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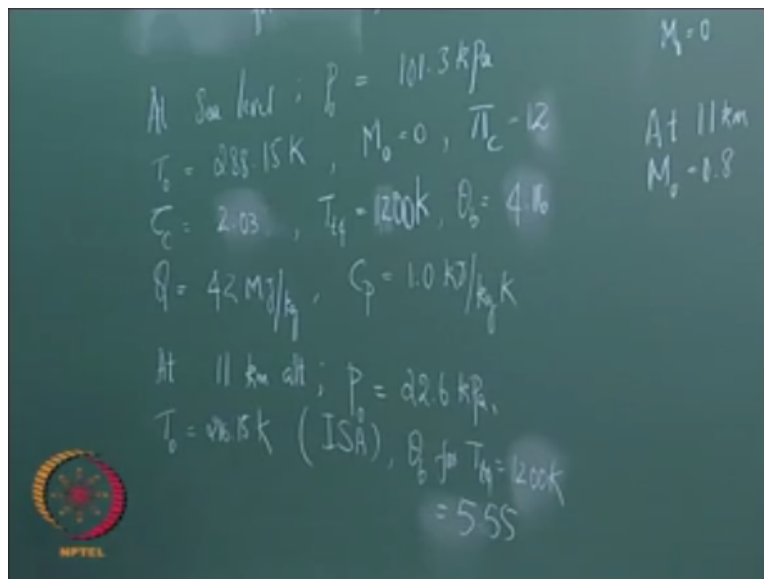
Aerospace Propulsion
Cycle Analysis – Turbojet Interviewee:

Lecture 14

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In the last class we had seen the performance of a turbojet we derive equations for the non dimensional thrust as well as we had derived expressions for ISP of a turbojet. Now let us put in some typical numbers and see what is it that we get out of it and we will see some interesting facts effect.

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So firstly we will look at optimal, optimal expansion of flow in the nozzle here there are two conditions that we need to look at 1 is sea level thrust or sea level is been what happens at cruise conditions so at sea level $P_0 = 1$ atmosphere 1011.3 mPa and T_0 to T if we assume an IS, IS a

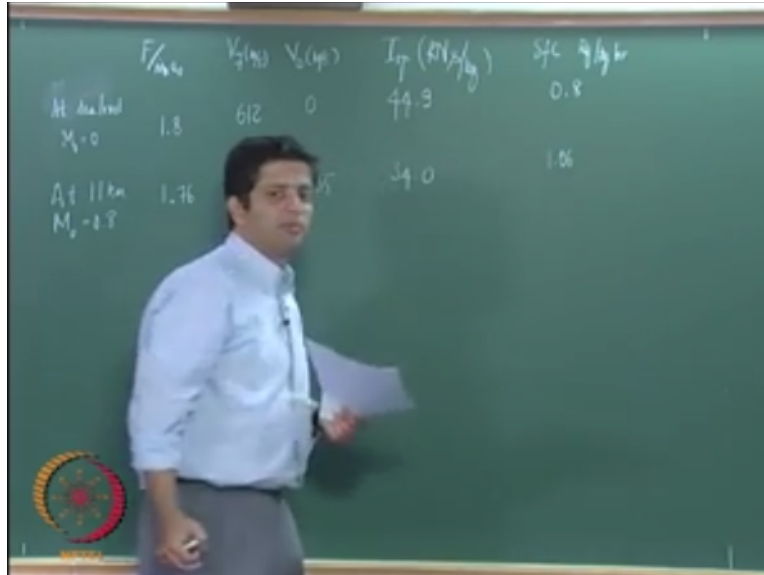
atmosphere model then it will be 288.15K and $\dot{m}a_0$ at sea level before take at take off conditions.

And now if we assume the compression ratio π_c to be very low that is spicy to be of the order of from 4 then my τ_c would be something like 1.484 and if we take T_{t4} to be 1000K then this makes data b is equal to something like 3.4 and we know the Q of the fuel this is kerosene so 42MJ/kg and let us take the C_p of air and flue gases to be the same and let us take it to be 1kJ/kgK.

So with this conditions now you have to find out what is it at the cruise altitude of around 11 kilometers, so altitude what are the things that are going to change the things that are going to change are these two only atmospheric conditions and at cruise conditions this Mach number will also change so we need to change the only three parameters P_0 would be lower and it would be somewhere around 22.6kPa and T_0 would be this is a you have taken ISA International Standard Atmosphere so therefore we get this.

