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Aerospace Propulsion

Rocket Nozzles – 1D Analysis I

Lecture 18

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In the last class we had looked at cycle analysis of everything engines now let us look at rocket engines in the next few classes here we will be trying to derive expressions for specific impulse and how mass flow rate varies through a choke nozzle how do we get thrust how do we get exit velocity what are the assumptions that we make and as a consequence of these assumptions what do we have to deal with or what is the error of these assumptions we will look at it a little later in the course okay.

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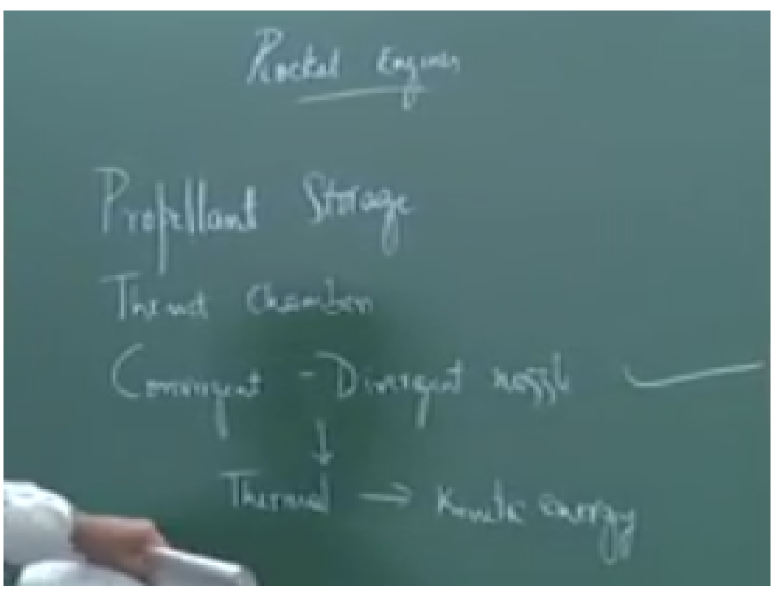

So now let us look at rocket engines now rocket engines all the three kinds of rocket engines that we discover discussed in the earlier classes that is solid liquid and hybrid all three of them have the following that is they have propellant storage then they have a thrust chamber and lastly a convergent divergent nozzle in the solid rocket both these two are in the same physical location whereas in liquids and hybrids all three of them are separate but one thing common to all of them is this part.

That has the nozzle part in the nozzle what you have as the terminal energy because of release because of chemical reactions is now converted into kinetic energy so in the nozzle you have thermal to kinetic energy conversion and as this is common to all three kinds of rockets we can study them exclusively okay and without having to pay any need to pay any attention to what kind of rocket to test whether it is a solid rocket or a liquid rocket or a hybrid rocket. So we will do that now we look at the converging diverging nozzle now before we go there what we are going to look at is quasi steady one dimensional analysis.

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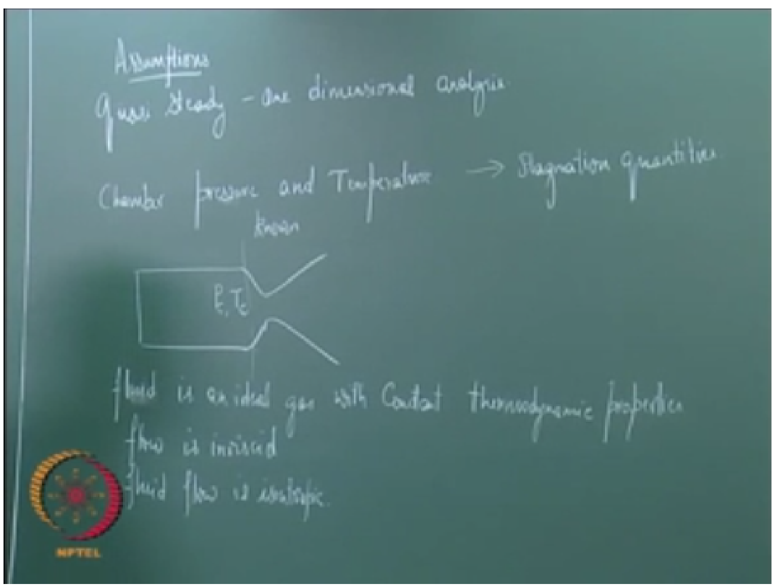

That is we are going to assume that all changes happen along the axis only there are no changes that are happening in the radial or the azimuth direction R and θ directions we do not assume any changes to be happening so all changes are happening along the axis so it is one done which is not strictly true for rocket engine nozzle okay we will see what error this bring about a little later first as we are going to make this assumption then we are going to assume that the chamber pressure and temperature are given to us.

