Indian Institute of Technology Madras
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NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

Aerospace Propulsion
Solid Rockets – Ignition, Quenching

Lecture 28

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In the last class we had dealt with how to obtain a grain design for a given thrust time curve right and towards the end of the last class we kind of found out what is it that is required of a grain design that is what are the desired features.

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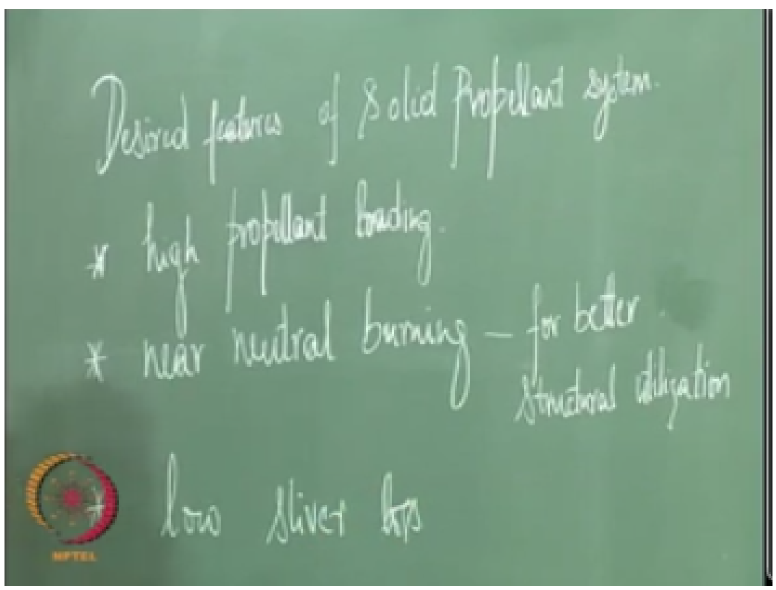

We discuss this that for a solid propellant system to be very good you need to have as high a loading as possible because then you are letting you or we are you are making an optimal use of the given volume right. So first is I propellant loading then we realize that it is better to have neutral burning right if you have neutral burning then your structural utilization is much better, so for better structural utilization then lastly it should have low sliver loss in addition to meeting the thrust time curve if there are three or four designs that meet the thrust time curve then we should look at one which has a high propellant loading and then near neutral burning and low slider loss okay.

There is a flip side to high propellant loading namely erosive burning which we will discuss in the future classes okay, we will come to that later. Now if you remember in the last class if you look at this figure here.

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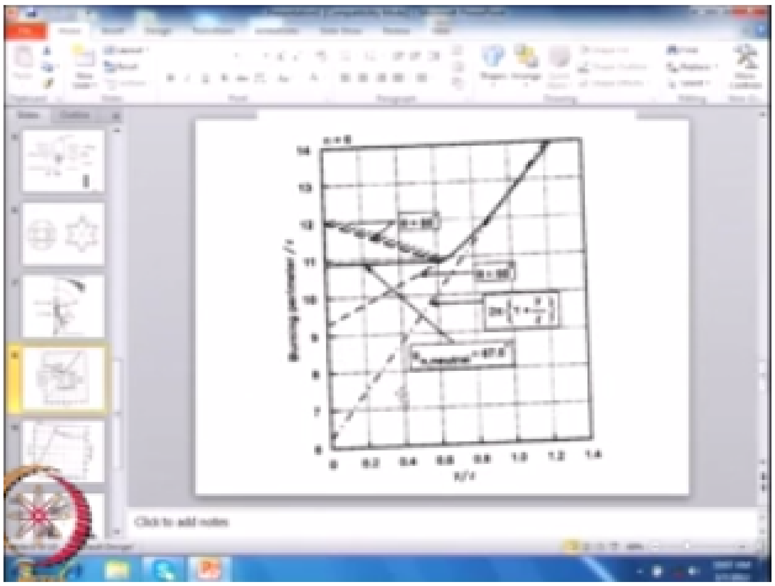

We had designed the grain in terms of Y or the extent of the propellant burnt and if you look at this graph then from this graph you can get for any given Y the burning perimeter and from the burning perimeter you can calculate the burning surface area a be right, so in essence we had got what we had got at the end of the last class was how to get burning surface area.

