Flight Dynamics II (Stability)
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Module No. # 11
Lateral-Directional Dynamic Modes

Lecture No. # 35

Spiral, Roll, Dutch Roll Mode Approximations

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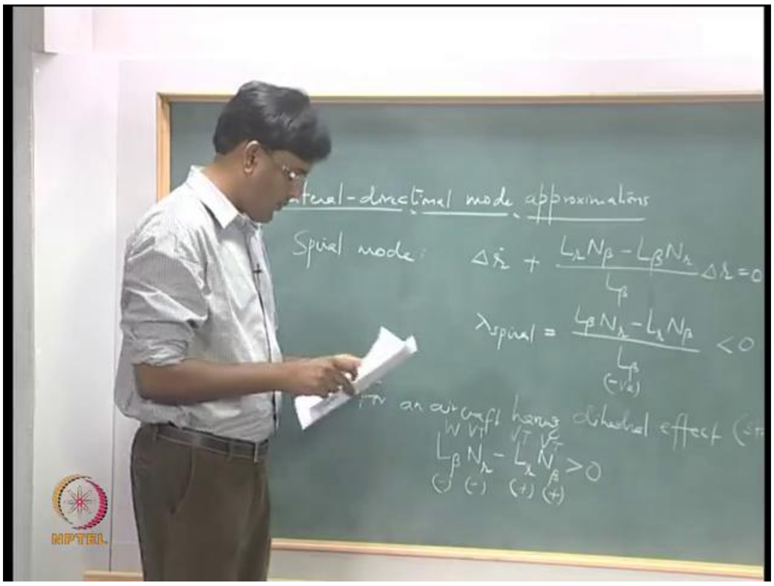

So, we are looking at lateral directional mode approximations, and in the last class, we looked at the spiral mode approximation, and we will be looking at the physics of the motion of aircraft in spiral mode. We said that, it is a slow change in bank angle and, so aircraft is having predominantly a motion in beta and r, where the roll rate is only giving the bank angle and bank angle is steadily changing.

$$\Delta\dot{r} + \frac{L_r N_\beta - L_\beta N_r}{L_\beta}\Delta r = 0 \Rightarrow \lambda_{spiral} = -\frac{L_r N_\beta - L_\beta N_r}{L_\beta} < 0 \qquad (1)$$

So, we came down to this ....Refer Eq(1) this equation in delta r, delta r is the perturbation in the yaw rate. The Eigenvalue for this linearized equation is the one corresponding to this spiral mode and, this is an .... approximate formula ..., this we arrived at yesterday. ... Further spiral mode of motion to be stable, this Eigenvalue must be lying in the left half (complex) plane. What it means is, this (λ_{spiral}) should be less than 0 .... For a stable aircraft in roll this L beta is negative, we have talked about this. .... (No audio)

3:42

For an aircraft having dihedral effect ... which is related to the stability in roll, C l beta is less than 0, ... and this L beta is ... function of C l beta ...

$$-\frac{L_r \overbrace{N_\beta}^{+Ve} - \overbrace{L_\beta}^{-Ve} N_r}{\underbrace{L_\beta}_{-Ve}} < 0 \qquad (2)$$

So, this is negative ... . So, this quantity should be positive for this whole thing to become negative. So, for stability in this spiral mode of motion of aircraft, L beta into Nr minus Lr into N beta must be positive .... Now, it depends upon the relative magnitude of this term and this term .... So, this aircraft that we are talking about is having stability in roll, so this term is negative, Nr is negative. What is Nr? Nr is damping in yaw .... When your aircraft is yawing, there is a contribution to the yawing moment coming from the vertical tail, which is trying to damp that yaw motion ... and it is usually negative... You understand what I am saying.

So, there is a force developed because of the yaw motion which is trying to oppose the motion ... You are yawing and a force develops because of the yaw, because the vertical tail is lying far away from the C g, that gives rise to a side force which tries to damp out the yawing motion... which is negative. So, this term is negative. Now, Lr is also coming because of the vertical tail. You identify these terms? You will find that major contribution to this L beta is coming from the wing, Nr is coming from the vertical tail, this Lr is also mainly coming from the vertical tail and N beta is also coming from the vertical tail ...

Vertical tail is providing the directional stability. So, N beta is coming from the vertical tail. So, there is only one term which is .. coming from, this contribution is coming from the wing, major contributions... This Lr is usually positive, N beta is positive for an aircraft which is having directional stability. Now, it depends upon the relative magnitude of these numbers, for your airplane you can figure out if this is true or not. If this is true then the spiral mode motion is stable . .....

