Flight Dynamic II (Stability)

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Module No. # 10

Longitudinal Dynamic Modes

Lecture No. # 31

Short period, Phugoid (Lanchester's Formulation)

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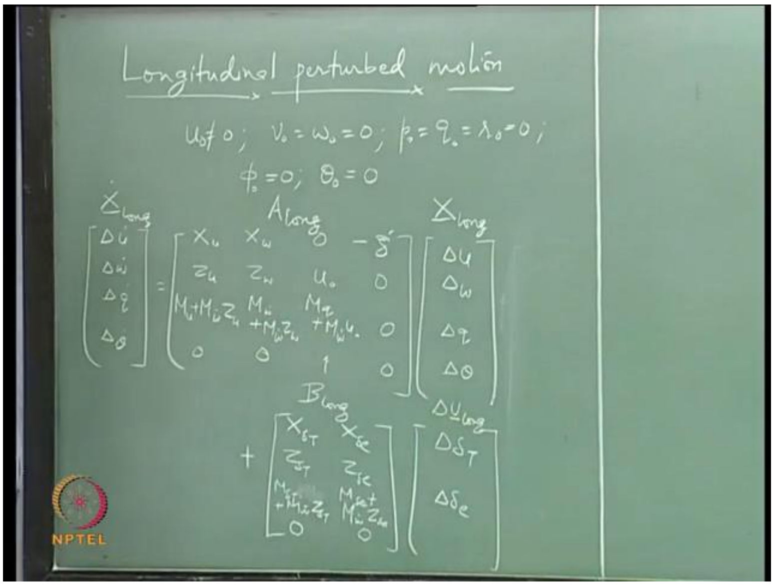

So, we are looking at longitudinal perturbed motion of an airplane about an equilibrium flying condition, which is …..Refer slide above and previous lecture.

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This is a cruise condition, and we have already derived the perturbed equations of motion.

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This we have written yesterday also, only for the sake of completeness, I am writing this again.

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And we call this matrix as A long, and this is the vector of the longitudinal perturbed variables.

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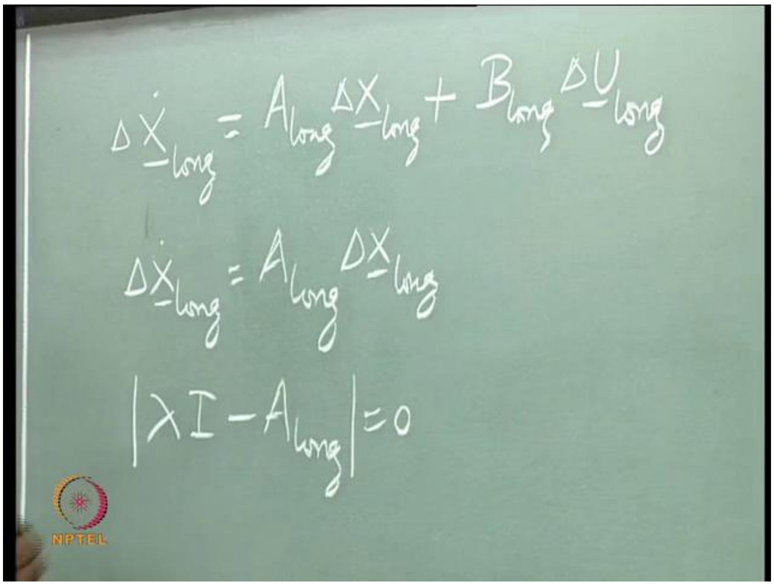

And we said that, if you want to look at the free motion of the aircraft with respect to the perturbed variables around this equilibrium ... flight condition, then actually we are solving an Eigenvalue problem. So, we are solving ..… this.

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... and this gives me a quartic equation in lambda. So, we will get 4 Eigenvalues.

