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Aerospace Propulsion

Cycle Analysis-Ramjet

Lecture 10

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This class will start with cycle analysis what we will do in cycle analysis is we will look at the brightening cycle which is mostly followed by all the air-breathing propulsion systems and look at what happens to non-dimensional thrusts ISP and other parameters if you vary the Mach number okay that is the primary motivation for doing this and we will be able to derive some simple relationships and show where the optimal light value of non-dimensional thrusts or specific impulse lies okay.

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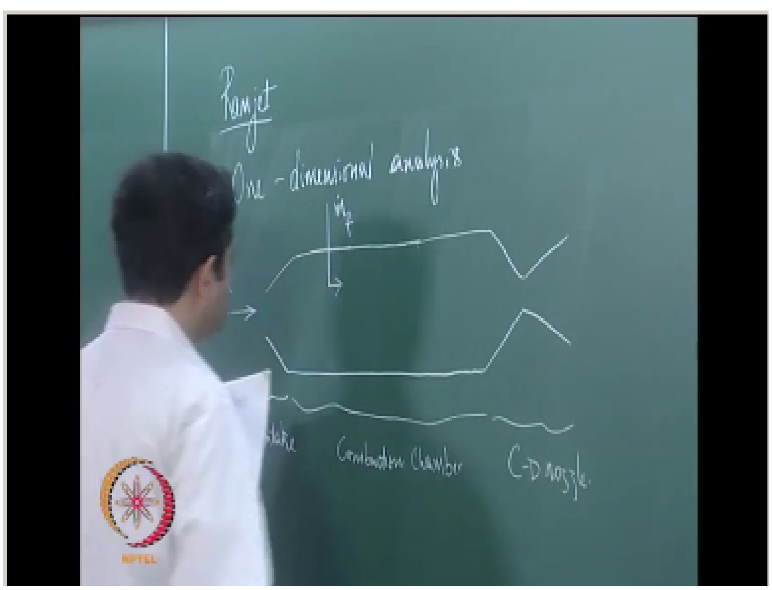

Now firstly we will start with ramjet as it is the simplest system with us with no moving parts and other things so we will start with this and this will be a one dimensional analysis that is the variation is only along the axis of the ramjet okay so just to refresh we have an intake for the ram jet okay there is a schematic of a ram jet you have air coming in from the left and you have fuel added this portion is the intake all this is the combustion chamber.

And this is the Sealy novel now let me call points let me indicate points here I will call this zero I will call this two three and four okay now correspondingly I will put this information on a TS diagram as to what happens to the cycle on a TS diagram let us look at it this is zero, zero two is compression isentropic compression we are assuming ideal processes here zero to do is isentropic compression two to three is heat addition in the combustion chamber here.

And three to four s expansion through the nozzle okay right now we had derived some time earlier the thrust equation and we had seen that for an air-breathing system the thrust is f is equal to m dot a into I will call it in terms of the quantity here I will indicate it by V 0 and not a into 1+ F V for that is the exit velocity - V0 plus the pressure thrust a 4 x T 4 – V0 okay so this is the exit velocity this is the entry velocity.

And this is the pressure thrust typically the pressure thrust part is very small compared to the convective flux thrust so in this analysis we will neglect this part okay so we will consider that the nozzle is optimal the flow in the nozzle is optimally expanded so P 4 = P0 so this part goes to 0 essentially we are left with only the convective flux now I can rewrite this as f is equal to m dot a V not okay right.

Now further I want to simplify it what we know is that velocities are related to Mach number and speed of sound so I will use that information I know V 0= M 0 V 0and V 4 = M 4 into a 4 I also know that V 0 is nothing but and a 4s okay so I know the relationship between the velocity in terms of Mach number and then in terms of temperature now we will make an assumption here if you see here a four corresponds to the velocity of sound at condition for at condition for you realize that we have added fuel.

And burned the fuel and therefore the composition of the gases at four and zero are not the same so typically speaking I should have written different notations for these two right but in this analysis we will assume that as the gases pass through the device there is no change to it to either γ or R okay so assuming R and γ to be the same for it roasts and exhaust gases and ambient air then we can do the further simplification of this.