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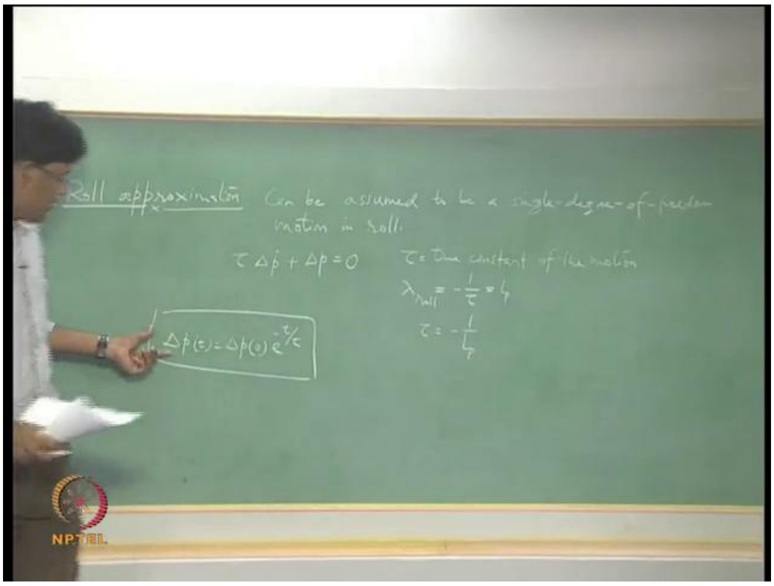

The roll mode can be approximated by the single degree of freedom motion in roll, what we have discussed already. So, the equation .... (Refer the slide above) can be, this roll mode can be assumed to be a single degree of freedom motion in roll, ... and this we have already discussed. So, the equation for this ....(Refer the slide above) corresponding to this particular mode is this, where tau is the time constant. (No audio) .....

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So, this is what we already derived for the constraint roll motion case which is quite a good approximation actually in this particular mode. ..... And this damps out quite fast, it is exponential convergence, exponential decay of this delta p. Let us look at the last one, lateral direction motion of aircraft, about the equilibrium point that we are talking about. .....

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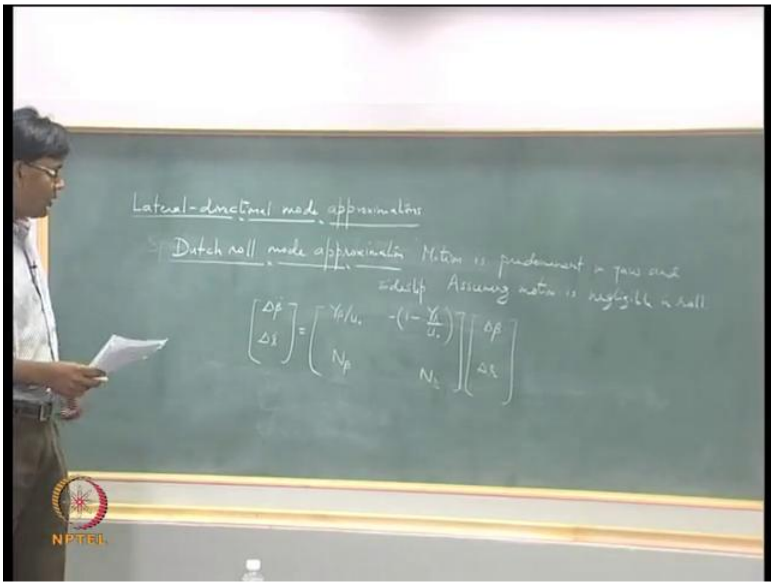

The assumption here is that the motion is predominant in beta and r, sideslip and yawing motion. ..... And roll is negligible which is not really a very good assumption, that is why you will probably end up with a formula which will also have error. So, if somebody wants to have a better estimate then you should also incorporate the rolling motion equation in this approximation. So, remember this is only an approximation, we assume that the motion is predominant in yaw (rate) and sideslip. So, we drop the rolling motion equation and what we get ..... So, this is after dropping the rolling motion of equation.