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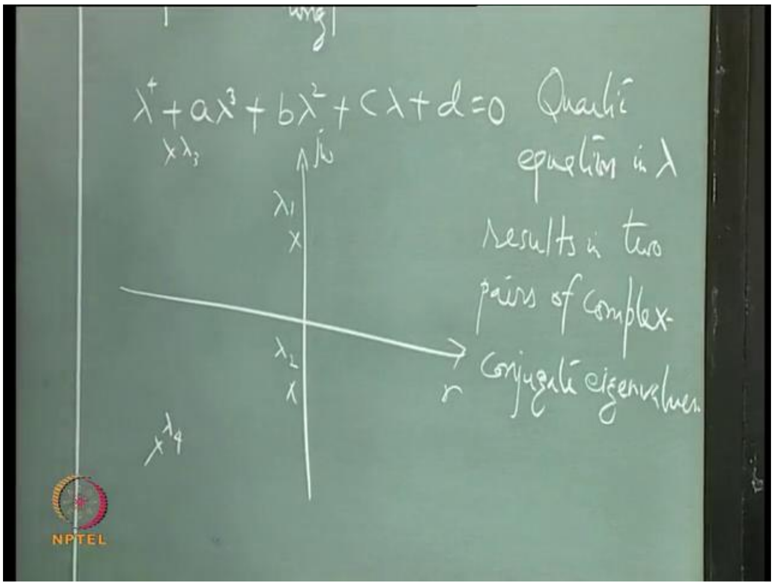

This quartic equation in lambda results into two pairs of complex conjugate Eigenvalues for this particular matrix. So, not that it will always happen, 4 Eigenvalues can be in different combinations. So, here what happens is now, for this particular matrix, you know, in the flying condition that I am taking, we get two pairs of complex conjugate Eigenvalues. And ... they are actually well separated in the complex plane.

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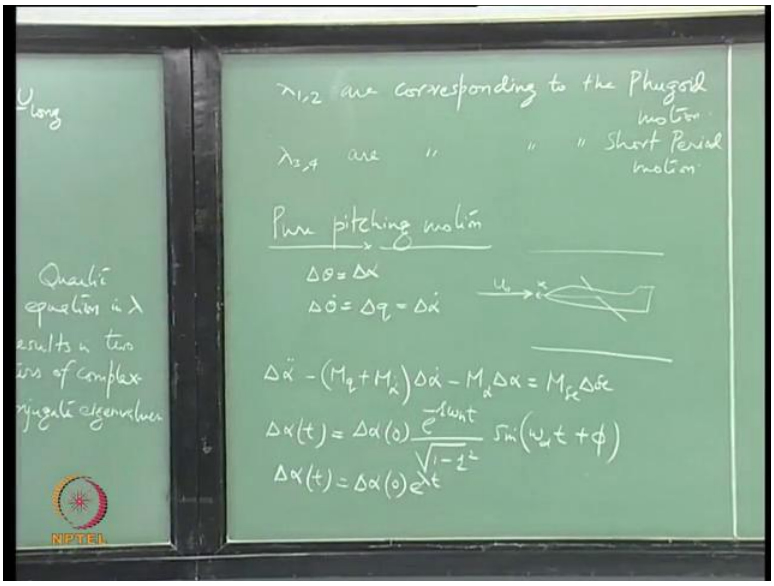

Lambda 1 and 2 are .... corresponding to the phugoid motion, and lambda 3 and 4 are corresponding to the …..

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Because of this separation of the Eigenvalues in the complex plane, you know, they are quite far located with respect to each other; we could make some assumptions to simplify this equation further. Solving a quartic is actually a difficult task, it is not a not an easy task, if you want to do it by hand ... So, you can, what you can do is, you can use this fact that the Eigenvalues are well separated in the complex plane, and make some assumption, so that you can arrive at an expression for the short period frequency and the damping and the characteristic of the phugoid motion.

So, what we did yesterday, we will assume the short period mode to be a pure pitching motion. So, we look at that pure …..

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Pitching motion of the aircraft about this flight condition... So, if you are doing a wind tunnel experiments, and want to measure the frequency and damping corresponding to this pitching motion, then place a model in the wind tunnel. So that you are satisfying this equilibrium condition. So, this X axis of the aircraft is actually aligned with the air flow.

