Flight Dynamics II (Stability)

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Module No. # 09

Perturbed (Linear) Aircraft Model

Lecture No. # 29

Contribution of Aircraft Components to Aerodynamic Derivatives

Let us try to understand what actually we are trying to do. So, we have been talking about the linearisation of aircraft equations of motion. And after that, I am also defining some derivatives...So, let us say where in this aircraft design cycle, this analysis fits. Why we are doing this? I want to look at that briefly, and then we can proceed from where we left yesterday.

So, you are asked to design an aircraft, given specifications, performance specifications or other requirements, for example, you have to sit 20 passengers. So, accordingly you will start planning your geometry of the aircraft ... and the weights you can calculate and then you can look at some of the performance parameters ... So, after you have done all that, you have come down to a basic configuration, is it not?

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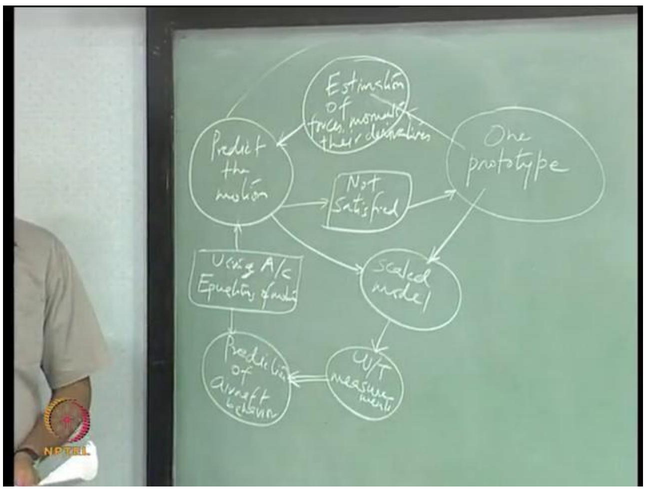

So, you have done a paper design of your aircraft; ofcourse, this will also require lot of repetitions. So, let us say, we have sort of arrived at a configuration, geometry I know, and other information like weight and inertia also we know. So, we have one prototype.

Now, this is on paper. We have got all the numbers on paper and what we want to start looking from here using whatever we have been doing so far is, how the aircraft is going to behave in motion, is it not? The performance of aircraft or the behaviour of aircraft in air is going to actually depend upon the configuration and the aerodynamics around this configuration. So, now after you have done this, either you build one scaled model and put it in the wind tunnel and compute or measure all the derivatives. That is a tedious task, is it not? Finding out all the derivatives at all flying conditions.

So, from here you can build a scaled model, .. and then go for wind tunnel measurements of the forces and the derivatives.... But when will you do this? Only after you have frozen your model, because this requires a lot of effort, is it not? You are not going to, you know, just like that put your scaled down model in the wind tunnel, start measuring something, and then you find that the aircraft is not giving you the desired performance in response, ... then again change this. That cannot happen. Only after you have frozen this (configuration), you are going to come down to this and then to this (wind tunnel measurements and mathematical modeling), and then, use this data to predict the aircraft motion using aircraft equations of motion, that is one thing.

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That is only when you have frozen your design. Let us say without doing this, without going through this route, now because after I have done this, I cannot go back and change something on my aircraft, not to a large extent. Only small variations, or probably, you have to design control systems to meet the specific requirements. So, this is one route and this will not involve cyclic repetition. We cannot afford that. So, instead what we would want to do is, we take this prototype, so design is there, paper design is there, and go for .... estimation of the derivatives of the forces, force moment and their derivatives.

Afterwards, once we have done this, we can now use the linearised aircraft equations of motion and look at particular dynamic behaviour with respect to that, we can look at stability and ... basically stability, and after meeting the stability criteria, look at some of the performance objectives. For example, whether your aircraft is giving you a good flying or handling quality or not.

