Indian Institute of Technology Madras
Presents

NPTEL
NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

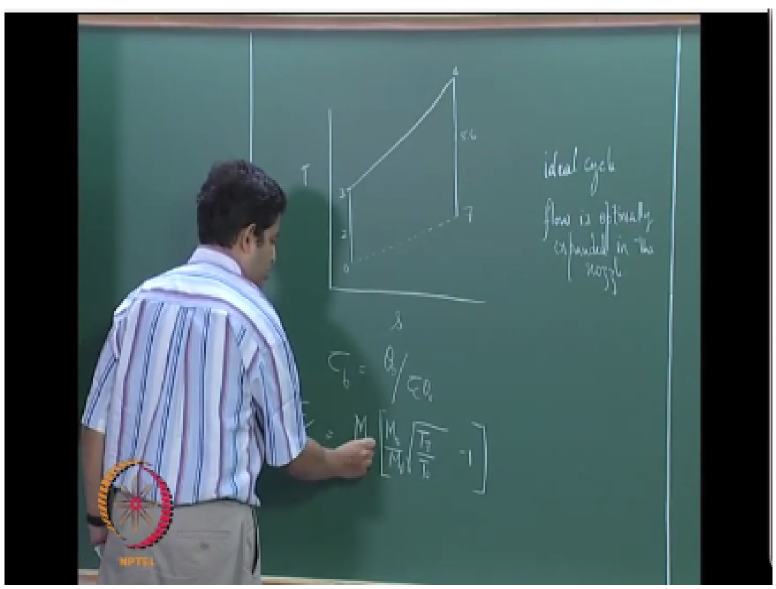
Aerospace Propulsion
Cycle Analysis – Turbojet II

Lecture 12

Prof. Ramakrishna P A
Department of Aerospace Engineering
Indian Institute of Technology Madras

In the last class we had looked at the cycle analysis for a ramjet and we had sat out the cycle analysis for a turbojet, let us continues the cycle analysis for the turbojet.

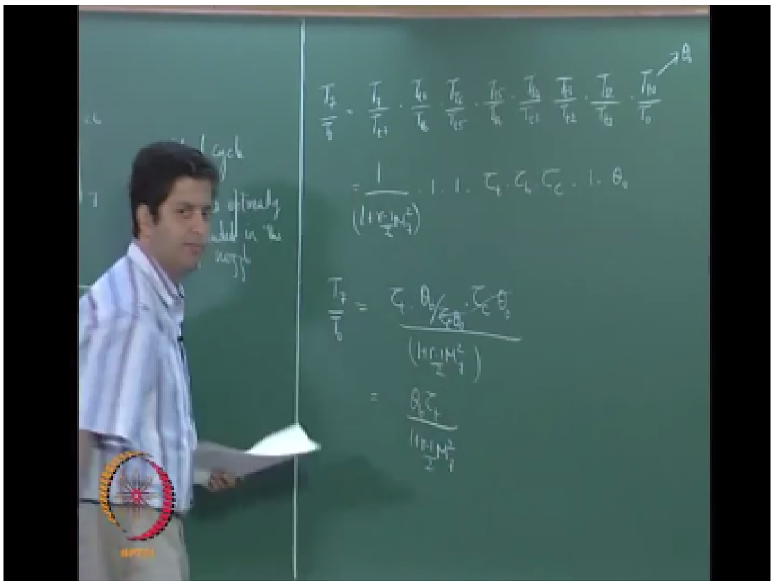
(Refer Slide Time: 00:29)



If you look at the Ts diagram firstly let us assume that all processes are 100% efficient that is we will assume an ideal cycle initially and we also started something, wherein the flow is optimally expanded in the nozzle. So these were the two things that we are going to look at if we take an ideal cycle then there is compression in the intake itself, so I will call 0 to 2 and then through the compressor, then you have heat addition then you have expansion through the turbine then you have expansion again through the nozzle this is the Ts diagram we had okay.

Now we had derived certain things in the last class we had derived that τ b is nothing but θ_b / ∂_C θ_0 and we have got this expression for F/ m. a b_0 this is the expression that we had okay. Now let us try and find out how to get these ratios just like the previous time when we had done this cascading in ramjets let us do the cascading and find out how we get these temperatures.

