Flight Dynamics II (Stability)
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Module No. # 06
Longitudinal Control and Maneuverability
Lecture No. # 16
Elevator per 'g', Maneuver Point

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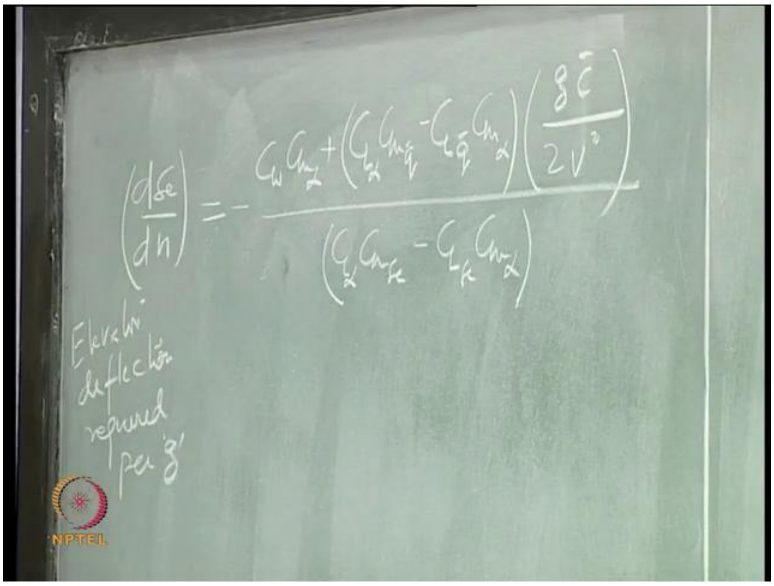

So, little bit more on the this maneuverability. We wrote this expression ...Refer Eq(1) this is elevator deflection required per g, right!

$$\left(\frac{d\delta e}{dn}\right) = \frac{C_W C_{m\alpha} + (C_{L\alpha} C_{m\bar{q}} - C_{m\alpha} C_{L\bar{q}})\dfrac{g\bar{c}}{2V^2}}{(C_{L\alpha} C_{m\delta e} - C_{m\alpha} C_{L\delta e})} \qquad (1)$$

Setting this to zero will give you, ... what does it mean? Setting this zero means you can get large acceleration even without deflecting the elevator, right. So, elevator becomes very sensitive, and any small deflection in elevator can give you large normal acceleration right, that is what small of this quantity means.

And it is a function of the c g location, right, Cm alpha is a function of c g. So, c g location where this becomes zero is the point which is called maneuver point, right! say stick-fixed ....

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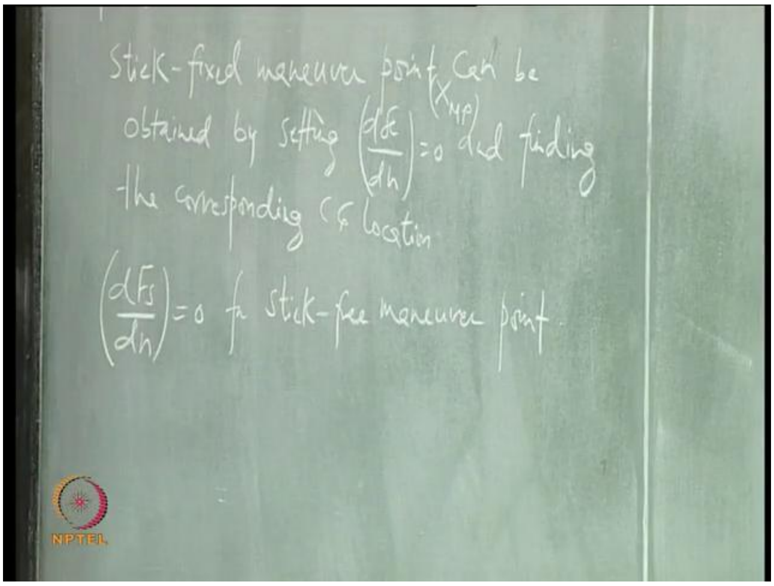

We also have an expression for. So, can you read this? Stick-fixed maneuver point can be obtained by setting this derivative to zero and finding the corresponding center of gravity location, right!

If I want to find out stick free maneuver point, then what is the derivative that I have to set to zero, set this dFs over dn to zero right. So, I will call this stick-fixed maneuver point as XMP and, if you want to see how its related to neutral point, stick-fixed neutral point .....

