Flight Dynamics II (Stability)

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Module No. # 06

Longitudinal Control and Maneuverability

Lecture No. # 17

Example Problems

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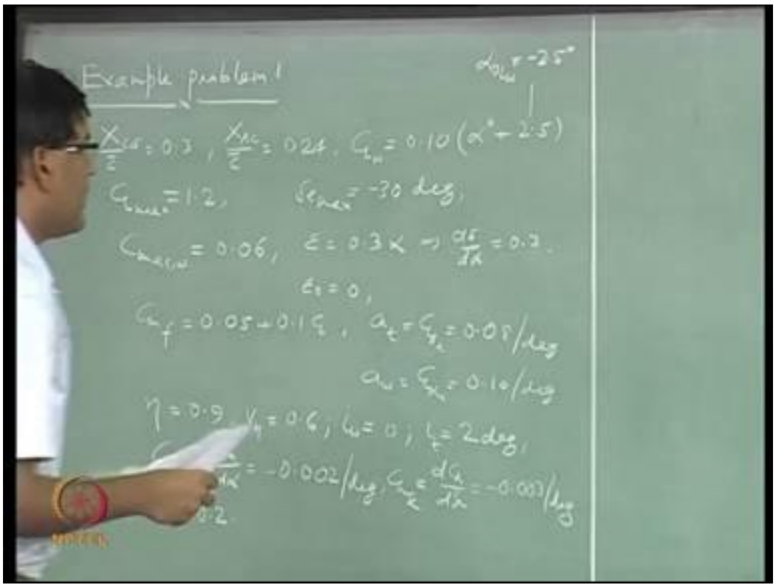

$$\frac{X_{CG}}{\bar{c}} = 0.3, \frac{X_{AC}}{\bar{c}} = 0.24, C_{Lw} = 0.1(\alpha + \overbrace{2.5}^{\alpha_{0w}=-2.5\deg}), C_{L\max} = 1.2, \delta e_{\max} = -30\deg$$

$$C_{mAC,w} = 0.06, \varepsilon = 0.3\alpha, \varepsilon_0 = 0, C_{mf} = \underbrace{0.05}_{C_{m0f}} + 0.1C_L, \underbrace{C_{L\alpha t}}_{a_t} = 0.08/\deg, \underbrace{C_{L\alpha w}}_{a_w} = 0.1/\deg \quad (1)$$

$$\eta = 0.9, V_H = 0.6, i_w = 0, i_t = 2\deg, C_{h\alpha} = -0.002/\deg, C_{h\delta e} = -0.003/\deg$$

So, we have an aircraft, for which, we have some data available. (Refer Eq(1)) This alpha is alpha at 0 lift, so alpha, your 0 lift angle of attack of the wing is minus 2.5 degrees. CLmax given as 1.2, and there is a CL max, what you get, so, we talk about CL of the wing right, and this CL max you get when you have maximum elevator up deflection right, because you are trying to move the elevator up and aircraft is pitching up, is it not? So, this CLmax is corresponding to the delta e max that is given to you.

So, delta e max is given as minus thirty degrees; everybody is getting what I am saying. So, it should not be, CL max should not be calculated as, CL max of the wing minus CL max or plus CL max of the tail. This C L max is the C L max for the airplane, and we are assuming that the most of the lift is provided by the wing, and delta e is only to cause a pitching moment mainly.

So, this gives me, Cm due to fuselage, lift curve slope of the tail, and lift curve slope for the wing, you can find out from here, (Refer Eq(1)) that is… And this eta, which is the ratio of the dynamic pressures, is given as 0.9; VH is the tail volume ratio, wing incidence angle with respect to the fuselage reference line is 0, tail incidence angle is given as 2 deg. This (Ch derivative Refer Eq(1)) is related to the hinge. Elevator effectiveness parameter given as ... So, this data is given and what we need to find out are these ....(Refer slide below).