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And this model is having only 1 degree of freedom, and that is in the pitching motion. ... So, there is a rod passing through this, which allows it only to have a pitching motion and nothing else .... And we assume that, to start with, this axis is aligned with the inertial frame of reference, so that any perturbation form that, delta theta, was equal to delta alpha, and delta theta dot which is delta q we have shown this already (())... delta theta will be equal to delta q even if it is not aligned, X is not aligned with the X axis of the inertial frame of reference. So, we arrived at the equation of motion for this pure pitching motion, and that was …

$$\Delta\ddot{\alpha} - (M_q + M_{\dot{\alpha}})\Delta\dot{\alpha} - M_\alpha \Delta\alpha = M_{\delta e}\Delta\delta e;$$
$$\Delta\alpha(t) = e^{\lambda t}\Delta\alpha(t=0) \;\; for \;\; \Delta\delta e = 0 \qquad (1)$$
$$\lambda^2 - (M_q + M_{\dot{\alpha}})\lambda - M_\alpha = 0$$

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Now, we are looking at the motion with respect to you know, disturbance coming from ... the wind; not because we are giving an input to create a disturbance. So, we will look at the response, you know, in alpha with respect to the disturbance which is coming from the wind, and not because of the control input ... Delta delta e is the perturbation in elevator ... So, if you want to look at the free response of aircraft, then you drop this to 0 and the homogenous part of the solution ... is this (Refer Eq(1)).

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So, what we have assumed here is that response, general form of the solution is something like this (Refer Eq(1))..

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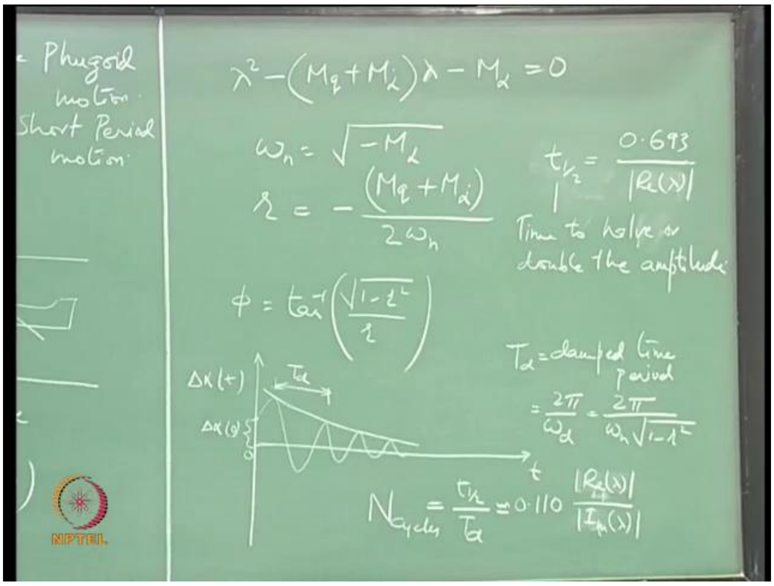

And then, we have solved an Eigenvalue problem ..... which is this (Refer Eq(1)).

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... So, corresponding to this motion, I can find out what frequency, and damping ratio are ....Refer to the slide above.

How will the response look like? So, clearly the response is consisting of two parts .. - one is the amplitude which is governed by this part ... that is also a function of time ... and this sinusoidal part, so, it is going to be oscillatory and the amplitude should be damping in time, because amplitude is being governed by this particular term. Is it not? So, here I am assuming that the real part of the Eigenvalue is negative, the way I have put it here. So, when t is increasing this part is going to decrease, and that is how the amplitude will go on decaying, (())…Refer to the slide above

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Let us look at this first. Many of these things, you can learn in many other subjects .. - mathematics, vibrations. But let us try to understand the flight mechanics or dynamics part. So, mathematics you can learn in much other courses .... So, this is governing the amplitude and this is the oscillatory part. So, if you want to plot the response …

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The response which is the perturbation, ... time response of the perturbation delta alpha, or time response of the perturbed variable delta alpha. So, I should not say that it is perturbation. So, you actually have a perturbation which is delta alpha at time t equal to 0, which is this .... .. and the response … Refer to the slide above

$$\Delta\alpha(t) = \Delta\alpha(0)\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \varphi); \omega_n = \sqrt{-M_\alpha}, \zeta = \frac{-(M_q + M_{\dot{\alpha}})}{2\omega_n}$$
$$\varphi = \tan^{-1}\left(\frac{-\sqrt{1-\zeta^2}}{\zeta}\right), \omega_d = \omega_n\sqrt{1-\zeta^2}, T_d = \frac{2\pi}{\omega_d}, t_{1/2} = \frac{0.693}{|\mathrm{Re}(\lambda)|} = \frac{0.693}{|\zeta\omega_n|}; N = \frac{t_{1/2}}{T_d} \tag{2}$$

Can look something like this, ... this is only a qualitative picture. So, what is happening? The amplitude is decaying, only because I have taken this part to be negative, otherwise it is going to, if this part is positive, then the amplitude is going to grow in time.

