Flight Dynamics II (Stability)

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Module No. # 07

Lateral Directional Static Stability and Control

Lecture No. # 18

Lateral - Directional Stability Derivatives, Fuselage/Vertical Fin Contribution

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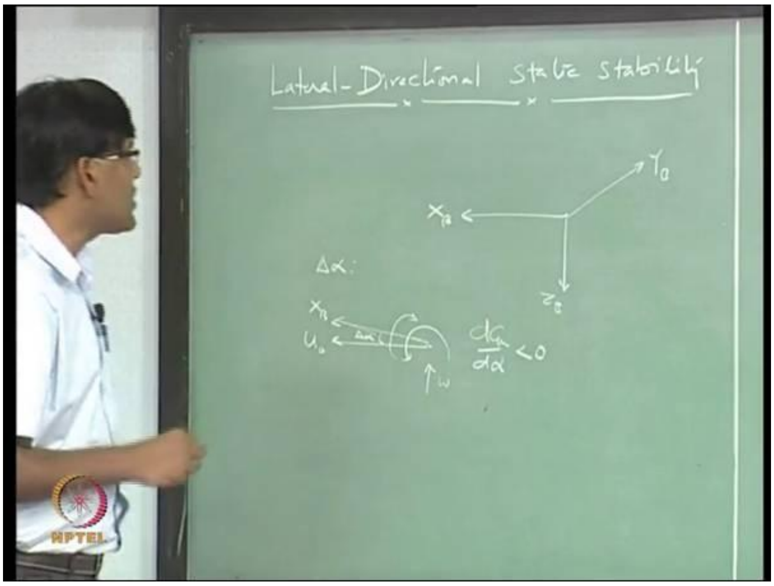

So we will start a new topic now; and that is lateral directional static stability, you know, so far we have been talking about stability of airplane in the longitudinal plane, right, that is the XZ plane. So, in this plane XBZB, you know, body fixed coordinates, right, XBZB is the longitudinal plane and we have been looking at stability and control only in this plane so far; there we were looking at what happens to the aircraft, when aircraft is disturbed by an angle of attack, so small change in angle of attack.

And first thing is, what is the effect of that on the airplane, and how airplane is trying to adjust to this. So, stability is defined by, you know, the stiffness that aircraft will automatically possess in order to kill this disturbance. So, we said, aircraft is flying level flight condition, you know, u naught and there is no other velocity, this u naught is, you know, going right straight.

What happens, when there is a change in angle of attack or there is a gust coming from downward, aircraft angle of attack is changed and that means, now this XB axis of the aircraft makes an angle, delta alpha, we are only talking about very small angles from the equilibrium condition. So, this is like a positive moment; the aircraft is having an angle of attack delta alpha now, which is positive, because of this gust coming from downward right. We are still flying this u naught remember that, this delta alpha is only a small disturbance, which is caused by this gust, right, and result of that is to create a positive pitching moment about the cg. And when I said the aircraft is stable in pitch, what it means is aircraft automatically will try to kill this delta alpha, is it not?

So, aircraft is actually, whenever it sees that there is a change in delta alpha because of the disturbance, it is trying to generate a moment in the opposite direction, so that it kills the delta alpha and that is when we say that the aircraft is statically stable in pitch, right, it is basically trying to kill the disturbance. And that is where we define this derivative. So, positive increase in angle of attack should automatically follow or be followed by a negative pitching moment.