(Refer Slide Time: 03:27)



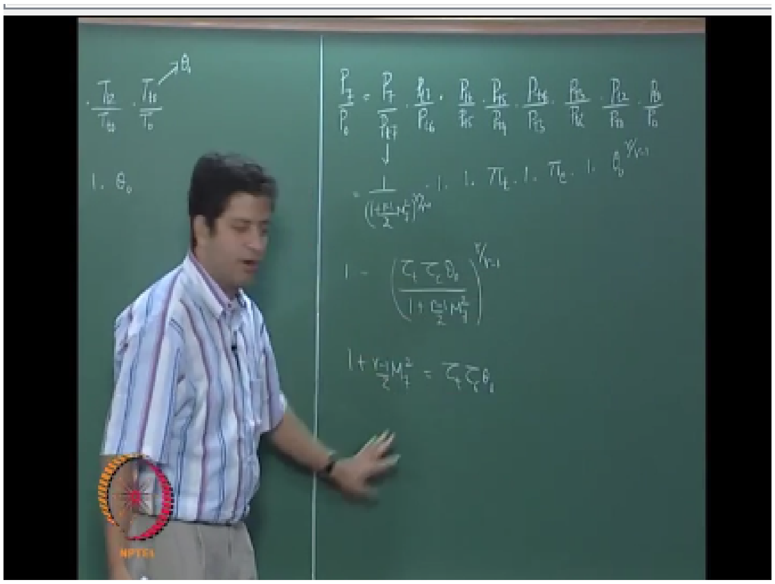
Now to get T_7/ T_0 can you cascade sub T_7 / TT_7 x TT_7 / Tt_6 / TT_5 / T_4 T_4 / Tt3 / okay again the subscript T here indicates stagnation conditions and if you take a look at this all these cancels out and you get $T_{E\ 7}$ / T_0 okay. So we also know that the last term is nothing but θ_0 from our previous class, so we have this is nothing but a ratio of stagnation to static, so we can express this in terms of Mach number as 1 + γ - 1/ 2 m7 ^{2} okay.

Now what is $T_{T\ 7}$/ T_{T6} this is flow through the nozzle again if the efficiencies are 1 the stagnation temperature ratio will be the same, so this is 1 and T_{T6} / T_{T5} this is flow through the afterburner anyway we are not considering in this analysis this analysis is without the afterburner. So this will again be 1 and T_{T5} / T_{T4} s flow through the turbine now flow through the turbine because it is again an isentropic process this ratio will also be what will that ratio be? T_{T5} / $T_{T4.}$

One you have forgotten something we derived in the last class that this is τ T and T_{T4} / T_{T3} is process through the combustor, so I will call it τ_B x T_{t3} / T_T to is flow through the compressor again this is τ_C x again you get the diffuser this ratio because the flow is isentropic is 1 and x θ_0 okay. So we know that τ b is this if we substitute it there we will get T_7 / T_0 =τ_T x τ_B is Θ_B / T_C

θ_0 x τ_C β_0 here is a ratio that we have this τ_C θ_0 and this τ_C θ_0 cancels off and we get θ_B τ_T divided / 1 + γ - 1 / 2 m 7^2 okay. Now similarly we will do for the pressures.

(Refer Slide Time: 08:03)



P_7 / P_0 now he has room all efficiencies to be 1so the first one this would be if you cascade it will be P_7 / P_{T7} x P_{T7} / P_{T6} P_{T6} / P_{T5} okay this is the cascading and if we plug in the values then this is the ratio of static to stagnation condition, so you get 1 + γ- 1 / 2 $m7^{2\ \gamma/\gamma-1}$ then this is flow through nozzle, so you get for all efficiencies being one you get one then you have flow through afterburner this is again one what does this flow through turbine you get P_{IT} okay, pressure ratio across turbine and what happens in an ideal cycle to the pressure ratio across the combustor.

For an ideal cycle the pressure ratio across the combustor it will be is isentropic isobaric process so the pressure is the same, so this ratio would be 1 + P_{T3} / P_{T2} this is again π_C ratio pressure ratio across the compressor and P_{T2} / P_{T0} is flow through diffuser for a ideal process this is 1 and lastly this is nothing but θ_0 $^{\gamma/\gamma-1}$ okay. Now I can change this to ∂t and we will get a new ratio, so I will do that I will get τ_T τ_C θ_0 divided / 1 + okay this is the ratio that we get.