That is let us say if this is the rocket motor we will assume that we know chamber pressure which is called PC and chamber temperature TC has known will also take these to be stagnation quantities that is if you look at velocities elsewhere in the nozzle and if you look at the velocity at the entry of the nozzle these velocities are very small compared to the velocities as you go along the nozzle so you can take them to be stagnation conditions at the entry of the nozzle okay.

And we are also going to assume that you are also going to assume that as I said these are known to us firstly and we are also going to assume that fluid is an ideal gas with constant thermodynamic properties that is the CP γ thermal conductivity all these do not change as we go from this portion of the nozzle to the exit of the nozzle then we will also assume that the flow is in viscid that is viscosities are very small and will also assume fluid flow is isentropic.

So with these assumptions let us see what we can derive and based on this we will also evaluate what are the short comings of some of these assumptions are these really valid and then we will try to estimate what is the error now from your basic gas dynamics you know that for a nozzle for a convergent.

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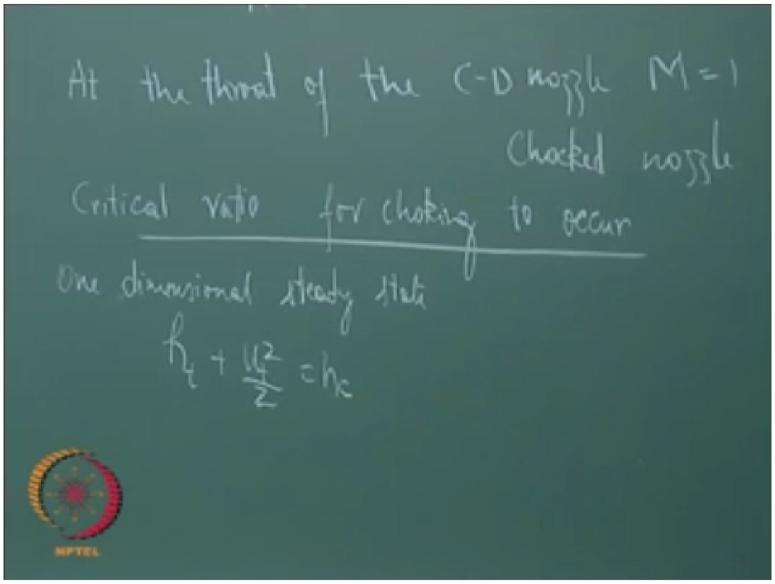

That was in nozzle I can write you buy you that is change in velocity as you move along the nozzle = now we know that if M is less than 1 what should be the dA / A if this has to flow it has to accelerate if M is less than 1 dA/ A has to be negative if fluid has to accelerate and if M is greater than 1 dA/ A has to be positive if fluid has to accelerate and we know that if at the throat at the throat of the C-D nozzle Mach number is 1 then we call it choke nozzle let us now find out what are the pressure ratio.

If you have PC here what should be the PA or what should be this ratio of PC / PA for the nozzle to be choked okay so let us find the critical ratio of this pressure for doping in order to do this we will solve the energy equation from here to here okay from the entry to the root section and I can write one dimensional energy equation as one dimensional steady state and for in viscid flow with constant thermodynamic properties I can write the energy equation as HT + = UT.