So I can now write F = m dot a I know that V0 is nothing but M0 import okay now if you remember our discussions earlier in the course in all the air-breathing propulsion we said that if kerosene is the fuel or any hydro carbon is the fuel then if the fuel air ratio that is m dot F / m dot e for stoichiometry is around six seven okay so this is pretty much less than one so we can neglect F in comparison to what okay.

So f is very much less than one so I can rewrite my equation as f is equal to m dot a M0 e0 so this goes to zero I will get me for into here I know that a four is γ RT four and a 0is γ RT 0 what I will do is I will take out the γ R part because of this assumption and I will be left with M 4 / M 0 into under root okays o we have been able to express this in terms of ratio of exit to In let conditions.

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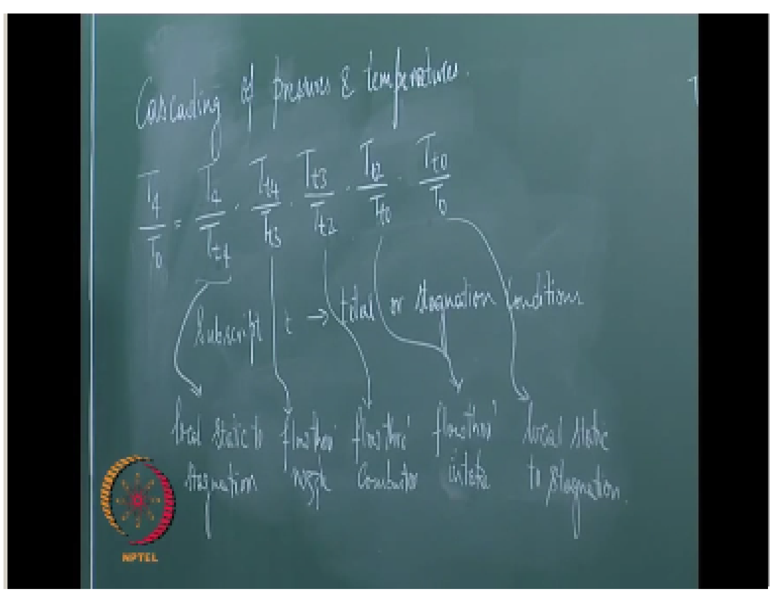

Now what we will do is what is known as cascading of pressures and temperatures we will try to cascade from the nozzle side towards the inlet and see what we can get out of this we will do cascading of pressures and temperatures and see what we can get out of it firstly let me take t 4 / D0 t4 /T not I can write it as t4 / TT 4 xTT 4 / TT 3x TTV /TT 2 x PT 2 / T0 or I call this TT0 2 TT 0 / T 0 the subscript T here indicates total okay so total or stagnation conditions so what we have done is if expressed it is a ratio of static to stagnation and then stagnation of 4 2 3 3 2 2 2 2 0 and then stagnation to static okay.

So all these will cancel out and you will get T 4 /T 0 so let us look at what are these values this is nothing but local static to stagnation condition which we can express in terms of Mach number this what does this represent TT 4 / TT 3years this by this it represents flow through the nozzle similarly et3 / TT two three and two t represents flow through the combustor this represents flow through diffuser or intake.

And this again is local static –stagnation okay now for an isentropic flow what happens to this ratio TT 3 / TT 4 if we assume the flow through the nozzle to be isentropic then both these temperatures are same so you get this ratio to be 1okay then again flow through intake this is also one fine you are left with now three things this flow through combustor.

And local static to stagnation this the first and the last one we can express it in terms of Mach number okay we will do that right now so this would be 1 by 1+ γ - 1/ 2 m 4^2 a x 1 and again this would be1 + γ- 1 / 2 m 0^2 okay so lastly we left with flow through the combustor.

