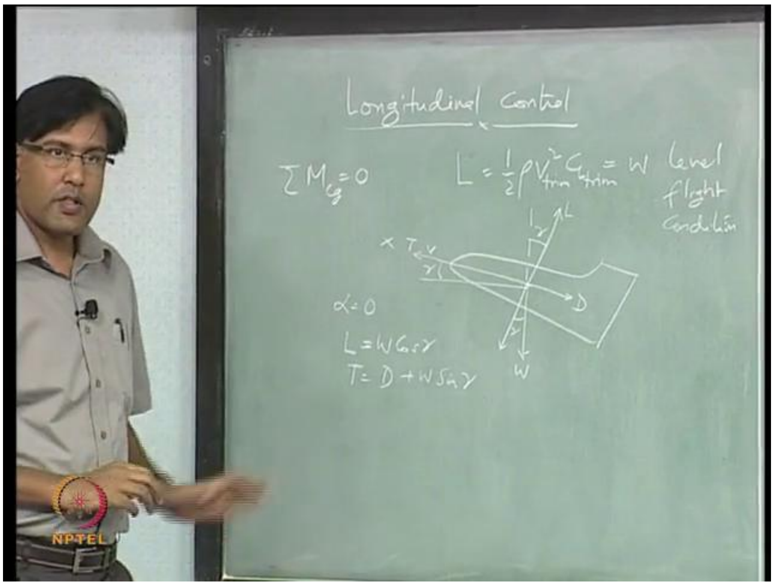
Flight Dynamics II (Stability)

Prof. Nandan Kumar Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module No.# 05.
Longitudinal Control
Lecture No. # 12
Elevator

(Refer Slide Time: 00:13)



So, we are now talking about longitudinal control. What do we want to do with this control? We want to change the trim condition. So, let us say I want to fly a different speed. So, I have to somehow adjust this part so that this (Lift) is equal to *W*. It is not only in level flight, you can also ... if you look at, let us say climbing flight, steady climb.

So, let us say this velocity is in this direction. alpha (α) is 0 ... and thrust is along body fixed X axis. So, if I want to resolve the forces along this body fixed X axis and the Z axis, then this L is what? L has to be, now, we are climbing. L has to be equal to $W\cos\gamma$ and thrust is equal to drag. So, not only in this level flight condition, but also in other longitudinal flights, you are actually trying to balance out lift with component of weight.

(( ))

No,no, I am assuming that the velocity is along this X axis. So, alpha (α) is 0.

Why because getting the sine component, sine component. So, it is going to standard.

What we make after that?

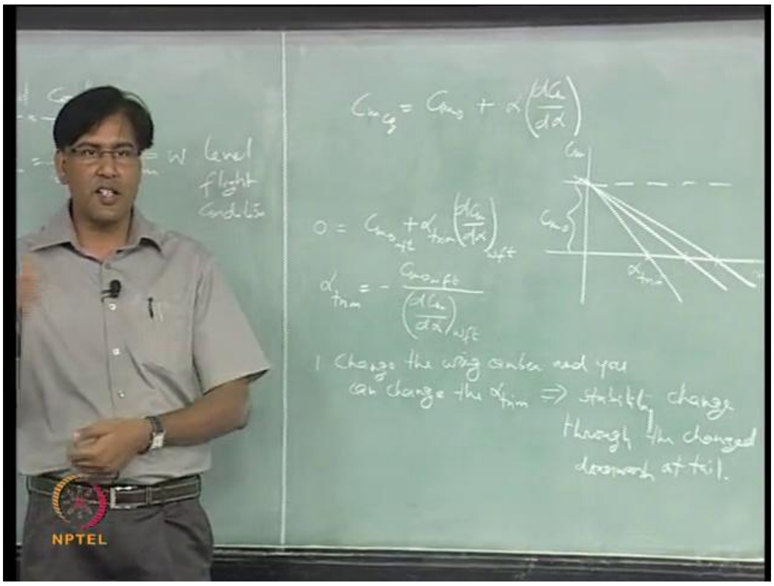
Which component is going to act in this direction?

