Smart Structures

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Week 11
Lecture No: 57
Analysis of Electro and Magneto Rheological Fluid Flow(continued)
Part 04

In today's lecture, we will discuss the flow of ER or MR fluids in the shear mode.

Now, all these topics including the constitutive relations of this ER or MR fluids, flow mode, shear mode, all these can be found in sufficient details in the book of Chopra and Sirohi. So, the learners are highly encouraged to read those relevant topics from the book. Now, if you want to talk about the shear mode, then the first thing is that - in shear mode, the flow is not driven by pressure unlike what was happening in the flow mode. So, in shear mode also we have two plates at the top and bottom. Now, the top plate is moved by a velocity u_0 and that can involve application of a force, F_0. And the velocity profile across the depth is a result of the movement of the upper plate, it is not due to any pressure difference. So, here we have x, here we have y, and the depth of this region we can call as d. Now, to analyze the flow, again we start with the governing differential equation, which we can write as del tau by del y is equal to del p by del x. In this case, I have no pressure difference. So, it is 0. So, del tau by del y is equal to 0 that is my governing differential equation.

$$\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x} = 0$$

So, the boundary conditions are - at y equal to 0, u is 0, and at y equal to d at the upper plate, u is equal to u_0, the velocity at which the upper plate is moving.

$$\text{at } y = 0 \qquad u = 0$$

$$y = d \qquad u = u_0$$

So, again we solve it under two conditions, one is 0 applied field. Now, under the 0 applied field, my constitutive relation is tau equal to mu multiplied by del u by del y and then if we put this relation here in the governing differential equation, the expression becomes mu multiplied by del 2 u by del y 2 is equal to 0.

$$\tau = \mu \frac{\partial u}{\partial y}$$

So, this equation is solved and these boundary conditions are satisfied. So, from this governing differential equation, we get mu multiplied by del u by del y is equal to C_1. And then, we have mu into del u by del y is equal to C_1 y plus C_2. Then, if we say that u at 0 equal to 0 that, sorry, there is one mistake here. So, if we integrate it once more, we get mu into u equal to C_1 y plus C_2.

$$\mu \frac{\partial^2 y}{\partial y^2} = 0$$

$$\Rightarrow \mu \frac{\partial u}{\partial y} = C_1$$

$$\Rightarrow \mu u = C_1 y + C_2$$

Then, if we put the condition that at y equal to 0, u is 0, that gives me C_2 equal to 0 and then, if we say that at y equal to d, u is equal to u_0, that tells me that mu into u_0 is equal to C_1 into d. On solving for C_1, we get C_1 equal to mu u0 by d.

$$u(0) = 0 \quad \Rightarrow C_2 = 0$$

$$u(d) = u_0 \qquad \Rightarrow \mu u_0 = C_1 d \qquad \Rightarrow C_1 = \frac{\mu u_0}{d}$$

So, if we put these two in this expression, in this expression, we get the velocity profile as - u as a function of y is equal to u_0 by d into y.

$$u(y) = \frac{u_0}{d} y$$

So, the velocity is a linear function of y. It starts from 0, here, and it becomes u_0 here. In between them it varies linearly. So, u is equal to u_0. Here, I have the u equal to 0. Now, if the velocity gradient is constant, because the velocity variation is linear. So, the velocity gradient is constant and that tells me that the shear stress is constant. So, we have shear stress, tau y equal to mu into del u by del y. From this expression, if I evaluate del u by del y, it is just u_0 by d. So, my tau as a function of y is mu into u_0 by d. So, tau is not a function of y anymore. So, in this case the shear stress variation is constant.

$$\tau(y) = \mu \frac{\partial u}{\partial y} = \mu \frac{u_0}{d}$$

So, beside the same figure if I draw the shear stress variation, it is just mu u_0 by d, mu u_0 by d, that is tau versus y graph for our case. Now, that we know the shear stress. We know the velocity. Now, we can find out the equivalent damping for this case. So, for that what we do is - first let us find out the force F_0, force F_0 is tau multiplied by L and b.

If we look at this diagram, so, we have z axis here, the dimension of this plates along the z axis is b. So, the inner surface of this plate which is in x z plane that experiences the shear stress tau. The value of tau at y equal to d, in our case, tau is constant, so, it experiences a shear stress of amount mu into u_0 by d. So, the corresponding force, if I multiply that stress by that dimension, by the area of the inner surface of this plate which is L into b, if we multiply by that we get the total force, F_0.