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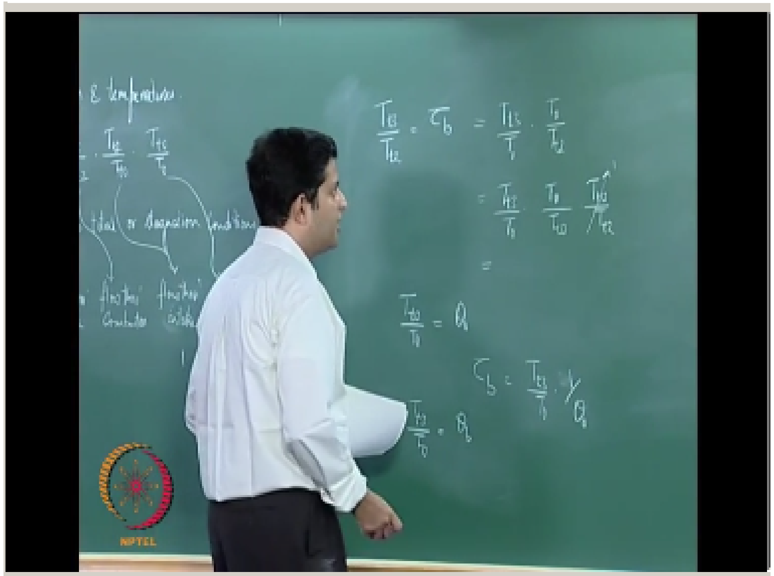

Now let me call this ratio 83 by I will call it tau B okay now I can also express that this tau BS 83 /T 0into right T0 T 0 cancels out you get the same thing this is equal to T0 / T 0/ T t2 this is flow through the intake which is one okay so this is this is one so you get this we can express in terms of Mach number okay and let me call TT 0 / T 0 as θ 0 okay so if I call TT 0 / T 0as θ 0 I will get τ B = TT 3 / T 0 x1 / θ 0.

I will also call this quantity I will define TT 3 / T 0 as θ B this is nothing but the ratio of the exit temperature of the combustor to the ambient temperature okay now this is those under the designers control right so therefore there is a very important parameter as we will see later on in the discussions.

So T TT TT 3 is temperature at the exit of the combustor and T 0 is the inlet temperature so we will call this θ B so I get tau BS this is nothing but θ b / t 0 okay so if i substitute back for this t4 / t 0 all this I will get the following expression.

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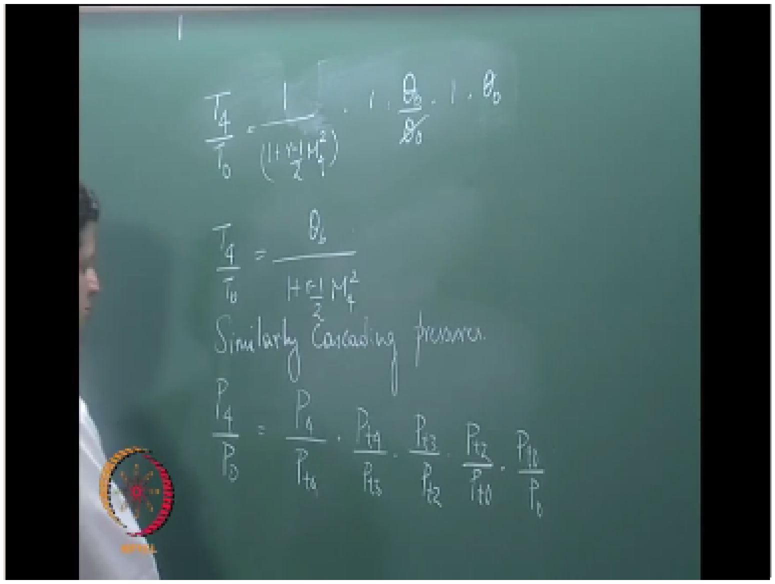

Let me put that as 1/ 1 + γ - 1 / 2 M 4^2 - 1 into if you see here TT 3/ 2 T 2 is nothing but tau B which is nothing but θ B/ θ 0 into flow through intake which is 1 into the last part 1 + γ – 1/ M0 square I have defined this ratio as θ 0 here so I will put that down so finally we get P 4 / T 0= θ B / 1 + γ- 1 /2 M 4^2 now we have been able to derive this expression for t 4 / t 0 by cascading temperatures.