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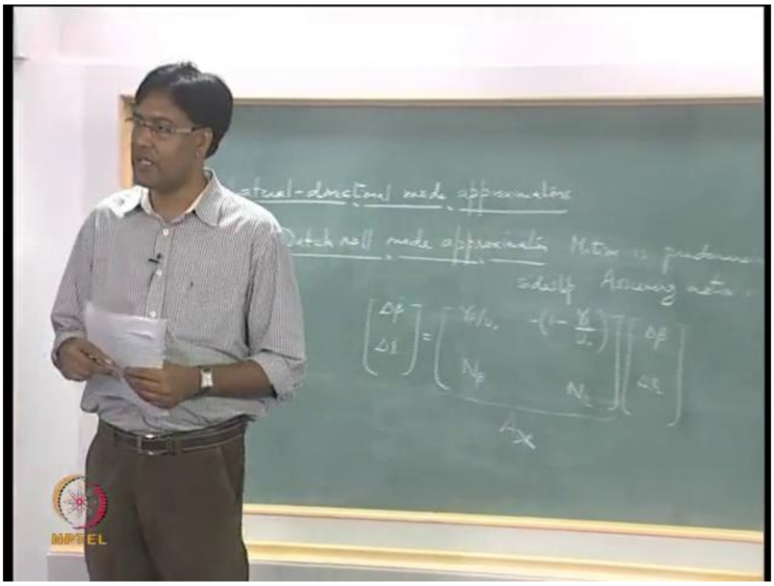

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Now, solve for the Eigenvalue of this matrix and you will get the frequency and damping of the Dutch roll mode motion. ... Let us call this ..... A Dutch roll and solve this ..... . (No audio).

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This gives me a quadratic equation in lambda, lambda squared minus lambda multiplied by Y beta over u naught plus Nr plus Y beta over u 0 into N r plus N beta .... So, you can find out from here the natural frequency of motion in this particular mode ..... and the damping ..... . So, the reason why we are deriving these expressions is that, once we have these expressions in hand when we are designing the aircraft, then I know which parameter I can play with at the design stage itself. But if the formula becomes complicated ... then we will not know which parameter ... can directly affect by changing some components of the aircraft. For example, here this frequency is depending upon Y beta, Nr, N beta, Yr N beta, right. So, most of these parameters, you see here are related to the vertical tail .... Y beta is the side force developed at the vertical tail, is it not?

This N beta is also because of the vertical tail, but it depends, know which way you place the vertical tail on the aircraft. If it is upside down, ... right now we are talking about conventional configuration when it is lying vertically up, if it is vertically down then this may give different, do you think this, this may give a different N beta if it is vertically ... upside down below the fuselage? (()) So, this will not be affected, but the roll may get affected ... because then you will have the aerodynamic center of the vertical tail lying below the CG. So, if we try to make any sense out of it, that is, which parameter you can actually play with at the design stage, so that you get a better frequency in Dutch roll and a better damping. If it is not well damped in Dutch roll, the aircraft, then it might lead you to motion which is called wing rock. So, damping, if it becomes 0, then you have a self induced oscillation which is called wing rock, it is a limit cycle oscillation.

So, let us try to now look at one example, we looked at this general aviation airplane in a flying condition and we found the Eigenvalues, if you remember in the longitudinal case. Here also we will do the same thing. We take the same airplane, same flying condition, and try to find out the values of these parameters using numerical methods and also using approximate formula and see how well they match. If they do not match then, you know that the approximate formula is not correct and you have to look for a better formula.

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This example problem, I have taken from Nelson's book chapter 5. The flying condition is this ...., so you are flying cruise at this trim speed, angle of attack is 0 and all other variables are 0 ... (No audio) ..... For general aviation airplane, Navion, for which the data is given. You look at the backside of the book, you will find data for lot of aircraft, at least six aircraft data in the low alpha range are given. So, the data that is given is not going to change much because we are talking about speeds which are low, ... only when speed is higher than or the Mach number is higher than 0.5 that we start worrying about the compressibility effects. Otherwise the data that is given is valid up to 0.5 Mach number, ... and it is in the pre-stall region. So, you can assume the data to be fairly constant, you know for the speed below Mach 0.5 and alpha below 10 degrees. (No audio) ......

24:34

Now, solve for the Eigenvalues of this matrix, this will result in characteristic equation which is quartic in lambda, ...(no audio). Remember this is at this trim, you know with trim this matrix will change .... We have evaluated this matrix at this particular trim condition, it is going to change with trim condition.

So, right now what we are trying to compare is the Eigenvalues of the exact solution with this alpha naught equal to 0. ... As you go on increasing alpha naught this approximation is going to get worse. ... You have to keep that in mind. So, this is right now at this alpha naught equal to 0 conditions.

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So, I will write here exact values for different modes and approximate values. The exact one is what you get by solving this, this gives you exact Eigenvalues. (No audio) ..... So, lambda spiral is, we have understood that this spiral mode Eigenvalue is lying close to the imaginary axis. ... Look at this number, it is very close to the imaginary axis, almost at the origin and look at this number 0.144. So, there is a huge difference in the two numbers. So, formula for this spiral mode Eigenvalue, approximate formula is not correct. That is what we can conclude from here, the roll mode approximation is fairly good. (No audio) ....