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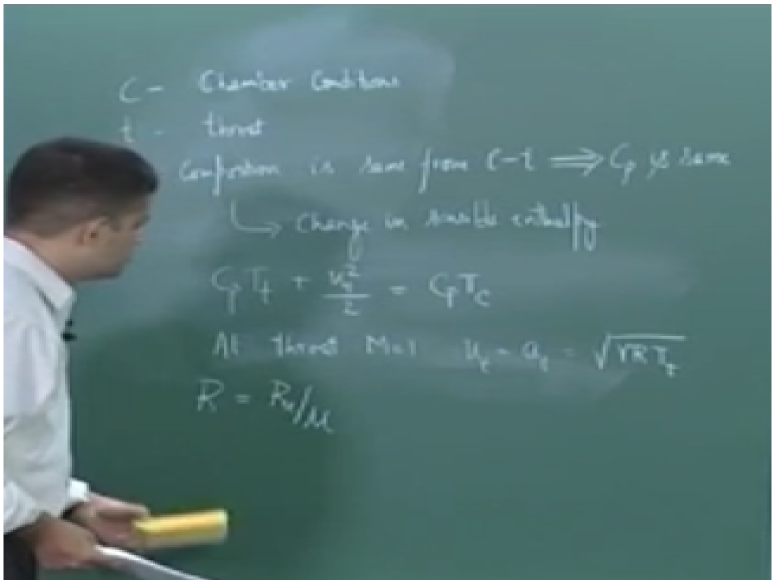

Where C were c there indicates our chamber conditions and T indicates throat conditions C indicates and T indicates throat that is we are looking at this to be see this to be okay now there is no heat addition that is taking place from here to here okay and the composition is the same so if the composition is the same then CP has to be the same so C to T that is from chamber to throat it means that CT is same and there is only change in sensible enthalpy and it also means that only there is change in enthalpy.

Because there are no reactions that are taking place so if all the change from C to T is only because of sensible enthalpy change then I can rewrite this equation as right that is I have replaced HT / CpTt and HC / CpTc we also know that if the throat is if the flow is choked at the throat then add throat M = 1and therefore U_t = 80 that is the speed of sound which is given by here Rs nothing but R = Ru universal gas constant /molecular weight okay now using all this we can simplify this equation further and write it as.

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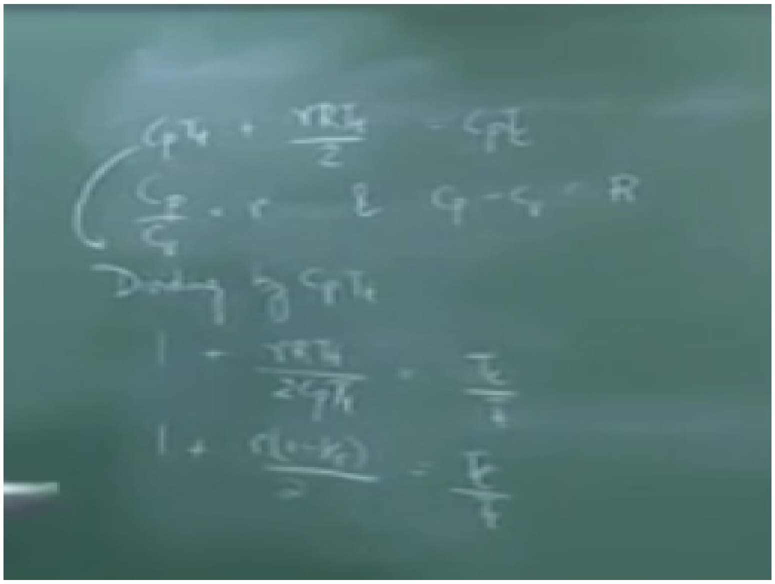

Because we know Ut = 80 and 80 is this I can write it bit like this = we know that CP / CV = γ and CP - CV = R so dividing the entire equation / CpTp that is dividing my CpTp we will get 1 + γRTT / 2 okay now we know that R and CP we can get through this so let me write that down this will then become 1 + γ x 1 / 1 - γ this and this cancels out so you get 2 = TC/ DT or simplifying.