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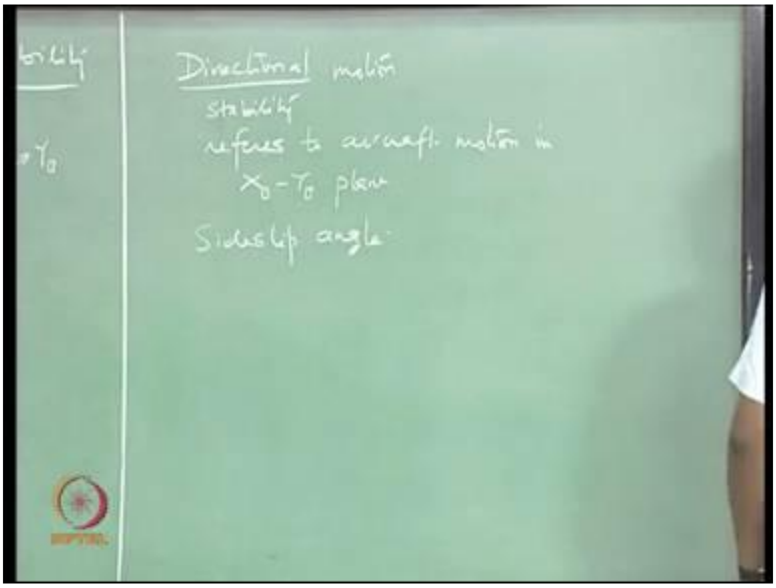

bility

Y_B

Directional motion
stability
refers to aircraft motion in
$X_B - Y_B$ plane
Sideslip angle

Now, let us look at what happens in other planes. So, there are two motions, which we are going to see now; one is directional and that is in this XBYB plane. So motion about the Z axis, right, or directional stability; and what is the angle associated with the wind in this plane? So far we have been looking at stability with respect to this angle of attack.

This angle, which is angle of attack. Which is the other angle associated with the wind. See, wind may be coming from any direction, may not be coming only in the longitudinal plane right, it can come from any direction span-wise right, but what is the angle associated with that?

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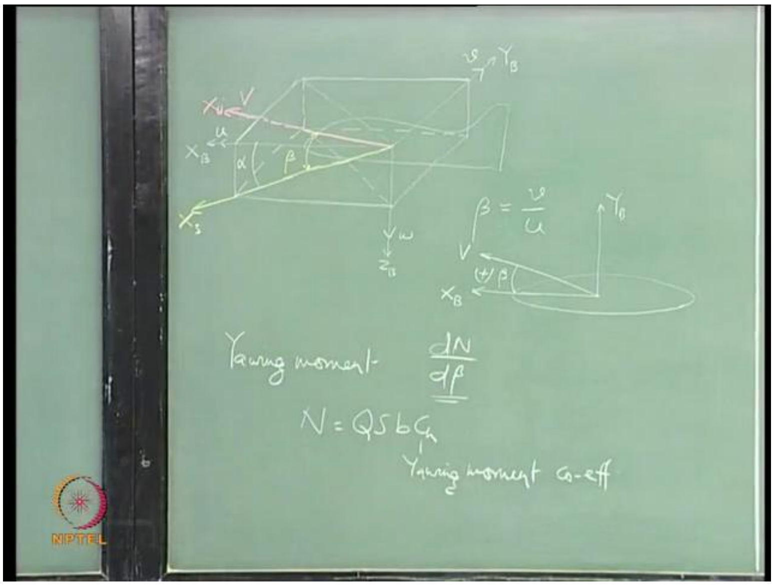
Side-slip, right. So, if you want to see what this angle is in three-dimension. So, let me draw one picture for you, and also draw ....



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There are some colored chalks. ..... So, this could be probably your velocity vector, right, how many angles it involves if you want bring it to say this axis system or wind axis, we want to find out the components along body fixed axis system, how many angles are involved? One angle is slide-slip and, the other is, angle of attack. So, let us try to define those angles, beta angle is actually the angle between this wind X axis, so, that is, that is the velocity, direction of the velocity vector, right, so your aircraft can be actually doing this, right, if it is side-slipping and also with some angle of attack, then it would be going something like this. So, this angle is called the side-slip angle, you see that? I am joining these two vertices, right, and drawing a line through that, diagonal, and that is the direction of the velocity vector, right, and wind is relative wind. So, If I am going like this, the wind is coming on to me in the same direction, right, the opposite direction actually. So, this is the angle beta.

