**Flight Dynamics – II**

**(Stability)**

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**Module No. # 10**

**Longitudinal Dynamic Modes**

**Lecture No. # 32**

**Short Period Mode Approximation**

In the last class, we looked at phugoid approximations using aircraft linearized equations of motions around an equilibrium state. Now, we will continue with our approximations, so-called literal approximations, and last class also we did phugoid approximation; we assumed there that, $\Delta\alpha \to 0$ ... when the aircraft is showing the phugoid mode, there is hardly any change in angle of attack.

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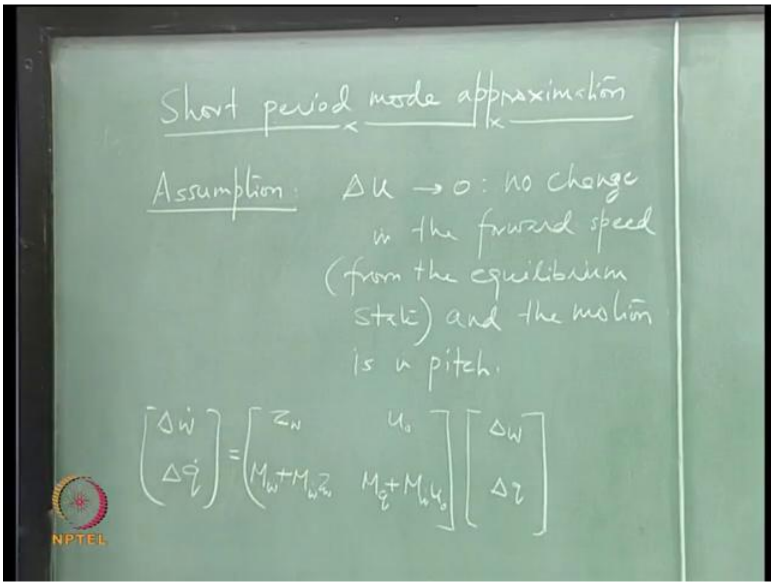


So, here in the short period mode approximation, the assumption is that, $\Delta u \to 0$..... remember, we are trying to come down to simpler model, so that we can work something out you know, analytically. Otherwise, you know, if you can solve the quartic equation, you can get exact expressions for phugoid mode Eigenvalues and the short period mode Eigenvalues, but it is not possible to solve analytically. And why we are doing it, because we want to relate the perturbed motions of the aircraft with some design parameters .... Otherwise, you can solve it numerically on computer; and probably, you will have to work with a lot of parameters in the A matrix, you know, change them see, how the roots are moving right? We have to carry out that exercise.

But the question is how would you do that? You are ... still trying to talk about the phase, when you are designing an aircraft ... and trying to see how the geometric or the aerodynamic parameters of the aircraft are affecting the motion right. This we are thinking at the design stage; that is where these simpler models help. Now, assumption involved here is that when aircraft is showing short period behavior, there is hardly any change in the forward speed. So, we can set this to 0 $\Delta u \to 0$.

(No audio from 02:55 to 03:01)

When we do that, so there is no change in the forward speed, you know, from the equilibrium state; this delta u is from the equilibrium state, is it not?

(No audio from 03:31 to 04:00)

$$\begin{bmatrix} \Delta\dot{w} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & u_0 \\ M_w + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} u_0 \end{bmatrix} \begin{bmatrix} \Delta w \\ \Delta q \end{bmatrix} \quad (1)$$

Right, and motion is ..... (No audio from 04:11 to 04:21) Can we say it is pitching? .... Now, with this assumption, I can reduce my 4 by 4 longitudinal linearized aircraft dynamic equations of motion to a 2 by 2, you know, the equation in 2 variables; and those are ....Refer Eq(1) - the variables involved here are delta w ... and delta q. (No audio from 05:04 to 05:15) And when we are looking at stability, we are actually looking at the free response; we are not applying any control, so model - the simplified model for short period mode approximation is this ….Refer Eq(1) (No audio from 05:31 to 05:37) Z w u naught, M w plus …. (No audio from 05:42 to 06:00) ... We can also write these equations in terms of delta alpha.