$W\sin\gamma$.

(( ))

So, you are getting this. We are trying to; this is the level flight condition. .... So, if this gamma (γ) is a constant, then it is a steady climb right, I have to balance this lift. This lift has to be at least equal to this to fly this condition. You know, one case of this is when gamma (γ) is 0; that is the straight and level flight condition. So, what I am saying is longitudinal control is required for changing the trim. Why? When you want to change the trim, changing the trim is, changing this (V) or this (C_L), you have to compensate for the decrease or increase in one of the parameters and we also have to satisfy this condition ($L = W$).

(Refer Slide Time: 05:00)



$$C_{mCG} = C_{m0} + \alpha\left(\frac{dC_m}{d\alpha}\right) = 0 \Rightarrow \alpha_{trim} = -\frac{C_{m0,wft}}{\left(\frac{dC_m}{d\alpha}\right)_{wft}} \qquad (1)$$

So far we have looked at,...we know these quantities so far. We found out expression for this $dC_m/d\alpha$ and for C_{m0}. And this is giving me... I want to plot this ... one α_{trim} (equilibrium alpha corresponding to C_{mCG} = 0). If I do not change anything, then this is what I am getting. Now, I want to fly a different speed; that means, I have to change the $C_{L\text{trim}}$ or α_{trim}. Now, the question is how do we do that? (Refer Slide Time 6:10) If you set this ($C_{m\text{CG}}$) to 0, you are going to get C_{m0} plus α_{trim} into $dC_m/d\alpha$.

Question is how do you do that? In how many ways you can get that? One is of course, we started with. The last class we said that we are going to use a flap at the trailing edge of the horizontal tail which is called Elevator. But let us say, we will go into that little later, change the camber of the wing airfoil itself. Wing camber if you change, this C_{m0} is going to change. So, you can change the α_{trim} .

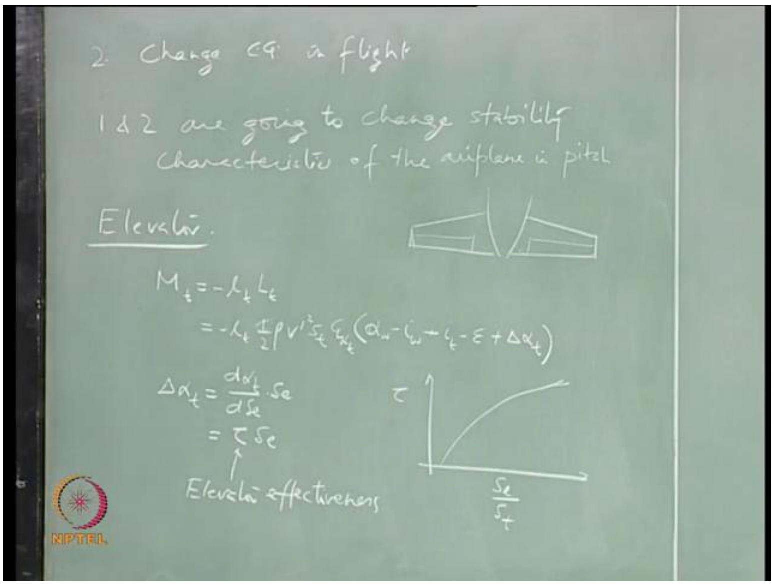
α_{trim} is this (Refer Eq(1)). C_{m0} naught is for the complete airplane. Wing, fuselage, tail. If you change the wing camber, this quantity is going to change so that you can get a different α_{trim}. So, that is one thing you can do. ... and that will have no effect on that stability.You are changing the C_{m0}, you are not changing $C_{m\alpha}$. It is not going to change the slope of this curve. So, we are only going to shift this curve parallel to this original one. How you can do that in flight? So, there will be flaps, leading edge flaps on the wing, and the trailing edge flaps also which you can adjust. So, you get a different camber and you can modify this, for same static margin or $dC_m/d\alpha$, you can get a different α_{trim}. That is one possibility, but would you want to do that?