What is alpha? Alpha is this. So now, If I am taking a component of this vector, velocity vector, on the longitudinal plane, right, that vector makes an angle with this body fixed X axis, and that is the angle of attack. And when we said that, we are only going to look at the longitudinal plane, we neglected that part. So, that is how, this is how you defined your angle of attack. So, this is the direction of your velocity in the longitudinal plane and the angle that it makes with the body fixed X axis. So now, if I am talking about this XBYB plane, I am actually assuming that alpha has become 0, right, and then you are in this XBYB plane and I am talking about stability only in that plane.

So, let us see what we are trying to do here. Right, ... and this beta is positive. Is it all right. So, now we are looking at directional stability and the stability is related with motion only in the directional or the XBYB plane. And this stability is related to the change in the side-slip angle. So now, we have to talk about derivatives with respect to this side-slip angle all right. So, similar to this condition, there is a condition also on the moment in the XBYB plane about the Z axis. So, what is that moment?

What is that moment called, moment about the Z axis, yawing moment. And I want to look at what happens to this moment with respect to the side-slip angle and that will define the directional stability. So, this is the derivative (dN/dβ) that I want to look at. If I want to write it in non-dimensional form, then you can use the yawing moment coefficient; ... this is how the yawing moment is written right, Q is the dynamic pressure, S is the reference area, which is the wing planform area, b is the wing span, and Cn is the yawing moment coefficient. We want to find out what is happening to this dN, when there is a small change in side-slip angle and this change is really small, you have to remember that.

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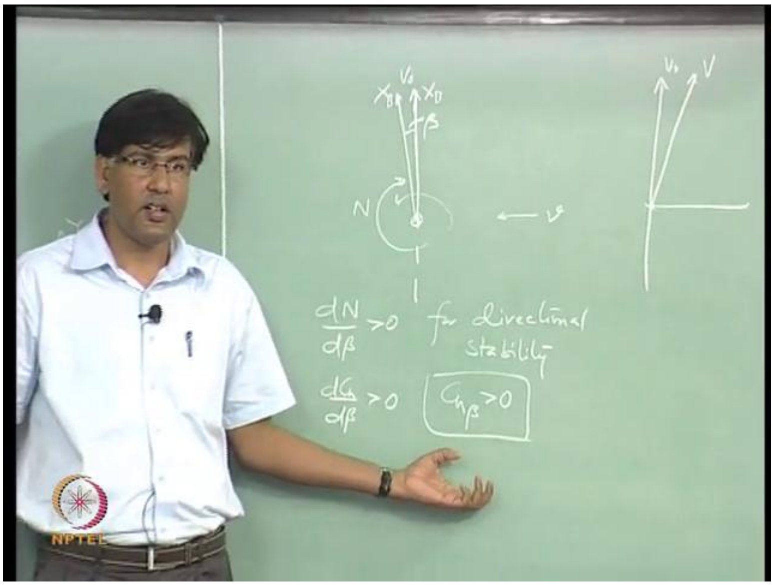


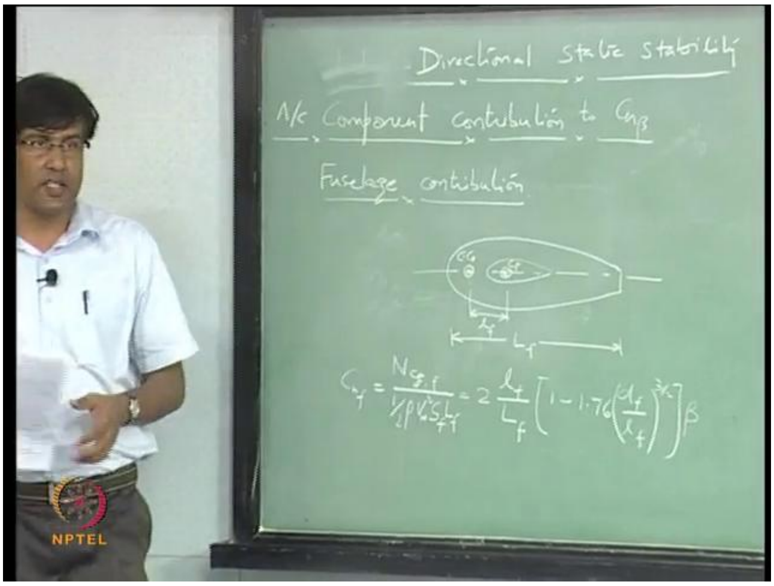
We are flying level condition. Now I am looking at what is happening only in this XBYB plane, so, we are neglecting the motion in the third direction. So, you are flying like this, now there is a velocity coming from the side that is like a disturbance; and that will have a tendency to introduce a beta, this is how the beta is defined. So, as soon as you see this, there is a beta introduced right, and that beta, so now you are still going with the same velocity in this direction, but there is a beta introduced like this, right, is this all right? So, you are going like this, right, and suddenly your Cg, looks like started moving in the other direction. So, my XB is this, but I am moving in this direction, right, many places, many books, you will find that they will use this kind of notation.