Let us now do the same exercise by cascading pressures try and find out what is the relationship for Mach numbers so similarly you get p4 / PT 4 PT 4 / PT 3 PT 3 / PT 2 PT 2 / PT0x E T 0/ T 0 okay again he indicates the subscript T here indicates total conditions or stagnation conditions.

So this is ratio pressures static to stagnation the first and the last terms are ratio of start local static to stagnation pressure one is at the exit one is at the inlet okay now this is flow through diffuser flow through combustor and flow through intake right now this pressure term here the pressure term here this indicates the efficiency across the nozzle okay this indicates the efficiency across the combustor.

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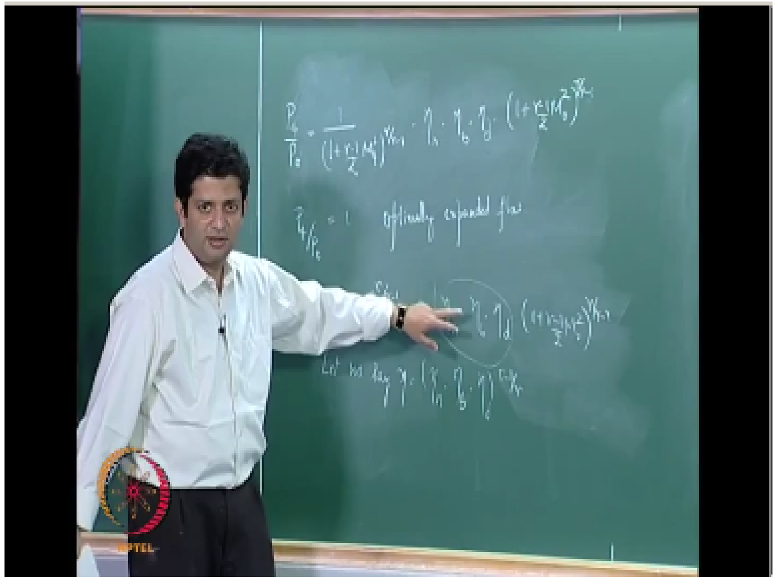

And this indicates the efficiency across the intake so I will call them as this is nozzle efficiency η and this is burner efficiency be η B and lastly this is diffuser efficiency η D okay the other two terms can be expressed in terms of Mach number okay so we get P 4/ P 0 = 1 / 1+ γ -1/ 2 $m4^2$ γ / γ - 1 xη nx η B x η diffuser or intake into 1 / γ -1 /2okay so this is the expression that we have what is P 4/ P not remember.

When we started out we made an assumption saying that P4/ P 0 =1 optimally expanded flow so P 4/ P notice so you get 1 + γ -1 / 2 + 4^2 okay this is the expression that we get by cross-multiplying now I will put all these three ratios into one value of η and an express let us say η = e turn into the tab θ b x β d ^{2} γ - 1 / γ if I do it this way then when I plug in for this it will all be a relation of γ/ γ - 1 so I can remove the powers and look at what we have yet okay.

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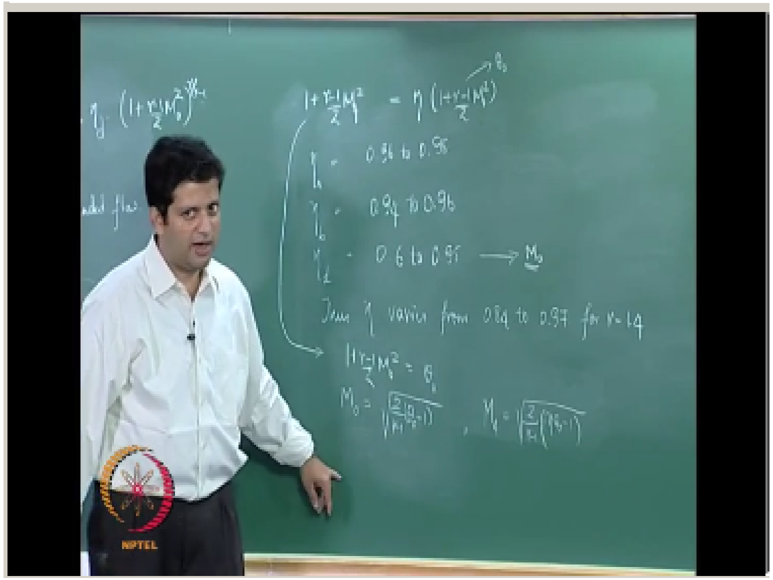