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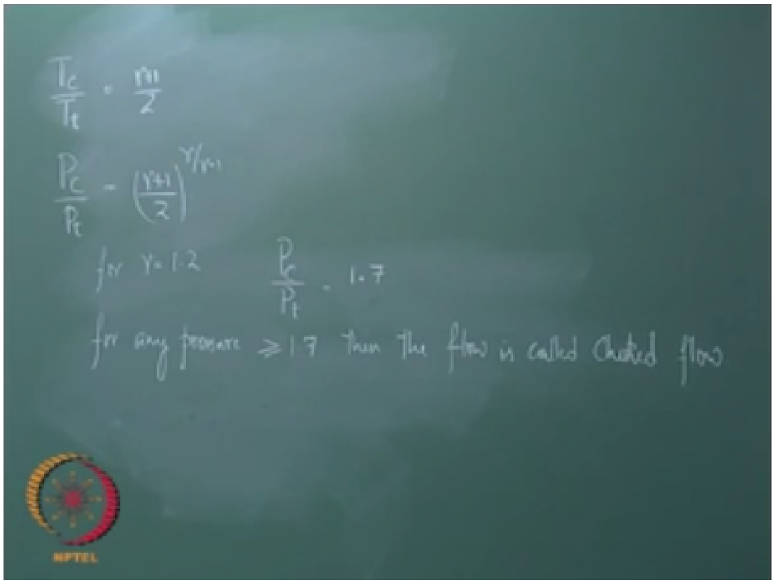

I can write Tc / Tt = γ + 1 / 2 from this and knowing the connection between CP 2 temperatures to pressure for an isentropic flow I can write Pc / Pt as = so now we can get the critical ratio that is required for choking okay and if you substitute for γ =1.2 which is usually the value of γ for burnt gases you get Pc / Pt to be 1.7 that is if the chamber ratio to the exit pressure or the pressure outside is greater than this then the nozzle will be choked okay for any pressure ratio P_C / or for any pressure ratio that is from Pc to ambient wear in it is greater than or = 1.7.

Then the fluid flow is all choked flow as I had said in the previous class the advantage of chalk flow is now it becomes independent of downstream conditions and it is only a function of upstream conditions okay so let us see how we can use this to calculate what is the mass flow rate through a choke nozzle we have calculated what is the ratio that is required for the flow to be choked let us now calculate what is the mass flow rate through a choke nozzle.

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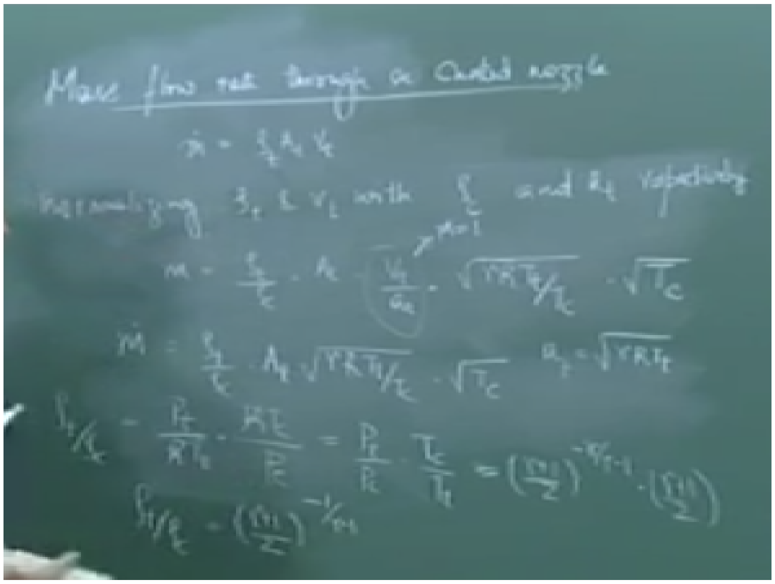

Now we know that m. which is the mass flow rate is given / ρtAt Vt where T as I said earlier indicates throat conditions. So this is the density into area x velocity now let me normalize ρT and VT with respect to chamber conditions why because I know for a choked flow that it is only dependent on the upstream conditions upstream of the throat of the chamber so if I normalize it with chamber conditions I will be able to derive some useful equations ρT and VT with ρC which is the density at in the chamber and At which is the acoustic speed or the speed of sound at the throat.

