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Aerospace Propulsion

Cycle Analysis – Turbofan

Lecture 17

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In the last class we had seen how efficiencies affect the performance of gas turbine engines that is turbo jet engine in all these classes in all our analysis and now there is an assumption that we made that CP is constant γ is the same right and if we take F into account also purely ratio we have assumed to be very much less than 1 and therefore we have neglected it now we have done all this and our analysis does not seem to include if there is bleeding from some of the compressor stage after the low pressure low pressure compressor stage if there is bleeding how do we account for it okay.

ICP's are different if γ the ratio of specific heats is different how do we account for that now what we do is we will do a step by step procedure that is we will go from the inlet to the outlet and we put down what do we know and what we need to get and try and see if we can get it even with all these complexities that are there in an actual engine now actual engine is not as simplistic as we assume that efficiencies are all one and all right. So we will see how we can still analyze an actual engine with this.

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Firstly the TS diagram would be this is for a cycle with all efficiencies being one now if you have an actual cycle this would be 2' 3' the .ted line shows the actual cycle right now what we are going to do in this analysis is if we know conditions at the inlet can we get whatever parameters we were looking for that is if we know P_0 T_0 M_0 this is the inlet conditions and then let us say we know the compressor pressure ratio πC then the other parameter that we know is the turbine Inlet temperature then the exhaust area a7 and all the efficiencies I will call it as η I if we know compressor efficiency intake efficiency then efficiency of the burner nozzle turbine efficiency is all this if I know it I wil put this in η I.

Okay now if we know this can we calculate now from this what is the parameter that we have interested in f that is the thrust of the engine then the ISP of the engine then the other non-dimensional quantities that we looked at so we will try and see how to get to this knowing this one okay so as a first step we also need to know in addition to this we need to know what else we need to know thermodynamic properties right.

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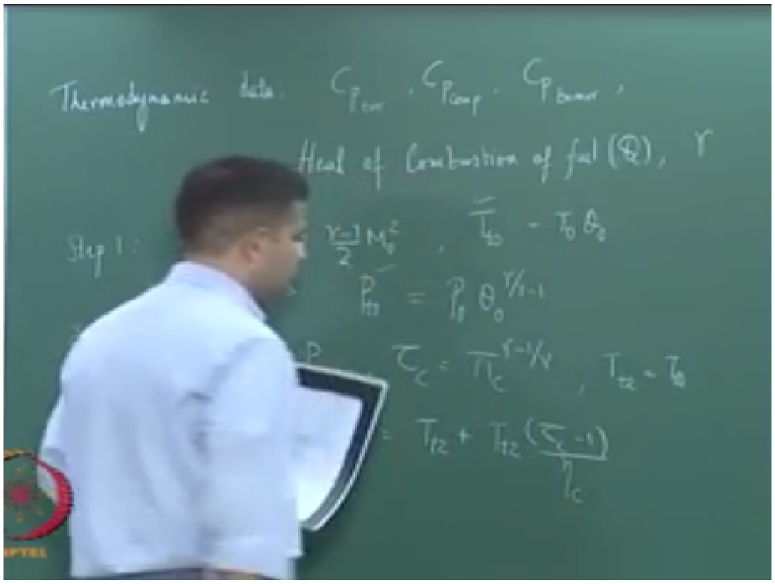

Like thermodynamic data that we require our CP of turbine then CP of compressor then CP of burner then heat of combustion of the fuel that is Q right then we need to know γ right if we know this and if we know this can we calculate thrust ISP non-dimensional thrust and non dimensional ISCB this is under the condition wherein you could have bleed you could have a patience ease being different from one and also all these cps b0 = 1 and γ being also different so as a first step.

Step 1 I know IT0 I know I N0 what can I calculate from these two stagnation temperature right I can calculate T T_0 and from that I can calculate it or not okay so I can write θ_0 if you remember was $1 + \gamma - 1 / 2\ m_0^2$ I know M_0 so I can get θ_0 okay right and I can also from this get TT_0 which is nothing but T_0 into θ_0 and similarly I know p_0 so and I also know θ_0 so I can get PT_0 the stagnation pressure at Inlet as p_0 into θ_0 to the power of $\gamma / \gamma - 1$ okay.