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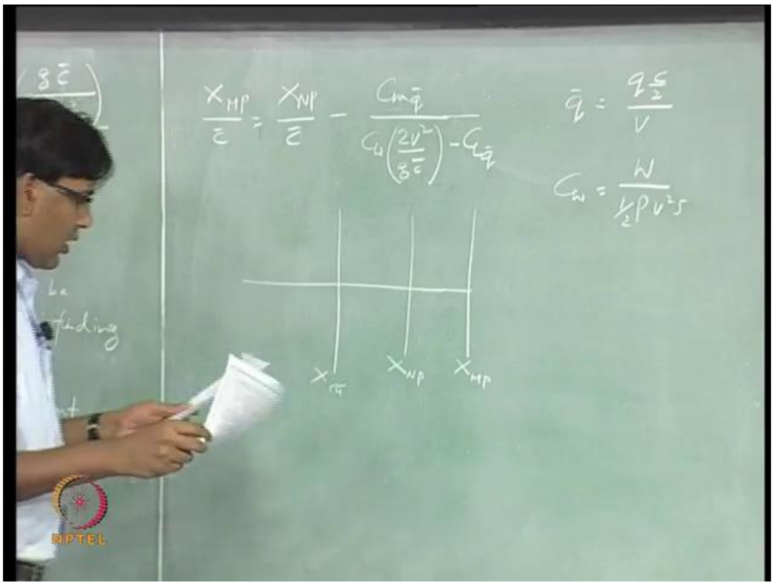

$$\frac{X_{MP}}{\bar{c}} = \frac{X_{NP}}{\bar{c}} - \frac{C_{m\bar{q}}}{C_W\left(\frac{2V^2}{g\bar{c}}\right) - C_{L\bar{q}}}; \bar{q} = \frac{q(c/2)}{V}, C_W = \frac{W}{\frac{1}{2}\rho V^2 S} \qquad (2)$$

So, clearly this is going to lie aft of the neutral point because we said that Cmq is negative for positive damping in pitch and this quantity is positive because Cw is the weight coefficient which is going to be very large, right! So, CW is, is very large as compared to this CLq.

So, this is where the location of the maneuver point is, and cg is here, right! So, I want to look at let us say what is happening to this elevator control requirement per g of normal acceleration with respect to this c g movement, right, c g can change and that can be a design criteria, right!

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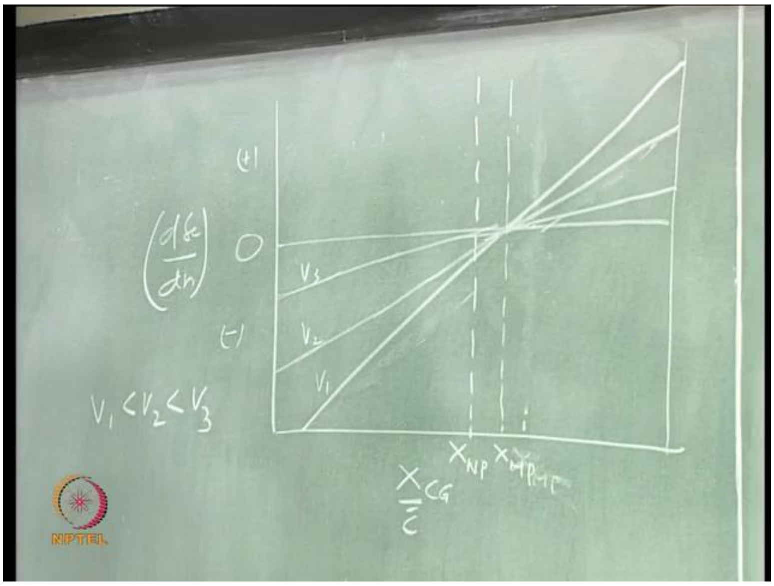

So, only a few more things that I want to say about this. So, my X axis is the c g location, let us draw the .... Refer slide above, and let us start moving the c g to see how c g is going to change, you know, this control requirement with respect to velocities. So, I can plot this against, this for different velocities. So, I am trying it for three different velocities.

So all of them actually pass through this point for different trim speeds, this V_1, V_2 and V_3. V_1 is less than V_2 and V_2 is less than V_3. You are flying a level trim when you want to initiate a maneuver, and velocity is a constant, in this maneuver, the maneuver that we are talking about, the pull up maneuver, right, from a level trim condition.