So, after we have done this, now everything is on paper. Paper or you can use computer, you know, to predict the motion and this will be using ..... you know both.

Of course, they are not connected like this but they are using the same set of equations, is it not? Now, if I have gone through this route, then I have a choice, because I have not done this expensive wind tunnel testing, is it not? Now, the question is how do you estimate these derivatives, and this is exactly what I am discussing right now in this, in the last few lectures.

... We want to estimate the derivatives, predict the motion, predict the stability or dynamic stability behaviour of the aircraft and see if I am meeting some of the flying and handling qualities or not. If I am not, then I have a choice. I have actually done nothing here, nothing that requires, you know, nothing that requires lot of money, is it not? So, if I am not satisfied, so if you are not satisfied, then you can (( )) and right here not satisfied, go back. If you are happy, if you are satisfied, then you can go through this cycle now... because finally, you would want to look at aircraft actual motion ... in air and that we can do using, by simulating aircraft equations of motion. So, you understand this part?

So, if you are satisfied, you can go from here to this point or may be not that .. but you can directly go for wind tunnel. So, if you are satisfied, you can go to build a scaled down model ... collect the data through wind tunnel measurements and repeat this. You will still have to do this, because wind tunnel measurements also are going to introduce some errors. That is one thing, and wind tunnel measurements will be for the scale down model. What here we are doing is, we are looking at you know different components and trying to find the derivatives by adding the effect of you know contributions coming from different components. But if you are putting the scale down model in the wind tunnel, then you are measuring actual data. Is this alright? Everybody gets what I am saying?

Now, the question is how do you estimate these derivatives? How do you go from this point to this point and that is what we are addressing right now. That is the question that we are trying to address, and because we cannot, you know in a normal class, you cannot do it for all kind of flying conditions. I am sticking to one flying condition that we have taken.

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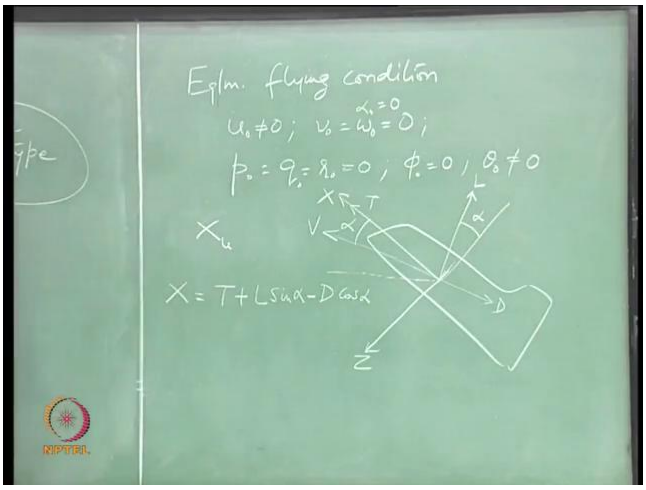

Equilibrium .... flying condition or equilibrium flying condition is, and this is one for example, is it not? I am doing this part only for one particular equilibrium flying condition; you may have to repeat it for several flying conditions.

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So, yesterday (in Lecture 28) we were looking at the derivatives of forces with respect to the state variables of aircraft. So, we already found an expression for X u... It came from .... it came from this balance of forces. Let us say, the thrust is acting along the X axis of the body and you have X equal to T plus L sin alpha minus ..... . So, finally we have found, if I include this thrust, we can also keep this thrust separately. Right now, I am including this thrust in this X force .... We have already separated the gravity, gravitational forces. I am not including that in this force because that is not changing aerodynamically. Thrust also we will have to see, ... whether it is changing with velocity or not, if it is, or angle of attack, right. If it is not, then we can drop this.