So we were able to calculate the stagnation conditions here right now the next step what happens between 0 and 2 there is the intake here stagnation temperature remains the same stagnation pressure changes because of diffuser efficiencies or efficiency of the intake right so if I know all the efficiencies right that is diffuser efficiency burner efficiency compressor efficiency turbine efficiency and then nozzle efficiency I can calculate these parameter so let us do that I need PT 2 that is pressure at this point.

So I will get that by η diffuser into PT_0 okay and I can get $\tau\ C = \pi\ C^{\gamma - 1 / \gamma}$ so I know PT2 I also know TT 2 is nothing but TT_0 so I know TT_0 so I know fresh stagnation quantities at 2 so I can calculate quantities at 3 right I know conditions at 2 now I can calculate at 3 / PT 3 is nothing but πC into PT2 right I know PT2 I can calculate PT3 and I want this temperature right for an actual cycle so I can calculate PT 3' as equal to okay.

So I know TT 2 I know τc know η c so I can get this TT 3' so I know now the conditions at the inlet of the combustor.

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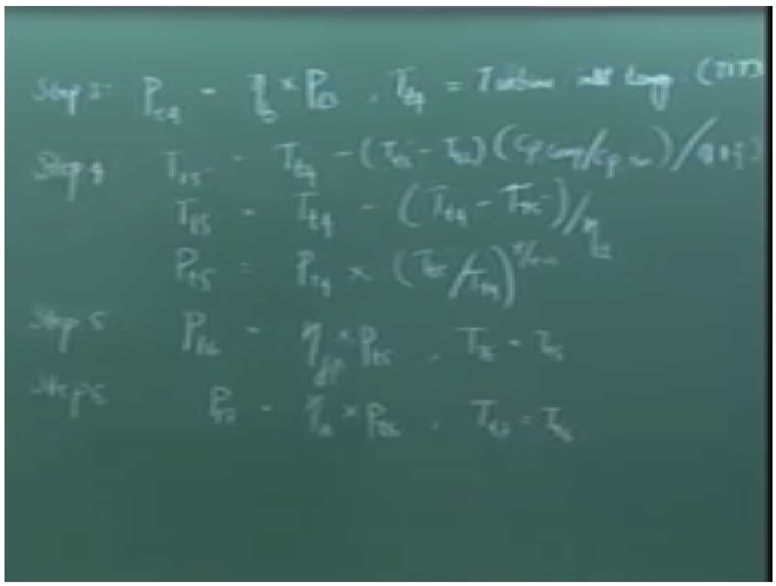

So next I need to know PT4 what is PT 4 there is a pressure drop as you go from the inlet of the combustor to the exit because of a rally process wherein you are adding heat therefore pressure will drop okay so I know the efficiency of the burner η B into PT 3 will give me PT 4 and what is TT4, TT4 is nothing but the turbine Inlet temperature that is a given condition so now we know conditions at this point that is temperatures and pressures we know efficiency of the turbine okay.

And we also know the compressor pressure ratio so using this we can calculate what is the condition at π okay what we have used here is that the CPs need not be the same okay CPs compressor CP of turbine that is the burnt gases need not be the same so we were counted for it and we have also taken a condition where an F need not be very small or if we do not want to neglect that part or be happy with that error we can take into account here knowing TT π' I can calculate TT5 as TT4 – efficiency of the turbine and from this I can calculate the PT5 PT5 is nothing but okay this quantity is IT sorry this quantity is Tt.

This whole thing is PIT so PT4 into πT is what gives me PT5 now I having known this PT6 in case we are using the after burner okay even otherwise also there is an efficiency there is a pressure drop across the jet pipe so we can take that into account here into PT5 gives me PT6 and PT 6 = TT 5 right if there is no afterburner if we switch on the afterburner then we need to take that into account here that will be specified in addition to TIT that will be specified to us that will be known to us.