But this schematic will not, will be slightly difficult to follow. So, that is why I am trying to use this. See, you are going straight and then there is a (side) velocity, you see, that tries to introduce a moment, which is in this direction, that is how the aircraft has, XB axis of aircraft is not, not same as the old one right, because of this angle. So, there is a moment, and that moment is what, direction of that moment is, sign of that moment is, YB going towards XB. So, it is a negative moment; so negative moment is generated, which is trying to introduce this beta; and what stability property means?

The aircraft should be able to kill that beta. So, you have to generate a moment, which is in the positive direction and this is restoring (()) force, the aircraft has to generate automatically.

This beta is positive; so positive change in side-slip angle has to be followed by a positive change in the yawing, yawing moment right, that is when you are going to kill that beta. So, what should be the condition of stability?

This is, what has to happen, to kill this beta....(N beta > 0) So, every aircraft should posses this property for directional stability, clear? Now if I want to write that derivative in terms of this yawing moment coefficient, then I can also say that, in the short form is written as .... (Cn beta > 0) So, you have to design your aircraft in such a way that you are satisfying this. Now similar to Cm alpha there will be contribution from each component of aircraft to this derivative also.



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$$C_{nf} = \frac{N_{CG.f}}{\frac{1}{2}\rho V_\infty^2 S_f L_f} = 2\frac{l_f}{L_f}\left[1-1.76\left(\frac{d_f}{l_f}\right)^{3/2}\right]\beta \qquad (1)$$

This is for aircraft, aircraft components contribution to Cn beta. So now, we start doing the same exercise, what we did for Cm alpha and let us look at each one of them, one by one. Fuselage contribution; now let us draw the same picture, which I have drawn for Cm alpha and so we have our fuselage looking like this .... length of the fuselage is L f. Let us say cg is located here, which is at a distance ... I am looking from the top. Let us call this some distance small L f right. So, this is the distance between the center of pressure and the center of gravity of the airplane.

And this yawing moment coefficient for this fuselage, it is, is with respect to the fuselage parameters, if I have to talk about delta Cn now for the whole airplane, then I have to take wing as the reference condition.

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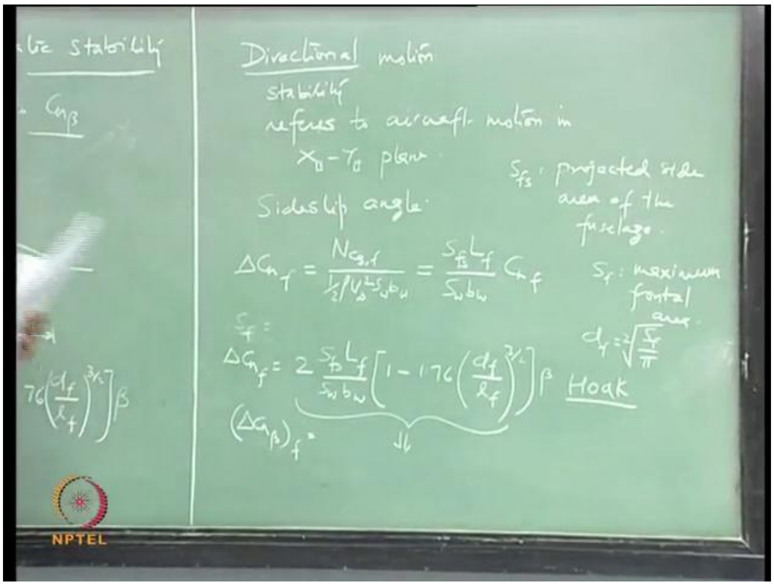