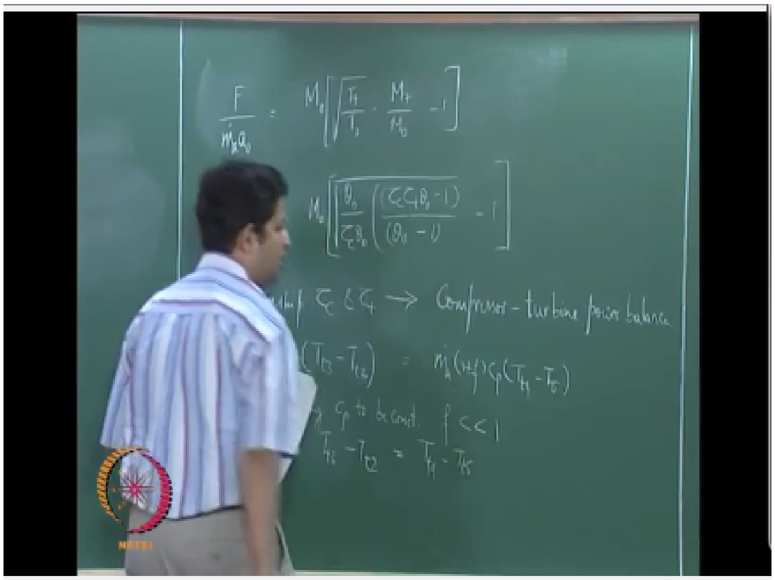
And if you remember we have taken the case where the flow is optimally expanded through the nozzle so what is P_7/ P_0, so we can rewrite this as this ratio as 1 + γ- 1 / 2 m 7^2 must be =τ_T C θ_0 all right I can take out the powers without any problem and therefore I can write it like this. Now remember in this expression T_7 / T_0 in the denominator I have 1 + γ- 1/ 2 m 7 ^{2} which is what I have got that, so if I plug in this value I will get $T_7 / T_0 = \theta_B \tau_T$ divided / $\tau_e \tau_C$ the $\tau_{0,}$ so I will get this is the temperature ratio.

Now to find the Mach number ratio we will have to take this expression and derive the ratio for Mach numbers.

(Refer Slide Time: 13:49)

So we know that $1 + \gamma- 1 /2\ m_7^{\ 2} = \tau_P \tau_C \theta_0$ and we also know $1 + \gamma- 1 / 2\ m\ 0^2 = \theta_0$ so from these two expressions I can write $M_7 = \tau_C \tau_D \theta_0 - 1$ okay and similarly M_0 we had done this earlier = okay and the ratio M_7 / M_0 will then become okay. So this is the ratio of Mach numbers and that is the ratio of temperatures that we were looking for we have got those two and if we plug them x the expression for thrust $F / M.A_0$ okay that is what we were looking for.

(Refer Slide Time: 16:08)



So F/ M . a_0 =what we got was M_0 x p_7 / T_0 x m_7 / M_0 - 1 and if we substitute the ratios 4_7/ T_0 and m_7 / M_0 we will get okay, this is the expression for non-dimensional thrusts that we were looking for, now is this expression complete is there something that is still missing, say here you have got τ_C & τ_T right but we know that compressor and turbine and Power Balance must be there, so they cannot be independent of each other they must be related to each other and therefore you can write one expression for connecting these right.