On some airplanes where there is no tail, tailless airplane. This is what you are going to do actually- you can change the wing camber. One thing is .... it would be wrong to say that it will not have any effect on the stability, because when you are changing the wing camber, you are going to disturb the flow field behind the wing, and through the downwash term, this stability is going to get affected.

When we say that we are changing the wing camber, we are also changing the flow field behind the wing and that is going to affect the downwash on the tail and that will have in turn effect on this $dC_m/d\alpha$, stability is going to change. So, this means stability change. .... So, we are actually changing these stability characteristics. Second strategy, throttle, throttle is a control. Let us not talk about throttle right now. Changing throttle means lot of things happening. Right now we are not, we are trying to look at the aerodynamic control.

(Refer Slide Time 11:18) Second strategy - look at this equation. What else you can change? How do you change the slope? So, I can change the slope ($dC_m/d\alpha$) right and get different trims, different trim conditions. How do you do that? You are changing this, everything else is fixed, now what is there in your control? This term ($dC_m/d\alpha$) will depend upon the CG location; neutral point location is fixed; CG location is something you can change. You can change the static margin of the airplane, and when you change this, we are going to change the α_{trim}. So, that is one possibility.

(Refer Slide Time: 12:44)



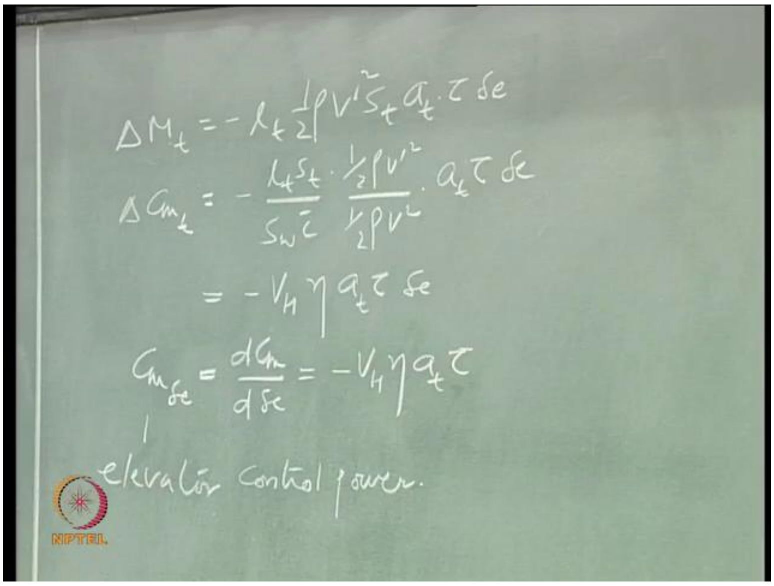
And this you have to do in flight because we are trying to change the trim condition in flight. But that is going to have an effect on the stability characteristics, both of them actually, would you want to do that? So, one (change the wing camber; will change C_{m0}) and two (change CG; will change ($dC_m/d\alpha$)).... I do not want to do that, I do not want to do that, and that is where we come to the third control that we talked about, that is use of Elevator. So, in the last class, we discussed how Elevator is going to change the camber of the horizontal tail and that is how we are going to get change in angle of attack α.

$$M_t = -l_t L_t = -l_t \tfrac{1}{2}\rho V'^2 S_t C_{L\alpha t}(\alpha_w - i_w - i_t - \varepsilon + \Delta\alpha_t); \quad \Delta\alpha_t = \frac{d\alpha_t}{d\delta e}\delta e = \tau\delta e \qquad (2)$$

So, if I want to find out what is the moment due to tail about CG when Elevator is also incorporated, then ... and this has to somehow incorporate the Elevator effect ... C_{Lt}. So, I will write this as $C_{L\alpha t}$ α_t .... plus a change in angle of attack due to the Elevator. And this $\Delta\alpha_t$ is ... this (Eq (2)). Depends, this quantity which is the change in angle of attack at the tail due to a change in Elevator deflection is depending upon the area of the control surface and the area of the complete tail (S_t). It ($\tau = d\alpha_t / d\delta e$) is called Elevator effectiveness and this depends upon S_e, platform area of the Elevator part, so, only the control part.