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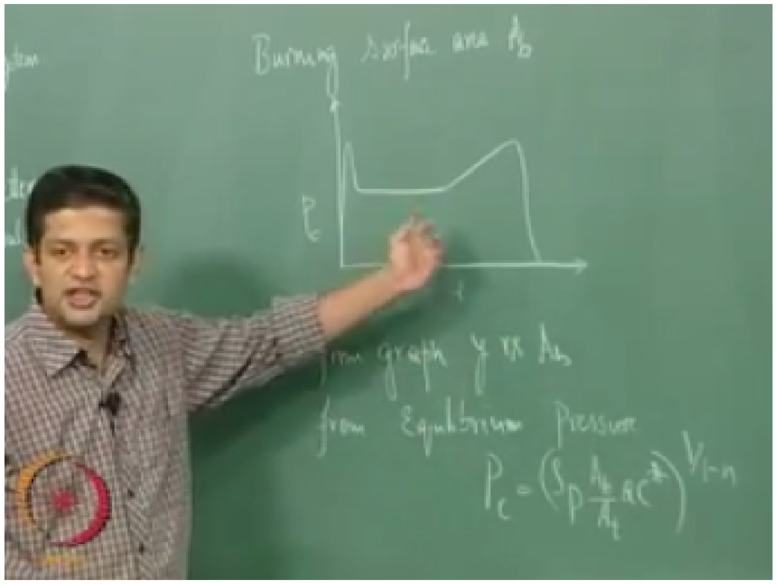

A b to meet R requirements now how do we cross check whether what we have designed is going to give us what we wanted in the first place right we have to cross check our design so to do that let us say we were given a pressure versus time curve like this right if you were given a pressure versus time curve like this now having gone through the grain design how do we know that we have been able to get as close to this as possible all the information that we know is we know how the burning surface area evolves right as some extent of web burnt we know y versus a b right from the graph.

Now we also know that from equilibrium relations equilibrium pressure PC ρ P a B / a t - n so if we know a b right we can calculate what is pc right the other terms are fixed 80 density of the propellant a and C star and n all these are fixed so we will get to know what is the chamber pressure variation with a b so we know why versus a be from there we can get y versus PC.

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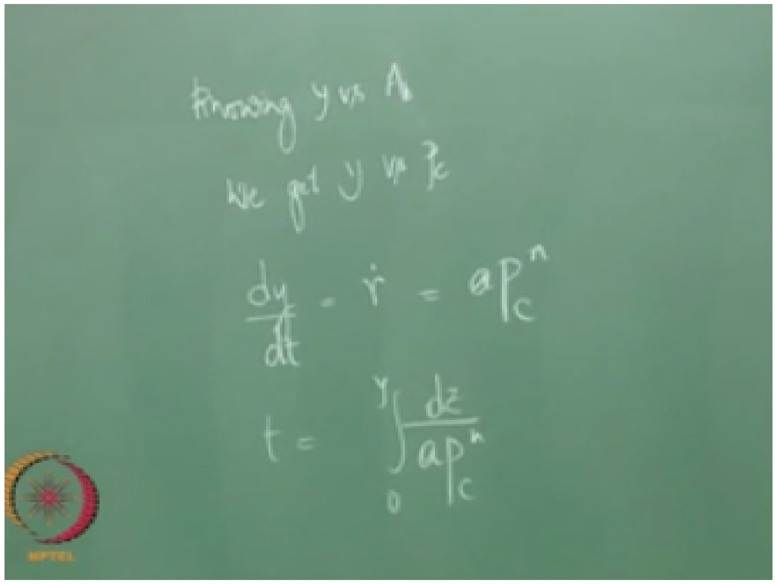

So knowing y vs Pc we get y versus DC but we are still not depth because we need x vs pc vs t right this is what we want so how do we get that very simple we know burn rate right burn rate we know is nothing but dy / dt is nothing but burn rate right. So this is nothing but a pc to the power of n so using this I can integrate for y and then get or integrate this equation and get a time, so you can rewrite this as T is equal to 0 to y DZ a PC to the power of n so we now know time we now know pc.