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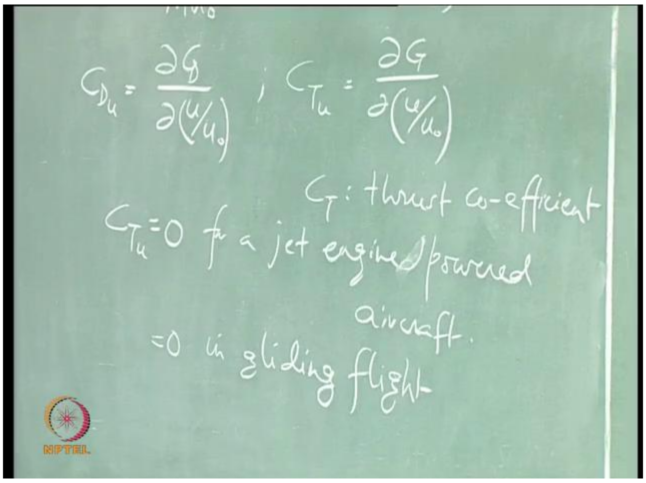

So, we derived expression for Xu ....(done in previous lecture) and that was for this equilibrium condition. So, you have to keep that in mind. ..... Where C_{Du} is ....(Refer slide above), So I want to find out, this is a constant, so I am non-dimensionalising this derivative with respect to u over u_0. C_D has no dimension and this will also have no dimension. So, we can find this as a constant. .....

C_{Tu} is actually 0 for an aircraft powered by jet engines. We do not have thrust changing much with forward speed, so C_{Tu} is actually 0 for .... and it is also 0 for ..... and it is also 0 in gliding flight. You have switched off your engine, or glider will not, you know, many gliders. Now-a-days we have motor gliders, which have engine also there, but there are gliders which are not powered or your aircraft may be gliding. You know, you have switched off your engine, when you are trying to land your aircraft.

So, C_{Tu} is going to be 0 in these two cases and for variable pitch propeller, for aircraft with variable pitch propeller engine, C_{Tu} is nothing but minus C_{D0}. This is again an estimate, it is only an estimate. If you want to get an accurate measurement, then you have to, you know, do a wind tunnel testing.

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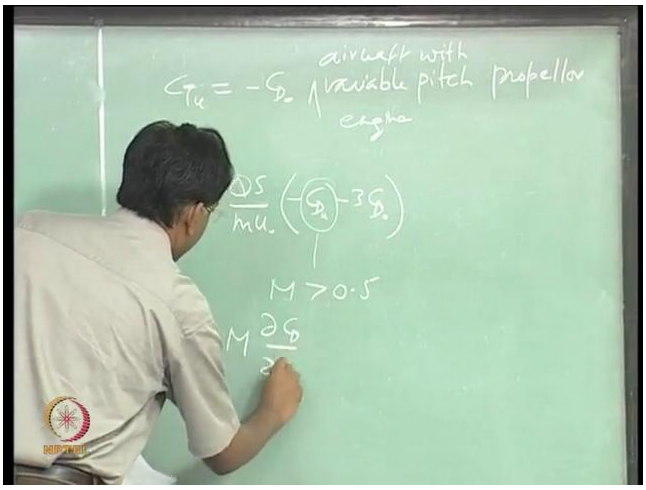

So, in this case when $C_{Tu} = -C_{D0}$, X u becomes ..... $X_u = \frac{\frac{1}{2}\rho u_0^2 S(-3C_{D0} - C_{Du})}{m u_0}$. What about this term C_{Du}? This term is change in C_D with respect to a change in non-dimensional forward speed... When is this going to be important? Only for the flight in the compressible range. Actually, it is going to be effective only when you are flying beyond Mach number 0.5. So, you have to remember that you have to actually include this in your equations when you, if you want to accurately predict the motion.

So, you can never actually predict motion using this, you know, 100 percent accurate .. but you want to get a close estimate, a close prediction.