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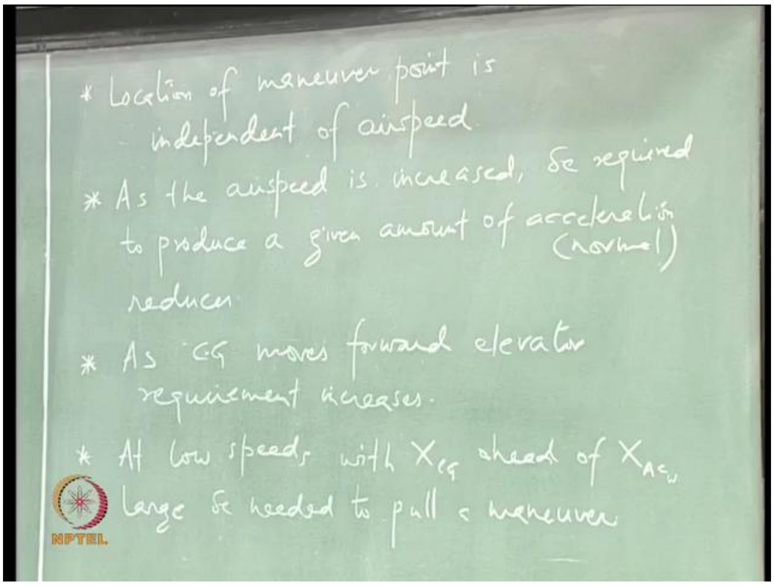

So, this is how it looks like. There are few things that you can note down, right, all because the curves are passing through this point and that point is the maneuver point, its independent of airspeed,..... right.

As the airspeed is increased for a fixed c g location, you can see that elevator required has reduced. Moving forward means going towards the aircraft nose, right....

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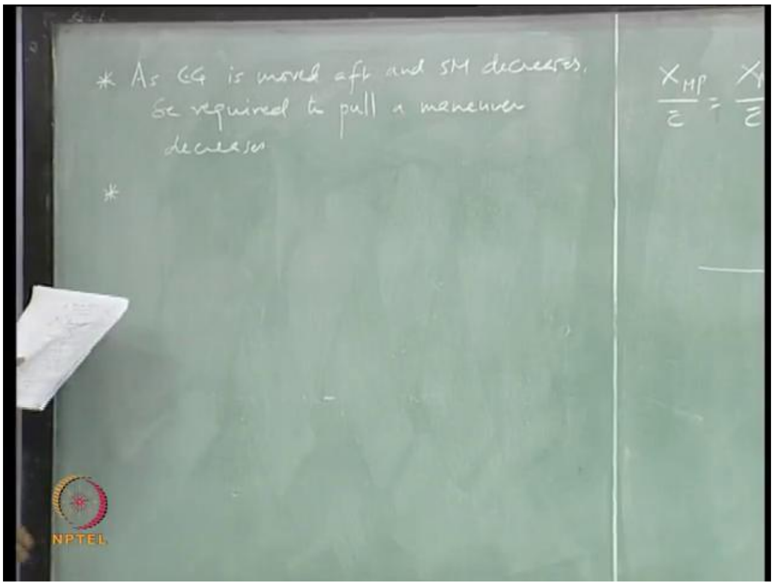

So, when we are moving the c g aft, towards the neutral point, static margin is, static margin is decreasing, what happens, less stability! So, I can maneuver better, right. Take any trim speed, and you are moving your XCG towards XNP, right. So, this curve is coming closer to this zero line right. So, elevator requirement is actually decreasing.

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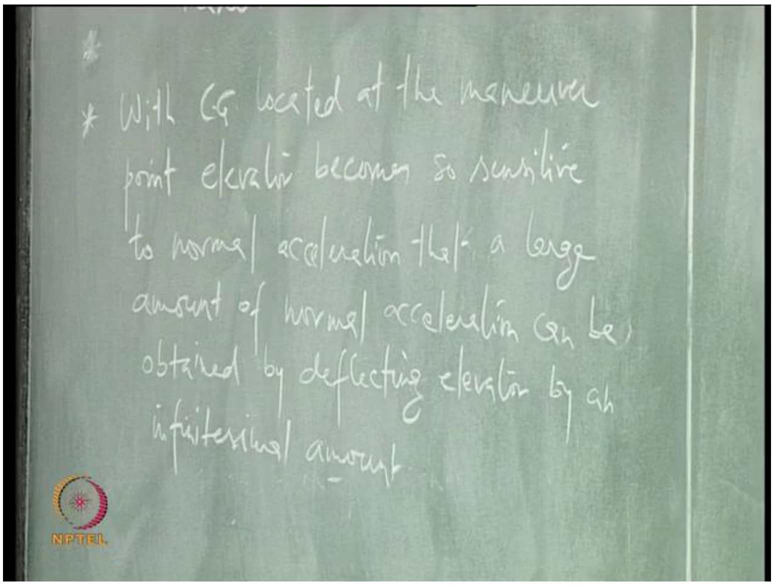