$$\Delta C_{n,f} = \frac{N_{cg,f}}{\frac{1}{2}\rho V_\infty^2 S_w b_w} = \frac{S_f L_f}{S_w b_w} C_{n,f}$$

$$(\Delta C_{n\beta})_f = 2\frac{S_f L_f}{S_w b_w}\left[1 - 1.76\left(\frac{d_f}{l_f}\right)^{3/2}\right] \qquad (2)$$

So, if I have to say, contribution to Cn due to fuselage for the whole airplane, then I have to use this (wing parameters) as the reference condition. What is Sf? Sf is the maximum frontal area, right, when you look at this cross section, you have to take the maximum frontal area, whatever you see as the maximum area. Wait a minute, wait, that is not correct; this Sf is the projected side area of the fuselage.

And this d_f is corresponding to the frontal, maximum frontal area that you see. So, if I say that area is Sf then d_f is ....Refer Eq(2).

So now, I have to take the derivative of this quantity with respect to beta, so, we get this expression. So, this expression is again given by Hoak in 1960s, but there is another relation, which is available in many books and let us look at that as well.

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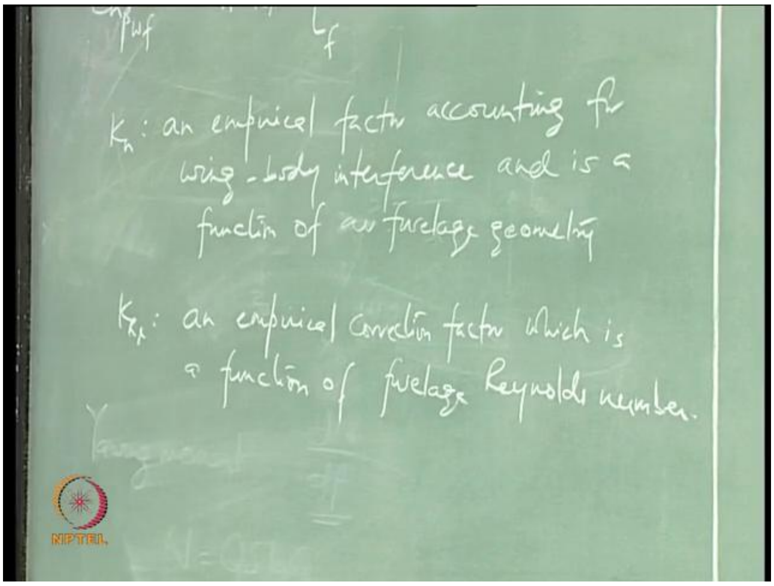

K_n: an empirical factor accounting for wing-body interference and is a function of fuselage geometry

K_{R_l}: an empirical correction factor which is a function of fuselage Reynolds number.

This is for wing fuselage combination, ...Refer Eq(2). What else, what else will contribute to this moment, or derivative. Which, which other component will contribute to this derivative C n beta. Vertical tail, vertical tail is a stabilizer.