So we can get pc versus time so from this we can get pc versus time after this we can evaluate whether the design that we have proposed is a is going to agree with what was given to us what was desired of us right you might not be able to meet with precisely at all the locations but in an overall sense if you are going to meet it that is good enough okay, so there is one doubt still that is some time back someday posed this are not we using equilibrium pressure here this relationship right.

But if you look at this portion right this portion is not constant pressure are we right and using this equation even to get something for this portion this is changing with time s not a constant pressure, so let us see how to find out that okay before we do that we have to derive the unsteady equation so let us go back to a few classes back wherein we had this control volume of the rocket motor.

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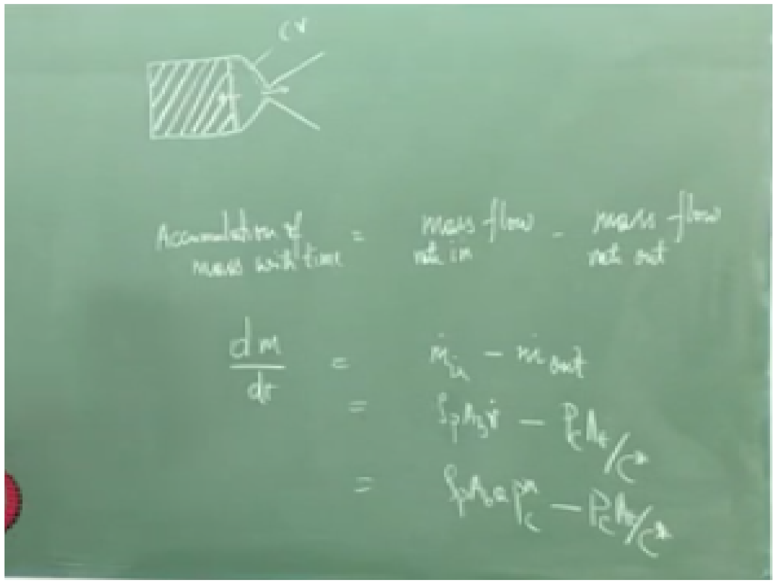

If we have a rocket motor like this then we know that there is mass coming in because of the burning of the propellant right no it is a Z to the dummy variable that, so if you have why you integrate it and it has no meaning here it this is a dummy variable so here it is burning in this direction so this is adding mass and the mass is going out through these throat so if we write the unsteady equation right then the accumulation of mass the time must be equal to mass flow in minus mass flow out right.

Now how do we express equation for this accumulation of mass travel time this is nothing but M DM by DT mass flow rate in M. in minus m. out m. n and m dot out we had learnt what they are this is ρ p a b r. minus p CA T / C star r. I can write it as a pc to the power of n so I will get ρ p a b a C star. Now what is this m? M is nothing but density into volume all right.

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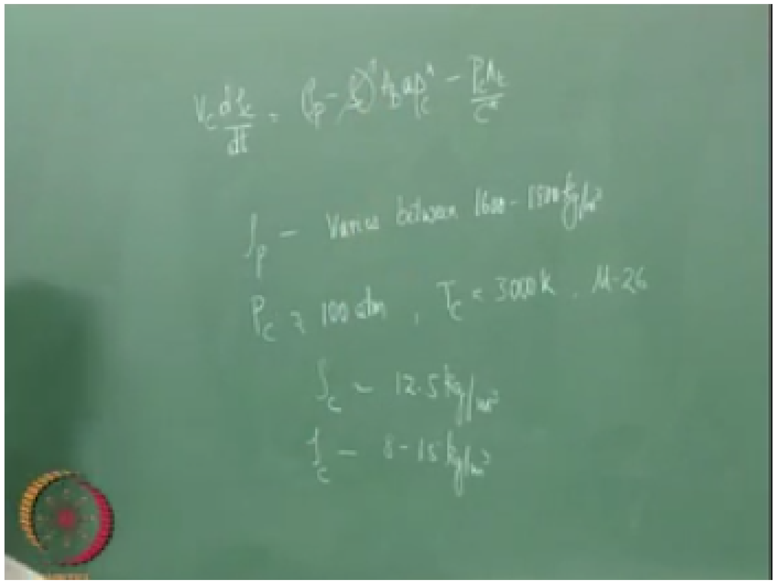

So we can write okay, ρ c is nothing but density of gas and v c is the chamber volume now we can write d ρc / ρ c v c / dt = ρ c x DV c / dt + vc x d ρ c / DT / chain rule right what is d vc / DT if we come over here and look at this finger how is the VC increasing a B into r dot s what is giving the increase in the chamber volume right so d vc / DT I can write it as a bee into a. so I write this as r. I can write it as a pc to the power of n now if i substitute this back in this equation I can rewrite that equation as we see d ρ c / de is equal to this goes to the other side ρ p - ρ c okay.