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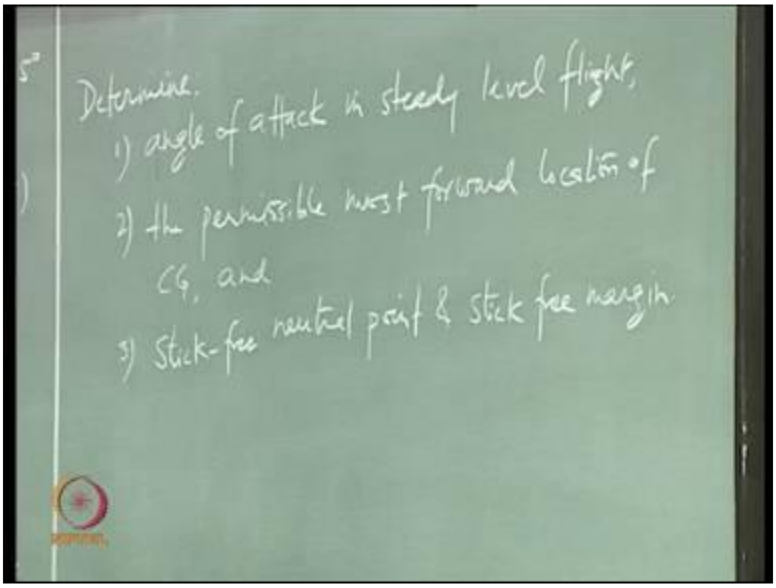

So, for this data given, you have to determine these three. So, let us first try to find some numbers; we know some formula which we should directly start using, and we can see, where we get, dCm over dCL.

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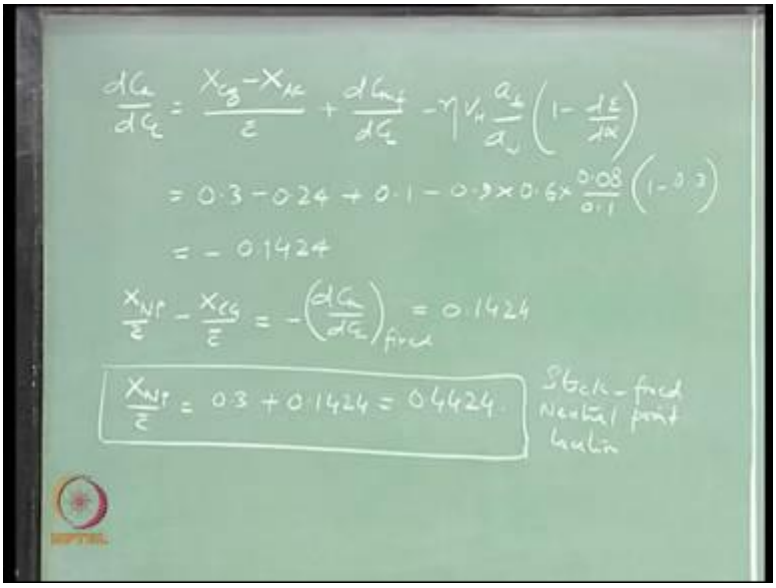

$$
\begin{aligned}
\frac{dC_m}{dC_L} &= \frac{X_{CG}-X_{AC}}{\bar{c}} + \frac{dC_{mf}}{dC_L} - \eta V_H \frac{a_t}{a_w}\left(1-\frac{d\varepsilon}{d\alpha}\right) \\
&= 0.3-0.24+0.1-0.9\times 0.6\times\frac{0.08}{0.1}(1-0.3) = -0.1424 \\
\frac{X_{NP}}{\bar{c}} - \frac{X_{CG}}{\bar{c}} &= -\left(\frac{dC_m}{dC_L}\right)_{fixed} = 0.1424 \\
\frac{X_{NP}}{\bar{c}} &= 0.1424+0.3 = 0.4424
\end{aligned}
\tag{2}
$$