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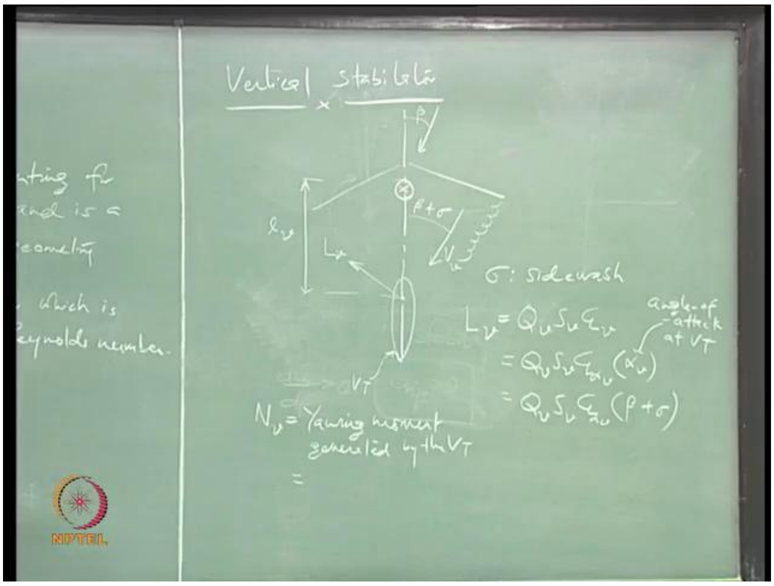

So, vertical stabilizer, by the name itself, it says that contribution coming from the vertical tail should be positive. Let us try to derive an expression for the contribution coming from the tail. So, this is your vertical tail, I am looking from the top. This is how wind is coming on to the aircraft now at slide slip angle beta. So, tail is definitely lying behind the engine. We should also accommodate the propeller, or the effect coming from the vortices, trailing edge vortices. And here the effect of the trailing edge vortices on the wind is to increase the angle of attack. beta is like an angle of attack for this tail. So, effect of the trailing edge vortices is to change the angle of attack of this free stream, and that change in angle of attack at the tail is positive. If there is a beta, so trailing edge vortices will be, you know, if you look at, let us say, this is your wing, right, and you are at some angle beta, so, you see trailing edge vortices coming like this, and, that is going to change the magnitude and also the direction of this velocity vector, which the vertical tail is seeing. And in this case, it is higher than beta, and the angle which we called the earlier downwash, here it is called side-wash.

And let us say this is the aerodynamic center of the vertical tail. So, if this is V, the small v is for vertical tail. So it is going to generate a lift at the tail right, and how do you quantify that? Lv is, is almost like half rho V square CL. So, same expression, because if you look at the section of the vertical tail, it is an airfoil, right, and the angle of attack is this beta plus the side-wash angle, ... angle of attack at vertical tail $\alpha_v = \beta + \sigma$. So, but that is not what we want to find out, we want to find out the moment generated by this force. So, let us say this distance is lv. So, what is the moment generated, let us also assume that these angles are small, actually beta is small, beta is a very small angle and side-wash angle is also small. So, I can assume that this Lv is almost perpendicular to this line, right, and then I have to take the moment of, you know, that force about the cg. So, this is roughly Lv into this l v right, capital L v into small l v. And what is the direction, what is the sign of this moment? Positive, right.

$$N_v = Q_v S_v C_{L\alpha v}\alpha_v . l_v = Q_v S_v C_{L\alpha v}(\beta+\sigma). l_v$$

$$(\Delta C_n)_v = \underbrace{\left(\frac{Q_v}{Q_w}\right)}_{\eta_v}\underbrace{\left(\frac{S_v l_v}{S_w l_w}\right)}_{V_v} C_{L\alpha v}(\beta+\sigma) \Rightarrow (\Delta C_{n\beta})_v = \eta_v V_v C_{L\alpha v}\left(1+\frac{d\sigma}{d\beta}\right) \qquad (3)$$