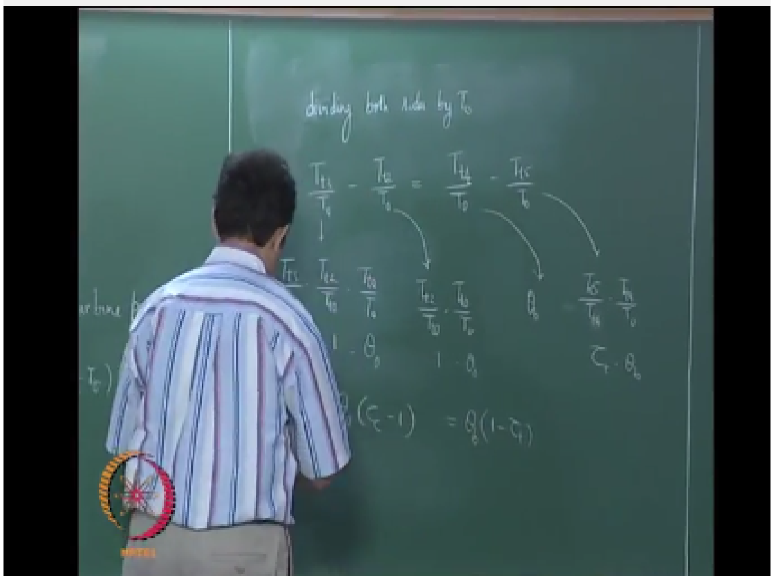
So there is a relationship for connecting τ_C n τ_T that is through the compressor turbine power balance, now if we do the compressor turbine power balance we will get this ratio connecting T_C to τ_T. Now we know that flow through compressor is M. a C_P T_{T3} - T_{T2} this must = M.a x 1 + F E T_{T4} - T_{T5}, now there are 2 things that we are going to make an assumption on if you really take a look at this is primarily air getting compressed in the compressor whereas for the flow through the turbine you have product gases.

If you remember earlier when we said γ and r are the same for both air and exhaust gases right if γ and our concert is the same for both air and exhaust gases then C_P what should happen to C_P should also be the same for both of them, so assuming C_P to be constant and also we will make another assumption that we have been doing all along that the fuel a ratio F is very much less than 1 when for turbojet, because typically this will be of the order of 0.02 2.04.

So even if you neglect it you want to make a 2 or 4 % error okay, so with these two assumptions this will simplify to T_{T3} - T_{T2} must be =T_{T4} - T_{T5}. So what we do from here is we know /

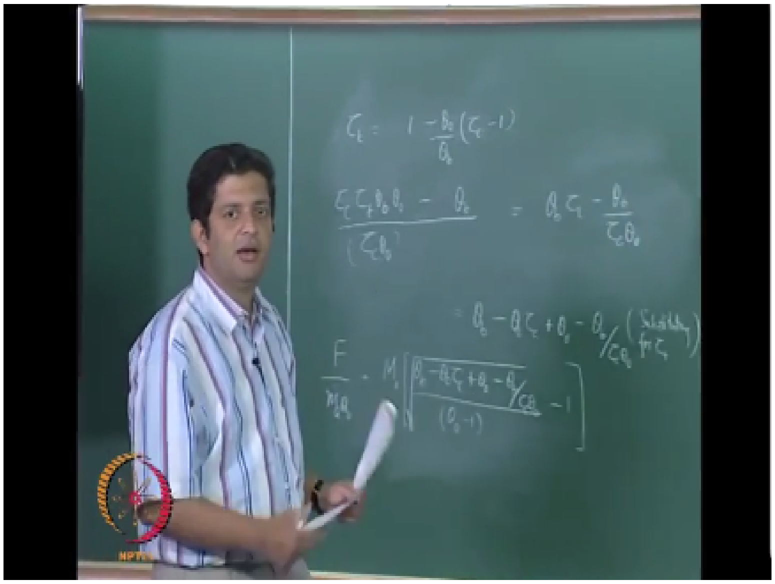
cascading what the ratios are so we just have two more divided both sides / T_0 let us divide both sides / T_{not} okay.

(Refer Slide Time: 21:52)



Now what does T_{T3} / T_0 again if we cascade I will get T_{T3} / T_{T2} x T_{T2} / T_{T0} x T_{T0} / T_0 what is T_{T3} / T_{T2} this is flow through compressor, so this will be τ_T so τ_C and this ratio is 1 across the diffuser. So and this is Θ 0 right T_{T0} / T 0 is Θ 0 and this would be T_{T2} / T_{T0} x T_{T0} / T_0 this again is T_{T2} / T_{T0} is 1 and this would be θ_0 right and what is T_4 / T_0 this is θ_B this is from our definition and T_{T5} / T_{T0} I can write it as T_{Tv} / T_{T4} x T_{T4} / T_0 what is this 85 / T_{T4} τ_T this is τ_T x θ_V. So I end up getting if I take out θ_0 as common θ_0 x τ_C - 1must be = θ_B x1 - τ_T, so therefore I can write the expression for τ_{TS}.