So we will get m.= ρT / ρC x At xVtL / At x what is At I know that At = right so I will normalize this also with the chamber conditions and multiplied /under $\sqrt{T_C}$ what is this ratio is the ratio of local speed fluid speak to acoustic speed which is nothing but Mach number Mach number at throat for choked condition is 1 so this is 1 so we get m. = ρT / ρC x At okay so now we need to find this ratio and this ratio we already know for choked flow what is this ratio we also know pressure ratio.

So density ratio is nothing but ρT / ρC is nothing but P t / Rt /tc so R cancels out you get Pt / Pc x Tc / Tt we have already derived them that is nothing but x γ + 1 / 2 this is the temperature ratio this is the pressure ratio so now if we because these two are the same we can add the powers and we can simplify it as ρT / ρC I will get it as I also note Tt / Tc which is nothing but.

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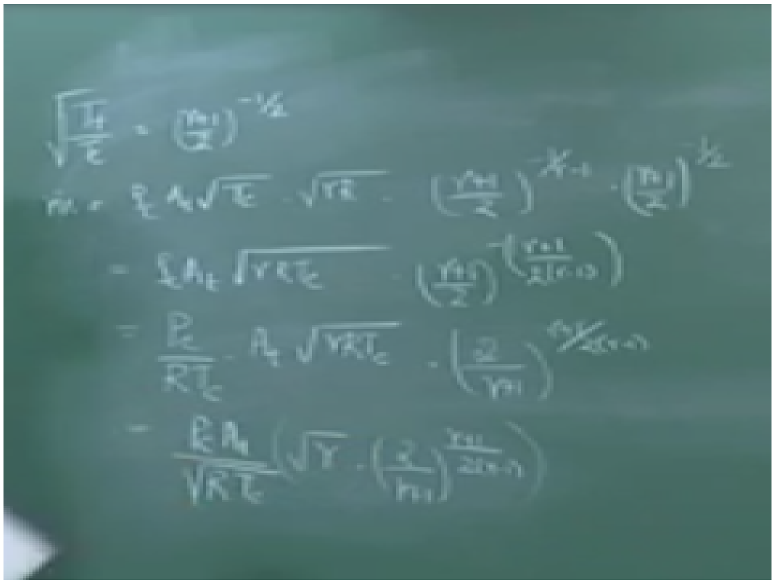

I need under √ so this would be- ½ now I know all the ratios that I wanted these two I can substitute them and get the relation for mass flow rate as M_0 = I am sorry you do not remind me I needed to multiply/ if I divide this / ρc I need to multiply / ρc again so this is ρc x √ y / ρc x this so I will get here ρc At okay now I can deal with them because their same base but different powers I can add the powers.

So I will get and further this - I can replace it as + and this ρc I can write it in terms of pressure so I will get PC /RTc x At γ RTc because I have a - sign here I can write this as 2 / γ + 1 x γ + 1 now I have RTc here and RTc here so I can find one of this and write it as okay this is very similar to something that you have already derived in gas dynamics where and you will not use Pc you will instead use P0 A* / T_0right this whole function this whole thing is a function of only γ so normally in rocket literature this is denoted as a γ of γ function.

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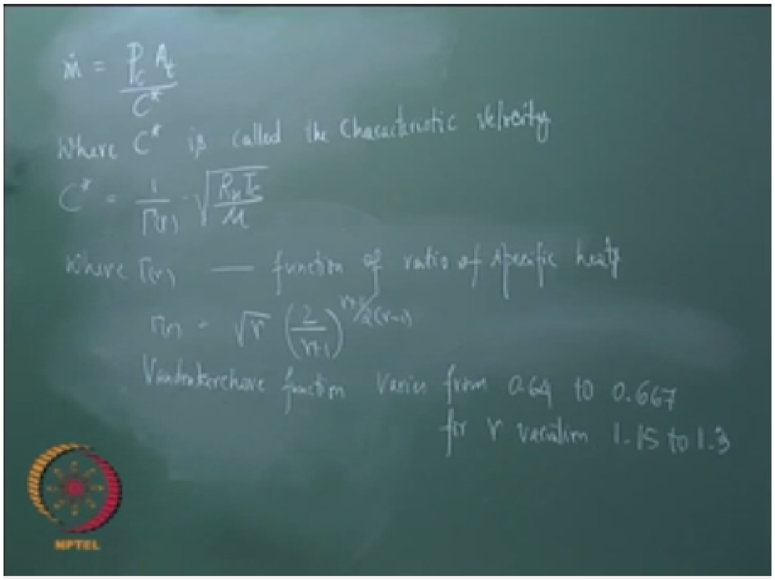