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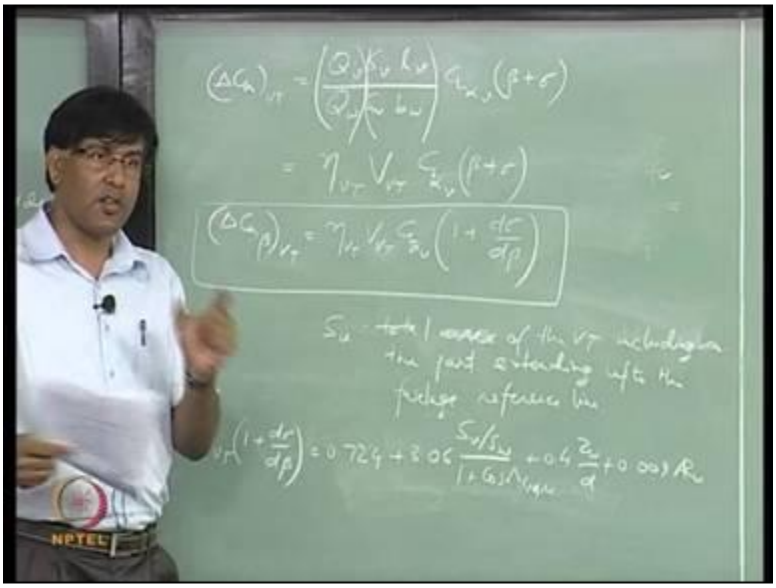

Now the total contribution to this airplane delta Cn due to vertical tail will be ...Refer Eq(3). right, this ratio is the ratio of two dynamic pressures .... Refer Eq(3).. you can call this tail efficiency (eta v) of the vertical tail. And what is this? Vertical tail volume ratio (Vv). What is this quantity (CLalpha v)?

Lift curve slope of the tail. So, vertical tail is supposed to provide stability to the aircraft in this plane. So, we are seeing here that, that is indeed true. The Sv here is the area of the vertical tail including that part which is going inside the fuselage. Vertical tail there will some portion, which will be in the fuselage. So, this is total, ...Refer Eq(4). there is a relation available for this part, which is … Refer Eq(4) So, you know, what is aspect ratio of the wing? Sv you know, Sw you know; this is sweep of the quarter card line on the wing.

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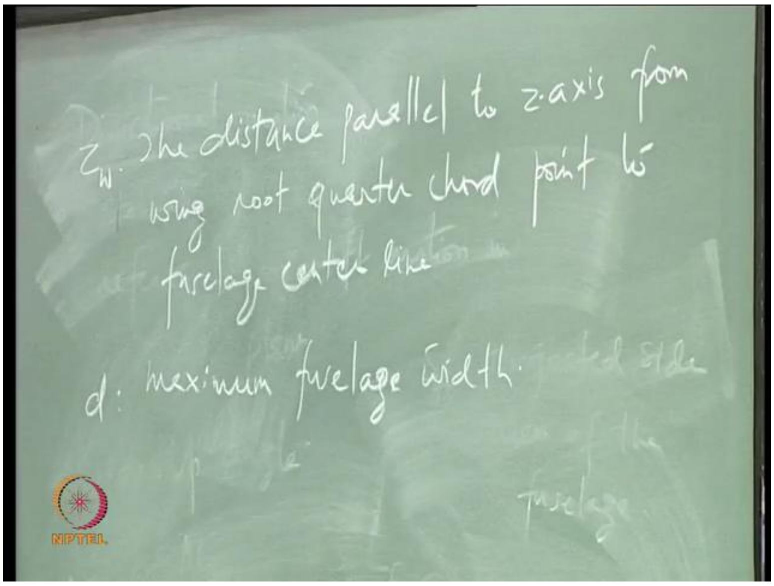

z_w: The distance parallel to z-axis from wing root quarter chord point to fuselage center line

d: maximum fuselage width.

$$\eta_v\left(1+\frac{d\sigma}{d\beta}\right)=0.724+3.06\frac{S_v/S_w}{1+\cos\Lambda_{c/4,w}}+0.4\frac{z_w}{d}+0.009AR_w \qquad (4)$$

And Zw .... right. Which other component, you have several components on the aircraft. So, what else, nacelle will be, nacelle is looking same as fuselage. So, if you want to calculate contribution of nacelle to this Cn, then you can calculate in the same manner, right. What about the wing?

Now, wing is located in a different fashion on the airplane, right, it is located on the top, in the middle, right, or at the bottom of the fuselage, and how else? Some wings are having wing sweep, right, some wings are having some elevation, right, like this or going down, diahedral, anhedral and, yeah so all those effects we have to see, right. We will look at all those effects in the next class.