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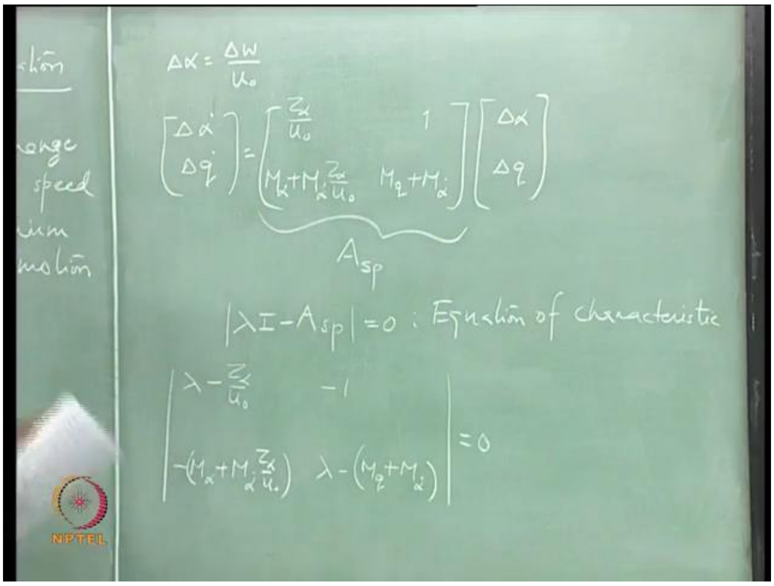


$$\Delta\alpha = \frac{\Delta w}{u_0}$$

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{Z_\alpha}{u_0} & 1 \\ M_\alpha + M_{\dot{\alpha}} \frac{Z_\alpha}{u_0} & M_q + M_{\dot{\alpha}} \end{bmatrix}}_{A_{SP}} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} \qquad (2)$$

I know what, how to write delta alpha in terms of delta w; and so, you can rewrite the equations .....Refer Eq(2)

(No audio from 06:29 to 07:04)

So, this A matrix is for the short period case. With this assumption in the background, now I have to find out the Eigenvalues of this matrix and that Eigenvalue will be corresponding to this particular mode.

$$|\lambda I - A_{SP}| = 0 \Rightarrow \begin{vmatrix} \lambda - \frac{Z_\alpha}{u_0} & -1 \\ -M_\alpha - M_{\dot{\alpha}} \frac{Z_\alpha}{u_0} & \lambda - (M_q + M_{\dot{\alpha}}) \end{vmatrix} = 0 \qquad (3)$$

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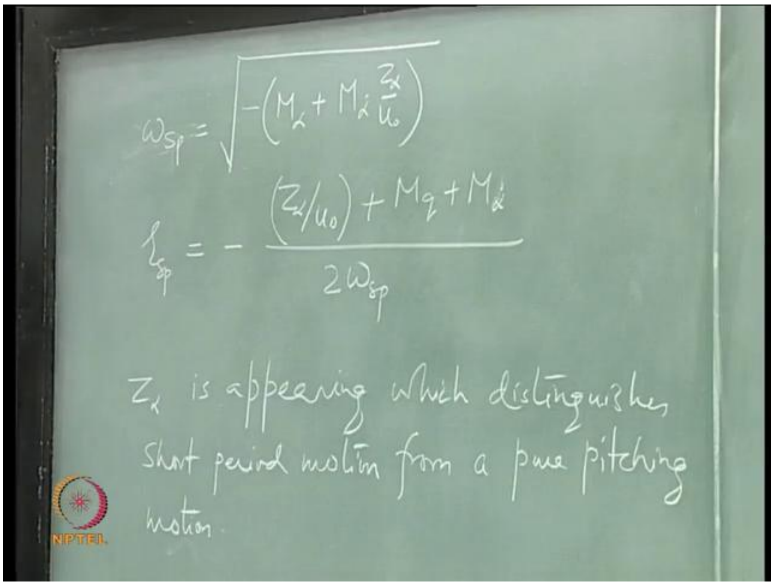


$$\omega_{SP} = \sqrt{-\left(M_{\alpha} + M_{\dot{\alpha}} \frac{Z_{\alpha}}{u_0}\right)}; \zeta_{SP} = -\frac{\left(\frac{Z_{\alpha}}{u_0}\right) + M_q + M_{\dot{\alpha}}}{2\omega_{SP}} \tag{4}$$

This will result into two values of lamdas, complex conjugate lamdas ... and it will come from this equation Refer Eq(3). So, what I do is, I directly write down the expression for the frequency .....Refer Eq(4) (No audio from 09:05 to 09:27) and the damping ratio.