Now if we use this and calculate $F/\dot{m}a$ and ISP we have all the parameters here to calculate both of them or we will get this kind of a table.

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$M_0=0$ then I will also put v_7 v_0 both these in m/s and lastly $ISP(kNs/kg)$ so if we take the sea level performance that is at sea level $M_0 = 0$ and using this we will get F/\dot{m}_a as 1.8 and this would be something like 612m/s and this would be 0 and ISP would be something like this. Now at 11km M_0 would be 0.8 and this would reduce slightly to 1.76 okay, now there are two things to notice.

One is that irrespective of the altitude you see that f/\dot{m}_a is nearly constant right, this happens because of two things, one notice that at sea level this is large the velocity differential is large okay, the mass flow rate is very small velocity differential is large mass flow rate is very small so you end up getting a high thrust because of the velocity differential.

At a higher altitude the velocity differential is reduced but you have air coming in now at this velocity therefore what you get as a larger mass flow rate, so you get a larger mass flow rate with a smaller velocity differential therefore you get nearly the same thrust or same non dimensional thrust .

What happens to I_{sp} , I_{sp} is come down which means that SFC is gone up if I were to write SFC also here SFC would be in kg/kg hr this would be something like 0.8 this would be like 1.06 so we are cruising at an altitude where an the SFC is low why do we do that, I think it would be more beneficial if we were to stay on ground or close to ground and carry out our operations in it from the looks of it what we see is that the I_{sp} come down or SFC has increased.

So why do you think this is happening what this is desirable not desirable what is the opinion you would not say something no. So what happens, no not really well actually the reason why we operate it at a higher altitude is primarily the drag is less at a higher altitude right, you are not looking at drag of the engine Percy you are looking at the drag of the entire aircraft where in when I was discussing about turbofan engines I said G goes in for a bypass ratio of 9 and says that the engine of the nasal drag is very small compared to the overall drag.

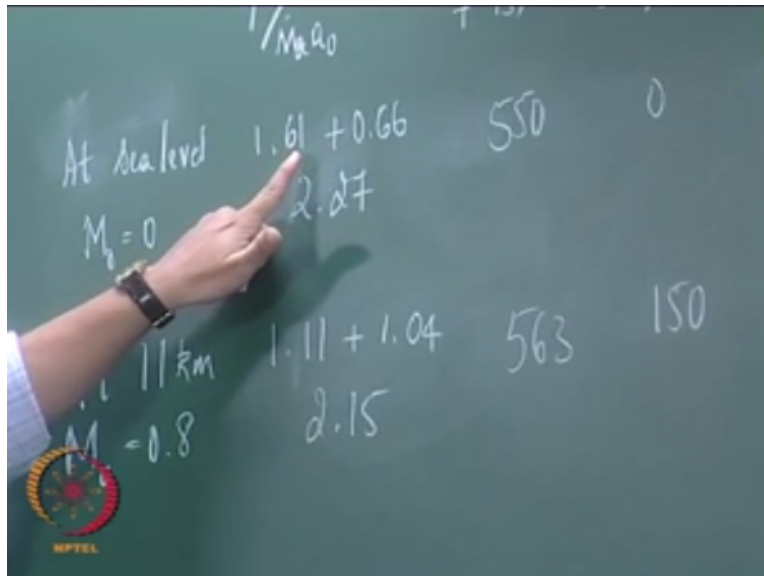
And therefore they go in for a large bypass ratio right, now what we are looking at is the drag of the entire aircraft goes down as altitude increases because the load goes down as you go up in altitude right. Now density decreases drag decreases but your SFC will increase because what we have set in here is if you notice I did not change this part T_{t4} , T_{t4} is the same at both the altitudes.

And if you look at θ_b in this case θ_b would be because T_0 is lower, so θ_b for $T_{t4}=1000K$ would be 4.63 that is what we are saying is you need more fuel to bring the air from a lower temperature after compression to 1000K because we have set that temperature so therefore we will find that the SFC will be higher, right, okay and that is the reason we see this now we can look at similar numbers for choked flow through the nozzle the only thing that changes would be I will just put them here itself.

Now here we will assume a slightly larger compression ratio of round 12 which means that how sea will also go up it would be as much we can also increase the turbine inlet temperature this means θ_b will go up to the rest of the quantities remain the same here again sorry, I had written under K here do not point it out it was to be 1000K earlier and that makes it 4.63 okay, and that is why we saw that change in ISP or SFC.