So if it is given then we have to take into account that part also and in the next step we calculate what happens through the nozzle okay so what we have done is starting from here and knowing efficiencies and CP we have been able to calculate all these parameters right now we need to calculate thrust right there are two conditions that can exist one is depending on the nozzle pressure ratio we could either use a converging diverging nozzle wherein we will assume P7 = P_0 or even in a converging nozzle a special case P7 = P_0 or nozzle is choked.

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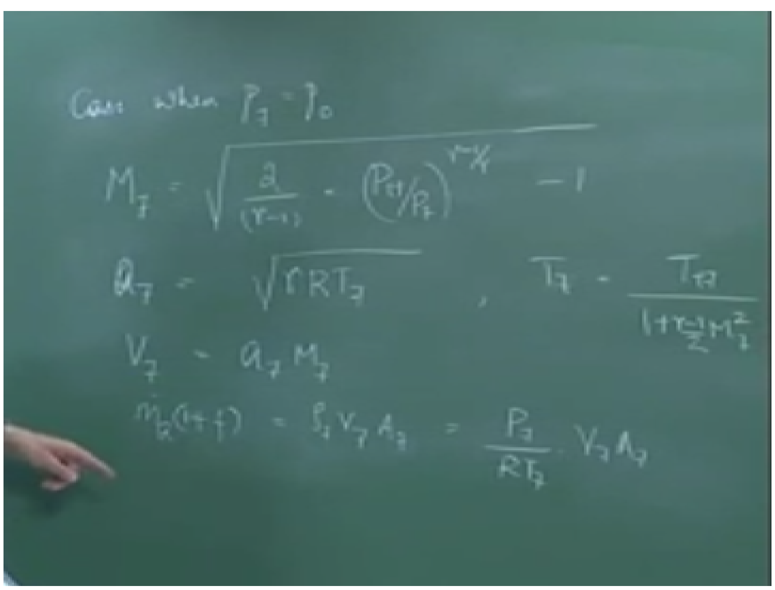

Let us first take the case where P7 =P_0 this can be for both CD as well as convergent nozzles so if P7 = to P_0 then I can calculate Mach number n7 is equal to okay so I know Mach number at the exit what can I find out I need also a7 right if I know a7 then I can calculate velocities right so a7 is nothing but under $\sqrt{\gamma}$RT 7 but I do not know T7 as yet so let us get T7 as what do we know TT7 / T7 = T7 is equal to okay.

So I know m7 I can get I know TT7 so I can get T7 and since I know T7 I can substitute it here I get as even now knowing Mach number and a7 I can calculate the velocity at the exit that is V7 also need to calculate the mass flow-rate okay so mass flow rate we can mass flow rate leaving the I can do two cases one is m. a = m. a into 1 + F = ρ7 right if I do not want to neglect F I can write this expression I know this to be P7 / RT7 into V7 T7 seven now here I know all the quantities I can get m . a into 1 +F right. I still do not know what is f we will get to that shortly so how do we get F if you remember when we did ISP calculations what did we do energy balance across the burner.

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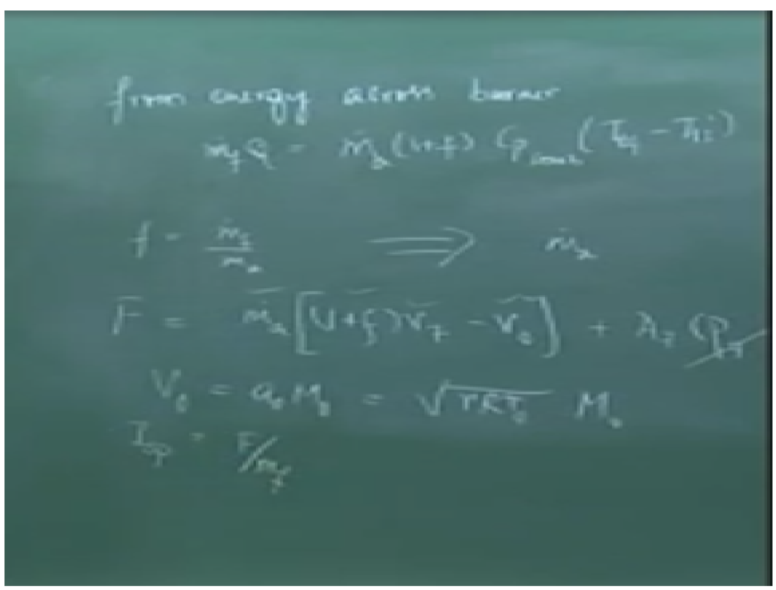