(()) Drag polar, yeah. Even for lower velocities it matters, how do we say only for Mach greater than 0.5 and different Mach.... but how is that happening? Because that that ~~learner~~ C_D. Yeah. Once another keeps increasing, the C_D will keep decreasing at least zero point (student's question not clear! Reference to drag polar not relevant here!). One should refer to drag vs Mach number curve for understanding.

So, only in this part it becomes, only beyond Mach number 0.5, it becomes, you know, .. I am not, remember I am saying that if it is not important, you should not include it. If the aerodynamics is telling you that this number is significant at your flying condition, then you have to include it to predict your aircraft motion accurately.

Now, we can repeat the exercise that we did yesterday using this balance of forces. We can find out what is X_w.

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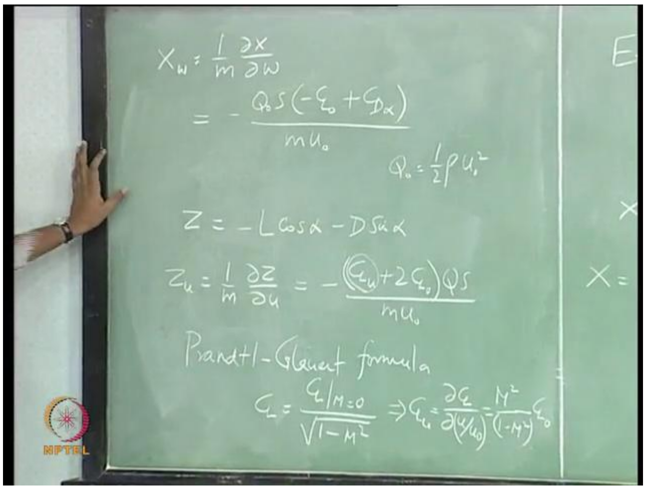

So, remember this 0 is the equilibrium flying condition. This C_{L0} is corresponding to that particular equilibrium flying condition, which is also alpha naught equal to 0 and Q naught is half rho u naught squared ....(Refer Eq(1)) .

$$X_w = \frac{1}{m}\frac{\partial X}{\partial w} = -\frac{\overbrace{\tfrac{1}{2}\rho u_0^2}^{Q_0} S(-C_{L0} + C_{D\alpha})}{m u_0}$$

$$Z = -L\cos\alpha - D\sin\alpha \qquad (1)$$

$$Z_u = \frac{1}{m}\frac{\partial Z}{\partial u} = -\frac{\overbrace{\tfrac{1}{2}\rho u_0^2}^{Q_0} S(C_{Lu} + 2C_{L0})}{m u_0}$$

Now, let us try to write down the equation in this Z direction. Then, we can find out the Z derivatives. ..... So, without repeating this exercise, now I am going to write down what these derivatives are. ..... (Refer Eq(1))

Actually, there is one more thing we could have done. You know, I already wrote force equations in two different axis systems if you remember. Now, you will think why I am doing this. If I had just written my force equations in the wind fixed axis system, I would not have really bothered about the derivative of Z with respect to u... I would have only talked about C_L and C_D, is it not? Because there we do not see this notation.

Let us look at what C L u is? Again, this is going to be only significant when you are in this range of speed ... Mach number greater than 0.5. So, there is a formula given by Prandtl and Glauert to account for the compressibility correction ......

$$C_L = \frac{C_{L,M=0}}{\sqrt{1-M^2}} \Rightarrow C_{Lu} = \frac{\partial C_L}{\partial (u/u_0)} = \frac{M^2}{1-M^2} C_{L0} \qquad (2)$$

From here, you can find out what C_{Lu} is. C_{Lu} is ....(Refer Eq(2)) Now, we are talking about derivatives which are non-dimensional. ..... So, as I said, now each of these forces are going to depend upon each of the variables; .... u, v, w, p, q, r. Now, it depends which flight regime you are designing your aircraft for. So, if you are designing your aircraft for supersonic speeds, then you have to see, in supersonic speeds, which are the derivatives which become important.