So, you have ... (Refer Slide Time 17:15) this tail and I am only talking about this area, so, this area plus this area divided by this total area of the tail. This parameter (τ) is going to depend upon the ratio of the two areas (S_e/S_t). It varies something like this, may be it is saturated, is it alright? Now, let us look at what is the change we see in ΔC_m, in the pitching moment because of this particular term.

(Refer Slide Time: 18:23)

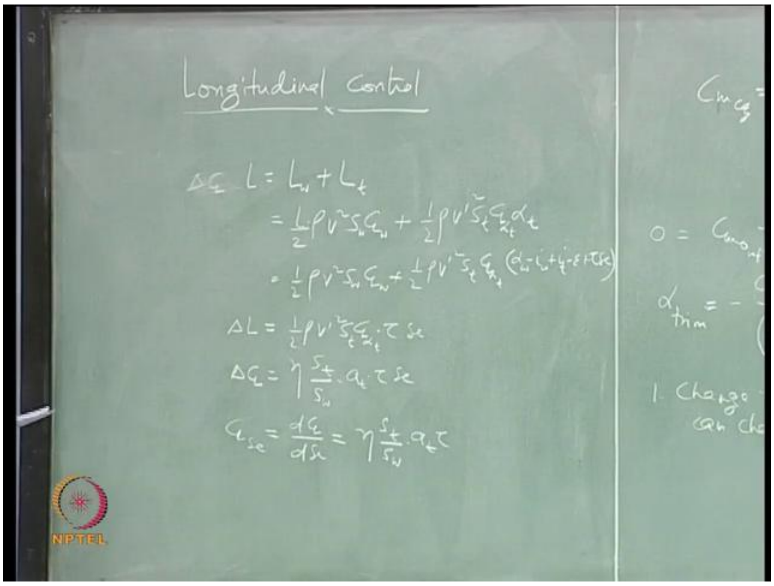


$$\Delta M_t = -l_t \tfrac{1}{2}\rho V'^2 S_t C_{L\alpha t}.\tau\delta e;\ \ \Delta C_{mt} = -\underbrace{\frac{l_t S_t}{S_w \bar{c}}}_{V_H}.\underbrace{\frac{\tfrac{1}{2}\rho V'^2}{\tfrac{1}{2}\rho V^2}}_{\eta}.\underbrace{a_t}_{C_{L\alpha t}}\tau\delta e = -V_H \eta a_t \tau\delta e; \tag{3}$$

$$\underbrace{C_{m\delta e}}_{Elevator\ Control\ Power} = \frac{dC_m}{d\delta e} = -V_H \eta a_t \tau$$

What is that change? .... into… (Refer Eq(3)) Now, I can find out the change in the pitching moment coefficient due to this parameter. ... So, this is equal to ... (Refer Eq(3)) change in pitching moment that you can achieve due to deflection in Elevator can be represented by this derivative.... So, effective change in pitching moment due to deflection of Elevator is this (Refer Eq(3)). You can properly choose this parameter and design your control surface in such a fashion that you may want to have large Elevator control power. This term (Refer Eq(3)) is called Elevator control power. Anything else you see here? Similarly, we will also have a change in lift, through this term.

(Refer Slide Time: 21:05)