29:22

So, Dutch roll mode Eigenvalue is, the exact Eigenvalue is this: minus 0.487 plus minus the imaginary part 2.335, and approximate value is minus 0.51 plus minus .... 2.109 imaginary part. So, here also there is a difference. So, there is some error involved in making an assumption for this Dutch roll mode motion. And, indeed, you know we said that there will not be any roll motion in this particular motion which is not really true. But if you include that then you have to solve a cubic equation in lambda. The problem would be, you will be able to solve a cubic equation, ... even by hand, but the problem would be you will see lot of other parameters coming into your expressions for frequency and the damping, and then that will not make much sense to designer. We want simpler relations, so that we can directly see the physics behind the effect of the components on these Eigenvalues. This formula actually gets worse with increasing angle of attack. Is this clear? Remember when we started talking about stability, we were first talking about static stability and there we were talking about terms Cl beta and Cn beta, ... you remember we talked about different cases when we can get more negative Cl beta ... and more positive Cn beta. So, that was based on the design of the wing or orientation of the wing with respect to flow, and location of the wing, whether it is dihedral anhedral, all that played a role in the roll stiffness of the aircraft and Cn beta was depending upon the vertical tail sizing.

So, now you would want to go back and see how these Eigenvalues .. move in complex plane when you change Cl beta or Cn beta.

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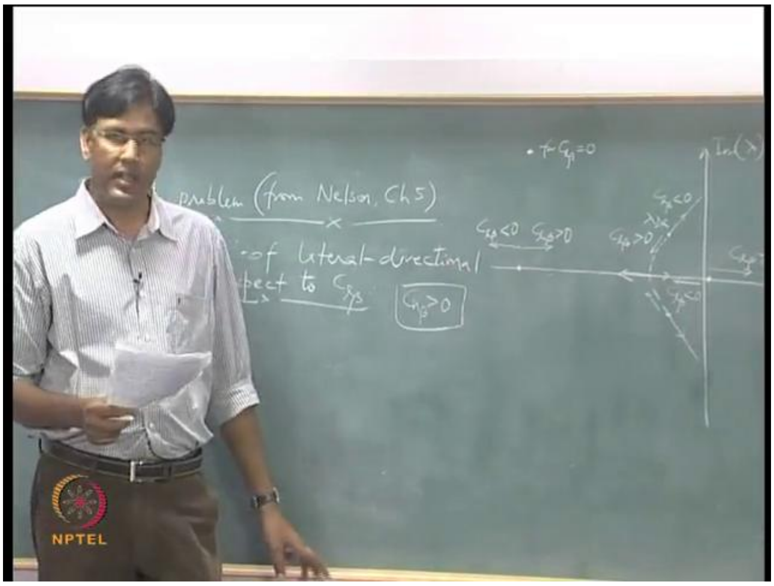

So, root locus is one technique, ... which is for tracking the roots or the Eigenvalues with respect to a parameter. ......

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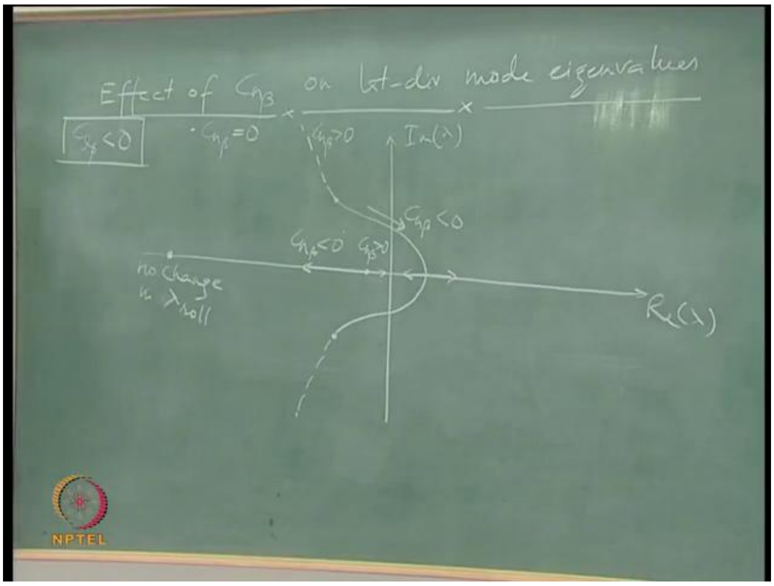

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So first look at, we assume that Cn beta is large and now we start varying Cl beta and see how the lateral directional mode Eigenvalues are moving in the complex plane. .....