(Refer Slide Time: 25:33)



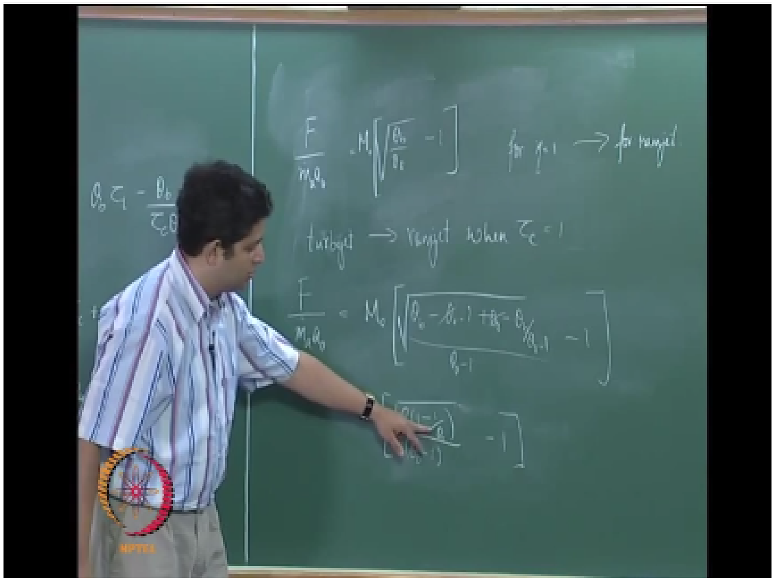
I can write τ_T as =okay, so this is the expression for τ_T now if I plug in this expression for τ_T in this expression for F /M dot a you note that is the non-dimensional thrust before we do that what we see here is we need this ratio all the in τ_C terms are only in this ratio so let us look at this first term in the under the square root sign and let us try to simplify that. So $\tau_C\ \theta_B\ \theta_0$ right / $\tau_C\ \theta_0$ - I can write the numerator combining these terms these are the terms containing τ_C and τ_T.

So if I combine these terms and rewrite it I will get it like this okay which is I can simplify this further and write it as $\tau_C\ \theta_0$ cancels off here, I will get $\theta_B – \theta_0$ and now if I substitute for τ_T in this expression okay I will get this as = $\theta_B - \theta_0\ \tau_C$, so that is now if I plug back this expression x that and rewrite my expression I will get the non-dimensional thrust as F / M.a a_0 must be =this is the final expression for a non-dimensional thrust totally.

See how T and τ_C are connected now θ_b is a controlling factor is okay with you because it is a turbine Inlet temperature, so that is fine with you what you are looking for is why is it that τ_t has gone out it out had to go out because the compressor and turbine there is a power balance between them and we have assumed the processes to be 100% efficient. So the compressor power must be = the turbine power, so when you do that you can eliminate τ_t and that is what we have done here.

So it will be only dependent on $\tau_C\ \theta_0$ and θ B now we have done all these calculations right what we need to do is cross check whether what we have got is correct, how do we do that? How do we cross check what you got is corrector not and still does not Ellison what do we know you will always assume previous class is true right or previous class is correct. So we know the results for ramjet right what is the result for ramjet the result for Ramjet.

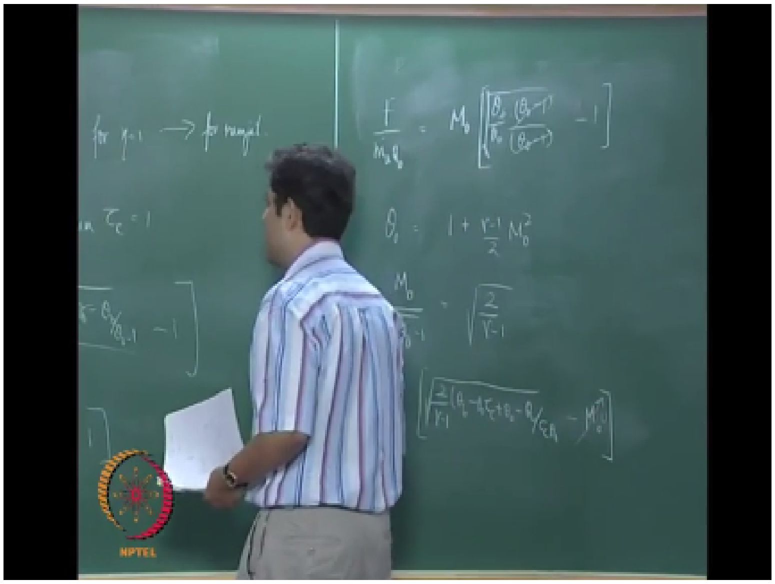
(Refer Slide Time: 31:43)