Now if we change it to 1200K θ_b would be 5.55 and this would mean correspondingly all these quantities will change, because now we have choked flow in the nozzle the thrust consists of two parts, one is the convective flux part and the pressure thrust part.

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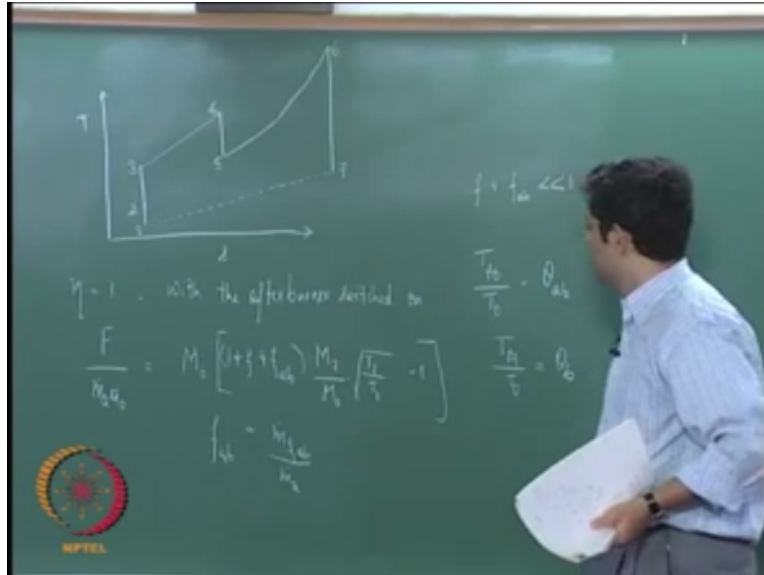


So this will again have two parts 1 point and at an altitude okay, this is what we see notice that as you go up in altitude the fraction of the pressure thrust is increasing it is a lower fraction here right or if you look at this number it is a lower fraction here compared to a higher fraction here right, the overall thrust why does this happen, why does pressure thrust part increase as you go up in altitude.

Ambient pressure if you see is $1/5$ th so the pressure cross part is going to increase if you go up in altitude and again you see the same feature here that SFC has increased and that is primarily because you need to heat the air from a lower temperature to a set temperature of 1200k here instead of 1000 earlier so therefore you will find that SFC is higher in this case okay right, okay. Then we have dealt with two things we have looked at the optimal fulfill optimally expanded flow through the nozzle and we looked at choked nozzle, okay.

Now let us look at what happens if you have the after burner on and the other method of increasing the thrust that is water-methanol injection let us look at these two methods and how to derive expressions for them. So firstly let us look at what happens if we have the afterburner on. I will retain these set of numbers because I want to use this to look at what happens without a afterburner with the afterburner conditions, so I will retain this numbers and we will get back to a similar table at the end of the discussion on afterburners.

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Now if we look at the TS diagram with the afterburner on we will again assume η to be 1 okay, so you have isentropic compression through the intake as well as through the compressor then you have heat addition in the main combustor expansion through the turbine so this is 0, 2 okay, so this is the TS diagram for efficiencies 1 and with the afterburner switched on okay, now what do we have to do if you have to analyze this okay.

Firstly our expression for $F/m_0 a_0$ the non dimensional thrust would be M_0 into in this case $[1 + F +]$ I will call, I will introduce a new parameter $f_{ab} = M_7/M_0(\sqrt{T_7/T_0} - 1)$ right, this is the expression for non dimensional thrust now if we put $f_{ab} = 0$ this is the expression that we had derived for a turbojet without the afterburner switched on right, when f_{ab} goes to 0 this is the expression that we get.

Now we have f_{ab} where a f_{ab} is nothing but M_0 fuel that is added in the afterburner divided by $m_0 f$ okay, so it is the fuel ratio in the after 1 so what do you think of the value of this plus this what is the maximum of F and $f + f_{ab}$ that it can go to it can maximum go to stoichiometric which is what is the value it is 0.067 which is the stoichiometric value you cannot burn more than that so it goes to something like 0.67.

Now if you put a 0.67 I can still use the condition that $f + f_{ab}$ is less than 1 right, with an error of around 6.7% so I will use that I will say $f + f_{ab}$ is very much less than 1 okay, so that we can take this terms of and I will also introduce another parameter here just like we had T_4/T_0 as a control

parameter we have a new control parameter that is T_{t6}/T_0 right, what is the maximum temperature in the cycle divided by T_0 the minimum temperature.