So you get m. = PcAC/ c* where C* is called the characteristic velocity and Z* is given / 1 /γ of γ x Ru/ is nothing but R so C* is given / this where γ of γ is only a function of function of ratio specific heats that is okay so this is the typical expression that we use in Rockets when we are discussing Rockets for mass flow rate through a tube nozzle this γ of γ is also called as a random capture function and varies from 0.642, 0.67 for γ variation of 1.3.

Now we know how to get the mass flow rate through a joke nozzle now if you are collect back we had derived expression for the thrust of the rocket motor when we were discussing about the thrust of the turbojet engine so or an air breathing engine.

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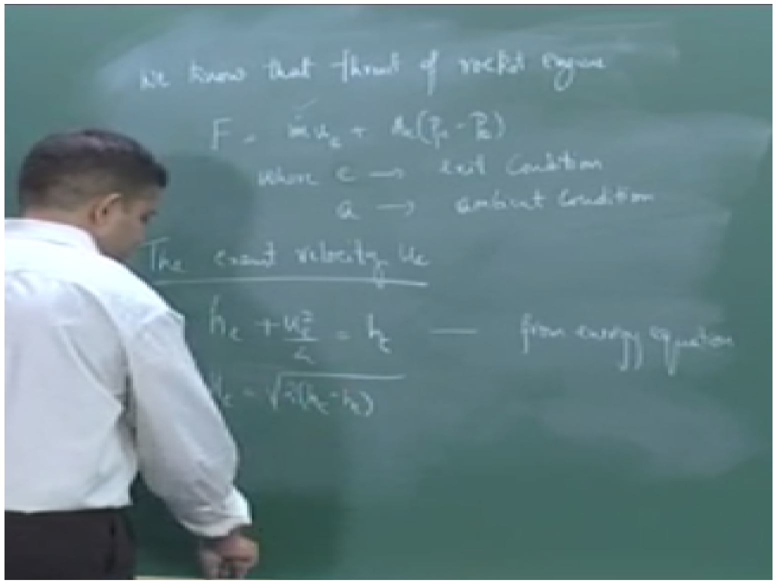

So we know that rocket engine f = m. Ue+ Ae Pe - Pa where e indicates exit conditions a indicates ambient condition so you have here Ue which is the exit velocity exit area exit pressure and ambient pressure now we in the thrust equation we already know how to calculate m. for a choked nozzles o this part we know we need to now find out how to get the exit velocity and then we need to get something about area ratios and other things we will look at it a little later firstly let us get an expression for the exit velocity.

How do we get this expression we know that the energy equation is valid from the entry of the nozzle to the exit of the nozzle so from that we know we can write from energy equation he + Ue^2 / 2 =hc we had seen a little earlier with respect to the throat this equation now we apply it to the exit of the nozzle okay so I can rewrite this as Ue = under √2 hc – he since we know that there are no reactions that are taking place inside the nozzle and the thermodynamic properties are constant from the entry of the nozzle to the exit of the nozzle all this must be sensible enthalpy.