Significant derivatives, you have to look at the magnitudes, and depending upon their relative magnitudes, you may want to keep them or you may want to drop them. But if I am doing a numerical simulation, I may just want to keep all of them. Even if the effect is smallest. But if I am doing something analytically, I want a simpler model and there I will only keep the significant terms. Is it not?

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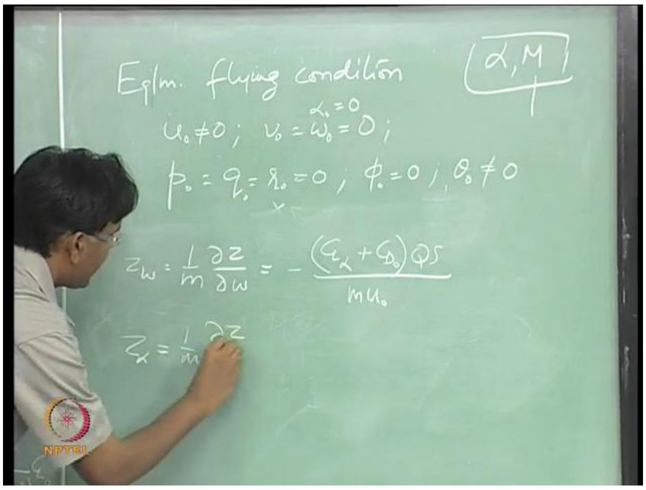

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See whenever we are talking about flight regimes, we are talking about two parameters. One is angle of attack and the other one is the speed ... So, alpha and Mach number are the two parameters defining the flight regime ... You can be flying at high alpha but in the subsonic speed regime ... At a higher Mach number, you may not want to try high alpha maneuver. You know, supersonic speeds you cannot actually change alpha the way you can change in the subsonic speed, is it not? Because large momentum in the forward direction, if it is flying at supersonic speeds. So, there these derivatives with respect to Mach numbers will become important and you can keep alpha as low alpha flights.

31:05

So, let us say somebody comes to you and asks you to design a four seater aircraft... So, you can figure out, you know, there will be many four seater aircraft, ... the geometry for which will be available, but aerodynamic data will not be available. That is the trade secret, because that is the one which requires the maximum amount of work and also classified. So, that is one thing which is classified about the aircraft. Nobody gives you the aerodynamic database just like that ... So, I am going to look at some more derivatives because it may not be possible to look at all of them.

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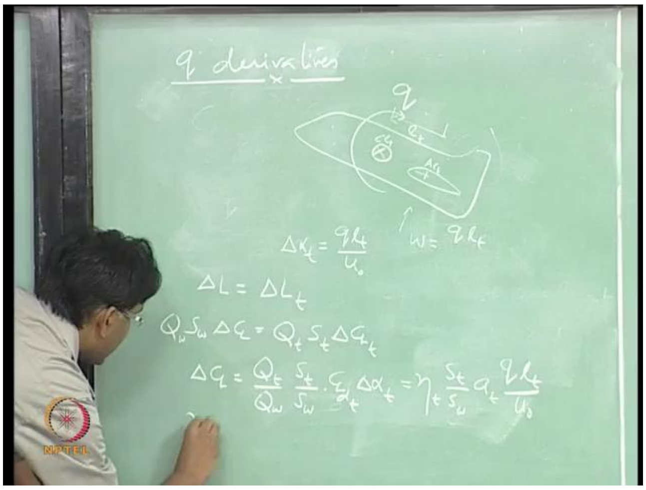

Let us look at q derivatives. So, the aircraft is having a steady pitch rate ... and now, we want to find out how that is going to affect the forces and moments ... So, what I am interested in finding out is this ..... How C_m is changing with respect to this steady pitch rate? How C_L is changing with respect to this pitch rate or how C_{zq}, ... if you want to talk about this normal force coefficient. So, your aircraft is steadily pitching up. ... We will only include the significant parts. We are only trying to get an estimate .... In general there will be contributions coming from other components also ... but we are looking at major contributions.