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And this picture Refer to the slide above is correct with respect to our assumption that short period mode Eigenvalues are lying in the left half complex plane. So, this is going to be negative term. So, this is what the response would look like.

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So, this is the damped time period and there are other parameters also, which normally we talk about when we talk about second order system response, and those parameters would be, you know, time to double or half.

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And that is given by ….Refer Eq(2)

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Or the real part of the Eigenvalues.

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So, because I have, its called, because I am looking at the oscillatory response are, not saying is decaying or growing, you can call this as .….

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Another parameter is, number of cycles that it will take in the time to half or double the amplitude.

(No audio from 22:40 to 22:58)

This (Ncycles) is roughly equal to $0.11\frac{|\mathrm{Re}(\lambda)|}{|\mathrm{Im}(\lambda)|}$

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Or .....

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We keep this here for a while now we are just looking at simpler motions. So, this was for pure pitching motion and we arrived at the expression for frequency and the damping. Now, let us look at phugoid motion, ... phugoid we said that it is an exchange of energy. So, you get a change in forward speed coming from somewhere, and then the aircraft will start going up, lose that extra energy and gain a height, so when you gained the height, you have actually gained extra energy compared to your equilibrium flying condition. So, you want to...

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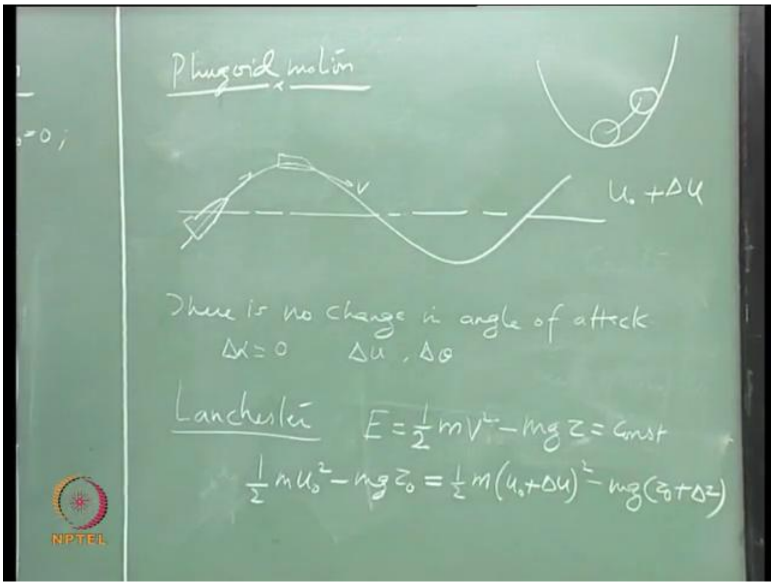

So, you have to remember this. Here, when you are flying you are sitting here. And now, there is a perturbation, and we are trying to find out if the system is stable in that particular equilibrium flying condition or not, your airplane. So, you have to see, if the ball is eventually back to this equilibrium condition or not .... And then only, we will say that the ball is stable in this equilibrium condition. So, similarly if you think of this phugoid motion, what I was saying is, you are flying along this line, constant altitude, you know, it is a level cruise condition and then there is an increase in forward speed. So, delta u naught is the equilibrium speed, and add that to this delta u.

So, because of that, there is a increase in kinetic energy, and the only way you can exhaust or the aircraft can exhaust this energy is by gaining some height... So, it is gaining some height, and now there is an increase in the potential energy. So, eventually it should settle down to this, if the aircraft is stable in phugoid, is it not?

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So, we assumed that there is no change in angle of attack in this motion ....