So we will do that right this is the expression that we get if we do energy balance across the combustor we know this quantity m. a into 1 + F what we have just now calculated this will be given to us these two things are what you have seen earlier TT3 - is what we calculated TT4is given to us so we can calculate m. F right now F is nothing but M_0f / m. a so I know m. f from this I know m0a + 1 / 1 + F and using these two I can calculate m . a right that clear so from here we get m. a.

Now I know I m0 a m. a into1 + F all these quantities so my thrust equation is f = m. a into 1 + f v7 - v_0 + a7 a7 - v $_0$ this goes to 0 because we assumed p7 = V $_0$ so here we know this we know this we know this now what we do not know yet is V_0 now I know M 0 right and I also know T_0 so I can calculate V_0 firstly V $_0$ is nothing but a_0 M_0 that is equal to okay so we know both T $_0$ and M_0 so I can calculate V_0 so I will know everything here right in this equation all these quantities are now known and therefore I can get thrust and once I get thrust I can also get ISP very easily ISP is nothing but right thrust per unit mass flow rate of fuel I know mass flow rate of fuel.

I have calculated here I know thrust so I can get is P now what are the other quantities the non-dimensional quantities I know I know it also so I can get is P / e_0 I also can get m . F / m . a a_0 so all these quantities we can get by doing this analysis going step by step one can also look at there are cases there are certain engines where in a portion of the compressor compressed bled to cool either the turbine.

Turbine blades or you use it for air conditioning the aircraft certain quantity of the compressed air is taken out especially after the low pressure stage so you can then suitably write the equation for power balance across the turbine and compressor taking that into account because there will be a reduced mass flow rate through the second stage of the compressors and also through the turbine.

So going by the step-by-step procedure you can calculate that and account for it also right so in this way we can do even for a realistic engine all these calculations it is not as if that what we did was something that is not applicable to any realistic engine this is applicable to any kind of realistic engine with bleed and other things also okay now we have looked at turbine it we have looked at turbofan sorry the turbojet and turbojet we have considered all the cases that is afterburner on then water methanol injection conversion sorry one minute I need to still we have only considered one case P7 = P_0 .

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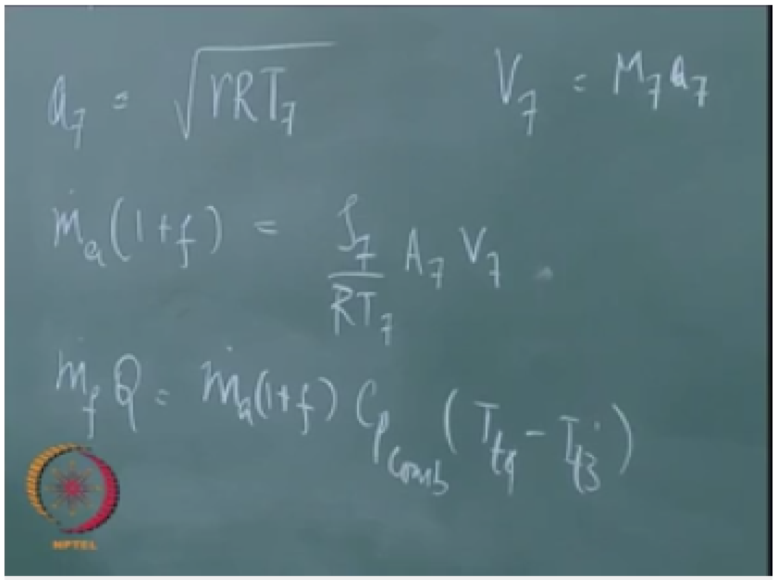