Of course, with large transport aircraft you do not want to maneuver, right, no larger transport aircraft will actually maneuver. So, this will be what category of airplane?

(( ))

Fighter and ah.

Military transport.

No military transport also.

(( ))

Aerobatic. Yeah.

Right! What happens when CG crosses this maneuver point? Control reversal, right! So, now, when you are taking your elevator up, or in this case, to get the normal acceleration that is demanded, you have to put your elevator down, right!

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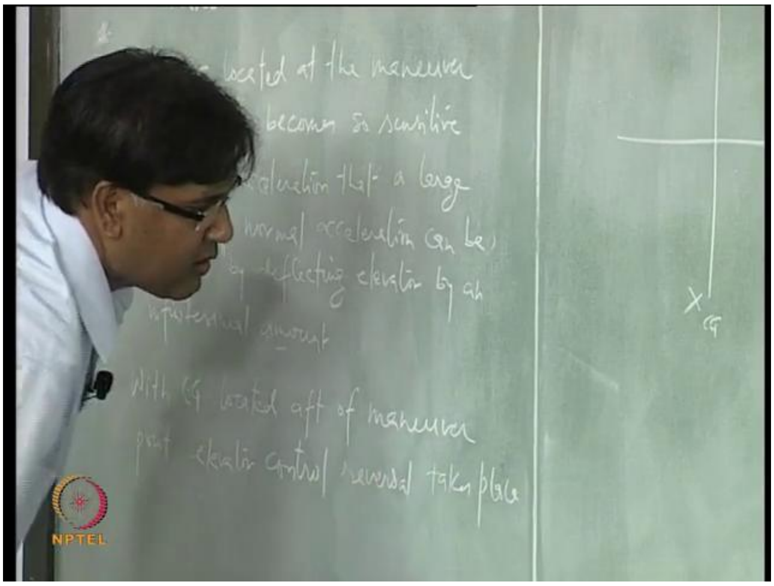

Of course, the aircraft also has become unstable, is it not? You are crossing this XNP, so, aircraft has become unstable, right, and this is what is happening. So, any any acceleration that you want in normal direction, this sign of delta e will be changed, reversed, that is what it means.

But normally you would not want to cross this XNP, right. What it is all telling you is that you have this margin which is the maneuver margin, right! So, you can, your aircraft can be marginally stable, you know, it can lie very close to this XNP or marginally unstable, when it is going to cross this XNP, it can still maneuver and in maneuver it will be stable, right, it can still have a stable maneuver, right.

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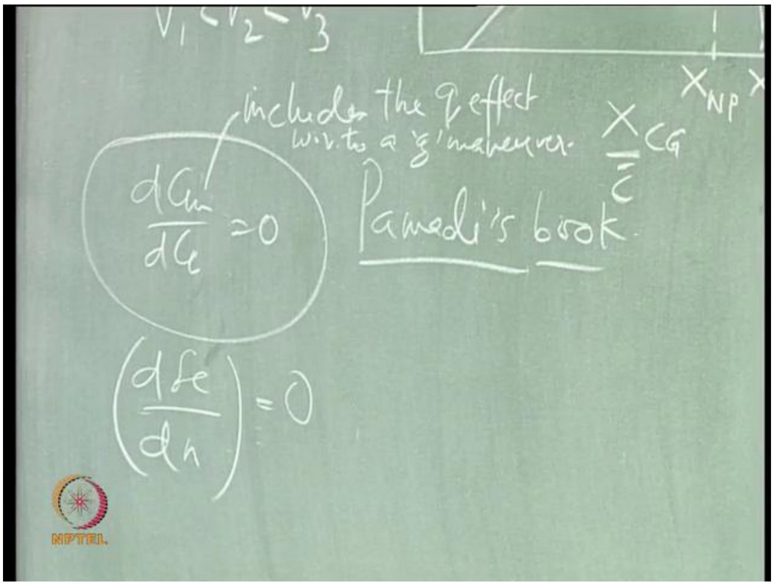

So, one of the ways to find the XMP, given in some other book, is also by solving, setting this to zero, but, with the q effect included, right. So, I would give you this as a problem for, know, homework problem. So, try to see what is the relation between....this and this. So, this dCm over dCL with pitch rate effect included, right, when you solve this for the c g location where this becomes zero is equivalent of finding c g location through this relation, and can somebody work this out? try this out.