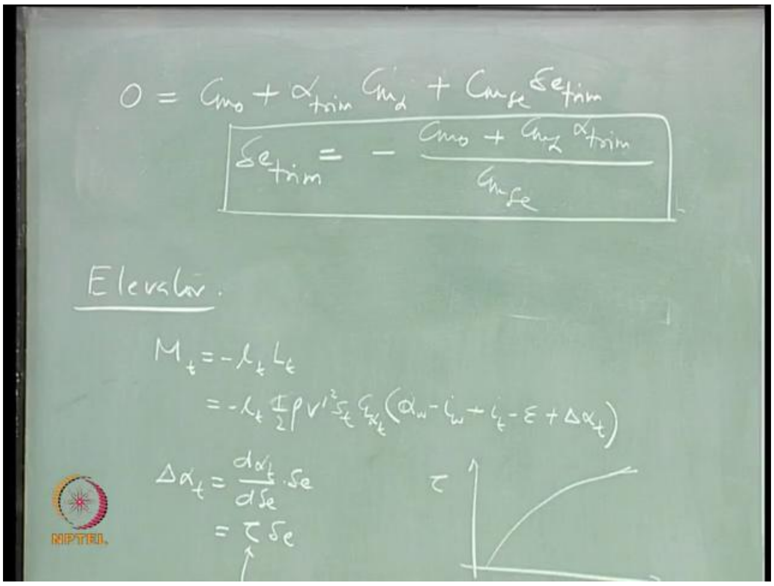
$$
\begin{aligned}
L = L_w + L_t &= \tfrac{1}{2}\rho V^2 S_w C_{Lw} + \tfrac{1}{2}\rho V'^2 S_t C_{L\alpha t}\alpha_t \\
&= \tfrac{1}{2}\rho V^2 S_w C_{Lw} + \tfrac{1}{2}\rho V'^2 S_t C_{L\alpha t}(\alpha_w - i_w + i_t - \varepsilon + \tau\delta e) \\
\Delta L &= \tfrac{1}{2}\rho V'^2 S_t C_{L\alpha t}.\tau\delta e \Rightarrow \Delta C_L = \eta\frac{S_t}{S_w}.a_t.\tau\delta e \Rightarrow C_{L\delta e} = \frac{dC_L}{d\delta e} = \eta\frac{S_t}{S_w}.a_t.\tau
\end{aligned}
\qquad (4)
$$

So, I can say ΔC_L due to Elevator you know is going to be (Refer Eq(4)). So, let us write the expression for this total lift first, ... and this is half rho V squared S plus C_L alpha tail into alpha tail (Refer Eq(4)). And alpha tail is now slightly modified, ... is not it? So, ΔL that you can get through this term is half rho V prime squared into ... S_t $C_{L\alpha t}$ into $\tau\delta e$ (Refer Eq(4)). Because of Elevator deflection, the change in total lift that you can get.

So, I can also write this as change in total lift coefficient, that is going to be ... $C_{L\alpha t}$ is a_t. The change in total lift due to change in Elevator deflection can also be given by this derivative. ... So, what are we doing finally? (Time 24:00) We are actually adding a term here which was not there. When the Elevator is not there, Elevator is 0. When I get this, when I add the Elevator, what I get is ... So, this is the new expression for the C_{mCG}. Is it making any sense? Is it all right? Yeah, any question? (Time 25:03) Now, I have a relation, relation which is telling me, you know, if I set this (C_{mCG}) to 0, that is the trim condition.

$$C_{mCG} = C_{m0} + \alpha_{trim} C_{m\alpha} + \delta e_{trim} C_{m\delta e} = 0 \Rightarrow \delta e_{trim} = -\frac{C_{m0} + \alpha_{trim} C_{m\alpha}}{C_{m\delta e}} \qquad (5)$$

(Refer Slide Time: 25:17)