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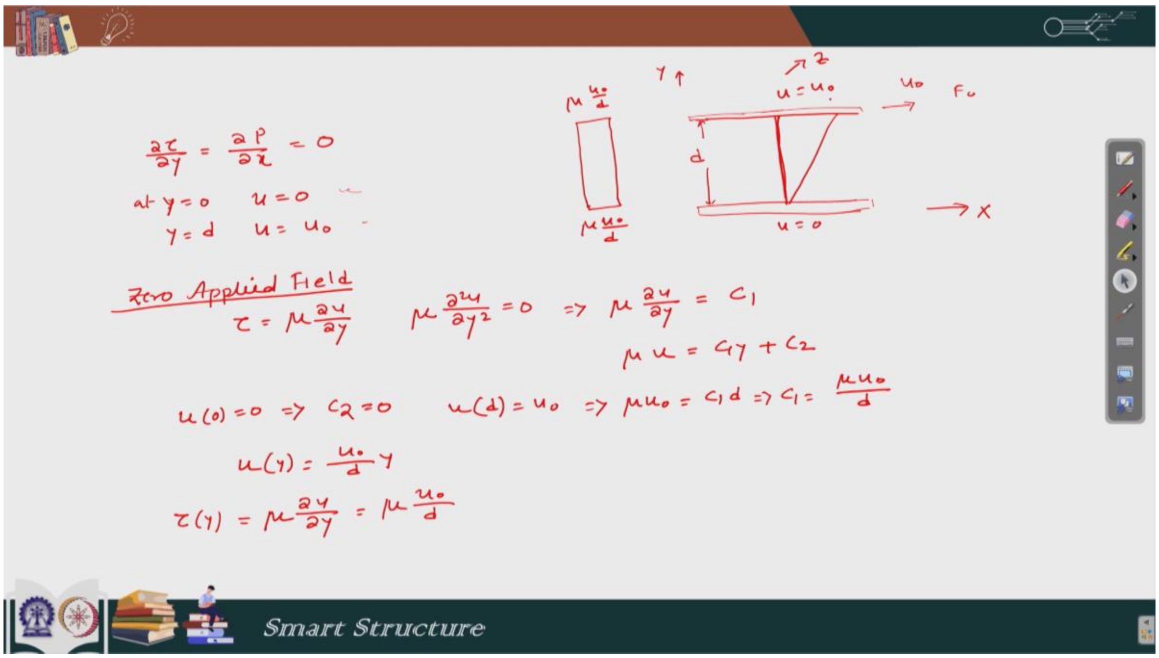

So, F_0 is tau into L into b which comes to be mu multiplied by u_0 by d into L b.

$$F_0 = \tau Lb = \mu \frac{u_0}{d} Lb$$

Now, the equivalent damping comes as C equivalent 0, means, when there is no field applied is equal to F_0 by u_0. So, if I divide F_0 by u_0, finally, we get mu L b by d and which we call mu multiplied by capital gamma.

$$C_{eq}^0 = \frac{F_0}{u_0} = \mu \frac{Lb}{d} = \mu\, \Gamma$$

So, capital gamma is a parameter which depends on these dimensions. Next, let us do the same thing for a non-zero applied field. So, in this condition, we have to write the constitutive relation considering the yield stress. So, tau as a function of y is τ_y multiplied by the sign of τ_y, sorry, multiplied by sin of gamma dot which can be plus or minus, plus mu into gamma dot and we know that gamma dot is del u by del y.

$$\tau(y) = \tau_y sign(\dot{\gamma}) + \mu\dot{\gamma}$$

Now, here if we differentiate this again, we get del tau by del y as just mu into del 2 u by del y 2. So, the governing differential equation remains same. And if the governing differential equations are same, boundary condition also same at y equal to 0, u is 0, at y equal to d, u is 0. So, that gives me the same flow profile. So, from here, we can find out u as a function of y as u_0 by d multiplied by y.

$$\frac{\partial \tau}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} = 0$$

$$u(y) = \frac{u_0}{d} y$$

So, here we can note that governing differential equation and boundary conditions are same as those of non-zero applied field, as those for non-zero applied field. So, we have the same solution. Now, we have to find out the stress. So, the stress as a function of y is just, we have τ_y plus mu into u_0 by d. So, only τ_y gets added here.