Now before we go further let us get an estimate of what is ρ P and what is ρ C ρ P is density of the propellant right depending on the kind of propellant it varies between 1600 to 1800 okay, what about ρ c how do you calculate gas density DC upon RTC so what typical chamber pressures can go up to 100 okay and chamber temperatures of the order of 3000, so PC around 100 atmospheres then TC around 3000 Kelvin molecular weight around 26.

So you will get typically ρ variation from for this case would be around 12.5 ρc that is and typically depending on this pressure if the pressure is higher you are going to get a higher density and the temperatures are higher you are going to get a lower density and pressure lower also you're going to get so typical variation of ρ C is from somewhere between 8 to 15 kg /m^3.

Compare this with ρ P it is somewhere around one less than 100 right, so if you neglect this term what is the error that you are going to make less than one percent right, so from an engineering perspective it makes sense to neglect this term okay.

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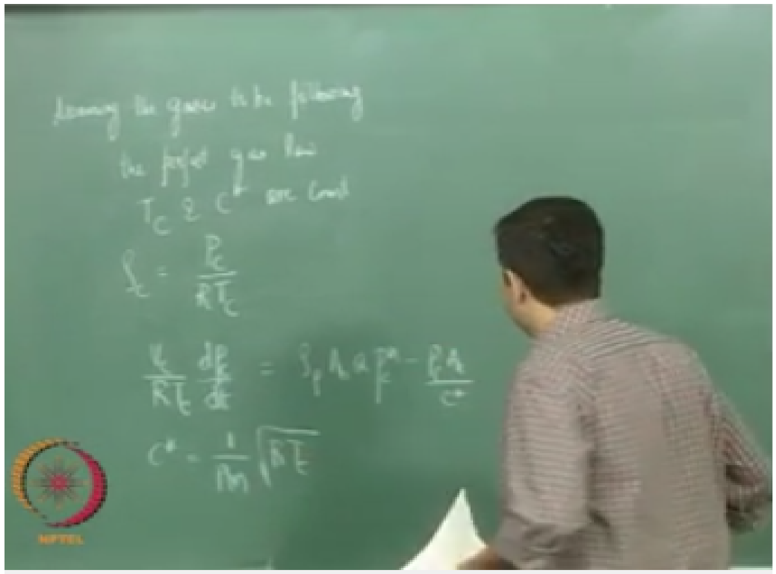

So this ρ c we can write it as PC / RTC right so here if we make the assumption that assuming the gases to be following the perfect gas law which we have already used and also making the assumption that TC and C star are constant okay, which is valid assumption in a sense even if pressure varies quite a bit beyond some level T C and C star do not change by a large magnitude they do change but by not by a large magnitude. So if you make this assumption then we can write ρ C is nothing but PC by RTC.

So if you include in that you will get VC these two one is a gas constant the other one we have assumed for the for this case to be a constant okay so then I can take out vc / RTC and I will get DPC / dt is equal to Rho P a be a pc to the power of n now we also know that C star what a C star Z star is nothing but 1 / λ of λ RT C right, or if you are going to use the universal gas constant r you TC by molecular weight.

So RTC that I have here I van replace it with c star into λ of λ there is a reason for doing that we will see that in a little time.

