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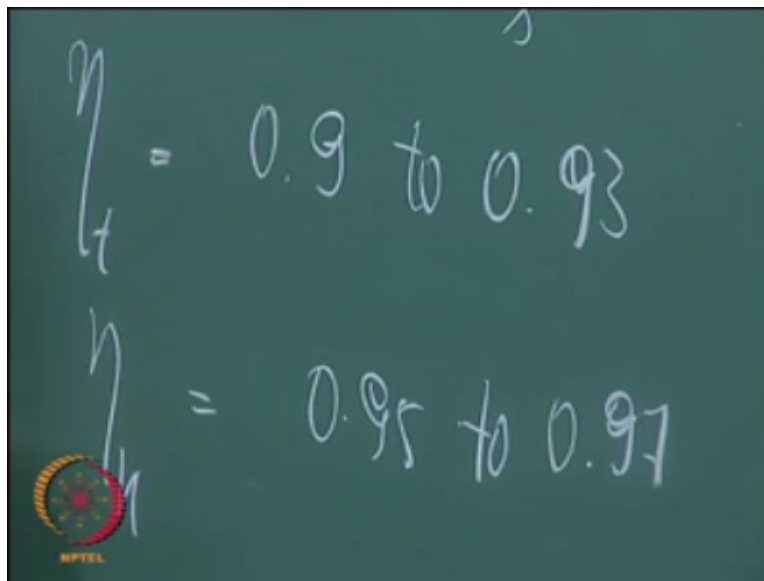
**Aerospace Propulsion
Cycle Analysis – Turbojet VI**

Lecture - 16

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In the last few classes we had looked at non dimensional thrust and ISP for cases where in the efficiencies was all unity right, let us now look at the case where efficiencies are not equal to unity and let us see how the equations change there.

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The image shows a chalkboard with two handwritten equations. The first equation is $\eta = 0.9 \text{ to } 0.93$. The second equation is $\eta = 0.95 \text{ to } 0.97$. In the bottom left corner of the chalkboard, there is a small circular logo with the text 'NPTEL' below it.

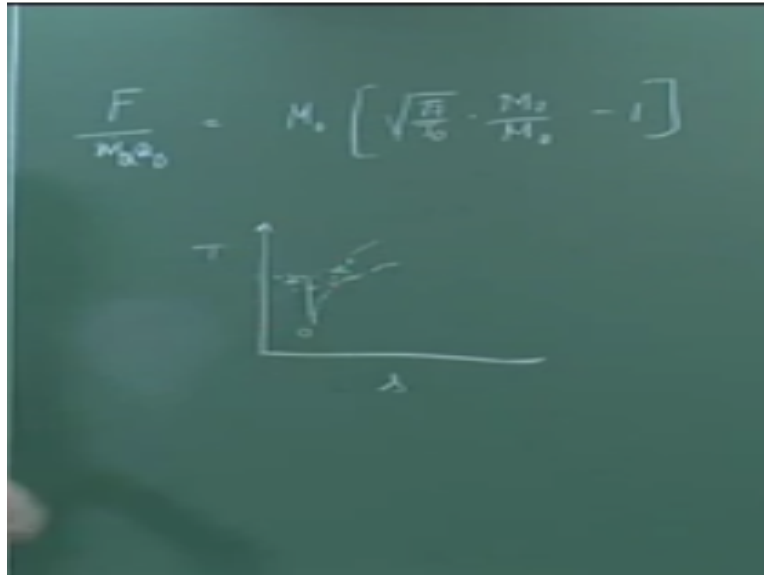
Now if you look at the TS diagram for a turbojet with efficiencies what you will let us consider only the simple cycle no after burner this is a cycle with the 100% efficiency, now if you have non unity efficiencies, now if you have non unity efficiencies you will get a cycle like this the dashed ones are for non unity efficiencies, now when we did the analysis for a ramjet we saw

that we were able to handle efficiencies of the non moving parts namely intake then burner then nozzle right.