We need to consider the other case wherein we have choked condition so let us do that first case when nozzle is choked we know that m7 =1. So if M7 = 1then I know that T7 = 2 x TT 7 / γ +1 right, so I also can similarly get P 7s now I know t7 using this I can get a7 = and I know m 7 = 1 so I can get V7 = m7 x a7 and similarly m. E x 1 + F is equal to ρ 7 / RT 7 a 7 v 7 and a similar expression for m. F Q = m. a x1 + f CP combustor so I know from here I can get F which is nothing but m . F / m. a I know m. a I know m. F I know F I know v7what else do we need V_0 we know has nothing but M_0 e $_0$ that is M_0 x so knowing all this I can get again f 0 okay.

I know p7 I am given P_0 m. a is known v7 is known V_0 is known a seven P 7 and P_0 so I can get thrust from here I can get is P as f / m. F and the other non-dimensional quantities that we wanted, so we have looked at the cases where in for ramjet as well as turbojet we have looked at cases where efficiencies are equal to 1not equal to 1 and for a turbojet we have looked at case wherein with after burner without afterburner choke nozzle optimally expanded flow then we have also looked at case with water methanol injection all this we have looked at.

Now let us look at the next two classes of engines that is turbofan and turboprop, turboprop is very simple we do not need to do any analysis except that all the power that is developed in the turbine must be equal to the power of the compressor as well as the power of the fan propeller right. So if we do that you will get the turboprop engine that is not a big problem but a turbofan engine needs some effort so let us look at the turbofan engine first.

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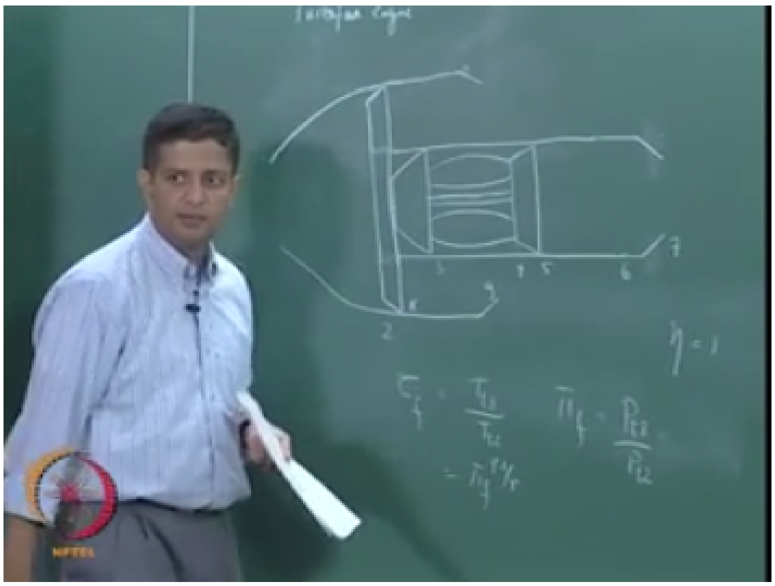

So this is a turbojet engine and to this we add the turbofan part both of them all of them are mounted on the same shaft right now our typical conditions where this was zero after the intake we had to write this was three, four, five, six, in this is seven whatever we had done for the turbojet remains as this now for the turbofan you have the fan in front also and there is a certain amount of air that is bypassing and is going through only the fan portion okay, so if you look at condition after the fan will call it as eight okay and we will call the exit condition here as nine at the exit of the finest nine the bypass exit as nine okay.

So you have 0 to 2, 2 to 8, 8 to 9 that is the additional part in the turbofan engine the rest of it to the turbojet engine right.

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So how does the thrust expression look like for a turbofan engine F is equal to m. a x 1 + F V 7 -V_0 + a 7 e 7 - P_0 all this is turbojet art now in addition to this you will also have α x m. v9 - V_0+ a 9 p 9 - V_0 this is because of the fan or the bypassed stand α is the okay now we will make an assumption that the flow is optimally expanded through both the nozzles there are two nozzles now one is this nozzle and the other one is this nozzle.