So, this is how in Pamadi's book, this is how they are trying to find out the maneuver point, ok. And see if you get the same expression using the both relations.

Here this Cm includes the q effect with respect to g maneuver and we are still talking about longitudinal dynamics, right, longitudinal plane, we have not yet talked about maneuver in any other, you know, plane other than the longitudinal plane. So, see if you get the same expression for X m p by solving this (dCm/dCL)... as what you get by solving this (dδe/dn)....

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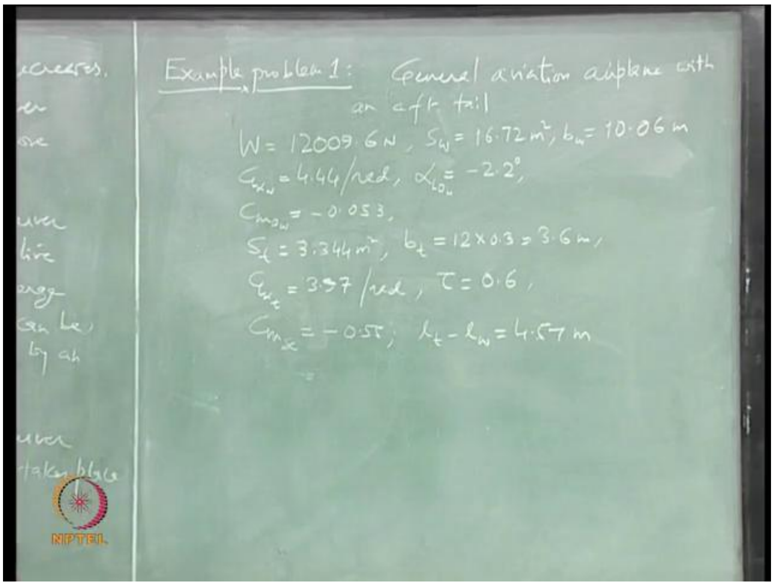

$$W = 12009.6N, S_w = 16.72m^2, b_w = 10.06m, C_{L\alpha_w} = 4.44/rad, \alpha_{L0_w} = -2.2\deg$$
$$C_{m0w} = -0.053, S_t = 3.344m^2, b_t = 3.6m, C_{L\alpha_t} = 3.97/rad, \tau = 0.6, C_{m\delta e} = -0.55 \qquad (3)$$
$$l_t - l_w = 4.57m$$

So we said we will solve some problems in this class, let us see. We have a general aviation airplane with an aft tail, ok. The weight of the airplane is this .. . So, I am converting all the units into SI units. Here they have given it in fps unit, all right, Sw is wing platform area, lift curve slope for the wing is 4.44 per radian. Zero lift angle of attack for the wing is minus 2.2 degrees, wing span .... horizontal tail area .... is given in feet square, trying to convert to SI units .... span of the horizontal tail – bt is 3.6576 roughly, CL alpha tail is 3.97 per radian. What is this now? Elevator effectiveness parameter, right, 0.6.

lt is tail arm length, lw is wing arm length, right. So, cg location from the aerodynamic center of the wing is this lw, right, and lt is the aerodynamic center of the tail, the distance between the aerodynamic center of the tail and the c g, right. ok.

Let us see what all we want to find out. This aircraft has a trim condition, level flight trim condition, which is at alpha equal to zero, and this alpha is alpha of the airplane, right!