Here in addition to these we have the compressor and the turbine okay, so in addition we have good thing about this is we are going to look at it in two different ways we are going to handle efficiencies with relate related with the non moving parts in the same way as we did in the ramjet analysis that is we are going to look at intake burner and nozzle same way as in done yet here we are going to look at it a little differently just to refresh your memory the intake efficiencies can be varying from 0.62 0.9 and is a strong function of the Mach number then we have compressor efficiency which can be of the order of 0.2187 then the burner is 0.932 0.95 then turban efficiencies and finally nozzle efficiencies of the order of okay notice.

One thing that both these two are lower than these two right there is a difference between the two of them that is both these have an adverse pressure gradient to cope with which is why they will have a typically diffusers and compressors will have a lower efficiency compared to our turbines and nozzles okay, now let us do our cycle analysis cycle analysis the expression for thrust per unit mass flow rate right.

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Or non dimensional thrust what does this expression now this is the same as earlier that is $T_7 - T_0$ into right this is for optimally expanded flow in the nozzle, so therefore $d7 = P_0$ that term vanishes out the pressure thrust term vanishes out we have to find expressions for these two ratios to find expression for T_7 / T_0 before we got here let us first look at what is different from the earlier duty cycle, now if you remember in the previous case when we analyze ramjet efficiencies we dealt with slightly differently.

We said for the non moving parts if you look at intake burner and nozzle we included the efficiency terms in the pressure with cascading the reason for that being that, if you assume that it goes to the same stagnation temperature, let us say we are looking at intake on a TS diagram if you are looking at only an intake process this is for a 100% efficient cycle, now the assumption that we made was an honest entropic process would go something like this to do dash where in the temperatures are the same.

The stagnation temperatures of 2 and 2 dash are the same only that these pressures are different right these are constant pressure lines these are different and therefore we looked at handling this terms with only efficiencies coming up in the pressure terms fine the flow is at most stagnation it is not standing, now soon then at the end of a diffuser intake do not 2.2 we are assuming that it is a stagnation temperature okay but we know st north is the content of energy in the flow yes okay if some losses are there.

So energy should not be equal to the ideal energy like H_T not of the ideal H_{idol} is not equal to H Dino dash true so only say in real systems what we look at is when we are looking at an intake okay if the process were isentropic you would have a certain pressure recovery okay at the end of the isentropic compression you would have a certain pressure recovery, if the process is real what is the pressure recovery or what is the pressure at the end of such a process is what you are looking at so from that perspective.

If you look at this here you are trying to capture what is the pressure recovery this efficiency would then indicate whether it is 100% efficient 90% 60% efficient the pressure terms contain the efficiency part right and here, if you look at this case we have only turbine and compressor to take care of right, now in the turbine and compressor what we do is we know that the power of the turbine the work or the power produced by the turbine must be equal to the power consumed by the compressor right because of the balance between the two.

So if you remember we then say τ_C and τ_T are two different things we try to combine them through one equation, so if we take efficiencies of turbine and compressor there we are going to complete the cycle, so even in this analysis we will carry out a cascading wherein we will deal with efficiencies in terms of pressure for the non moving parts and when we come to the compressor and turbine we look at what happens between the power of the compressor and the turbine with efficiencies okay.

Now if you look at the compressor turbine and power balance or before we go that our efficiency is defined for the compressor and turbine.

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The image shows a chalkboard with two equations written in white chalk. The first equation is for compressor efficiency, $\eta_c = \frac{T_{t3} - T_{t2}}{T_{t3}' - T_{t2}}$. The second equation is for turbine efficiency, $\eta_t = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5}'}$. In the bottom left corner of the chalkboard, there is a small circular logo with a red and yellow design and the text 'NPTEL' below it.

η_c is defined as $\frac{T_3 - T_2}{T_3 - T_2}$ okay and similarly the turbine efficiency is defined as $\frac{T_4 - T_5}{T_4 - T_5}$ okay this is obvious from this figure where this is T_{t3}' this is T_{t2} okay, so let us now try and do the cycle analysis and try to get these two ratios, so from here if you are going to do a similar analysis to what we did in Ramsey okay would whatever we had derived in the previous classes $\frac{47}{T_0}$ be any different see compressive turbine power balance will consider efficiency is there but otherwise in the temperatures while cascading we do not look at efficiency true.