That was F/ M.a A_0 =M 0 θ $_B$ mine / θ_0 - 1 θ =1 this is for okay this was the result that we had for R and it, now what is the difference between ramjet and a turbojet? That you have a compressor right compressor and therefore a turbine, so what happens if you put τ_C to be =1 compressor pressure ratio or PI_C is 1 or τ_C is one compressor turbine D re temperature ratio is one or compressor pressure ratio is 1 that is what is a ramjet right.

So if you put τ_C =one here what happens θ_0 θ_0 cancels off and you get τ C this is Θ_B / θ right, so let us do that the budget becomes a ramjet 1 now C = 1 so when I substitute τ_C =1 in the expression for ramjet or turbojet, you get θ_B - θ_0 x 1 + θ_0 - θ_B / θ_0 τ_C is again 1. Now it is obvious that you can cancel out this and you get you can take out θ_B common so you get M_0 and if you take out θ_B as common what you get is 1 - 1 / θ_0 okay, this is anyway the same as you can take one / θ_0 out you will again get.

(Refer Slide Time: 35:24)

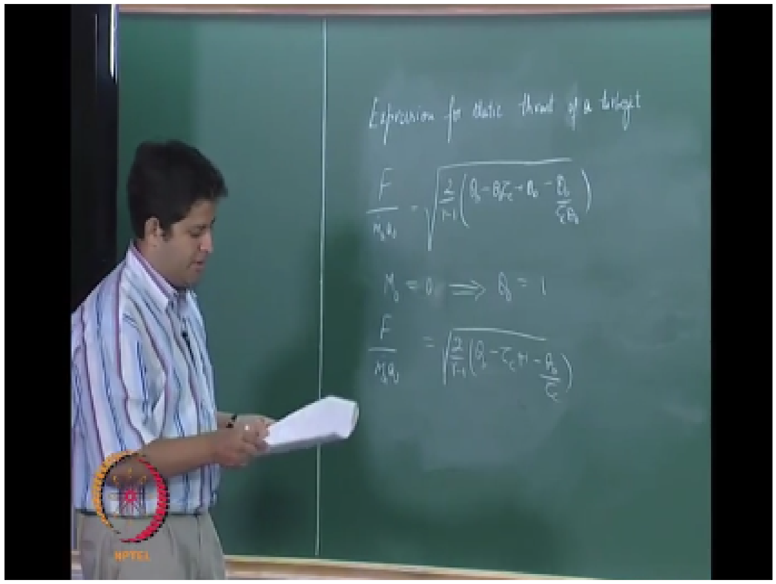


So I get which is the same as rammed it so in that sense it is consistent okay what we have derived is consistent with when we put $\tau_Z = 1$ we get the ramjet result okay. Now there is another thing that we need to look at what is that? We know that turbo Jets have a static thrust right the budgets do produce a static thrust, now we need to find out whether the expressions that we have derived do show that.

Now in this expression here what happens what is the condition for static thrust, if you put $M_0 = 0$ then you get 0 new suddenly we have done all this extravagant calculations and suddenly found out that we are on the wrong side, our equations do not bring out that fact that it still can produce you know static thrust looks like it produces zero thrust or is there a catch to it let us see. We know that what is $\theta 0$ one $+ \gamma - 1 / 2 m 0^2$ okay.

So if you can you have M_0 and $\theta_0 - 1$ so $M_0 / \sqrt{\theta_0 - 1}$ = what you get here √right, so we can use that here and rewrite the expression let us do that, so I get okay is this correct right. We have substituted for this in the earlier equation and now using this if you put here $M_0 = 0$ this goes to 0, so the expression for static thrust would be static trust for turbojet.