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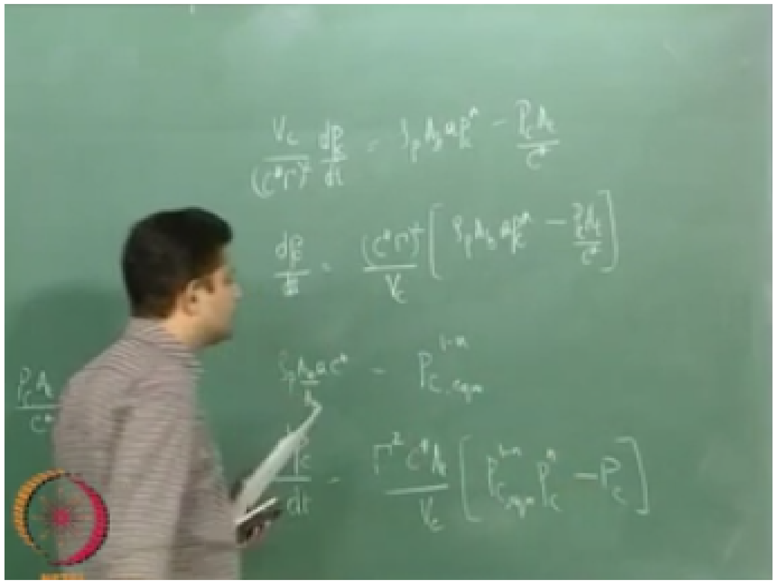

VC / okay I can take this to the other side write and rewrite this expression as Yeah right where is it a vc vc sorry fine between these two right I can what is ropey a B AC stark if I divide this by 80 and multiply this term by C star what do I get this entire term ρ p a be a C star by 80 what is this is PC to the power of 1 / n right. So now I can do that here there is a C star by 80 that I can take out and rewrite this expression as follows, so if I take out 80 by PC star right this term will be pc this pc is nothing but pc equilibrium pressure right pc equilibrium to the power of 1 minus n so I will get okay.

There was a reason for doing this which will notice in a short time what is the unit of this is pressure right and the left hand side is pressure by time so in some sense this is one by time right so let us look at how this is 1 by time we will call vc / a t right vc / a t is timber volume by a throat area we will call that a quantity named as L star okay, sorry.

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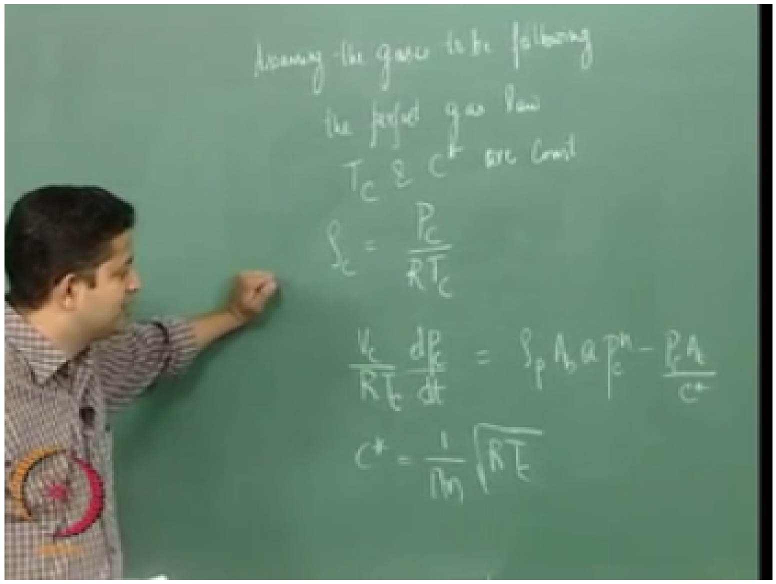

So if we call vc / a t as L star this is the Ln scale okay if you look at this has meter cube when meter square, so this is a length scale is the characteristic length scale of the motor. Now if you divide length by velocity what do you get time, so L star / C star λ^2 is nothing but TC or the characteristic time scale this is also known as the residence time this gives us an idea about what is the time that the gases are going to be there in the chamber okay. And L star right side is a length scale or L star assumes importance in all the three rocket motors that we are going to decide even in liquids and in hybrids.

And this is related to there is an instability called as L star instability also now if you look at this time characteristic time if you have a short motor right we have a small motor this should be smaller if you have a large motor there should be larger. So typically variation of TC is it is around 1.5 to 2 milliseconds for small motors and 20 to 40 milliseconds for large motives this other implication on what is known as combustion efficiency that is how much of the combustion is being completed inside the rocket motor also depends on the residence time right.

If the residence time is very small then you would expect if the reactions take a little longer those would not be completed right so the slowest process in solid propellant combustion is aluminum oxidation and that takes something like 5 to 20 milliseconds so if you have a very small motor then probably a combustion efficiency is are not going to be very high whereas if you have a large motor your combustion efficiencies are going to be very high because there is ample residence time to complete the reactions.