But they appear as τ_c and τ_t right if you look at the expression there like appear as τ_c and τ_t when you include that power balance Gaussian τ_c τ_t cannot be independent terms, so when you include them in the power balance that is when you get the real expression for both of connecting both of them okay, so let us do the cascading.

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Handwritten equations on a chalkboard:

$$\theta_b = \tau_b \cdot T_c \theta_0 \Rightarrow \tau_b = \frac{\theta_b}{T_c \theta_0}$$

$$\frac{T_7}{T_0} = \frac{\tau_7 \tau_c \theta_0 \theta_7}{\tau_0 \theta_0} = \frac{\theta_7 \tau_7}{\tau_0}$$

$$\frac{T_7}{T_0} = \frac{\theta_7 \tau_7}{\tau_0} = \frac{\theta_7 \tau_7}{\tau_0} \cdot \frac{1 + \frac{\gamma-1}{2} M_7^2}{1 + \frac{\gamma-1}{2} M_0^2}$$

NPTEL logo is visible in the bottom left corner of the chalkboard image.

So firstly cascading temperatures okay we want an expression T_7 / T_0 not similar to the previous cases T_7 / T_0 into T_7 by T_0 by T_0 by T_0 by T_0 not okay now as in the previous case what this is a ratio of stagnation to static, so I can express it in terms of Mach number this is flow through jet pipe this is one now this is flow through nozzle one is flow through jet pipe again one this is τ sorry τ and this is flow through the main combustor one and this is, now see this is one flow through in teak and this is θ not right.

So it is similar to what we had earlier derived that is sorry this has to be T_4 / τ_B and then if we put it in terms of θ_B the expression that we get this we know that τ_B and θ_B are related that is so from here I get τ_B would be equal to θ_B by $\tau_c \theta_0$ and if i substitute it finally I will get T_7 / T_0 is equal to $\tau_D \tau_c$ listen this cancels out I get okay, now I need an expression for mach number ratio which I will get by cascading pressures.

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Cascading pressures

$$\frac{P_7}{P_0} = 1 = \frac{P_7}{P_{T7}} \cdot \frac{P_{T7}}{P_6} \cdot \frac{P_6}{P_{T6}} \cdot \frac{P_{T6}}{P_5} \cdot \frac{P_5}{P_{T5}} \cdot \frac{P_{T5}}{P_4} \cdot \frac{P_4}{P_{T4}} \cdot \frac{P_{T4}}{P_3} \cdot \frac{P_3}{P_0}$$

$$1 = \frac{1}{(1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma}{\gamma - 1}}} \eta_n \cdot \eta_{JP} \cdot \eta_b \cdot \eta_c \cdot \eta_d \cdot \eta_0^{\frac{\gamma}{\gamma - 1}}$$

$$\eta = (\eta_n \eta_{JP} \eta_b \eta_c \eta_d \eta_0^{\frac{\gamma}{\gamma - 1}})^{\frac{\gamma - 1}{\gamma}}$$

Okay, so we know that $P_7/P_0 = 1$ is equal to $P_7/P_{T7} \cdot P_{T7}/P_6 \cdot P_6/P_{T6} \cdot P_{T6}/P_5 \cdot P_5/P_{T5} \cdot P_{T5}/P_4 \cdot P_4/P_{T4} \cdot P_{T4}/P_3 \cdot P_3/P_0$ okay now when we cascade pressures, if you remember what we did with ramjet will get efficiencies here so this is 1 is equal to 1 by $1 + \frac{\gamma - 1}{2} M^2$ into P_{T7} by P_6 this is flow through nozzle, so this has an efficiency that is the nozzle efficiency okay, now a flow through the jet pipe also has an efficiency let me call it η_{JP} , if it is on it will be different value if it is off be a slightly different value.