See, everything is available, right; we know, this, we know this, we (Refer Eq(2)) also have fuselage pitching moment expression, and looks like everything else is there. So, we can find out what this derivative is, right. And what do we get from here? This is for stick fixed condition. So, we can find out, what is the neutral point location in this stick fixed case. Let us also try to use the trim condition, and find out what is CL trim.

$$
\begin{aligned}
C_{mCG} &= 0 = C_{m0} + \left(\frac{dC_m}{dC_L}\right)_{fixed} C_{Ltrim}, \\
C_{m0} &= C_{m0w} + C_{m0f} - \eta V_H a_t(-\varepsilon_0 + i_t - i_w) \\
&= 0.06+0.05-0.9\times 0.6\times 0.08\times(-0+2-0) = 0.1316
\end{aligned}
\tag{3}
$$

So, CmCg, ... for the whole aircraft and Cm naught is ...(Refer Eq(3)) So, angles have to be in degrees, right, because this is per degree. So, what is CL trim? We can find out, from this expression, CL trim is .... (Refer Eq(4)) now what can you find from this? CL trim is given, now we have found it, right.

$$C_{mCG} = 0 = C_{m0} + \left(\frac{dC_m}{dC_L}\right)_{fixed} C_{Ltrim} \Rightarrow C_{Ltrim} = -\frac{C_{m0}}{\left(\frac{dC_m}{dC_L}\right)_{fixed}} = -\frac{0.1316}{-0.1424} = 0.9242 \qquad (4)$$

And what I am saying is, most of the lift is being provided by the wing. So, we are not taking into account the lift produced at the tail, which is very small as compare to the lift produced at the wing. And if I use that, I can find out what alpha trim is, right.

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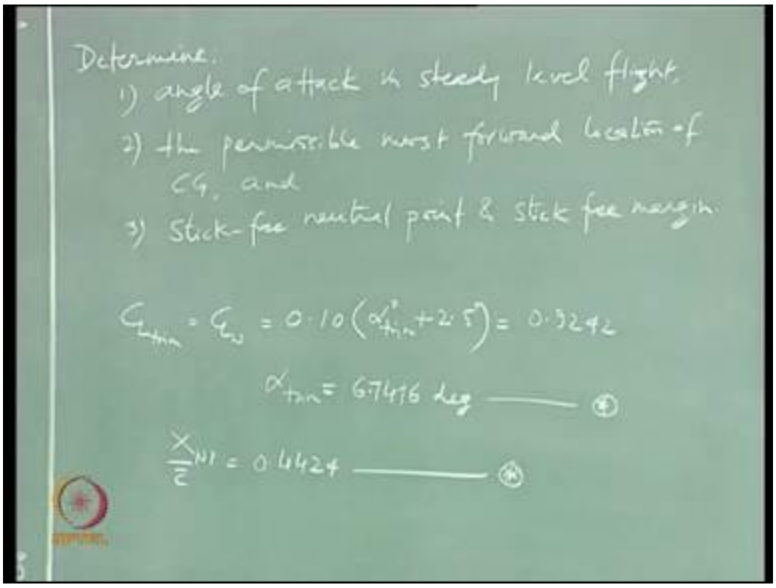

$$\underbrace{C_{Ltrim}}_{0.9242} = C_{Lw} = 0.1 \times (\alpha_{trim} + 2.5) \Rightarrow \alpha_{trim} = 6.7416\,\text{deg}$$

$$\frac{X_{NP}}{\bar{c}} = 0.4424 \qquad (5)$$

So, C L trim is, ...Refer Eq(4,5). Any question? Let us try to find out the elevator control power; Cm delta e.