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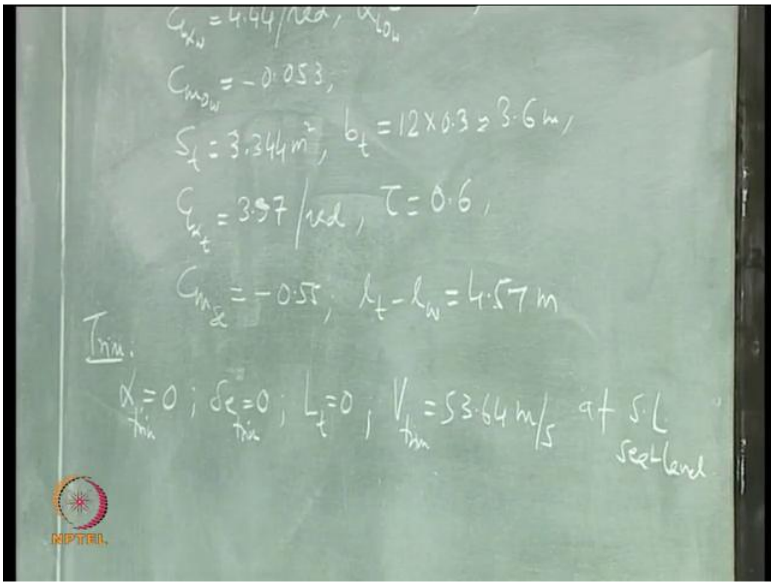

$$Trim: \alpha_{trim} = 0, \delta e_{trim} = 0, L_t = 0, V_{trim} = 53.64 m/s @ SL \qquad (4)$$

So, fuselage reference line and this we can obtain without changing the elevator, let us say, there is a condition like that, and at this delta e 0, there is no lift produced on the tail.

So, all of them are trim values at sea-level condition, we will also assume that the aerodynamic centers of tail and wing are lying on the fuselage reference line. So, there is no moment created because of the vertical distance and cg is also lying on the on the fuselage reference line. So, all that we are assuming.

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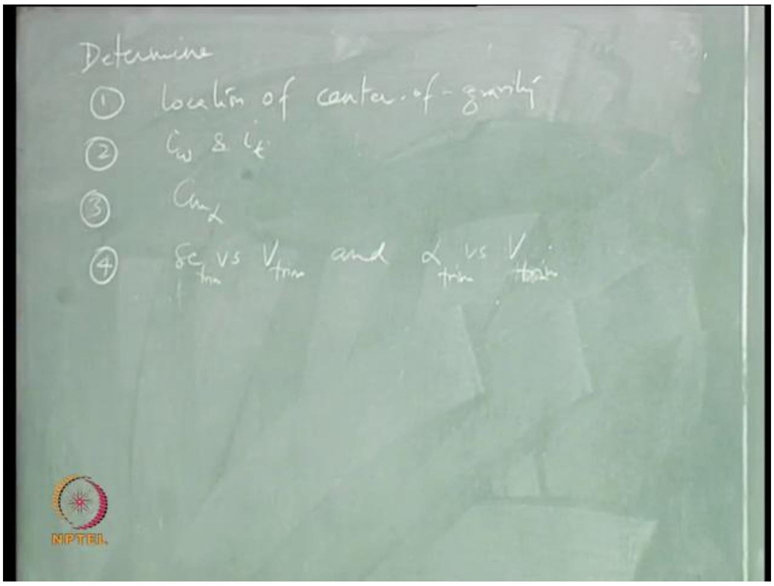

And the problem is to determine one location of gravity, cg! The wing incidence angle and the tail incidence angle, pitch stability derivative and delta e versus V... I think you are familiar with this one problem atleast, it is not so difficult to solve this. So, I will quickly, you want to find out the location of center of gravity and what is given is this ....

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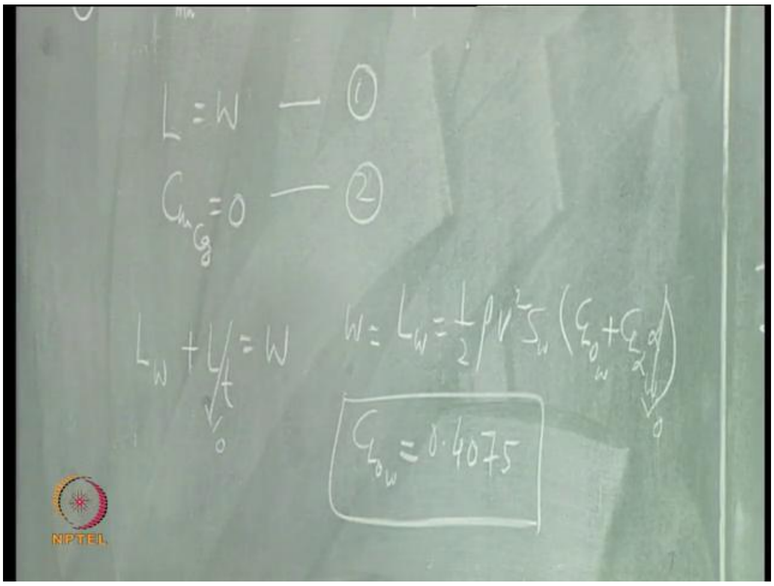