The typical combustion efficiencies for large motors would be of the order of 99% the most of the reactions are complete before they go out of the throat now coming back to the question that we had in our mind right that is when we were trying to look at pc versus T variation for a given grain design we had assumed that it is equilibrium pressure relations we had used is that valid is what we asked ourselves right.

So let us look at whether that is so or not so to do that we will take this equation here okay when we take this equation and if you how do we decide whether this term is large or small one way to look at it is if these two terms are nearly the same then this term would be smart right, so one way to get a good estimate is to divide this term by one of these two and find out if the fraction is large or small if it is very small then the error that we are going to make by neglecting it is going to be small to.

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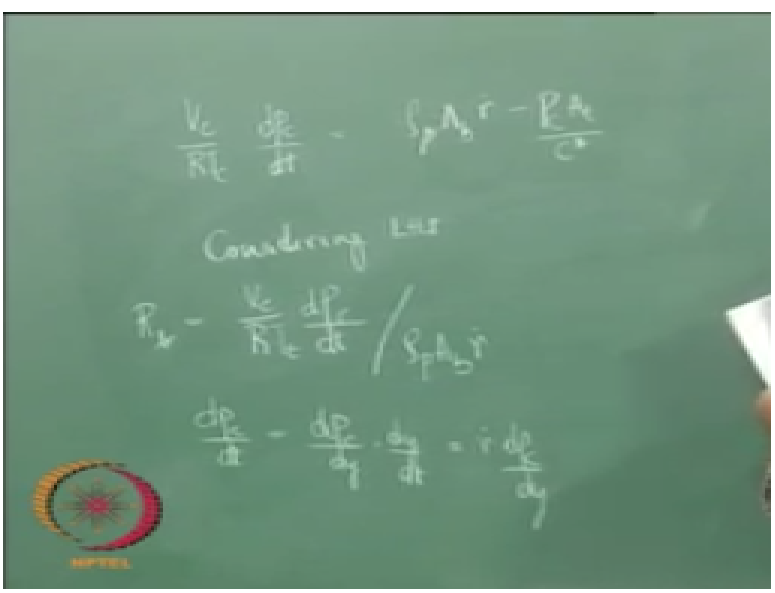

So let us take the term VC / RTC DPC / DT = ρ P. Now we will only need to consider the left-hand side of the equation okay as I said we will divide the left-hand side of the equation by this term what happens to the right-hand side of the equation is not our concern right now, we are trying to estimate this term in reference to one of the terms here right so I will take the let me call this term as r lr okay. So now what is d pc / DT I can rewrite this as d pc / x dy / DT right, and dy / DT we know is nothing but r dot so we will get this is equal to r dot into DPC / dy.

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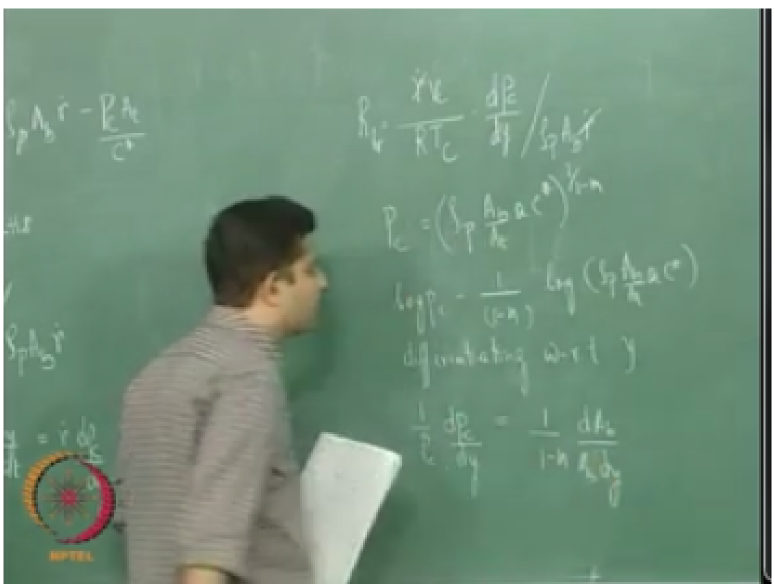

So if you substitute this back into this equation you will get r. v c / r t c RL r = x, so I can take out this r. sorry thank you now I need to get a relationship for DPC /dy right let us use the equilibrium relation we know that TC = ρ P a B / a t right if you take the logarithm of this you will get log p c = 1 - 1 / 1- n we know that a B is a function of Y right, so differentiating both sides with respect to Y we get 1/ 1 - n in 2d a B / a b sorry a b dy okay.