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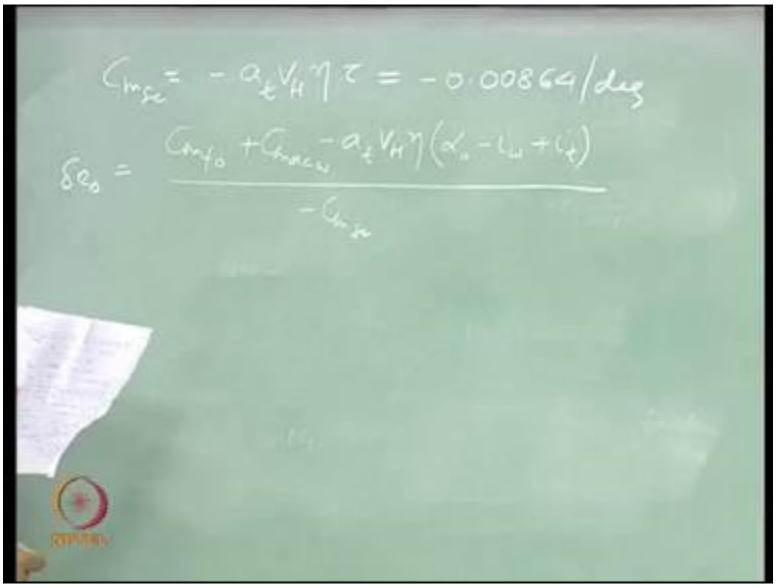

$$C_{m\delta e} = -a_t V_H \eta \tau = -0.00864/\deg$$

$$\delta e_0 = \frac{C_{m0f} + C_{mACw} - a_t V_H \eta (\alpha_{L0w} - i_w + i_t)}{-C_{m\delta e}} = 15.2315\deg$$

$$\delta e = \delta e_0 + \left(\frac{d\delta e}{dC_L}\right) C_L = \delta e_0 + \left(\frac{dC_m}{dC_L}\right) . \frac{C_L}{(dC_m / d\delta e)} = \delta e_0 - \frac{\left(-\frac{dC_m}{dC_L}\right) C_L}{C_{m\delta e}} \qquad (6)$$

$$\delta e_{\max} = \delta e_0 - \frac{(X_{NP} - X_{CG,\max fwd})/\bar{c}}{C_{m\delta e}} .C_{L\max} \Rightarrow \frac{X_{CG,\max fwd}}{\bar{c}} = 0.1167$$

We are going to use now, this information to find out the forward most location of the Cg. So, we have to see, what is the effect of deflecting elevator on the trim condition. So if you remember, Cm delta e is this ....Refer Eq(6). Everything you know here. So, you can find out the number. Now delta e naught which is the trim value of the elevator, you know, to give you alpha trim 0, right. So, trim at angle of attack which is 0, you have to deflect the elevator by delta e naught right, and that you can find out again from the relation, which is CmCg equal to 0.

Any question?

So, this is .....Refer Eq(6); I am setting the alpha trim to 0, but still have this 0 angle of attack of the wing. alpha w here is now also including that alpha 0 l w.

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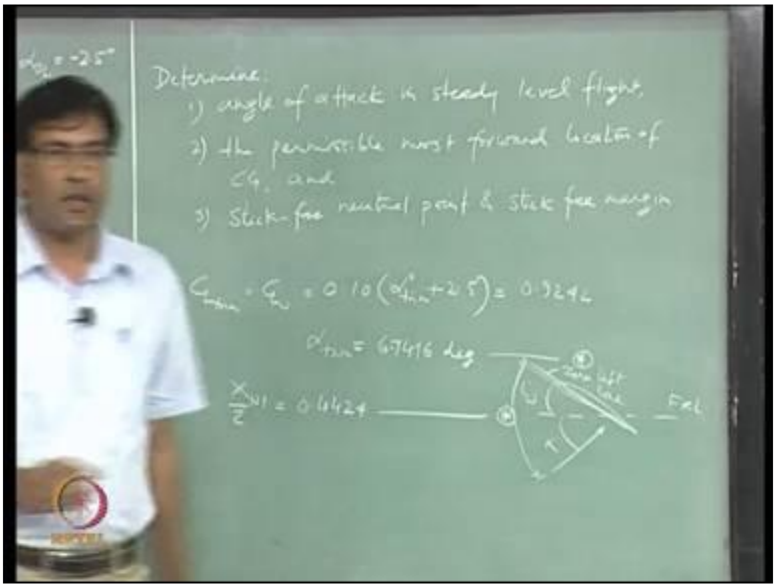

Remember we are talking about, so this is the fuselage reference line, and your alpha is this, right; what is alpha w? So, wing is, lets say,..... So, one angle is joining the two points, the chord, that is making an angle with this fuselage reference line is i w, incidence angle of the wing.