(No audio from 09:30 to 09:59)

Remember, we first of all started looking at a pure pitching motion ... and we got the second order equation from which we derived the expressions for frequency and the damping, of aircraft in pure pitching motion. And here, we are making the approximation saying that, the ... change in forward speed is negligible and motion is in pitch... What is happening here? So, is there any difference that you find between the two expressions?

(No audio from 10:41 to 10:49)

The only difference is the appearance of this term. So, earlier when we talked about the pure pitching motion, we did not have this Z alpha term... Of course, because the aircraft was constrained only to have pure pitching motion and not any other motion. So that was an assumption that we used there. So, here the … (No audio from 11:26 to 11:34) Which

… So, this Z alpha is appearing which distinguishes short period motion from a pure pitching … (No audio from 12:05 to 12:14)... An effect of that is, that .. even though you have damped out the motion - the short period motion due to .. these terms M q and M alpha dot; M q and M alpha dot are damping terms. Is it not? Because rates are involved here; see the pitch rate is involved; here alpha dot is involved.

And those terms, which are having rates involved, they are damping terms. So, even after this short period mode is damped out in pitching motion, because of this particular term, this term is like ... vertical motion with respect to change in angle of attack; that remains as a residual and there are people arguing that this will probably result into an ensuing phugoid motion. So, there are other models, new models for short period and phugoid motions, where the phugoid is seen as an ensuing motion, when the short period ends...

... So, what we have done is, we have, based on some approximations, we have come down to simpler models, from where we have found expressions for damping and frequency ... for phugoid and short period modes. Now what we are going to do is, we are going to take an example, where we have actual model with some numbers ..., and we are going to solve that on computer, find the Eigenvalues; and see how close they are to the Eigenvalues which we obtained from the approximations .... We will .... carry out this exercise now.

(No audio from 14:40 to 14:49)

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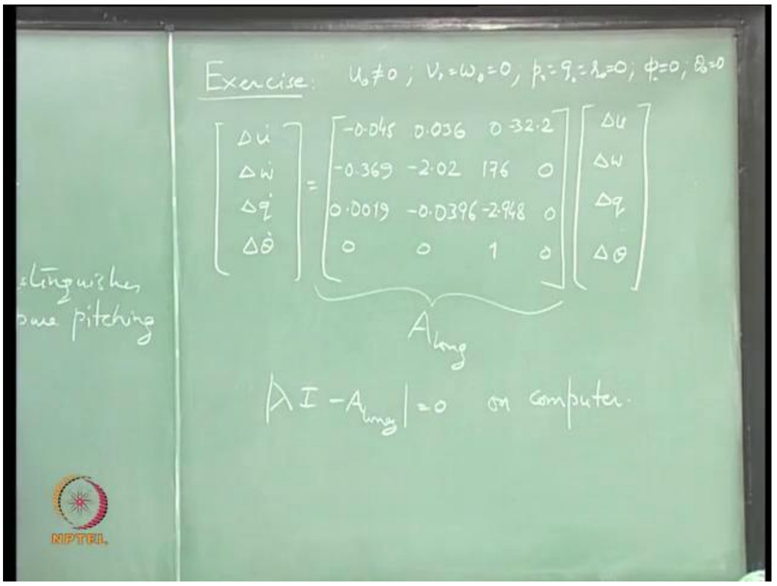


So, we have this ….

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This matrix given, ... so A matrix is given; remember A matrix includes the geometric and aerodynamic properties of the aircraft about that equilibrium state. So, equilibrium state here is the same what we have been using so far.

(No audio from 16:33 to 16:52)

So, ...Refer Slide above. this is my equilibrium state, about which we have been given this linear model to study the perturbed behavior of the aircraft. So, this is our A matrix ... So, we have to find out the Eigenvalues of this matrix, and those Eigenvalues are going to be exact. Now, I am solving an Eigenvalue problem and on computer without doing any simplifications. I am including all the parameters of the aircraft here. So, I will get exact Eigenvalues from this procedure. So, I am solving this Eigenvalue problem on computer;... mathematical tools are available for doing this.