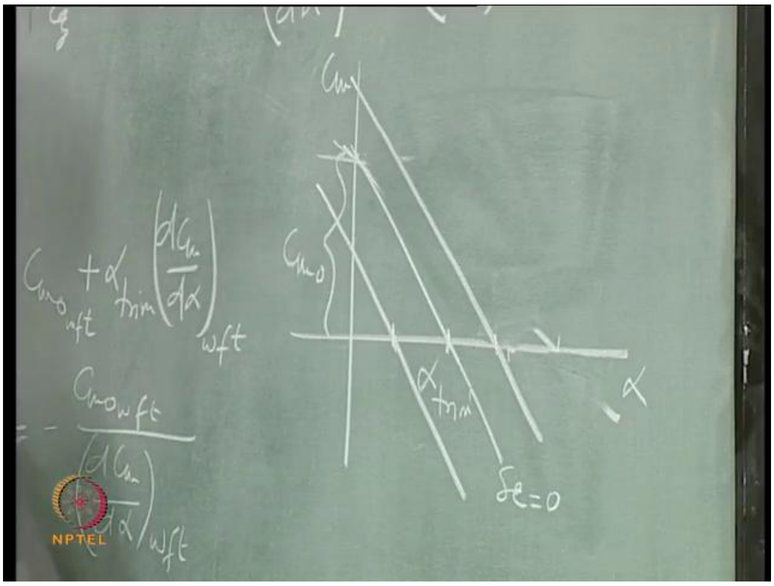


Now, if I want to find out, what is, ... what should be the Elevator for a particular alpha? That relation, we can find out from here, and this is without affecting the stability, is not it (Refer Eq(5))? Do you see that? For stability, we have to take the derivative of this (C_m) with respect to angle of attack, is not it? This quantity ($C_{m\delta e}\delta e_{trim}$) is not going to change.

It ($C_{m\delta e}\delta e_{trim}$) is only going to change when you apply Elevator. It ($C_{m\delta e}\delta e_{trim}$) is not going to be dependent upon the angle of attack that the aircraft is going to see, is it not? So, this is not going to affect the stability of the aircraft. But what is it going to change? If it ($C_{m\delta e}\delta e_{trim}$) is not changing the stability, what is it changing? It ($C_{m\delta e}\delta e_{trim}$) is changing the C_{m0}. Anything here which is not changing alpha, I can always add that part to C_{m0}. So, that is, it ($C_{m\delta e}\delta e_{trim}$) is changing C_{m0}, but it is not changing the stability.

So, this is where there is an advantage of using this Elevator. Many airplanes where you do not see tail, there, there are airplanes where actually they change the CG to change the trim. But definitely it comes with other considerations like the one probably is this Concorde which used to fly earlier and it was taken off. There they used to change the CG to change the trim, but of course, it was supersonic aircraft. So, there would be some other issues there. So, let us try to find out what is δe_{trim} required for getting any particular α_{trim}. .... (Refer Eq(5)) So, clearly it is going to change the trim, but not the stability because this ($C_{m\delta e}\delta e_{trim}$) part is only going to add to C_{m0}.

(Refer Slide Time: 28:58)

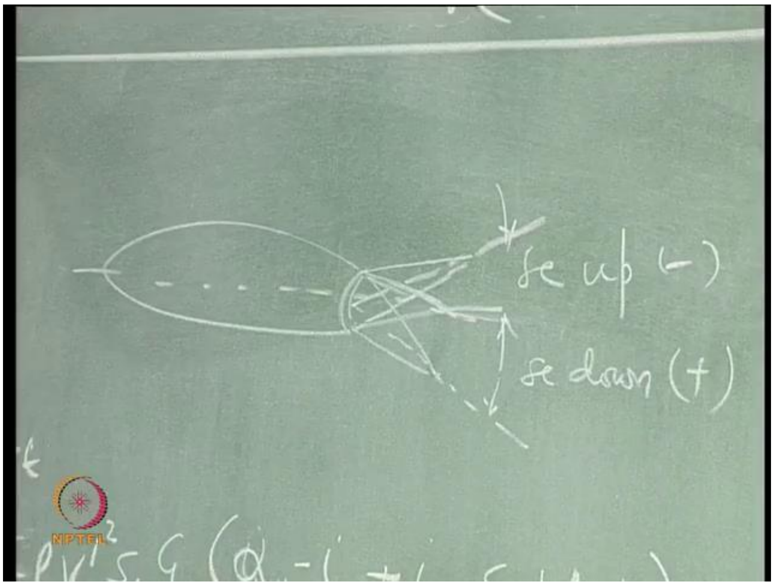


So, if you want to draw a corresponding $C_{m\alpha}$ curve, you will see this. Now, can you tell me, if this is for δe = 0 (neutral position of elevator), for what δe you will get this curve and for what you will get this?