And there will be another angle, which will be associated with the line, you know, which is giving you the 0 lift line, and that can be different, a different line than the chord. So, you have to account for that. So, total angle of attack at the wing will be this (Refer Eq(6)).

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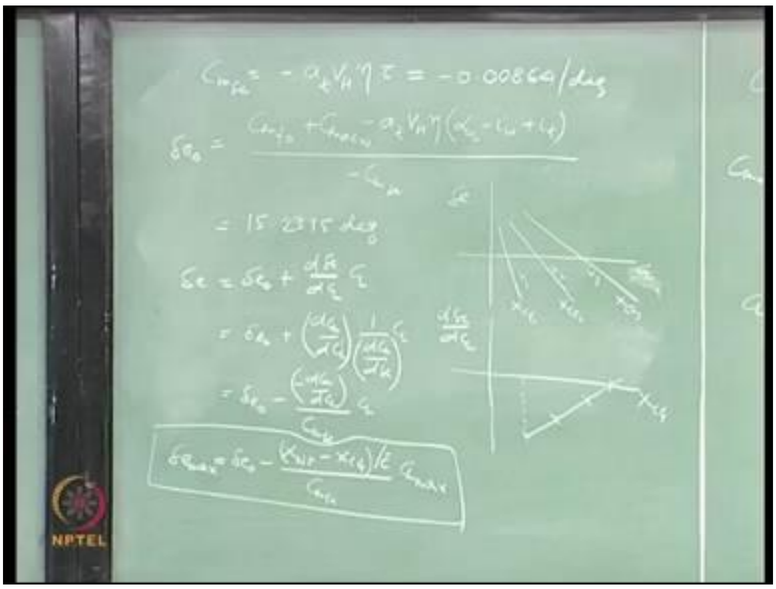

So we have all the numbers here, we can find out what is delta e naught. What I need to find out is the permissible most forward location of CG. So, I use this simple relation which is …Refer Eq(6). If you remember I had drawn this plot of elevator deflection versus CL, right, and for different velocities and different Cg locations, and what I did next was, plot, to get the neutral point location.

So, now, clearly, the most forward location of the Cg is somewhere here, which is going to depend upon what delta e is, right; and then we joined this and found the neutral point. A similar plot of delta e versus XCg can also be drawn. So, different velocities for different Cg, you find this derivative and extend this line, so that it will cut this X axis somewhere, that will give you the neutral point location.

So, this came from where? This came from flying different velocities for different Cg locations and plotting delta e trim versus CL trim. And we found that this relation is linear, so, we can actually write this expression in this form.

So, lets try to do some algebraic manipulation, and so, writing this as, ...Refer Eq(6). So, corresponding to this delta e max, there is going to be a CL max, is it not? And that CL max is of the whole airplane; ..... what is this quantity? XNP minus XCg (refer Eq(6)).

But now, this XCg is different; what is this XCg? Corresponding to delta e max, this XCg is the most forward location of the Cg. So, I can put the small f there to indicate that this Cg location is the most forward, location of the C g (refer Eq(6)).