When you are having a steady pitch up, what is happening is, this tail is going down. It is going to push air down and then you see a relative wind upwards. So, if this distance is l_t, then this tail is going to see upward wind, which is ...... $w = q.l_t$.

So, aircraft is flying at this forward speed and then you see a steady pitch, ...and this can be around your equilibrium condition ... We are talking about both, flying equilibrium condition and also the perturbed flight condition. So, this q may just come from because of the perturbed condition, is it not?

So, what is going to change here because of this? (())

So, what is going to change here because of this *w*, is the angle of attack. Angle of attack at the tail is going to change, it is going to increase, is it not? If I have positive pitch rate, then this alpha, increase in angle of attack at the tail is going to be positive $\Delta\alpha_t = \frac{w}{u_0} = \frac{ql_t}{u_0}$. So, there is a change in overall lift of the airplane and that is because of the change in lift at the tail due to this increase in angle of attack.

$$\Delta L = \Delta L_t \Rightarrow Q_w S_w \Delta C_L = Q_t S_t \Delta C_{Lt} \Rightarrow \Delta C_L = \frac{Q_t}{Q_w} \cdot \frac{S_t}{S_w} C_{L\alpha} \Delta\alpha_t = \eta_t \frac{S_t}{S_w} a_t \frac{ql_t}{u_0} \qquad (3)$$

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So, this eta t is the ratio of the dynamic pressures... St is the tail platform area, Sw is the wing platform area, a t is the lift curve ..slope of the tail, into this increase in angle of attack due to pitch rate. So, the change in lift because of this pitch rate is eta t .....(Refer Eq(3)).

Now, this is not a non-dimensional quantity ... Q is, C_L is a constant and q has some unit. So, I want to non-dimensionalise it. ... I want to find out an expression for C_{Lq} actually and this has to be a constant.

So, now I am introducing some parameter which is q c bar over 2u naught, which will have the same, if you look at the unit of 2u naught or u naught over c bar, it will have the same unit as this q, q is in radian per second. So, that is how you non-dimensionalise the rate derivatives.

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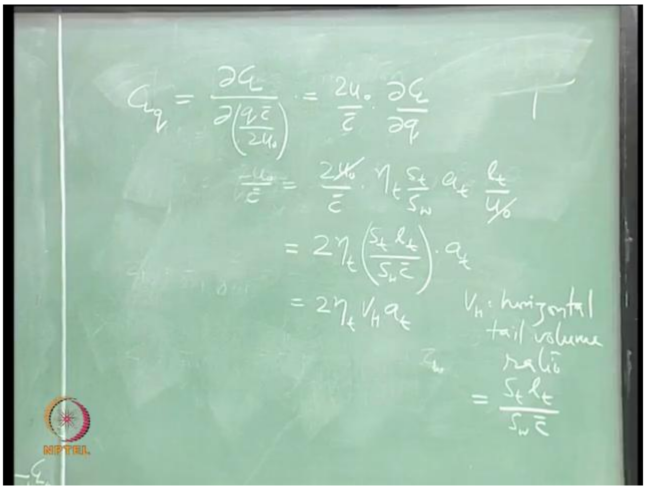

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$$C_{Lq} = \frac{\partial C_L}{\partial\left(q\bar{c}/2u_0\right)} = \frac{2u_0}{\bar{c}}.\frac{\partial C_L}{\partial q} = 2\eta_t V_H a_t; \quad V_H = \frac{S_t l_t}{S_w \bar{c}} \qquad (4)$$

Remember, if you are not talking about these derivatives which are damping derivatives, then you are not talking about dynamic stability. We already did a static stability analysis where we talked about the stiffness term ... What we were missing there are these derivatives. So, in this particular condition, when I am doing everything around alpha naught equal to 0, I am trying to find out derivatives only for this trim condition. Then, C_{Lq} is also C_{Zq}. There should be a minus sign here because lift is acting upward and Z force is acting down.