(( ))

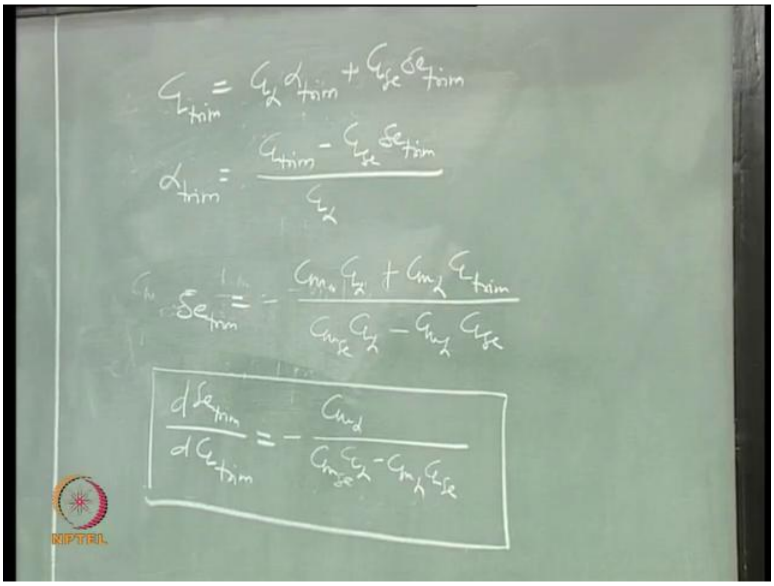
Remember, this ($C_{m\delta e}$) is a negative term, minus, it is going to change C_{m0}, this minus negative term. So, what δe? δe up is, up is negative, down is positive and this convention has been adopted only to indicate that $C_{m\delta e}$ is negative. So, δe down, that is going to give you lift up and, that is going to create a pitching moment down. So, $C_{m\delta e}$ is negative, is it not?

(Refer Slide Time: 30:31)



So, this curve is for what δe? Convention is that ... up is a negative δe . ... This one is bringing this down, changing the C_{m0} by a negative quantity. δe is positive, so, it should be down, right. Let me write another expression.

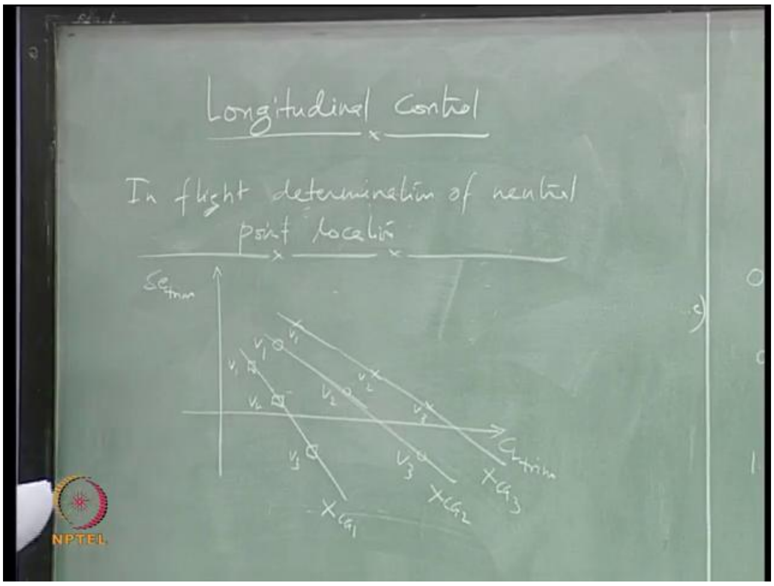
(Refer Slide Time: 31:59)



$$C_{Ltrim} = C_{L\alpha}\alpha_{trim} + C_{L\delta e}\delta e_{trim} \Rightarrow \alpha_{trim} = \frac{C_{Ltrim} - C_{L\delta e}\delta e_{trim}}{C_{L\alpha}}; \frac{d\delta e_{trim}}{dC_{Ltrim}} = -\frac{C_{m\alpha}}{C_{m\delta e}C_{L\alpha} - C_{m\alpha}C_{L\delta e}} \tag{6}$$

So, I can also write CLtrim. CL trim is, CL alpha into, this CL alpha is for the whole aircraft right plus CL.... (Refer Eq(6)) So, what is alpha trim? ... Now I can write this delta e trim in terms of CL trim (Refer Eq(6)). That expression is this. ... Let me also take derivative of this with respect to CL trim so that you get ... , is it not (Refer Eq(6))? So, you clearly see here that this term is going to be dependent upon the stability again, it is going to depend upon the stability margin that you have.