So we get finally $1+ \gamma - 1/ 2\ m4^{2} = \eta$ into okay the powers get cancelled out so you are left with this equation just to remind you we had done this exercise earlier the relative values of all these efficiencies eat iron typically it is flow through the nozzle it varies between 0.96 to 0.98 and eat a burner this varies between nine four two nine six and diffuser remember we had indicated this to be a strong function of Inlet Mach number if you have a large Inlet Mach number you are bound to get lower efficiency because of flow being processed through oblique η normal shocks okay.

So you will get a lower efficiency if you have high Mach number but as you go down and Mach number you will get a higher efficiency so if you combine the worst case scenario for this and remember this is a multiplication sign you will end up getting radiation η varies from point eight four two point nine seven four $\gamma = 1.4$ okay so the worst case scenario is this 0.84 and the best is 0.99.

We will discuss about this when we talk about what is the lowest Mach number in which a ramjet can operate now coming back to this equation here this was our relationship that we were trying to find in terms of Mach numbers what is this term here this is nothing but θ0 right so we get $1 +\gamma - 1 /2\ m\ 0^{2}$ is nothing but θ 0 so from here you get M 0 =- du / γ - 1 θ 0- 1.
And similarly if you put this θ 0 here you can get an expression for M 4 as okay so now we have the relationship for M 4/ M 0and T 4 / T0 in a in our earlier thrust relationship we were searching for a relationship for d 4 / T 0 and M 4 / M0 we have got both of them let us plug it in and see what happens.

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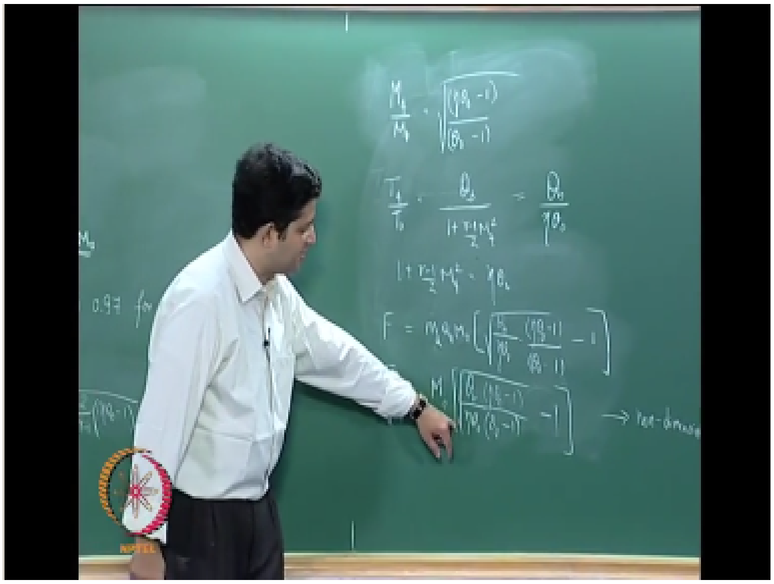

So our relationship M 4/ M 0 = θ θ 0- 1/θ 0 - 1 and similarly D 4 /T 0 was θ b2 1plus okay what is this term there is nothing but this term here right so it is nothing but η θ naught so you get η B = TT 4 t4/ T 0=θ B/ η θ 0 okay it will because we assume that 1 +γ -1 / 2 m 4^2 is nothing but it has it an order.

So we have both the ratios that we were looking for or our thrust equation becomes f = m dot a a 0 M 0 we had m4 / M not that is nothing but θ B I will firstly put t4 / D 0 θ θ 0t into this is the expression that we get fort hurts in a ramjet now we like to have non dimensional numbers okay we can easily see that we can non-dimensionalize this thrust if we divide by m dot a knot throughout okay.