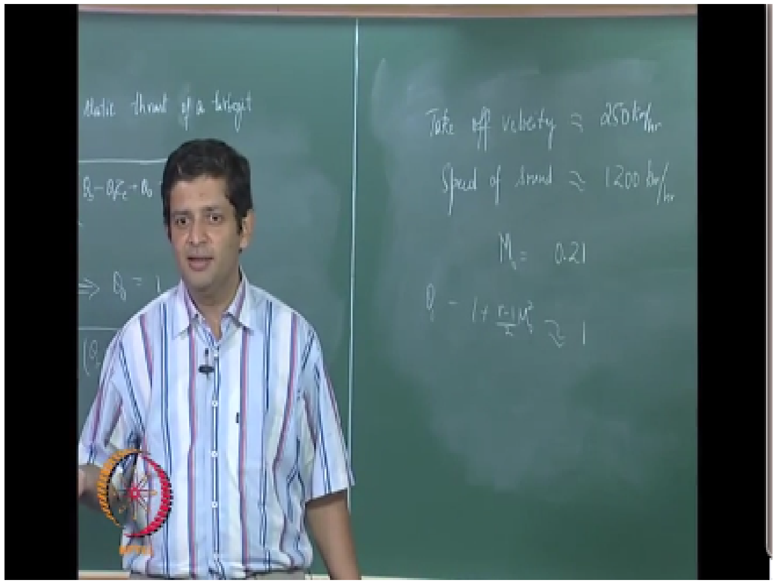
(Refer Slide Time: 39:17)



So you get okay so this is a nonzero quantity, so therefore we can safely say that whatever we have derived is correct and it produces nonzero static thrust okay, even when M_0 =0 what happens when M_0 = 0what happens to θ_0 θ_0 sorry when M_0 = 0 it means that θ_0 should be 1, so you substitute it here again you can rewrite this expression θ_0. So this cannot go to zero and therefore we have a positive static thrust okay.

The static thrust itself is some air moving in the compressor itself some air force sucked in the compressor said quote some velocity yes is it M_0 really 0 okay what does they takeoff velocity of an aircraft okay let us do that.

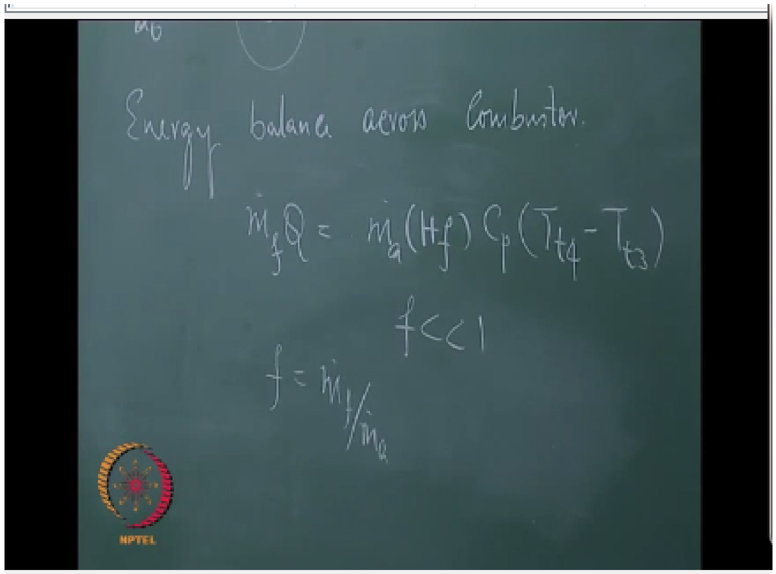
(Refer Slide Time: 42:30)



See so you have takeoff velocity around 250 km/h what is the speed of sound at that condition m/s so you convert it x kilometers per second kilometers per hour one. So I will assume it to be fine around 1200 km/h, so you calculate the Mach number based on this what will be. So what does this sacrosanct about 0.3Mach number which we say is the regime where we differentiate that the flow is incompressible and then the flow becomes compressible what is this limit of 0.3duration?