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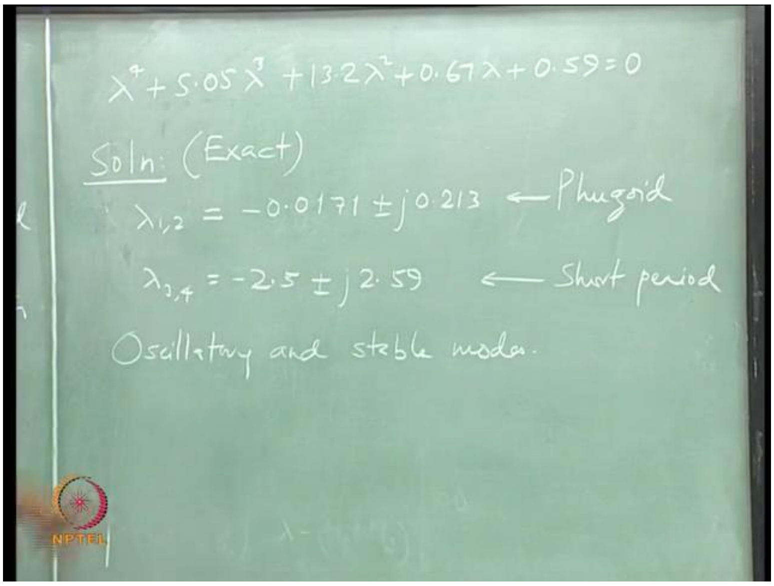


So, the equation, quartic equation that … you arrive at is this (No audio from 18:25 to 18:33) .... Lambda raised to 4 plus 5.05 lambda raised to 3 plus .. 13.2 lambda squared plus 0.67 lambda plus 0.59 equal to 0... Solution of this, you know is a pair of complex, 2 two pairs of complex conjugate Eigenvalues. So, one pair is

(No audio from 19:11 to 19:30)

This lambda 1 and 2 minus 0.0171 plus minus this imaginary part 0.213, ..... minus 2.5... (No audio from 19:48 to 19:56) plus minus 2.59, imaginary part. These are exact values. So, exact solutions; can you identify which one is short period mode and which one is phugoid? (No audio from 20:18 to 20:24) The one lying to, lying close to the imaginary access is the phugoid; we said short period is highly damped. So, it should be lying far to the left in the complex plane.... So, by looking at these Eigenvalues, magnitude of these Eigenvalues, I can say that this set is corresponding to the phugoid mode, and this is corresponding to the short period mode...

(No audio from 21:06 to 21:16)

So, these are exact numbers, by solving that matrix for Eigenvalues we have obtained this. So we find that both these modes are oscillatory, because you have these Eigenvalues which are complex conjugate, oscillatory and stable.... (No audio from 21:51 to 22:03) So, let us try to… What is the other way of identifying that this set is corresponding to phugoid and this corresponding to short period? (()) So, short period is right now, only related to short period.., the literal meaning of short period is short period. So, frequency is high. Is it not? Phugoid is long period motion. So, this Eigenvalue actually, results in a long period, long time period.

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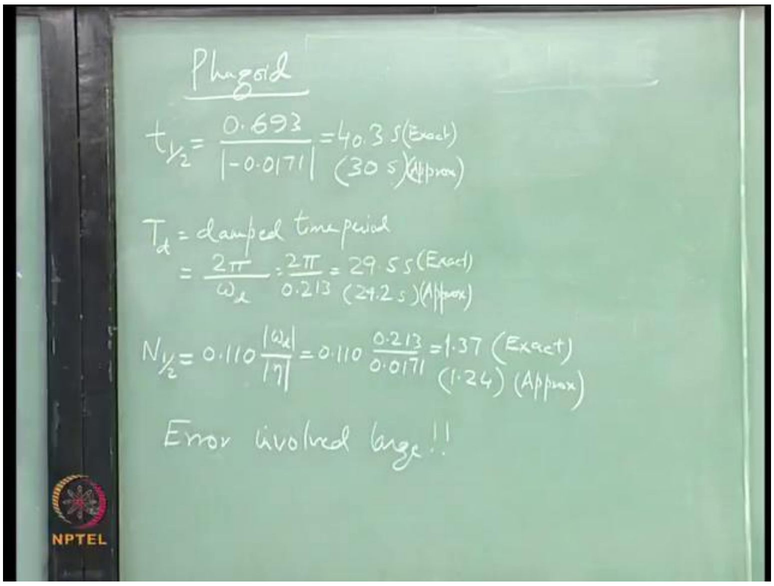