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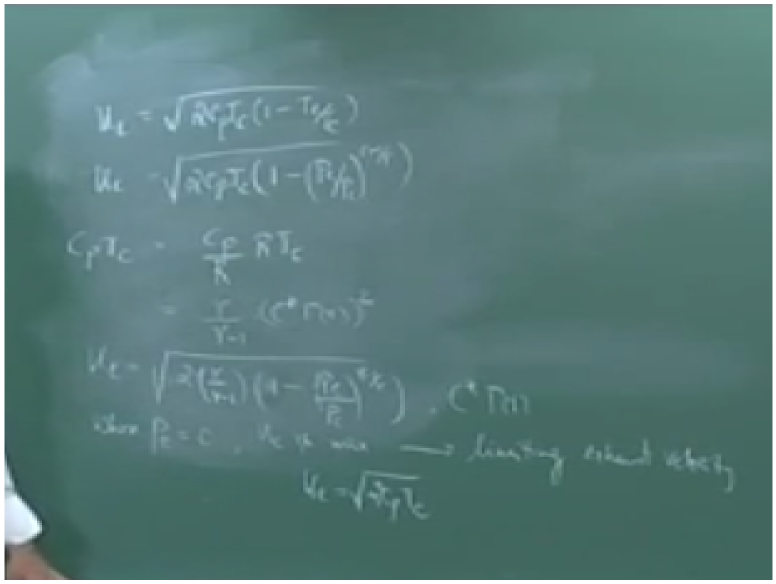

So therefore we can write this in terms of Ue = 2CpTc x 1 - T/ tc + e / tc we can express it in terms of pressure as = under √ γ – 1 this part CpTc part I can use e Ue CpTc I know = Cp / r x r Pc now this RTc what is C * C * is nothing but RuTc / molecular weight or RTC right this is nothing but = 1 / γ of γ x under√ RTc so this part here RTc must be = C^{*2} γ^2 okay so I can put this what is Cp / up this I can write it as γ / γ – 1 x so if I substitute back this into the expression for u I will get V = because this has square here and this is in this √ I can take it out and write it like this.

Now what this tells us is that if we know the characteristic velocity and the pressure ratio we can get the exit velocity right so if Pe goes to 0 what happens if this will Ue will be a maximum when Pe goes to 0 so when e = 0 U is maximum and is called limiting exhaust velocity that is it will only become a function of the chamber temperature the becoming 0 here means in this case if P goes to 0T U also has to go to 0 so the Ue would this then become under √ that is it becomes only a function of the chamber temperature but in reality.

We cannot get a condition where in the exit velocity this is not the ambient pressure this is the exit of the nozzle being at zero pressure this is not possible this will be some finite number so therefore you cannot get this condition but this is the maximum that one can get with a convergent divergent nozzle.

Now we need to also get one more parameter that is how does we still do not know how a and PE are connected that is if we know the area ratio there from the throat to the exit how does how does one obtain PE from if knowing PC how can we obtain PT.

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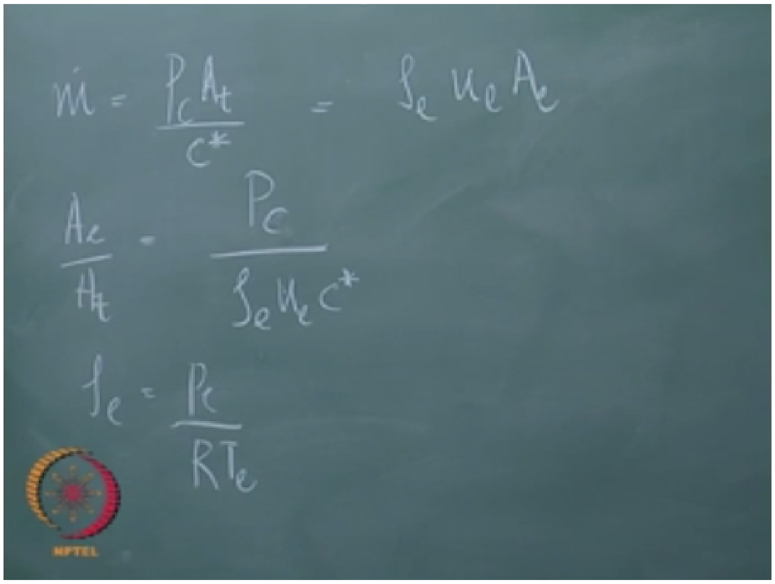

So let us do that so let us relate Pe / Pc to Ac/ At to start this what we know is we know that mass flow-rate once the nozzle is choked and the flow is steady the mass flow rate at any cross-section is the same and cannot vary so we know m. = Pc At / C* and is also = ρe Ue Ae where e indicates exit conditions so from this I can get an expression for Ae / At as Pc okay now I know ρe is nothing but ρe Pc/RTE and if I substitute for that here I will I also know the expression for Ue which we just derived okay.