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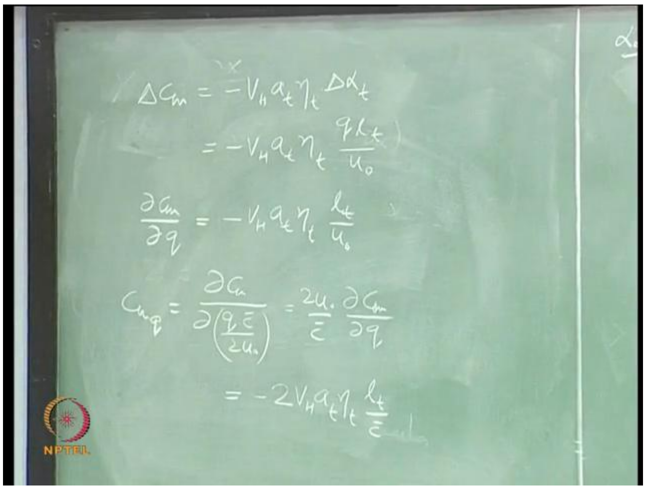

What is happening to the pitching moment, because of this pitch rate? ......

$$\Delta C_m = -V_H a_t \eta_t \Delta\alpha_t = -V_H a_t \eta_t \frac{q l_t}{u_0}; \ \frac{\partial C_m}{\partial q} = -V_H a_t \eta_t \frac{l_t}{u_0}$$
$$C_{mq} = \frac{\partial C_m}{\partial \left(q\bar{c}/2u_0\right)} = -2V_H a_t \eta_t \frac{l_t}{\bar{c}} \qquad (5)$$

So, I am including this angle which, you know, this expression we have written already, expression for Cm. Now, I am writing down the expression for change in this C m because of change in angle of attack at the tail due to the pitch rate ...(Refer Eq(5))

So, this was the steady pitch rate. It is going to change the angle of attack at the tail, but there is another derivative, which looks similar to this, is due to the unsteadiness in the flow.

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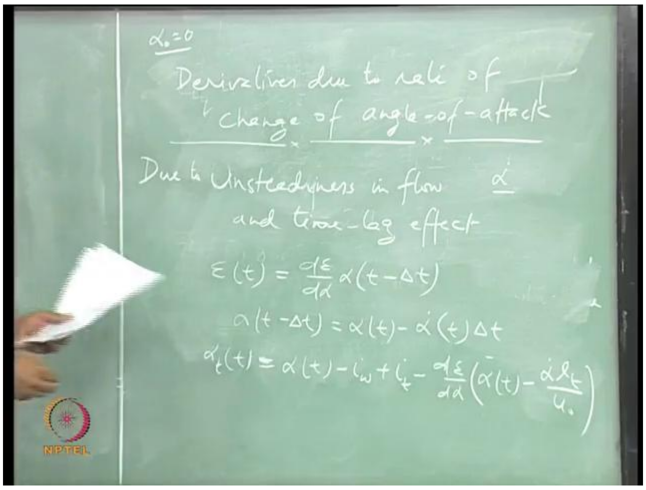

So, it is because of .... the (time) rate of change of angle of attack.

So, you are in a perturbed condition. Imagine where all you will need these derivatives? You are flying in equilibrium condition and then a gust will hit your aircraft. Let us say you are flying a cruise condition. Gust will hit your aircraft. So, you are going to see a perturbed motion ... Steady condition is when you are sitting in your aircraft, you actually feel nothing, is it not? You will feel as if you are sitting in this room, unless you see something outside moving relatively. Otherwise, you know you are actually sitting in a room and whenever there is a gust hitting the aircraft, the aircraft does some motion and that is the motion that we are talking about. So, derivatives due to rate of change .... And this is primarily because of the unsteadiness in the flow.