Then what is this P_{T5}/P_4 for this is πT into η burner this is through the combustor then this is through the compressor, so this is πT see this is intake into θ_0 to the power of right, now just like when we did ramjet analysis we will club all these efficiencies into one and we will define $\theta \eta$ if I define it this way I can express all the terms here as a power of γ by $\gamma - 1$ okay is as the intelligent way of putting it here, so that we will get the same powers so πT I know is nothing but τT to the power of γ by $\gamma - 1$.

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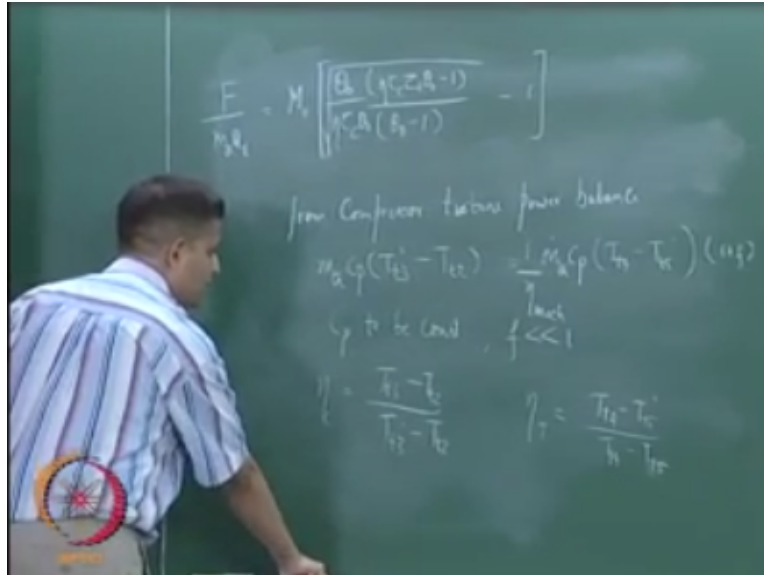
$$\frac{T_7}{T_0} = \frac{\theta_0 \tau_t}{\eta \tau_c \theta_0} - \frac{\theta_0}{\eta \tau_c \theta_0}$$

$$1 + \frac{\gamma-1}{2} M_0^2 = \theta_0$$

$$\frac{M_7}{M_0} = \frac{(\eta \tau_c \theta_0 - 1)}{(\theta_0 - 1)}$$

I will put all of them so I will get $1 + \frac{\gamma-1}{2} M_0^2$ now θ_0 not if you look at this expression we wanted in the denominator that value, so we have got this so now I can write T_7 by T_0 as equal took which I can cancel the τ terms write it as θ_0 now we are interested in Mach number ratios and the other thing that I know about us $1 + \frac{\gamma-1}{2} M_0^2$ is equal to θ_0 so using these two expressions I get the mach number ratio as $\frac{M_7}{M_0} = \frac{\eta \tau_c \theta_0 - 1}{\theta_0 - 1}$ now we know both the ratios temperature as well as Mach number we substitute them in the equation and find out how it looks like.

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So when we put them in this non-dimensional thrust equation $\dot{m} \cdot a$ not we get $m_0 \eta$ be by η okay, if we put efficiencies is equal to one here we recover back the turbojet equation right you will put this is equal to 1 which is what is the turbojet equation right now we need to find an expression connecting τ_C n τ_T right, so this comes from t from compressor turbine power balance we get $\dot{m} \cdot c_p T_{t3} - \dot{m} \cdot c_p T_{t2}$ this is the actual power consumed by the compressor this must be equal to $\dot{m} \cdot c_p T_{t4} - \dot{m} \cdot c_p T_{t3}$ this is the actual power delivered by the turbine okay.

So here we have assumed c_p to be constant yes you can take a mechanical efficiency which will turn out to be that you will have to take one by mechanical efficiency here you can include that you can assume it to be one, if you want to include it you can put η there so c_p is equal to constant and the other assumption is that I am sorry to write here $1 + F$ is because there is a larger mass flow rate and the other assumption that we will make is F is very much less than 1 okay.