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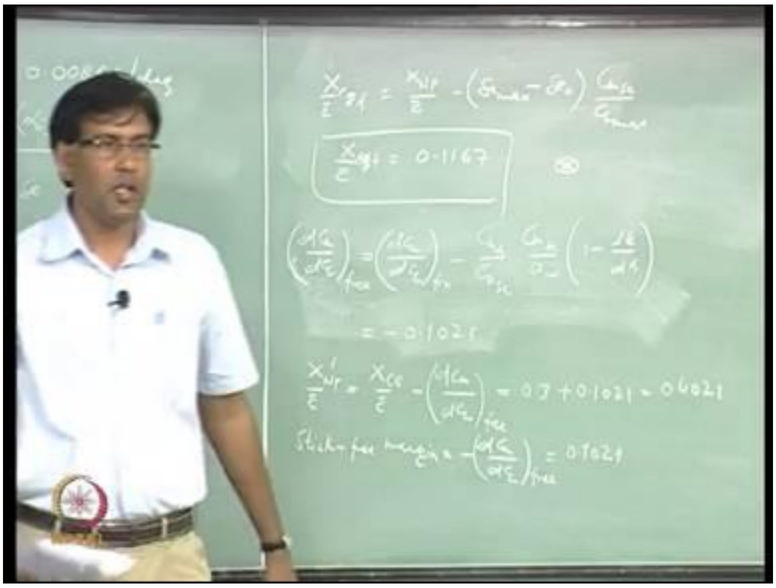

So, I can find out ... the third problem is finding the stick free neutral point and stick free margin. So if I find this stick free neutral point, then I can find out what is stick free margin,..... So, this is a relation between the dCm over dCL free, and dCm over dCL fixed, which includes the derivatives coming from the hinge moment, right.

So, we know all these numbers here, you can find out what is this quantity. So, XNP free, is stick free neutral point ...... Refer Eq(7).; and the stick free margin is, this minus this, right, which is ..... which is ten percent of the mean an aerodynamic chord. Yeah, if you are going back to your, the same question, I will answer that later.

$$\left(\frac{dC_m}{dC_L}\right)_{free} = \left(\frac{dC_m}{dC_L}\right)_{fixed} - \frac{C_{h\alpha}}{C_{h\delta e}} \cdot \frac{C_{m\delta e}}{a_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) = -0.1021$$
$$\left(\frac{X_{NP}}{\bar{c}}\right)_{free} = \frac{X'_{NP}}{\bar{c}} = \frac{X_{CG}}{\bar{c}} - \left(\frac{dC_m}{dC_L}\right)_{free} = 0.3 + 0.1021 = 0.4021 \qquad (7)$$

(( ))

Yes that.

Sir, it should have been a single minus sign sir.

Which which point you are?

That delta e naught minus minus dCm by dCL; you have written second line sir (( )), it should be delta e naught plus minus (( ))

Minus minus – plus; I am writing this expression, right.

That is your (( )), but previously you have given (( )).

We can talk about that.

And also if CL is maximum, then if you substitute CL max, then delta e is will be minimum; you put a minus sign there, if you see that boxed expression.

Delta e is maximum up, up is what, up is negative, right; up deflection of elevator is negative right.

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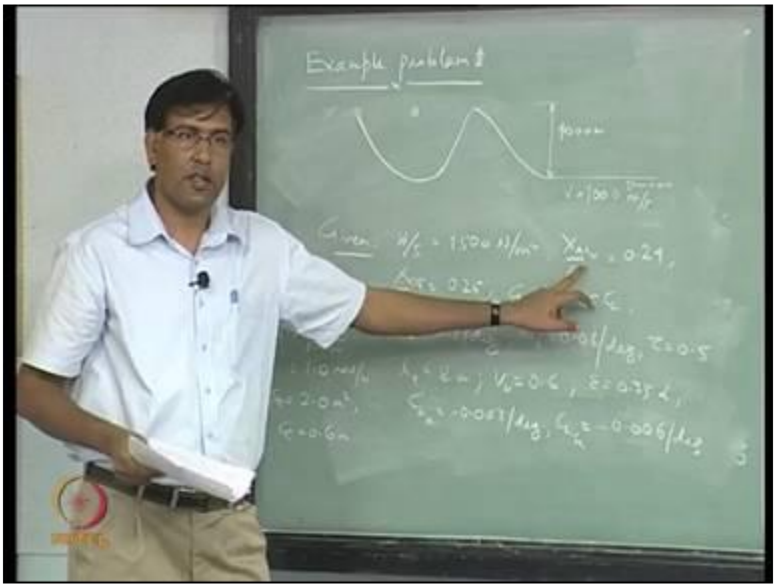