So, let us list down now the characteristic parameters ... which define the aircraft perturbed motion corresponding to phugoid and the short period modes and we will list down here the exact and the approximate values. Approximate values are based on the formula that we derived, using simpler explanation of physics. So, some of the parameters that normally people … this is the language you know, in which you talk about the behavior of the motion.

(No audio from 23:48 to 23:57) Time to half the amplitude, because it is a stable mode, so amplitude is going to decay, is going to decay in time. So, this is time to half the amplitude, that is equal to this (No audio from 24:20 to 24:29)...; 0.693 over the real part of the Eigenvalue; approximate value of this parameter is 30 seconds, which we obtained by using our formula ... that we derived for the phugoid mode.

(No audio from 25:04 to 25:30)

Omega d is the damped frequency right; frequency of the damped motion, and that will be given by the imaginary part ..... (No audio from 25:39 to 25:51) So, this is the exact value; ..... (No audio from 25:54 to 26:06) exact value, approximate value, this is exact. Whatever you see in the bracket is the approximate value, and the one with no brackets is the exact number.

(No audio from 26:24 to 26:41)

Number of cycles to half the amplitude is given by ..... (No audio from 26:48 to 26:56) This eta is the real part and omega d is the imaginary part of the Eigenvalue. So, this is the language, in which you talk .... how many cycles to have the amplitude right, so, you have to speak in that language.

(No audio from 27:20 to 28:02)

So, you see quite a bit of difference between the ... exact value and the approximate value. So, what it tells you? What it tells you is that the approximation is not so good. Is it not? You made some assumptions while arriving at the approximate formula for the phugoid mode, frequency and damping; from where we are getting all these numbers ... The approximation involved was that delta alpha is, change in angle of attack is 0, and that is resulting into giving me an error; ... this is like an error; this is the exact value and this is the approximate value.

So, there is a huge amount of error involved. What is the order of the error? So, roughly it is more than 20 percent, is it not? So, this approximation was not so good,...it was not good. Remember, approximations we are ... making assumptions and we are arriving at approximate formula to find the relation between these parameters ... and the design parameters. You want that to be very accurate. If it is not 100 percent, very close, so that you know which parameter is going to change the phugoid motion ... after the aircraft is made.... Then only, we can change that parameter at the design stage itself. So, the error involved here is quite large. (No audio from 30:07 to 30:19)

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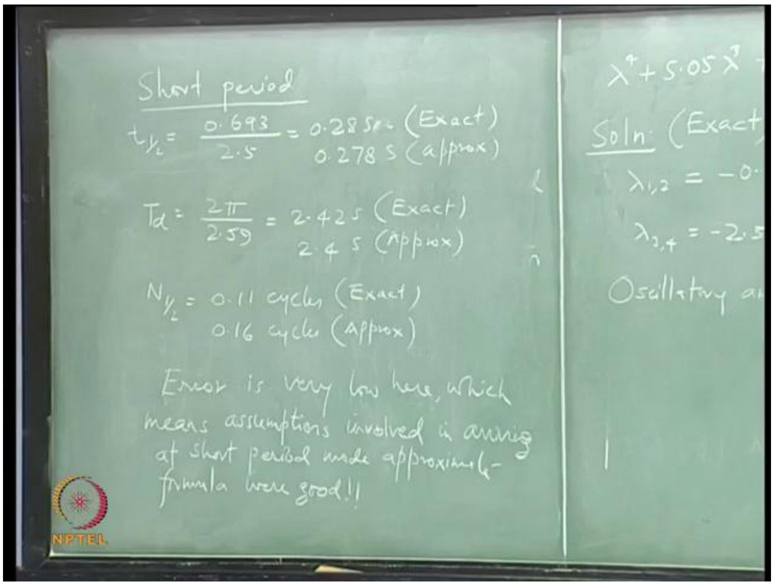