And Mach number is not a function of I mean it is a non dimensional value and all the other things data be η and θ note or also non dimensional values okay so I get F by so this is the expression for non-dimensional thrusts now we have found out what is the expression for non-dimensional thrust in terms of θ B which is nothing but the ratio of combustor exit temperature to ambient temperature and a function of Mach number and efficiencies θ naught is a function of Mach number an Δ is efficiency and M 0 is again Mach number itself.

So the only input parameter that we have under our control is through θ B and the flight Mach number okay the rest of it is system determined that is θ or the efficiency system determined okay now we have been able to do this we also need another parameter right what is the other parameter of interest to us specific fuel consumption in this case we look at ISP which is one over specific fuel consumption nothing.

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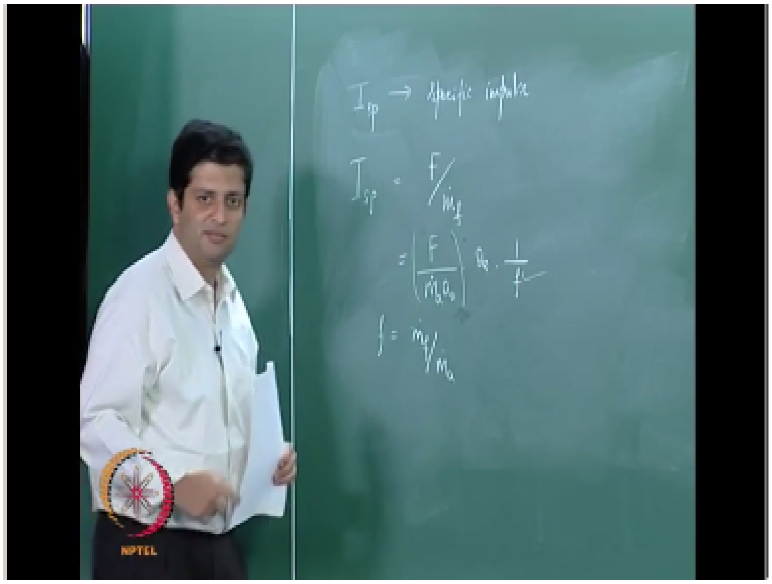

But specific impulse and it is one over SFC we define is PS is P is nothing but mass per unit mass flow rate of fuel okay so what we have got is an expression for F / m dot a and now I have a definition for ISP like this and what we have to look at is we spent a considerable effort trying to derive that expression you have to take a little more benefit out of it so what we will do is we will multiply this suitably and find out if we can use that expression so I can write this as f by m dot e 0 x E 0/F where F is nothing but purely ratio m dot F / m dot a

So if you put m dot F / m dot a here this is n dot a and this M no t cancels out and a 0 and a not cancel out you get f / m dot f so we have been able to get this expression that we derived here okay so if you want ISP now the only parameter that we need to derive an expression for s this F here right okay right we will do that right.

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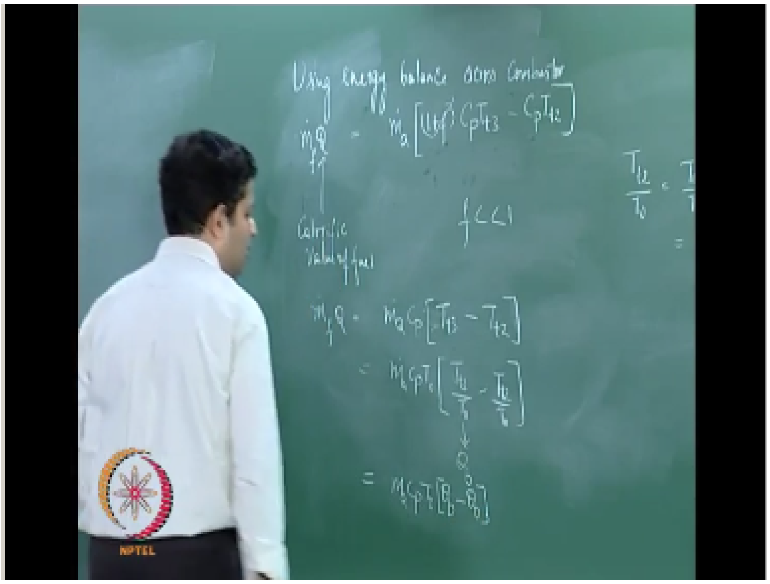