So, let us quickly discuss another problem. These problems are actually example problems available in (( )) Pamadi's book; you know Pamadi's books. There is one copy in the library. I will give you the name of the book after the class. So, here this problem: one trainer aircraft is flying level, and then it starts, this is at five kilometer above sea level, then starts climbing.

It climbs here, this height is now .....; it goes up to one kilometer extra while climbing, and then, it takes a dive; from this point, it takes a dive, so that, at the bottom, it touches this 5 kilometer height, and completes a semicircle. So, you are getting what I am saying, there is a trainer aircraft which is flying level at 5 kilometer, and then it starts climbing; and after reaching this additional one kilometer, it takes a dive.

$$W/S = 1500 N/m^2, \frac{X_{ACw}}{\bar{c}} = 0.24, \frac{X_{CG}}{\bar{c}} = 0.25, C_{mf} = 0.15 C_L, a_w = 0.1/\deg,$$
$$a_t = 0.06/\deg, \tau = 0.5, l_t = 8m, V_H = 0.6, \varepsilon = 0.35\alpha, C_{h\alpha} = -0.003/\deg, C_{h\delta e} = -0.006/\deg, \quad (8)$$
$$\eta = 0.9, G_1 = 1 rad/m, S_e = 2.0 m^2, c_e = 0.6m,$$

And it comes to the bottom of this path which is semicircular, and the bottom of this path is at the five kilometer level. And then, it completes this semicircle. So, what its doing? It is taking, it is pulling up. So, first it climbed, and then it took a dive, and after, after it reaches here at the bottom, its trying to pull up. So, this is level flight cruise, at velocity which is hundred meter per second.

So, given some data (Refer Eq(8)) which is, which are like this; wing loading is 1500 Newton per meter square, ... Can you read everything? This wing loading which is 1500 Newton per meter square, location of the wing aerodynamic center, Cg location, pitching moment due to fuselage.

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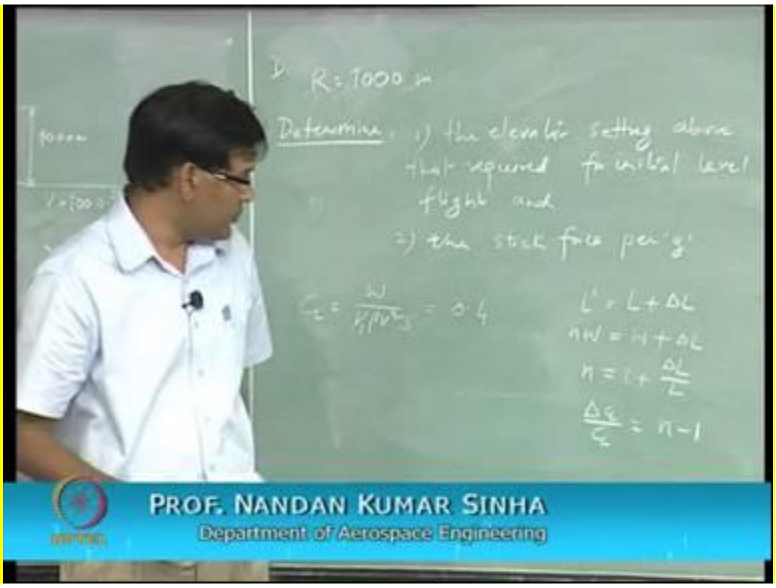

Lift curve slope of the wing, lift curve slope of the tail, elevator effectiveness parameter, this is tail arm length, right, which is eight meter, tail volume ratio 0.6, down wash is 0.35 into angle of attack alpha, this is Ch alpha and, Ch delta e.