.... Let us look at the short period motion (No audio from 30:22 to 32:09) .... right; so, t half here is the time to half the amplitude, which is 0.693 over the real part of the Eigenvalue, and that is giving me 0.28 seconds ... that is the exact number, and this is the approximate value. This is very close, 0.28 and 0.278 are very close. Look at the damped time period, which is 2 pi over omega d, omega d is this 2.59, and that is giving me 2.48 seconds, which is the exact value, and 2.4 second is the approximate value; cycles to half the amplitude, here we get 0.11 cycle exact, 0.16 cycles approx right. So, this is looking very close. Is it not? So, the assumption that we made while arriving at approximate formula for the short period mode was ... quite good, I would say. So, error is … (No audio from 33:31 to 33:40) very low here, which means assumptions involved in arriving at short ....

(No audio from 34:13 to 34:43)

Right. So ... if you want to find exact formula for both of them together, then you know that you have to solve a quartic equation ... using symbolic tool. But you cannot get an idea of, if I, let us say I am able to retain all the parameters not exactly giving them any number, is it not? And then we want to arrive at expressions for Eigenvalues for short period mode, and the phugoid mode, then you know that you are going to get large expressions. You know, you will have all kinds of terms here X u, X w and all that. In terms of those parameters .... if you arrive at expressions for Eigenvalues using exact method, then the expression is going to be large, it is a huge expression, and it will depend upon, Eigenvalues will depend upon everything.

Then a designer will not be able to ... understand which parameter is affecting which Eigenvalue the most, .. that is what we can play with at the design stage. That is where arriving at simpler expressions is useful right. But here we see that phugoid mode approximation was not good, there is too much of error involved here. If the approximate ... people have till recently also, even after 100 years ... of flight, been working on the approximate formulas, and the one recent article which talks about this says that we cannot really look at the two motions as decoupled motions. .. To start with, we assumed that they are decoupled based on the location of corresponding Eigenvalues in the complex plane right, then we started looking at the simpler models.

So, there are arguments based on that. What the people have come down to, is that, this phugoid mode actually is an ensuing motion; you know it comes from the short period. So, when short period ends, then you have this motion starting. (No audio from 37:39 to 37:52)

Just one more thing. Let us try to find out the Eigenvectors... Eigenvectors are going to tell us about the directions in which the system is going to evolve right, that is what we understand by Eigenvectors. Let us look at the Eigenvectors, and then try to find out some reasons for the errors that is involved here. So, I am trying to find out the Eigenvectors corresponding to the short period mode and the phugoid mode. And let us also try to non-dimensionalize the Eigenvector ... we have to keep some scale. Eigenvector is a general Eigenvector ... there is no exact number attached to it, it is always with respect to something right.

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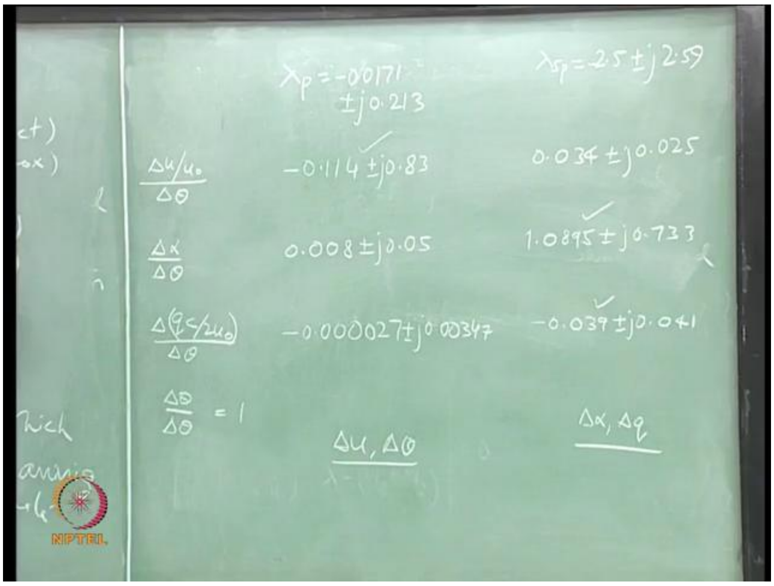


So, what we do is, we do some non-dimensionalization and look at the Eigenvectors. So, this phugoid ….. (No audio from 39:11 to 39:27) Short period Eigen values, .... (No audio from 39:30 to 39:46)

What I am doing is, I am assuming this delta theta to be 1, and looking at other variables with respect to that right. The size of this vector is one, and with respect to that, I am looking at the size of other variables.