So I will call T_{t6}/T_0 as equal to η be remember we had $T_{t4}/T_0 = \theta_b$ burner similar to that I am introducing a new control parameter to account for the after burn okay. So using this I need still these ratios T_7/T_0 and m_7/m_0 .

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The image shows a chalkboard with the following handwritten content:

$$T_7 = \frac{1}{1 + \frac{\gamma-1}{2} M_7^2} \cdot \theta_b$$

$$\frac{T_7}{T_0} = \frac{\theta_b}{1 + \frac{\gamma-1}{2} M_7^2}$$

Cascading of pressures

$$\frac{P_7}{P_0} = \frac{P_7}{P_{t7}} \cdot \frac{P_{t7}}{P_{t6}} \cdot \frac{P_{t6}}{P_{t5}} \cdot \frac{P_{t5}}{P_{t4}} \cdot \frac{P_{t4}}{P_{t3}} \cdot \frac{P_{t3}}{P_{t2}} \cdot \frac{P_{t2}}{P_{t1}} \cdot \frac{P_{t1}}{P_0}$$

NPTEL logo is visible in the bottom left corner of the chalkboard image.

So I can get T_7/T_0 as $T_{t7} \cdot T_{t6}$ by okay, T_{t7} sorry this has to be T_{t6} okay, fine these two cancel off and I will get T_7/T_0 what is T_7/T_{t7} it is the ratio of static to stagnation so I can express this in terms of Mach number this would be $1 + \frac{\gamma-1}{2} M_7^2$ what is $T_7/T_{t7}/T_{t6}$ this is flow through the nozzle right.

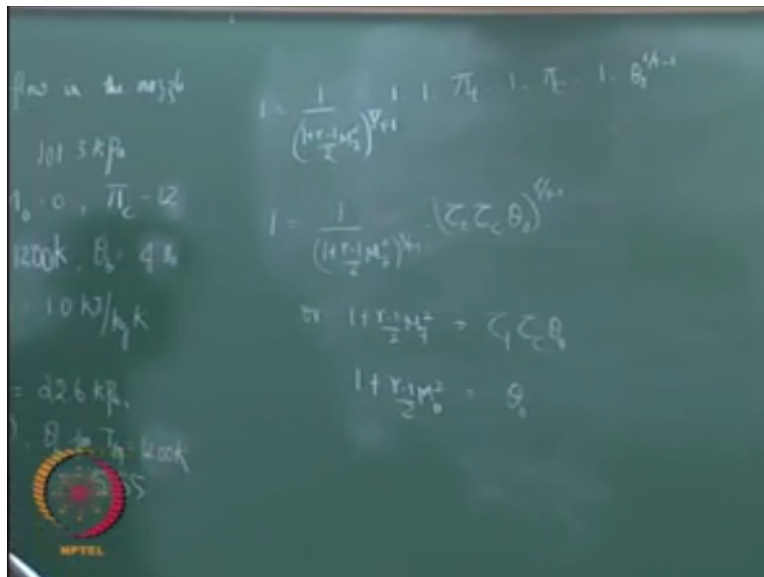
So flow through the nozzle we have assumed the flow to be isentropic so T/T_0 the total temperature does not change so this is 1 and T_6/T_0 we have defined it as θ_{ab} , so I get T_7/T_0 is equal to θ_{ab}/M^2 . Now we need to do cascading of pressures to get the other parameter yes, we always choke yes so while we are doing for the optimal because in case of optimally expanded if you use the conversion also it is still choke.

We will do both cases okay, first we will take the optimal expanded flow then we look at what happens if the nozzle is choked the nozzle being choked does not essentially mean that $P_7 = P_0$ it can be that P_7 is greater than P_0 even if the nozzle is choked right, so we are taking a special case of choked flow wherein the exit pressure is equal to the ambient pressure.

Now if we do cascading of pressures I get P_7/P_0 is equal to okay, this is what we get and the first term here is ratio again ratio static to stagnation pressure this is flow through a nozzle flow through jet pipe and then flow through a turbine, flow through the main combustor flow through compressor and this would be flow through intake and that is $\theta_0^{\gamma/\gamma-1}$.

So we will put that down here and we know because we are considering optimally expanded flow what is P_7/P_0 this would be equal to unity so we will use that.