Now using this will get we also know how to connect efficiency of the compressor to this right do you remember what was the efficiency T_{t3} by T_{t2} /okay so I get I want this term I want this term sorry so I will get it as $T_{t3} - T_{t2} / \eta_c$ and similarly a turbine efficiency was $T_{t4} - T_{t3}$ dash divided by, so if I take these efficiency terms into account these equations will be able to rewrite as follows okay right.

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I think this mechanical efficiency needs to be included here and not here because if mechanical efficiencies are non unity which is less than 1 then the compressor requires more than what the turbine can give so you have to include it here so et be if you cross multiply it will be a fraction compressor will only get a fraction of what the turbine develops okay fine so where does mechanical efficiency come in here in this equation right.

So we are interested in τ_C and τ_T so to get that we divide both sides by T_0 $\frac{T_3 - T_2}{T_0} = \eta_C \frac{T_3 - T_2}{T_0}$ what is $\frac{T_3 - T_2}{T_0}$ this is one for flow through diffuser or intake this is what is this now see and this is η_0 not so this term becomes η_0 and similarly on this side what is $\frac{T_4 - T_5}{T_0}$ this is η_B and this term would be $\frac{T_4 - T_5}{T_0}$ into $\frac{T_4}{T_0}$ okay so $\frac{T_4 - T_5}{T_0}$ is η_B this is so now T into η_B .

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$$\frac{p_0}{p_{0B}} (\tau_c - 1) = 1 - \tau_t$$

$$\tau_t = 1 - \frac{p_0}{p_{0B}} \cdot \frac{1}{\eta_c \eta_h \eta_{mech}} (\tau_c - 1)$$

So we get either not by η_B into η see you tally the mechanical right or $\tau_T = 1 - p_0 / p_{0B}$ okay now if you look at this expression what happens if efficiencies are not equal to 1 then this term increases right and you will get a lesser pouty fine that means that you will have lower and lower pressure at the end of the turbine right that correct sorry if you have unity you will have three turn out by η_B into $\tau_C - 1$.

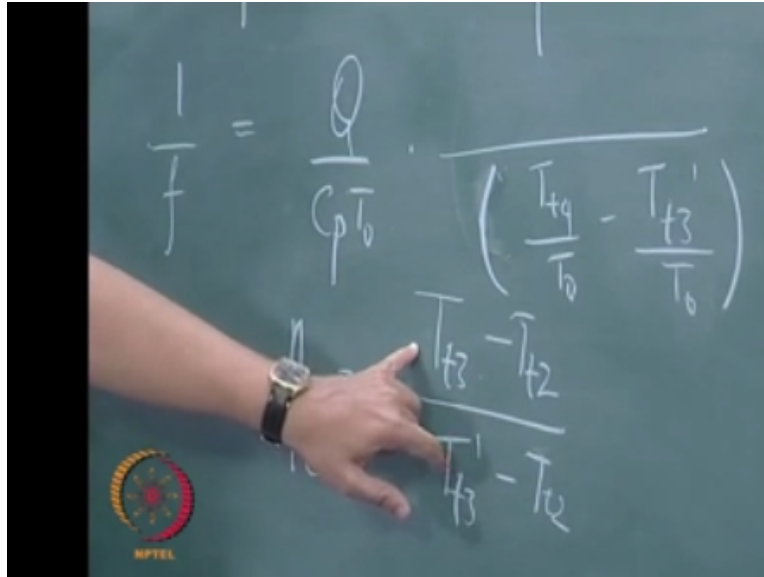
But if you have no η unity then what happens this term is large and therefore 1 minus this will be small which is again what I said earlier that is correct so you will get pouty to be smooth right and then τ_T is small the pressure ratio the pressure at the end of turbine is low and sometimes if the efficiency is a very poor one might actually end up having a turbine at the end of which there is no power left for X or there is nothing left for expansion through the nozzle which means there will be no thrust that is produced.

So bone needs to be careful about efficiencies here so that we do not end up in such a situation now we have been able to derive expressions for non dimensional thrust for all situations that is efficiency is not equal to 1 efficiency is equal to 1 and optimally expanded flow as well as nozzle being choked and we have also looked at afterburner without afterburner and what is the other mode of water methanol injection also we have looked at.