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So, delta alpha is 0. There is a change in forward speed, and the flight path angle will change then only you will get this height. You will attain this height, because of the disturbance.

$$E = \tfrac{1}{2} mV^2 - mgz = const$$
$$\tfrac{1}{2} mu_0^2 - mgz_0 = \tfrac{1}{2} m(u_0 + \Delta u)^2 - mg(z_0 + \Delta z) \qquad (3)$$

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So, there is no pitching motion involved here,.. only change in the attitude. So, earlier models the first one you know which Lanchester gave …Refer Eq(3)

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was based on this physical understanding. So, he said that energy is conserved. So, whatever energy you have …..

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thats conserved. I just write...Refer Eq(3).

(No audio from 29:03 to 29:19)

So, finally, when ... the disturbance is being killed, you are actually trying to come down to the initial energy level. Which was, you know, you have a forward speed. So, kinetic energy at the equilibrium condition is half m u naught squared ... and it will be flying at some height ....

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So, if there is an increase in the kinetic energy that is being compensated by .. a decrease in potential energy.

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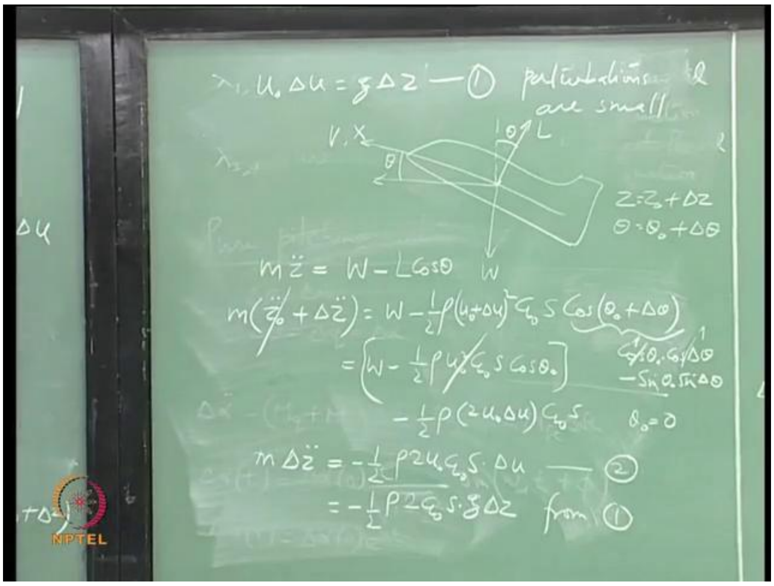

$$u_0 \Delta u = g\Delta z;$$
$$z = z_0 + \Delta z; \theta = \theta_0 + \Delta\theta$$
$$m\ddot{z} = W - L\cos\theta \qquad (4)$$
$$\Rightarrow m(\ddot{z}_0 + \Delta\ddot{z}) = W - \tfrac{1}{2}\rho(u_0 + \Delta u)^2 SC_{L0}\cos(\theta_0 + \Delta\theta)$$
$$\Rightarrow m\Delta\ddot{z} = -\frac{1}{2}\rho 2C_{L0}Sg\Delta z \Rightarrow \Delta\ddot{z} + 2\underbrace{\left(\frac{1}{2}\rho u_0^2 C_{L0}S\right)}_{W}\frac{g^2}{Wu_0^2}\Delta z = 0 \Rightarrow \omega_{nph} = \sqrt{2}\frac{g}{u_0}$$

So, that is what this expression means. So, balancing these two, you can find out .... this expression by assuming that these perturbations are small. So, I am dropping delta u square term .... Refer Eq(4).

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Now, let us look at the equation of motion in the ...

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in the normal direction .... Refer Eq(4).

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So I am writing down the equation of motion for this case in the normal direction.

(No audio from 32:24 to 32:30)

So, this acceleration is because of imbalance of force in the vertical direction. Let us also try to linearize this.

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Now, because there is no change in angle of attack, ... there will not be any change in the C L ... So, this C L value is the C L at the equilibrium flight condition.

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....Refer Eq(4) Is this alright?

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Now, if you assume this theta naught also to be 0, which is my flying condition, then this is 1 and this angle is small. So, this becomes 1, sin theta naught is 0. So, what you have is this into 1,.. now there are two components if you expand this and then drop delta u squared because the perturbation is small. Then what you can get by doing this ..... is the perturbed motion in the vertical direction.