So we will make an assumption where we say flow is optimally expanded, so therefore we get p7 = P 9 = P_0 okay, so this part remains the same as what we did for turbojet analysis we now have to only look at this part because of our assumption here this goes to 0 this goes to 0, so I have to only take care of this term all the rest of the analysis that we have done for this for a turbo jet with optimally expanded flow is valid.

So in order to do this what do we do what are the things that we need V 9 / V_0 is what we will get so we need T 9 / T_0 and V 9 / V_0 that is from here we will get m9 / M $_{0,}$ so knowing these two we can also get this portion non-dimensional is this portion and get this value okay so what is T 9 / T_0 you have teen T 9 / TT 9 TT 9 / T eight okay right now I know this quantity is η not fine this is ratio of static to stagnation conditions, so I will put this as 1 + 1/ γ - 1 M 9^2 next is flow through this duct here okay this is similar to the jet pipe.

So if we assume all efficiencies to be 1 which is what we will do so here we are assuming also η = 1 so TT 9 / T T 8 would be 1 and TT 8 / TT 2 is process across this fan I will define a new ratio that is I will define τ F is equal to TT 8 / DT 2 and π F = PT 8 / PT2, so this would be tau F would be equal to PI F to the power of γ - 1 / γ okay coming back here I will get this is tau F into this is T T 2 by T not it is 1 so I get tau F η_0 by 1 + γ - 1 by 2 m 9^2 okay when we cascade pressures will get this ratio also so let us do that.

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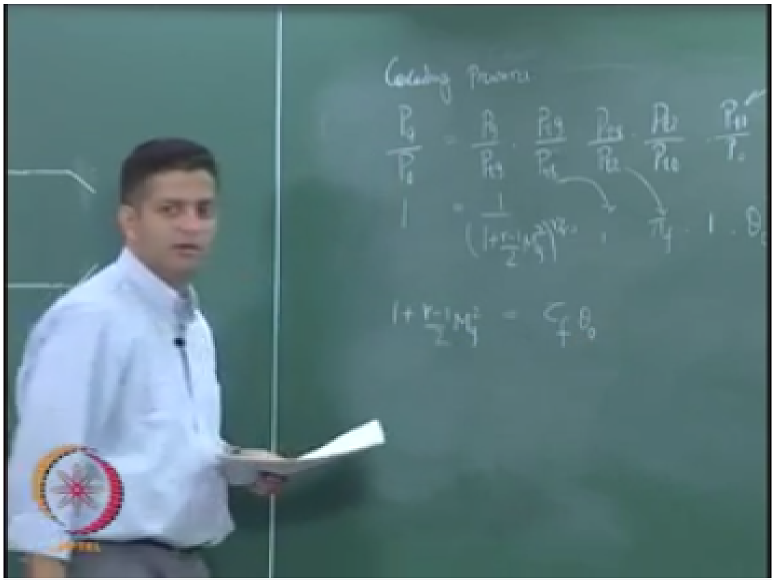

This will be p9 / PT 9 - PT 9 / PT 8PT 8 / PT 2 PT 2 / PT_0 P_0/ P_0 okay this quantity is nothing but θ_0 to the power of γ / γ - 1 this is equal to 1 and what is this is ratio of static to stagnation what about this PT 9 by PT 8this is 1 right so this is 1 and PT 8 by PT 2 this is PI F and again PT 2 by PT $_0$ is one because we have assumed all efficiencies to be 1 into η_0 to the power of γ / γ - 1 so if I change this to tau F I will get all of them in powers of γ by γ - 1 so I will get 1 + γ - 1 by 2 m 9 ^{2} is equal to τ F η_0.

So I can come back here and I know that this is again tau F η R $_0$ / τ F η_0 so this is 1 which means that if all efficiencies are 1 t 9 will be t 8 rights.