(No audio from 40:06 to 42:01)

Now, look at the magnitude of these quantities .., with respect to this delta theta, which is of the size 1. Now, and also look at the corresponding values, two different values corresponding to two different modes. You see that magnitude of this is very low, short period mode corresponding to delta u..., this number is very low. Take the magnitude of that value, this number ... Look at the magnitude of this number, this is very large as compared to this. So, this gives me a reason to keep this delta u as non zero for the phugoid mode, and we can assume this delta u to be 0 for the short period mode... And these are exact numbers; ... I am solving an Eigenvalue problem, and then I am arriving at this set of Eigenvectors ....

So, this was a correct assumption; we can keep the delta u as a variable for the phugoid mode and probably, neglect that delta u here, in the short period mode ... because this magnitude is very low as compared to this magnitude. Look at this delta alpha; delta alpha here is very low, here it is very large. So, keeping delta alpha as the variable in the short period mode was a ... good approximation. This delta q which is non-dimensionalised with respect to c over 2 u naught, again in phugoid, it is very low... In short period, you know as compared to phugoid, it is having a larger magnitude...

So, using delta alpha and delta q for (excuse me) short period was kind of good approximation right and dropping delta u. See, you can get better approximations, if you have more and more terms in your equations ... but solving an Eigenvalue problem becomes difficult, ... and even if you can solve them, you will arrive at an expression, which is you know, very difficult to interpret after that; or make any sense out of that, it will be more difficult. .. In this column, the Eigenvector corresponding to the phugoid mode Eigenvalue, you see the magnitude of delta u and delta theta are large right; and that was the reason ... why we kept delta u and delta theta as important variables in phugoid mode right.

And here, delta alpha and delta q. .. You understand the physical reason behind this? Now, this information is coming from the exact analysis. (No audio from 46:09 to 46:17) Any question? Try to look at now the variation of these Eigenvalues in the complex plane, when you change something on the aircraft... And one of the things that you can easily change is the CG right. So, sometimes there will be block of masses, you know, just for balancing out the aircraft. Dead weights inside the aircraft, which can be used to change the stability property of the aircraft. .......

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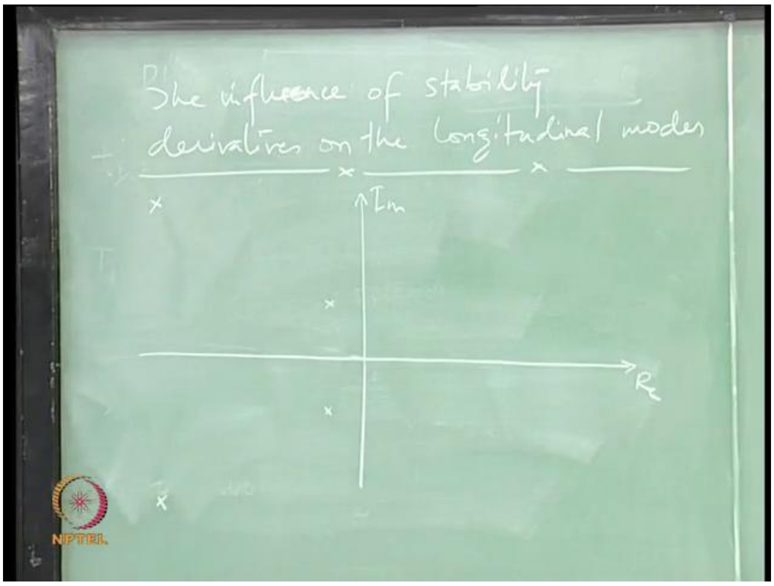


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48:03

So, to start with my Eigenvalues are located .... phugoid mode, short period mode right; and we want to change the CG... So let us see, what is the effect of changing CG on the Eigenvalues? So, CG is moving aft .. in this particular case, which means static margin is ..… What we will do is, we will stop here; and we will continue from here tomorrow.