Now we will derive an expression for F by looking at the energy balance across the combustor okay so using energy balance across combustor that is what you are putting in is chemical energy associated with the fear that is given by m dot F which is he mass flow rate of fuel into the calorific value of the fuel okay you indicates this must be worth must this be equal to the sensitive sensible enthalpy rise across the combustor okay so that is given by m dot ex the 1+ 1/2 here indicates that you are looking at the combined mass flow rate of fuel.

And air when you are looking at conditions at 3 okay now again using our this is a little weird you are trying to get an expression for F and what we are trying to do is we try and see if we can eliminate this, this just make sure that our algebra is a little more cleaner we will do this if you can compare it with one always in engineering approximations you tend to compare it with a known value.

And then if it is very small compared to that value you can neglect it okay so you cannot neglect it arbitrarily but you have to make a comparison with some other quantity so if you compare F with respect to 1 we have seen that F is very much smaller to 1 so you can neglect this part here so this goes out we get M 0 F Q = CP is common for both the burnt gases as well as the incoming air so I can take it out okay.

Now I need to let me divide both these by T 0 okay so I can write this as m dot e CP T not right I have divided and multiplied by T0 so what is TT 3/ T0 when we derived expression for F by m dot a what is this expression this is nothing but θ B right and what is this TT 2/ t0 is nothing but

TT 2 / TT 0x TT 0/ T0 right what does this ratio this is flow through intake this is 1 and this is what is this θ 0 so you get if you substitute back in here you will get m dot a CPT0x θ B - θ 0.

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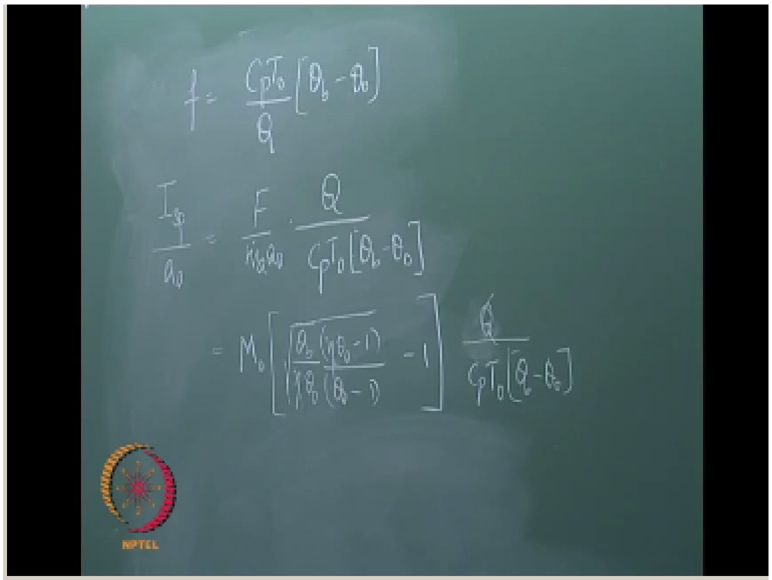

So finally we can get an expression for F by combining these two we can write F = CPT 0/ Q into θ B - θ 0 and therefore our expression for is B you see here this expression for ISP that I have if I divide the ISP by a naught I get anon-dimensional quantity what is the unit of ISP is P is unit is what Newton per kg second right what is Newton, Newton is kg meter per second square you get kg by per second.

So what you will get this meter per second okay so if you split it if you simplify further you will get ISP unit as meter per second a naught is nothing but speed of sound which is also in meters per second so you will get anon-dimensional quantity if you divide it by a naught and if we substitute the expression that we derived for f / m dot a not we will get this as η V ETA η not do AR not minus one this is the expression for is p by a not okay we will stop here we will continue in the next class thank you.

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