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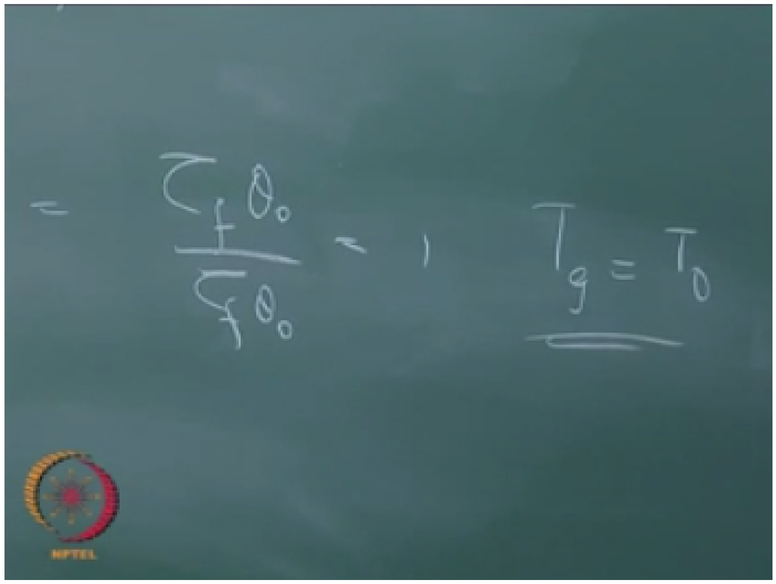

Because here in the intake you are compressing it then you have a fan to increase the pressure and again you are expanding the flow if there are no efficiencies involved there is no heat addition here so T9 must be equal to T_0 okay and I also know that $1 + \gamma - 1$ by $2m_0^2 = \eta_0$ so using these two I can get the ratio of Mach numbers which will be m9 by now F $\eta_0 - 1$ divided by $\eta_0 - 1$ so finally I can write the expression for thrust.

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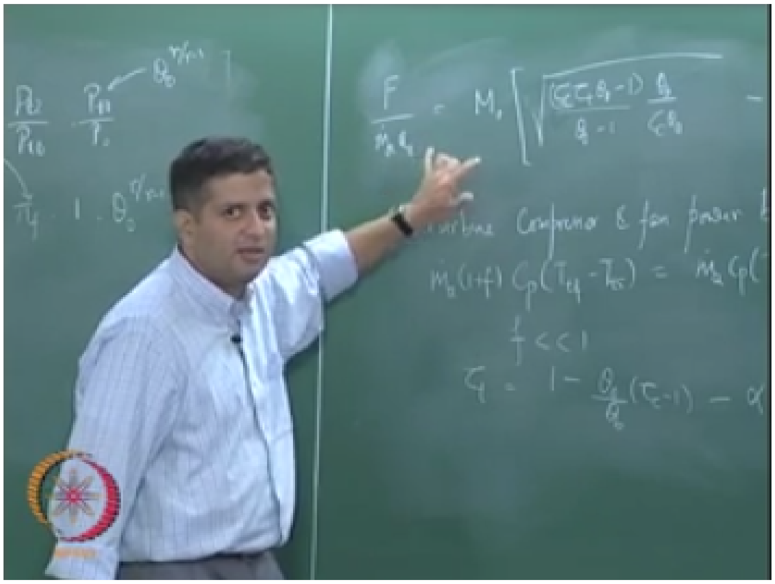

We non-dimensional is trust you do not need this F is less than one very much less than one, so this part is the turbojet part that we had already seen tau C τ T η_0 – 1 okay this is the expression that we get now we need to do the compressor turbine power balance because tau F τ C and τ T are all related okay, so let us do that turbine compressor and fan + α m. a PT 8 - TT - if we deduce from this similar to what we had done if you remember we had found an expression for tau T in terms of tau C by cancelling out m . a with assuming that F is very much less than 1 and CP being the same.

We can reduce this to τ T = 1 - η_0 / 0B τ C - 1 this was if only there was compressor turbine power balance in addition you have the other term – α okay, so if we plug back this into this expression here we will get the overall expression for thrust non-dimensional thrust okay, we will stop here and continue in the next place, thank you.

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