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So, you have an equation, now in delta z Refer Eq(4).

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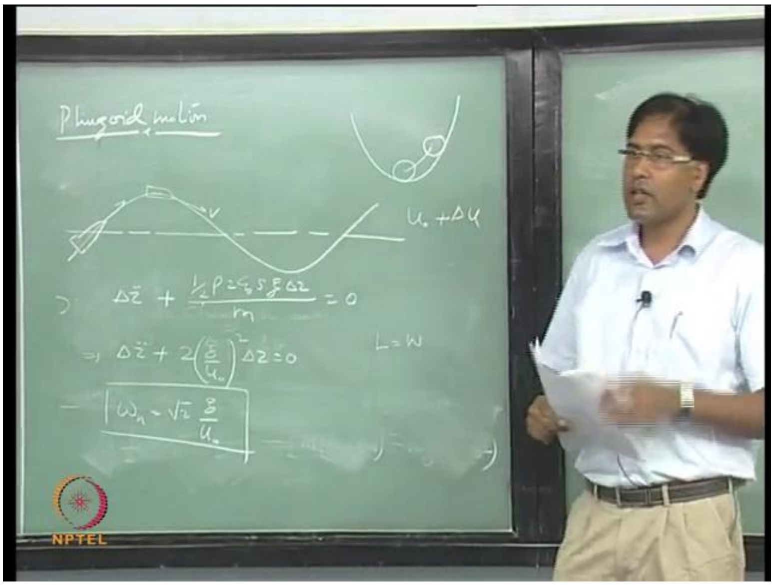

This comes from lift equal to weight, which is true in this flying condition.

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So, finally, what you get is an expression for frequency form here. So, it is a second order equation with no damping. So, you can find an expression for the frequency of this motion... ..... Refer Eq(4).

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So, there is an assumption involved here... and that is, there is no change in angle of attack. There is .. no pitching motion involved in this. But what we have neglected here is the damping.. Somehow it has got neglected, because of the assumptions that we have followed. So, Lanchester could only arrive at an expression for the frequency of the phugoid motion, and he could not give any expression for the damping and therefore, he could not comment on the stability of this motion. So, if you want to look at the stability or dynamic stability of the motion ... around the equilibrium flying condition, then you have to solve an Eigenvalue problem. So, if I now use some of these physical assumptions and try to look at how we can simplify our linearized equations of motion, then we can probably be able to find out expression for frequency and damping both.

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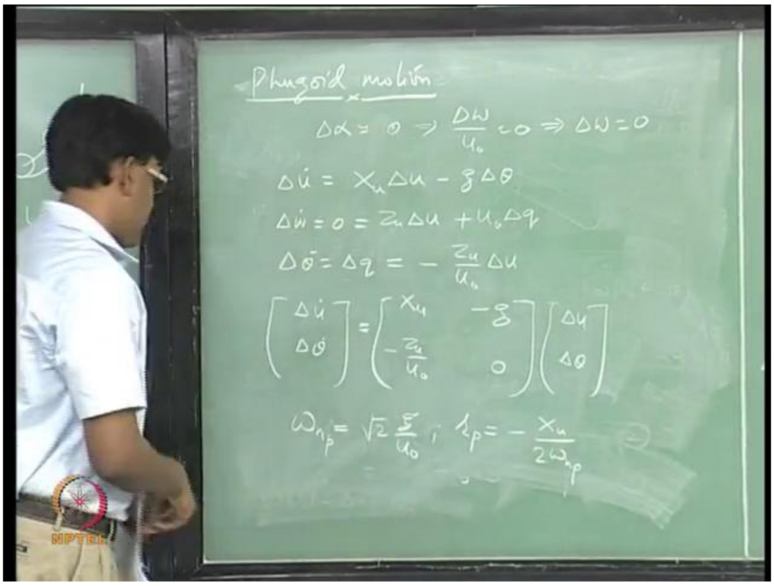

$$\Delta\alpha = 0 \Rightarrow \frac{\Delta w}{u_0} = 0 \Rightarrow \Delta w = 0$$
$$\Delta\dot{u} = X_u \Delta u - g\Delta\theta$$
$$\Delta\dot{w} = 0 = Z_u \Delta u + u_0 \Delta q \Rightarrow \Delta\dot{\theta} = \Delta q = -\frac{Z_u}{u_0}\Delta u \qquad (5)$$
$$\begin{bmatrix}\Delta\dot{u} \\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix} X_u & -g \\ -\frac{Z_u}{u_0} & 0 \end{bmatrix}\begin{bmatrix}\Delta u \\ \Delta\theta\end{bmatrix} \Rightarrow \omega_{nph} = \sqrt{2}\frac{g}{u_0}, \zeta = -\frac{X_u}{2\omega_{nph}} = \frac{1}{\sqrt{2}(L/D)}$$