So if you look at this number here Mach number is 1 to 1 so if we do 1+ γ- 1 / 2 this is $M_0{}^2$ is nothing but θ_0 how different will it be from 1 this will be anyway 1 so even while the aircraft is taking off with very high velocities of around 240 250 km/h the Mach number is around 0.2 to 0.3 which means that it is still θ 0 is still around 1 okay. So therefore this is consistent what we have done here is consistent right okay. Now let us look at what happens to the other quantity of interest to us that is B okay.

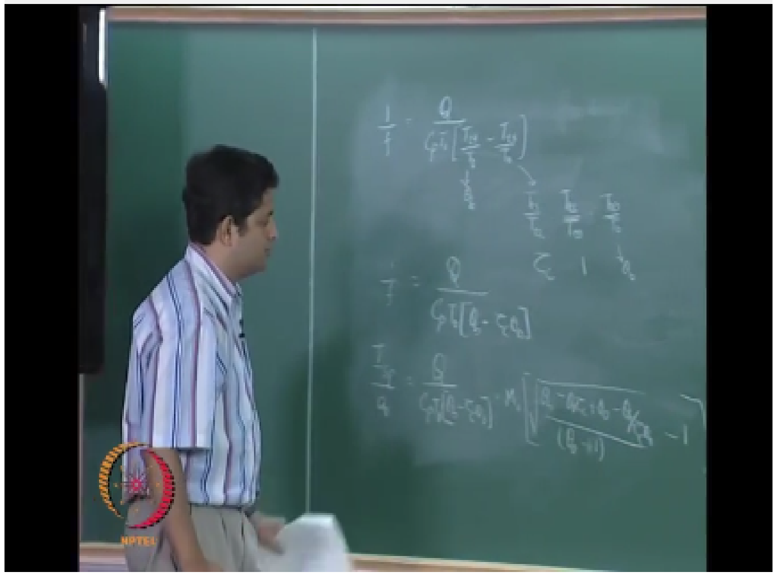
(Refer Slide Time: 45:19)



In the previous class when we were looking at ramjet we had already done this exercise and we noted that I_{SP} / E_0 which is the speed of sound we can write this as 1 / F x F / $M.e_0$ right and we said that we put in a lot of effort to find out this expression, so it is meaningful to use this to get the I_{SP}. So we know this part and we need to evaluate what this quantity right is so let us do that or we can evaluate this / again looking at the power balance in the combustor from energy balance across combustor.

I can write M. F_Q = M. x 1 + F right and what we will again do is we are trying to find an expression for F we will say that F here in comparison to 1small, so therefore we will neglect that we can do that even while trying to derive this expression for F. So f is nothing but M. F / me.

(Refer Slide Time: 47:51)



So using this so I can rewrite my expression as 1 / F must be =Q / C_P I will take out T_0 as common then I will be left with T_{T4} / T not okay, what is T_{T4}/ T_0 this is θ_B what is this T_3 / T_{T2} x T_2 / T_{T0} T 0 this is θ_0 this is 1 this is τ_C, so you get 1 / F = Q / $C_P T_0$ x θ_B - τ_C T done okay. So this is the expression into now we have got 1 / F there so I can write the complete expression for is P / E_0 as =θ_0 τ_C + θ_0, so this is the expression that we have for I_{sp} by you know we will stop here and will continue in the next class thank you.

Online Video Editing /Post Production

K.R Mahendra babu.
Soju Francis
S.Pradeepa
S. Subash

Camera

Selvam
Robert Joseph
Karthikeyan
Ram Kumar
Ramganesh
Sathiaraj

Studio Assistance

Krishnakumar
Linuselvan
Saranraj

Animations

Anushree Santhosh
Pradeep Valan .S.L

NPTEL Web Facilities, Faculty Assistance Team

Allen Jacob Dinesh
Bharathi Balaji
Deepa Venkatraman
Dianis Bertin
Gayathri
Gurumoorthi
Jason Prasad
Jayanthi
Kamala Ramakrishnan
Lakshmi Priya
Malarvizhi
Manikandasivam
Mohana Sundari
Muthu Kumaran
Naveen Kumar
Palani
Salomi
Senthil
Sridharan
Suriyakumari

Administrative Assistant

Janakiraman. K.S

Video Producers

K.R. Ravindranath
Kannan Krishnamurty

IIT Madras Production

Funded By
Department of Higher Education
Ministry of Human Resource Development
Government of India

www.nptel.ac.in