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So, this is for an aircraft which is having directional stiffness, directional stability. Cn beta is positive and let us look at the effect of ..... So, these are my 4 Eigenvalues and this is for .... Cl beta equal to 0 case. Now, you start changing Cl beta towards negative side.

What do you see here, you see that the Dutch roll mode Eigenvalues are moving towards the imaginary axis and what does it mean? It has lesser and lesser stability. The aircraft is having lesser and lesser stability in Dutch roll ... So, these mode Eigenvalues are Dutch roll Eigenvalues. In the other direction it moves like this, when you are making Cl beta positive. So, the aircraft which is more stable, statically stable in roll, that is what Cl beta less than 0 means, is making the aircraft dynamically unstable,... is making the Dutch roll mode unstable.... Not really unstable, but lesser stability in Dutch roll.

Lets look at what happens to this, we really do not worry much about this spiral mode Eigenvalue or spiral motion which is long, you know the motion is taking so long time that you do not worry about that. You can control it by using some control effort in flight.

This ... moves in this direction when Cl beta is greater than 0 and in the other direction, when ..... Remember we cannot just change one parameter. Cl beta if you are changing then you have to change lot of other parameters in the aircraft equation, because this Cl beta is coming from some component,.... and that component is also having an effect on other derivatives. So, if you are changing this alone and want to plot the root locus that is not going to be sufficient. You also have to see how other derivatives are changing, ... that and the effect of those derivatives on the Eigenvalues is also important

So, this is one single case when only Cl beta is changing, Cn beta is fixed at some positive value. Once the two Eigenvalues join together at this real axis they split like this. There is an effect on the roll mode Eigenvalue also. So, going to be moving towards left, becoming more stable, when Cl beta is less than 0. When Cl beta is greater than 0, then it is going to move towards the imaginary axis.

So, .. this gives some idea ... to the aircraft designer, how you are going to place your wing, what should be the size, size of the wing will be decided by the performance parameters, not really by the stability. If you need extra stability then you have to augment the stability using stability augmentation system, ... that is the control system which provides artificial stability to the aircraft. ......

Lateral directional mode. Lets look at the effect of Cn beta now. .... Clearly this Cn beta is coming from the vertical tail, here major contribution is coming from the tail, Cl beta, the major contribution is coming from the wing. So, these two are two different components which you can design separately. So, it makes sense to talk about them separately and see the effect of them on the, but still the motion is coupled, roll and yaw motions are always coupled. So, whatever is happening here is going to have an effect also on the, ... the effect of the roll motion is also going to be seen in the yaw motion...

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So, this is some Cl beta negative for which the spiral mode Eigenvalue is here. So, remember in this case the Cl beta was 0 at this point. So, it is lying on the right hand plane. Here Cl beta is ....., so there is some stability in roll. So, the spiral mode Eigenvalue is lying to the left of the imaginary axis .... When you start, so, this is for the case when Cn beta is 0 and you start changing now Cn beta.

You increase the stiffness in yaw and you get higher frequencies, because you remember the ..... second constrained motion that we talked about was yaw motion where we found that the frequency depends upon the N beta term which is proportional to the Cn beta... If the velocity is constant then that N beta depends on Cn beta. So, this is how the Dutch roll mode Eigenvalues move, when you start decreasing Cn beta, making it negative, it starts moving like this. Remember this is for a typical, the general aviation airplane that we are talking about, may not be true for all aircraft. For all aircraft, the only truth is the equations of motion. Trim conditions, and such a root locus plot are going to be different for different types of aircraft. ....

There is not really any change in the roll mode Eigenvalue, ... you know with respect to Cn beta. The spiral mode Eigenvalue ... moves to the left when Cn beta starts decreasing, becoming negative, and this direction, when Cn beta is increasing in the positive direction.... Such plots actually help you to size the appropriate components on the aircraft. Is this clear? So, we can just summarize what we did today. We found approximate formula for spiral mode Eigenvalue, Dutch roll mode Eigenvalues and roll mode Eigenvalue, and we also compared the exact values with the approximate values and we found that there was error in the Dutch roll mode Eigenvalue and the spiral mode Eigenvalue. It is mainly because we are not considering the roll in the approximation.

So, if you include also the roll dynamics, you will get an improved formula. But, that formula will depend upon ... lot many more parameters .. and then it will be difficult to understand which one is playing what role. So, we will stop at this.