So, let us look at ....Refer Eq(5) now the phugoid motion using this set of equation. And use the assumption that people followed. So, assumption here is that delta alpha is 0 ... Refer Eq(5). which also means delta w is 0.

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So, delta w dot is 0, delta q dot is 0. So, try to extract a simpler model form this. So, first equation is now with this assumption ..... Refer Eq(5).

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And this will be ..... Refer Eq(5)

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This equation is not going to give you anything, because delta q dot is 0, delta q is 0, delta w is 0, and the forth one gives you ….. Refer Eq(5)

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So, this I use to arrive at this. Form this equation, you can find out what is delta q, in terms of delta u. So, the model that you have is this .... Refer Eq(5).

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And solve an Eigenvalue problem now. So, you will arrive at the frequency of the phugoid mode, which is interestingly the same as what Lanchester obtained. Frequency is the same, what we achieved by doing this analysis. And the damping ratio turns out to be …. Refer Eq(5).

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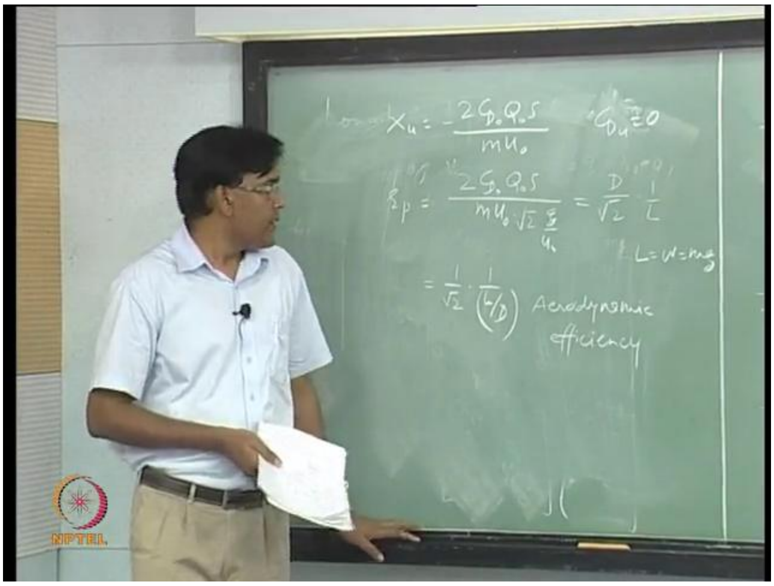

And X u is …..

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This, you know, this we have derived. If we assume C Du to be 0. Which is alright you know. So, a good assumption; you know if we assume that we are not accounting for the compressibility effect. So, the damping in phugoid is .... Refer Eq(5).

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So, again I am using L equal to W, equal to m g in this. So, what you see is …

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The damping in phugoid is inversely proportional to L over D; what is L over D? Lift over drag Refer Eq(5).

(No audio from 46:56 to 47:05)

Which is called aerodynamic efficiency of the aircraft. So, what it means is that if you have L by D ratio which is high, the damping in phugoid is low... And phugoid, it is a slow motion, ...you may be able to control it, because the time period involved is large. So, you have enough time to act, but let us say you are very close to the ground you know, this kind of motion is going to ... take the aircraft up and then hit the ground... It is very close to the ground, it can give you a bumpy ride hitting the wheel on the ground and so on ... The frequency is inversely proportional to the trim speed .... So, let us say ... you would still want to keep this L by D high. Now, what we get is this zeta P, damping ratio in phugoid, goes down, and let us say it is not meeting the requirement. So, there will be some requirement on these parameters, ... like how much values they should be, if it is not meeting that, then you have to go for control systems. So, in the next class we will look at short period approximation.