And now here in this case is P by a not that does not change the expression does not change but F / m dot a knot will be different because you now have efficiency terms coming the expression for is p by a naught would be I sorry I think we need to derive this see there is efficiency term

involved and therefore the T_4 because of which we need to relook at what is the η expression.

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So if we look at η by a T_0 we had in the previous classes looked at this expression this is $1 / \eta$ $\times F / m \dot{e}$ a not okay now how do we get $1 / \eta$ we get $1 / \eta$ by looking at the energy balance across the burner so from across the burner I get $m \dot{F}$ into $Q = m \dot{F} (T_4 - T_3)$ remember in the earlier expression that we wrote when the efficiency is where one we were looking at T_3 only right.

Because if you look at what happens due to non isentropic process this temperature would be higher than or different than the earlier temperature so you have a situation where η is different so therefore that will come here so we make the assumption that we are done earlier that is η is very much less than 1 so I can take out this quantity and I get my expression for $1 / \eta = T_4 / (T_3 - T_2)$ okay now we know that compressor efficiency is $\eta_c = (T_3 - T_2) / (T_3' - T_2)$ okay.

So using this three dash material will be one by one by as this society thank you this goes to the denominator so it will be thank you we know efficiency is denoted in this fashion so I can connect from T_3 to T_3' here so $1 / \eta = Q / (C_p T_0)$ not before I do that I will just do a small manipulation with this expression $(T_3 - T_2) / (T_3' - T_2)$ into $T_3 / (T_3 - T_2)$ I can write this expression rewrite this as this.

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$$\frac{T'_3}{T'_2} = \frac{(C - v)}{C} + 1$$

$$\frac{T'_3}{T_0} = \frac{T'_3}{T'_2} \cdot \frac{T'_2}{T_0} \rightarrow \theta_0$$

$$\frac{T'_3}{T_0} = \left[\frac{(C - v)}{C} + 1 \right] \theta_0$$

And T'_3 by T'_2 this in this cancels out I am interested in this expression so I will get T'_3 by $T_0 = T'_3$ by T'_2 T'_2 by T_0 would be 1 minus what is this T'_3 by T_0 this is see a so very 1 plus this goes here you get this now we have been able to get the expression for T'_3 by T_0 we can use it here and we can write T'_3 by D_0 is equal to this expression is what we have derived just now what is this, this is nothing.

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$$\frac{1}{f} = \frac{c}{v} \cdot \frac{1}{\left[v - v_0 \left[1 + \frac{(C - v)}{C} \right] \right]}$$

$$\frac{T_y}{a_0} = \frac{c}{v} \cdot \frac{1}{\left[v - v_0 \left[1 + \frac{(C - v)}{C} \right] \right]} \cdot \frac{F}{m_a a_0}$$

But η not so I get gt^3 by $T_0 = \tau C^{-1} / \eta_0$ so finally my $1/F$ term comes out to be $\theta_B - \eta_0(1 + 0 C^{-1})$ okay this is the expression that we get so what happens to if you plug this in by a not expression you get is $p / e_0 = q$ by cpt not okay this is the expression that we get for is p by a not now if you see in this expression what happens if $\eta C = 1$ this part increases and therefore η_B -this would be small right.

So can I now say it is advantageous to have a non efficient compression process if you look at this expression it is a it is in a sense it disguises some things you have not written the full expression for f by $m \dot{A} E$ not where inefficiency terms are also there right if you look at this it appears as if this part will be reduced will be increased and therefore the entire term will be smaller.

And as a consequence is P will be higher but if ηC becomes large means is a is a low value not large it is a low value then what happens is the pressure ratio across the compressor or the pressure at the end of the computer turbine becomes smaller and smaller they will not have any power left for expansion through the nozzle right so although it looks very deceiving here it is not the actual case you need to plug in the values for ηC and other things into this expression and then see how it looks like we look at it in the next class thank you.

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