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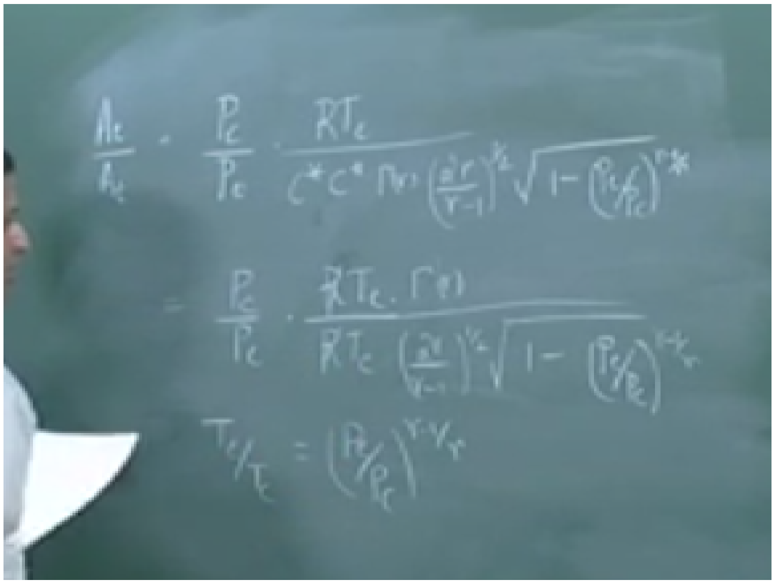

So if we substitute for both of them here ρe and Ue we will get an expression for A/ At eA this is the expression that we get once we substitute for Ue and ρe now here what is again C* x if we take a γ of γ here so you get C* x γ of γ^{2} which is nothing but RTC right so we get Pc /Pc x RTe divided / RTc that is we need to have a γ of γ in the numerator to account for this x 2 γ / γ – 1 okay I can cancel the R's and Te / Tc I can again express in terms of pressures I already have a Pc / Pe.

So I know that Te / Tc is nothing but Pe / Pc to the power of γ - 1 / γ so if I substitute for all this for this here and simplify or I can.

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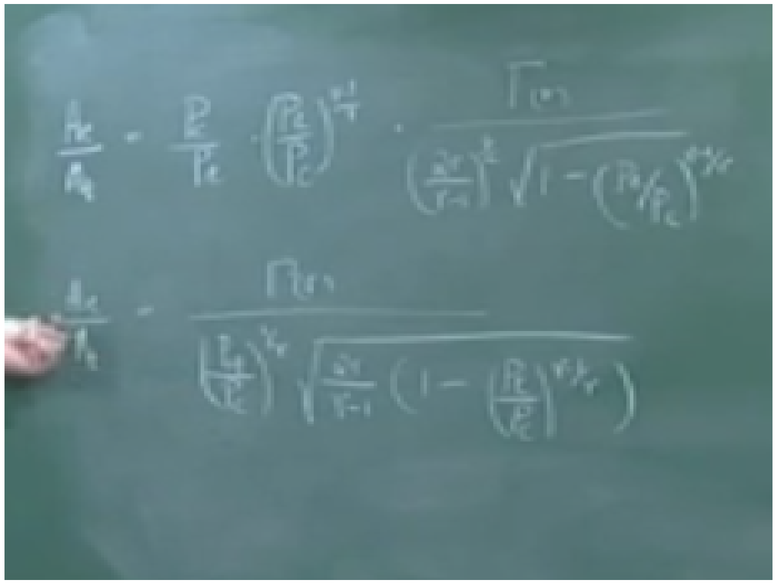

Because I can put them together and rewrite this as now if you notice this is a relation connecting area ratios to pressure ratios mostly we will know what is the geometric area ratio of the nozzle that we are taking and to get the pressure ratio from knowing the nozzle nice nozzle area ratio using this equation is very difficult so therefore we have gas dynamic tables which will give you this or there are plots that will tell you how T / Pc varies if you vary Ae / At okay we look at it and then next class thank you.

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