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So we can write $1 = 1/M^2$ into flow through nozzles we are assuming all efficiencies to be 1 then flow through afterburner again 1, then flow through compressor you get π_t then sorry flow

through turbine then you have flow through combustor which is 1 then you have flow through a compressor π_c and you have flow through the intake which is 1 and lastly $\theta_0^{\gamma/\gamma-1}$ and here also π . So I get I can rewrite these two in terms of π so I will get $1 = 1/$ or $1 + \gamma - 1/2 M_7^2 = \pi \pi_c$ eternal okay, and I know from the definition of θ_0 that $1 + \gamma - 1/2 M_0^2 = \theta_0$. So using these two I can get my expression for M_7/M_0 .

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$$M_7 = \sqrt{\frac{2(\gamma-1)}{\gamma}}$$

$$M_0 = \sqrt{\frac{2(\gamma-1)}{\gamma}}$$

$$\frac{T_7}{T_0} = \frac{\theta_0}{\pi \pi_c}$$

$$F/m_0 a_0 = M_0 \left[\frac{\gamma(\gamma-1)}{(\gamma-1)} \frac{\theta_0}{\pi \pi_c} - 1 \right]$$

$$= M_0 \left[\frac{\theta_0}{(\gamma-1)} \left(1 - \frac{1}{\pi \pi_c} \right) - 1 \right]$$

So I can write M_7 as equal $\sqrt{2/\gamma-1}$ $\pi \pi_c \theta_0^{-1}$ and M_0 similarly would be and we required in our expression for T_7/T_0 this term so I can get that too I can write T_7/T_0 as equal to $\theta_0/\pi \pi_c \theta_0^{-1}$ okay, so sorry no -1 sorry, so this is the expression that we have now we can substitute back and get our expression for the non-dimensional thrust which is $f/m_0 a_0 =$ or I -, I can rewrite this part by taking this out as common I can rewrite this as okay.

So this is the expression that we get for the non dimensional thrust, now there is still 1 part missing we know that there is a turbine compressor power balance so these two are not independent variables and they are connected, okay so let us get that expression also.

We had derived that expression for a case without the afterburner on do you think it will change with the afterburner being switched on, because this is something that happens downstream of the turbine it does not get affected and therefore whatever we are derived earlier holds good.

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from Compressor turbine power balance

$$\tau = 1 - \frac{\theta_b}{\theta_c} (\tau_c - 1)$$

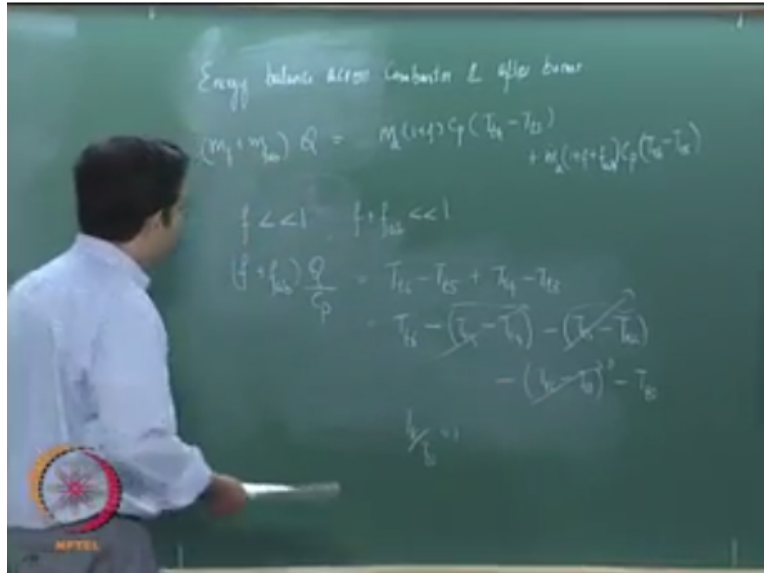
$$\frac{F}{m_0 a_0} = \sqrt{\frac{\theta_{t2}}{\theta_0 - 1} \left(1 - \frac{1}{\left(\tau_c \theta_0 - \frac{\tau_c^2}{\theta_b} (\tau_c - 1) \right)} \right)} - 1$$

So from we get $\tau = 1 - \theta_b / \theta_c \tau_c - 1$ and if you substitute this expression back then we will get F by okay, this is the overall expression that we get. Notice here that θ_b is now a parameter of very small consequence right, the temperature at the end of the combustor is not such an important parameter whereas for the thrust the temperature at the end of the afterburner is a more important parameter okay. Now let us look also at the ISP part if you have to calculate ISP we know from our previous expression for non dimensional specific impulse that I need $1/F F/m_0 a_0$.