$$L = L_w + \underbrace{L_t}_{0} = W \Rightarrow W = L_w = \tfrac{1}{2}\rho V^2 S_w (C_{L0w} + C_{L\alpha w}\underbrace{\alpha}_{0}) \Rightarrow C_{L0w} = 0.4075 \qquad (5)$$

So, we have to solve lift equal to weight for trim condition, right. So, one condition is this ... and second is that Cmcg must be 0, right. What is L? L is ..Refer Eq(5), right, lift at the wing and then you have to add this tail part also, but, here we are saying here that we are flying a condition when alpha is 0 and delta e is also 0 and the lift produced at the tail is 0.

So, that is given in the problem. See you are setting this to 0. So, W is L_w....of the wing. So, alpha is again 0 here, trim condition, and now I can find out what CL naught w is using this, is it not? I know everything here. I know the weight, I know all these quantities. So, I can find out what CL naught w is, right.

You also have to find out the downwash angle.

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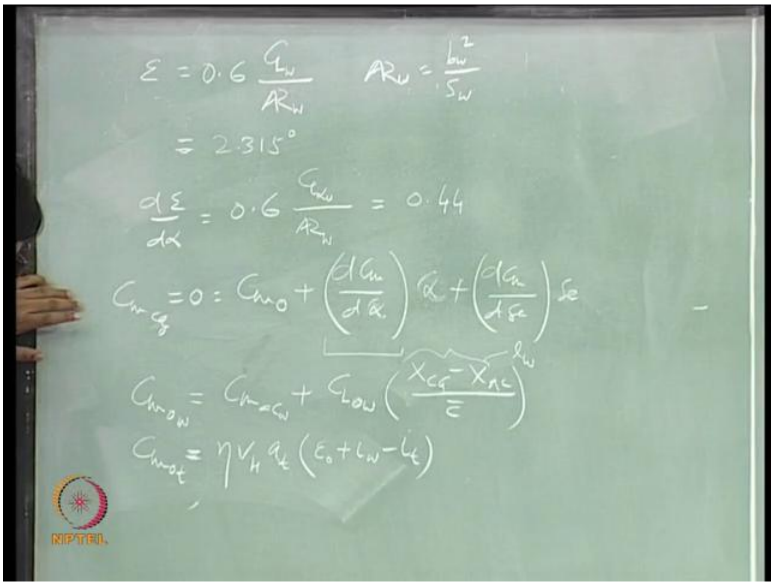

So, the expression for that is....

$$\varepsilon = 0.6\frac{C_{Lw}}{AR_w}; AR_w = \frac{b_w^2}{S_w} \Rightarrow \varepsilon = 2.315\deg; \frac{d\varepsilon}{d\alpha} = 0.6\frac{C_{L\alpha w}}{AR_w} = 0.44$$

$$C_{mCG} = 0 = C_{m0} + \left(\frac{dC_m}{d\alpha}\right)\alpha_w + \left(\frac{dC_m}{d\delta e}\right)\delta e \tag{6}$$

$$C_{m0w} = C_{macw} + C_{L0w}\left(\frac{\overbrace{X_{CG} - X_{AC}}^{l_w}}{\bar{c}}\right); C_{m0t} = \eta V_H a_t(\varepsilon_0 + i_w - i_t)$$

Let us try to solve this, two, what is two? This Cm naught is the total Cm naught, right, plus, or, Cm naught tail is what? Plus ... a t is CL alpha t. Now this Cm naught w will have two components, right, one is Cm acw ....what is given here actually plus CL naught w. So, this is, actually this quantity is that, and I will say is CL naught w into ....