So, there is a change in angle of attack, rate of change of angle of attack is involved and the ..... flow becomes unsteady for various reasons. When your aircraft is pitching up, you know it is almost like pitching up when I am changing, when I am having a rate like, you know, rate of change of angle of attack, then actually the aircraft is pitching up. That is not a steady pitching. You know it is a change in angle of attack. So, what happens is, the flow field gets disturbed ... and there is a time lag involved in the movement of the flow. So, the flow, that tail is going to see is going to be after time lag.

$$\begin{aligned}&\varepsilon(t)=\frac{d\varepsilon}{d\alpha}\alpha(t-\Delta t);\alpha(t-\Delta t)=\alpha(t)-\dot{\alpha}(t)\Delta t\\&\alpha_t(t)=\alpha(t)-i_w+i_t-\frac{d\varepsilon}{d\alpha}\left(\alpha(t)-\frac{\dot{\alpha}.l_t}{u_0}\right)\end{aligned}\qquad(6)$$

What this time lag does is, it introduces a rate in the change of downwash angle with respect to time. Earlier, we have assumed that, in steady conditions, downwash angle is only going to depend upon the steady alpha, but because of this unsteadiness in the flow and due to this alpha dot, ... (Refer Eq(6)). this downwash angle becomes a function of .... (Refer Eq(6)). alpha which is delayed. So, tail is not going to see the flow at the same angle as the wing is going to see, and you know, it is going to see a delayed change in angle of attack .....(Refer Eq(6))

So, alpha at the tail is going to be a function of time now. ..... (Refer Eq(6))

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So, I will just write down now the expression for the derivatives. ..... (Refer Eq(6)).

This you can find out from this expression. So, it is going to be

$$C_{L\dot{\alpha}} = \frac{\partial C_L}{\partial(\dot{\alpha}\bar{c}/2u_0)} = \frac{2u_0}{\bar{c}}.\frac{\partial C_L}{\partial \dot{\alpha}} = 2a_t V_H \eta_t \left(\frac{d\varepsilon}{d\alpha}\right)$$
$$C_{m\dot{\alpha}} = \frac{\partial C_m}{\partial(\dot{\alpha}\bar{c}/2u_0)} = -2a_t V_H \eta_t \left(\frac{d\varepsilon}{d\alpha}\right).\frac{l_t}{\bar{c}} \qquad (7)$$

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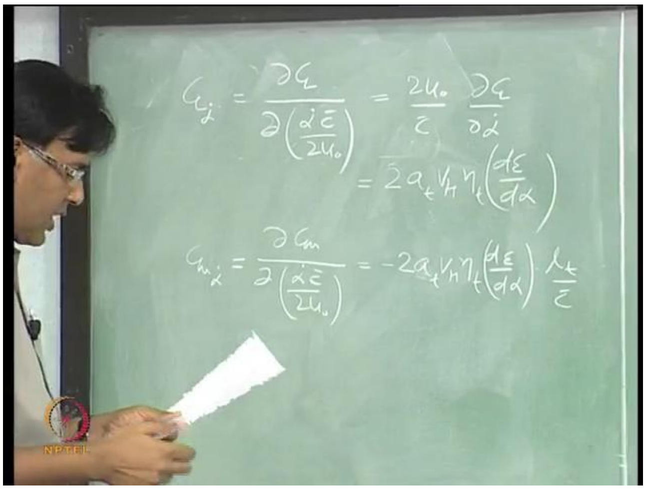

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So, with this backdrop now, I am going to move forward. I am not going to, you know, keep doing this. So, now whatever we have done, the derivatives, there are some more derivatives, the physical explanation of arriving at any formula for those derivatives are given in the book. .... I would not go into describing all of them. I will assume that we are knowing how to find the expressions for them .... and then move on from here.