So here this was the expression that we had without the afterburner how would this change with the afterburner I need to include here $f + f_{ab}$ for the case with the afterburner okay, so with the afterburner on you have this additional term here okay, now how do we determine this quantity, how do we determine this quantity what do we need to do, what did we do to get $1/F$ energy balance across the combustor.

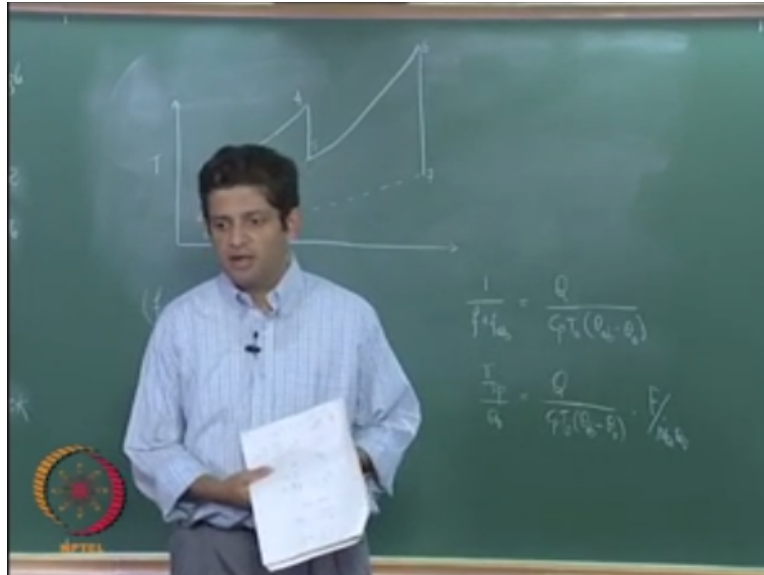
Now what do we need to do, energy balance across the combustor plus the afterburner so let us do that.

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By taking energy afterburner we get $(\dot{m}_f + \dot{m}_{fab})Q$ okay, must be equal to $\dot{m}_a(1+f)C_p(T_{t4} - T_{t3}) + \dot{m}_a(1+f+f_{ab})C_p(T_{t6} - T_{t5})$ okay, this is the expression that gets added on in the afterburner into $C_p(T_{t6} - T_{t5})$ okay, this is the expression that we get. Now we know that we can use f is very much less than 1 and we also know that $f + f_{ab}$ is also very much less than 1, so using this I can eliminate these two and therefore I will get $f + f_{ab}$ that is you take it here and divide this by \dot{m}_a you will get this into Q/C_p must be equal to $T_{t6} - T_{t5} + T_{t4} - T_{t3}$. Now if you look at the TS diagram.

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If we look at the TS diagram okay, this is the TS diagram we have and what we have here is $T_6 - T_5$ that is this minus this plus $T_4 - T_3$ right, now I can add this part what you will get is you will get as though it is a continuous rise from T_3 to T_6 okay, if I add this part but we know that compressor turbine power balance C_p being the same this portion in this portion is same you have to subtract 1 of them.

If you are adding this you to subtract this okay and I also know that the T_2 must be equal to T_0 right, because flow through the what do you have flow through the intake what kind of process is that it is an isentropic process because we are assuming all efficiencies to be 1, is an isentropic closest total temperature does not change okay, so you get $T_2 = T_0$ so we will use that here and do a little bit of jugglery as I said I will add $-T_5 - T_4$ okay.

If you look at this I have not done anything this is what is this part turbine, what in the turbine work in the compressor these two must be equal right. So I can take this out these two are equal now I know that $T_2/T_0 = 1$ so this goes to 0 okay, so by doing this manipulation have been able to or get this as $T_6 - T_0$ which is nothing but you are looking at raising the temperature from here to T_6 okay, so this was TS diagram so I can right now $(f + f_{ab})Q/C_p = T_6 - T_0$.

Now if I divided by T_0 on both sides I get what is T_6/T_0 this is θ_{ab} and what is this, this is θ_0 , so we get a new expression for we are looking for this expression $1/(f + f_{ab}) = Q/C_p T_0 (\theta_{ab} - \theta_0)$ okay, and following this I can now write $ISP/a_0 = Q/C_p T_0 (\theta_{ab} - \theta_0)$ into okay, so this is our expression for ISP with the afterburner switched on, I will stop here and continue.

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