This quantity is ....Refer Eq(7) So, this, let us try to expand this.

$$C_{mCG} = C_{m0w} + \underbrace{C_{m0t}}_{0} + C_{L\alpha w}\left(\frac{X_{CG} - X_{ACw}}{\bar{c}}\right)\alpha - \eta V_H a_t\left[\alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) - \varepsilon_0 + i_t - i_w + \tau\underbrace{\delta e}_{0}\right]$$

$$\frac{dC_m}{d\alpha} = C_{L\alpha w}\left(\frac{X_{CG} - X_{ACw}}{\bar{c}}\right) - \eta V_H a_t\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{7}$$

$$C_{Lw} = C_{L\alpha w}(\underbrace{\alpha}_{0} + i_w - \alpha_{0Lw}) \Rightarrow i_w = \frac{C_{Lw}}{C_{L0w}} + \alpha_{0Lw} = 3.1\deg$$

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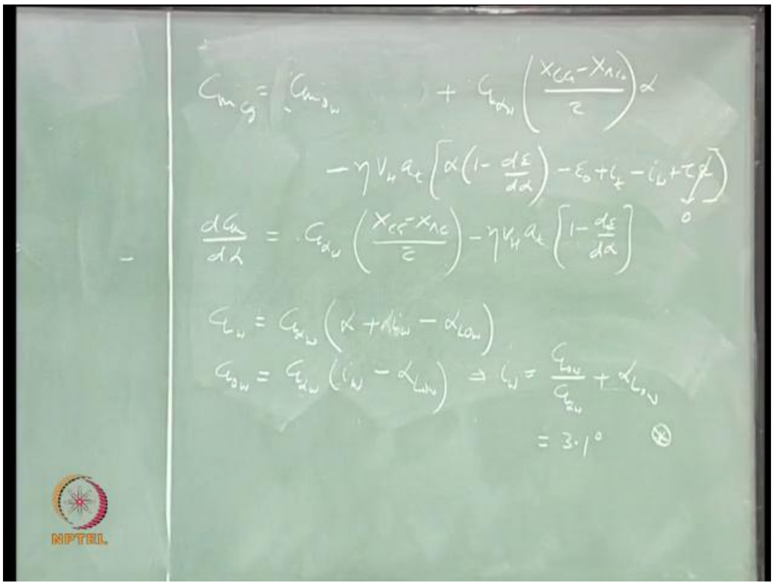

So, I am setting this alpha to 0 and that is CL naught w and that will give you .... Refer Eq(7). So, let us try to mark the answers that we are getting. When we set delta e to 0.

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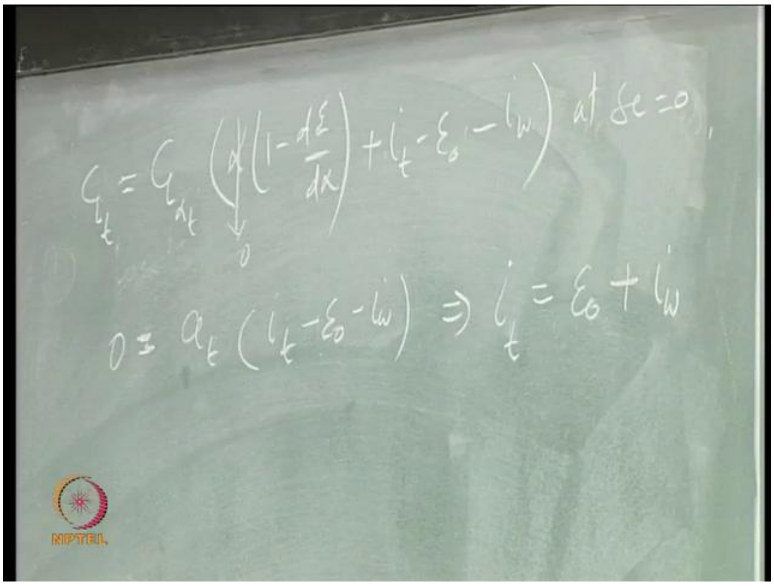

$$C_{Lt} = \underbrace{C_{L\alpha t}}_{a_t}\left(\underbrace{\alpha}_{0}\left(1-\frac{d\varepsilon}{d\alpha}\right)+i_t-\varepsilon_0-i_w\right) @ \delta e = 0$$
$$0 = a_t\left(i_t-\varepsilon_0-i_w\right) \Rightarrow i_t = \varepsilon_0 + i_w \tag{8}$$

And alpha is also 0, and this CL t is 0, right, when, this is the condition, when delta is 0 alpha is 0, what we have said is that lift produced at the tail is 0, right. So, this CL t is 0 and what you get is this .... Refer Eq(8), a t is not 0. So, you can set this 0 and get what i t is.

We will stop here actually, we will stop.