Tail efficiency, G1 is the gearing ratio, which is connecting the stick force with the gear. So, that gearing ratio is this 1 radian per meter, you are getting, so, you are deflecting the stick by some amount that is in meter and that is going to rotate the elevator in radians.

Se is the planform area of the elevator part, behind the hinge line, Ce is the chord, which we are measuring behind the hinge line, chord line of the elevator.

So, clearly if the radius that it is trying to pull through is thousand meter, its clear from here. So, let me tell you what we are trying to determine, one ......

So, we are pulling the g maneuver which is above the level flight condition, that extra g that you are trying to pull .... I will quickly run through the solution, you can …

So, what is happening? We are pulling a maneuver, right, this pull up. So, alpha is changing at the tail that we have discussed earlier, and so at the level flight condition what is the CL how do you find that?

You have the velocity. So, you can find out, you have velocity, you have this wing loading given, right. So, I can find out what is CL which is .... this rho is at... 5000 meter.

So, what are we doing, we are adding some lift, right, when we are trying to pull the aircraft, right, this L prime is n into W right, and this L is W. So, I can write this as ..... .

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S} = 0.4, \quad L' = L + \Delta L \Rightarrow nW = W + \Delta L \Rightarrow n = 1 + \frac{\Delta L}{L} \Rightarrow \frac{\Delta C_L}{C_L} = n - 1 \tag{9}$$

So, delta CL that we need right to pull this maneuver is related to this C L which is at the level flight condition and like this .... . So, extra delta CL that you need to execute any maneuver which is… So, this n is greater than 1.

$$\frac{d\alpha_i,t}{dC_L} = \frac{d\alpha_i,t}{dn} \cdot \frac{dn}{dC_L} = \frac{l_t g}{V^2 C_L} = \frac{l_t g \rho S}{2W} = \frac{1}{2\mu_1}$$
$$\mu_1 = \frac{W}{l_t g \rho S} = 25.4842 \tag{10}$$

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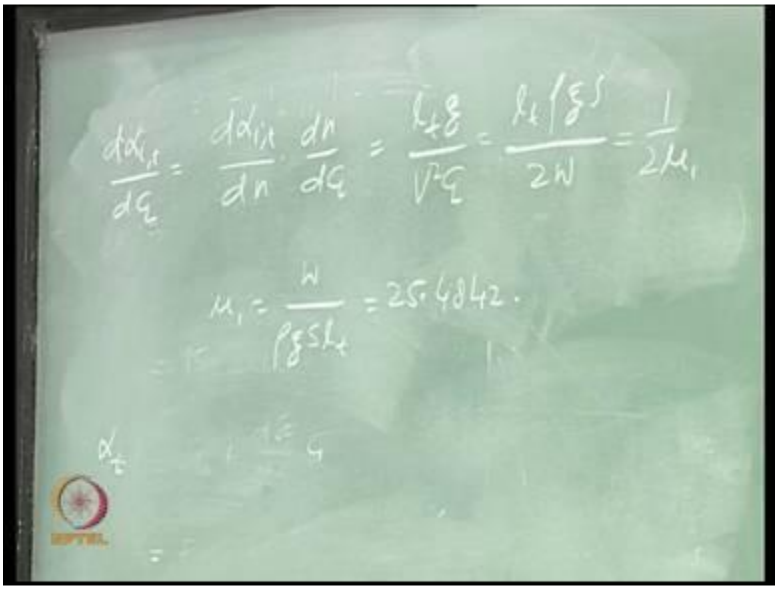

Now, this change in angle of attack that you see at the tail will also depend upon the CL that you want.

Let us call this, this, this is like some density parameter, let us call this mu one. So, mu one, I can find out and that is ....Refer Eq(10), let us also find out what is... α_t

Both these problems are solved in Pamadi's book.