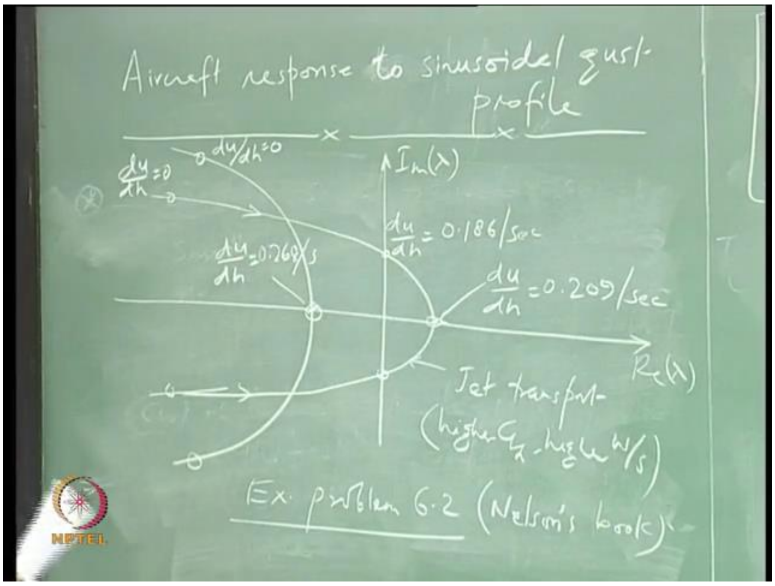

Here I am plotting the phugoid mode eigenvalues. We are starting at this point when (du/dh) is 0. When you start changing this gradient, this pair of eigenvalues will start moving and it becomes real at this point, and you want to know what is the magnitude of the gradient at this point, when it is lying on the imaginary axis, this (the gradient) is 0.186 per second. And this is for airplane B. This is likely that the phugoid mode of this aircraft will become unstable when you have this gradient having large value. You do not want this to happen when you are close to the ground, on an approach to land, you do not want this to happen.

So, this is for the aircraft which has higher $C_{L\alpha}$ and higher wing loading. For aircraft A which has lower $C_{L\alpha}$ and also lower wing loading the profile looks something like this... This is example problem 6.2 in Nelson's book. We will stop at this point as far as this course is concerned.