(Refer Slide Time: 34:28)

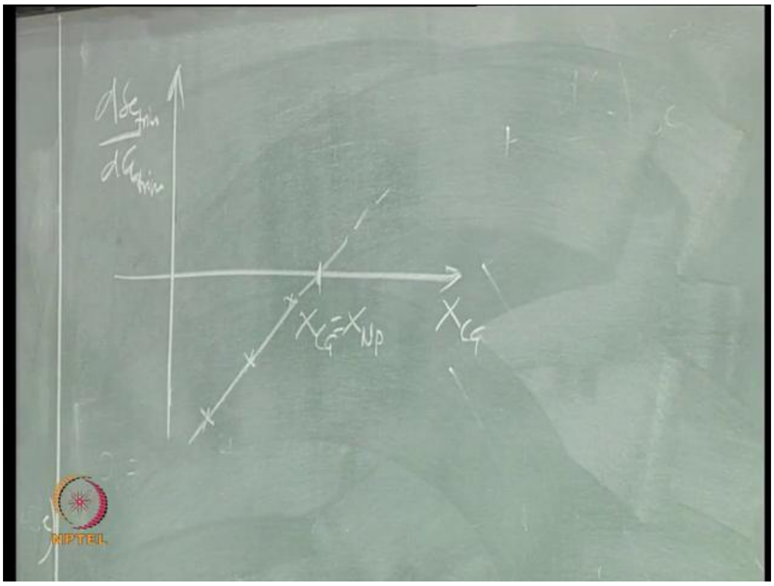


And this helps in locating the neutral point location on the aircraft in flight. ... Just look at this point, you are going to use this.This is one test that you are going to do.We are going to go for flight test lab. There is one test you have to do. ... Let us try to fly the aircraft at different speeds. So, we will you know how much Elevator I want at trim, how much C_L that is going to give me, and, so let us say you are flying one speed.

So, one CG condition, CG is fixed, that $C_{m\alpha}$ is fixed, and now, I am flying one velocity. So, I am at this δe_{trim}, this C_{Ltrim}. change the speed. Another point. One more change in speed V_1, V_2 .. all trim values. And for one CG location, you can actually find such a curve. Change something on the aircraft which will change the CG, and then, again fly different velocities speeds. So, you get 1, let us say 2 and 3 and you can join these. V_1, V_2,V_3 three different trim speeds (Refer Slide above).

Another CG location and then you get this. So, for three different CG locations, I have flown three different speeds. So, you get corresponding δe_{trim} as $C_{L\text{trim}}$. That is how we have obtained these three curves. Now, I can find out the slope of each of these curves, is it not? I can find out $d\delta e_{\text{trim}}/dC_{L\text{trim}}$ from these curves.

(Refer Slide Time: 38:41)



Refer Slide Time 38:40

So, now, I am plotting. Remember, I said neutral point location is the location of the center of gravity where $C_{m\alpha}$ becomes 0, is not it? And $C_{m\alpha}$ becoming 0 also means this slope ($d\delta e_{\text{trim}}/dC_{L\text{trim}}$) going to 0 (via Eq(6)). If I have the knowledge of how this is going to change with CG location, then I can find out where the neutral point is.

So, now, I have done these nine experiments I would say. For three different CG locations, I have found the slope ($d\delta e_{\text{trim}}/dC_{L\text{trim}}$) and I am getting three values of this quantity, and passing a line through these three points extrapolated is going to cut this X axis somewhere and that is the location of the neutral point. Because at this point, $C_{m\alpha}$ becomes 0. So, that is how you are going to find out neutral point location in flight.

Now, how it is going to be different from the one we, we found a formula for the neutral point location. Is that going to be different from this one? That was an estimate, remember that. What, that was, that was at the initial design stage. Now, you want to find out whether that neutral point location was correct or not, we have to do this experiment. I want to stop at this point.