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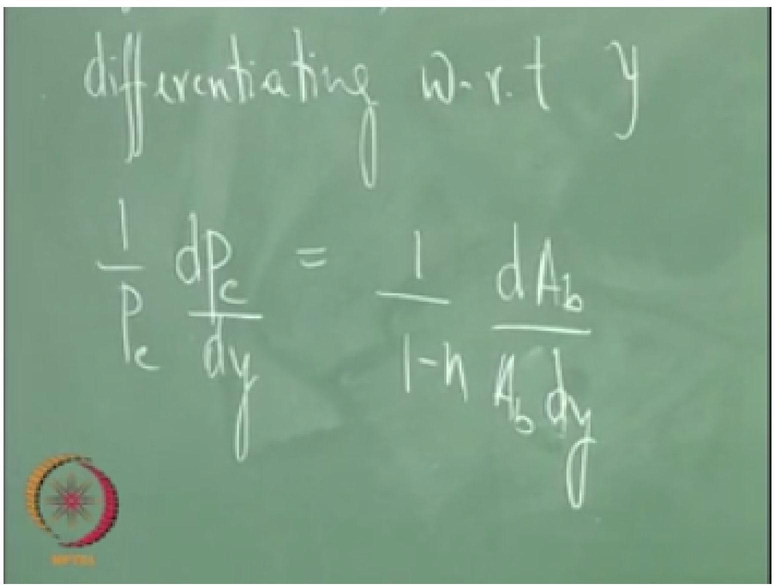

So or in other words I can get the expression for DPC / dy from this right and which is as follows ec / a bee okay, so if I substitute this back in the equation here right da be / dy then I will get Rlr to the vc by R_{TC} to pc there is an a/b here so I will get a b^2 1 - n okay we still need to know how the burnings of his area varies with why let us take a progressive burning rain that is let us take a cylindrical grain burning from inside to the outside okay so if you take a cylindrical grain burning from inside to the outside.

So at any given let me call this small distances why so at any given instant why if D_P is the port diameter right, so the burning surface area this is nothing but for any instantaneous burning surface area you need to add to why the diameter is D_P + 2y so the burning surface area would be π D_P + 2y x L the length of the grain into the board and a be at y = 0 would be okay and therefore, you can calculate da / dy for this what will it be 2 π l right they take the derivative of this you will get okay.

So let us substitute that back into the equation that we have here this equation again we can simplify it further let us see how to do it and substitute it back into that we know here pc by RTC is nothing but density right.

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So I can rewrite this as r l r = ρ c vc / ρ p a b ^{2} X 1 - n x da / dy and we know is nothing but 2 π l vc row c remains the same vc is nothing but what is the chamber volume it is the burning surface area into the length so a be at 0 that is π DL by D P ^{2} / 4 this burnings of this port area into the length is the volume and da be by this is 2 π l ropey remains as is a b square sorry this is π DP so we can cancel out DP ^{2} DP ^{2} here l square and L here right.

So after cancelling out and you have π ^{2} again here so you have two and four so you will have one by two so you will get 1 / 2 1 - n ρ c /ρ p okay we had calculated the what is Rosie in term and ropey right we had seen that a little earlier that Rosie varies from eight to 15 kg per meter ^{3} where as ρ P is around 1600 2,800 kg per meter ^{3}. Now if you take n even if you take a large n let us say of n of 0 point seven what you will get is our lr will vary from 0.01 to 19.

So what it tells us is even if the port is burning in a progressive manner or in any other manner even if you take equilibrium relations there you are not making a very significant error primarily because of the density difference between the gas and the solid being of the order of around more than 100 right and that is the reason why you can afford to take a steady case also and you will get very accurate result okay thank you we will stop here will continue in the next class you.

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