$$\tau(y) = \tau_y + \mu \frac{u_0}{d}$$

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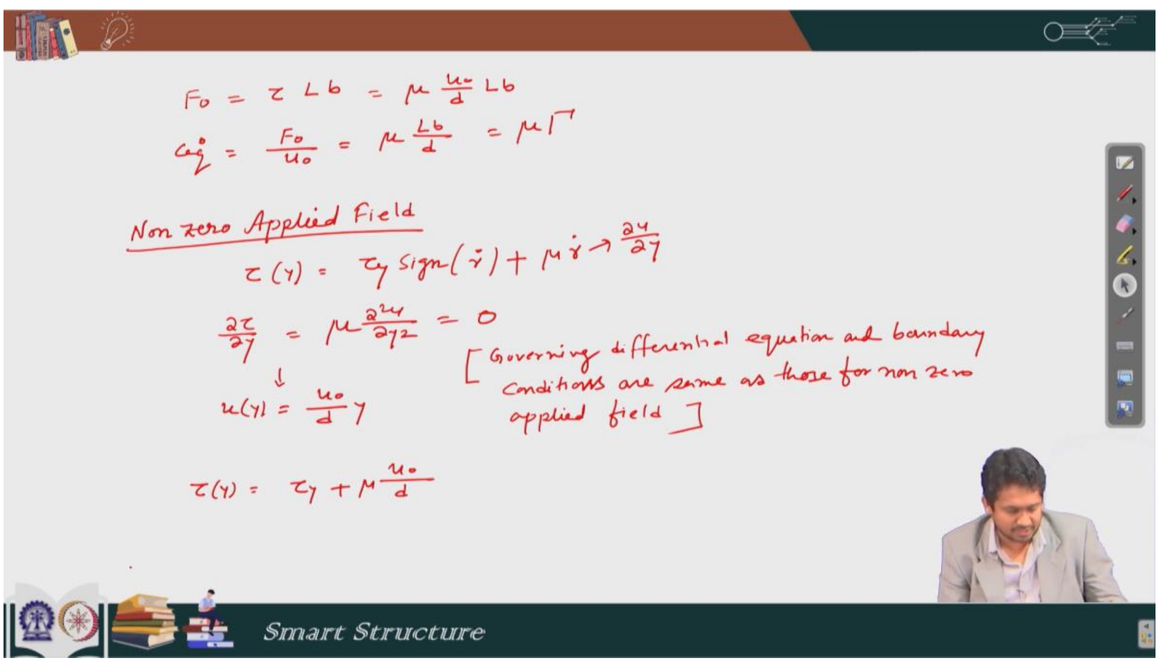

So, with this new stress, now we have to find out the equivalent damping for the active case. So, first let us find out the force. Now, the force is F_0, which is tau multiplied by b L. So, finally, the expression comes as τ_y plus mu into u_0 by d, multiplied by b L. Now, these things can be written in a somewhat different form. We can write tau_y multiplied by d by u0 mu plus 1, multiplied by mu u_0 by d, L b. So, with that finally, the expression is and that we equate with - C equivalent for the active case multiplied by u_0. So, on doing that we get C equivalent as tau_y d u_0 mu plus 1 multiplied by mu L b by d. Now, this quantity can be expressed as mu multiplied by capital gamma, we know that L b by d is capital gamma, and then this entire quantity is multiplied by 1 plus Bi.

$$F_0 = \tau bL = \left(\tau_y + \mu\frac{u_0}{d}\right)bL = \left(\frac{\tau_y d}{u_0\mu} + 1\right)\mu\frac{u_0}{d}Lb = C_{eq}^a u_0$$

$$\Rightarrow C_{eq}^a = \left(\frac{\tau_y d}{u_0\mu} + 1\right)\frac{\mu Lb}{d} = \mu\Gamma(1 + Bi)$$

We can see here that this quantity is again a ratio of yield stress and viscous stress. tau_y is yield stress, mu_0 by d is the velocity gradient which is gamma dot, and that is multiplied by the viscosity gives me the viscous stress. So, it is tau_y d divided by u_0 mu is our Bingham number.

$$Bi = \frac{\tau_y d}{u_0\mu}$$

So, in other words we can say that this quantity is equal to C equivalent 0, because we have seen that mu into gamma is C equivalent 0, C equivalent for the inactive case for the nonzero field case. So, that is C equivalent multiplied by 1 plus the Bingham number.

$$C_{eq}^a = C_{eq}^0(1 + Bi)$$

So, here the Bingham number tells me that how much damping we achieve by making the fluid by making by applying electric field across the fluid layer. So, more the Bingham number is - more I have the active component of the damping.

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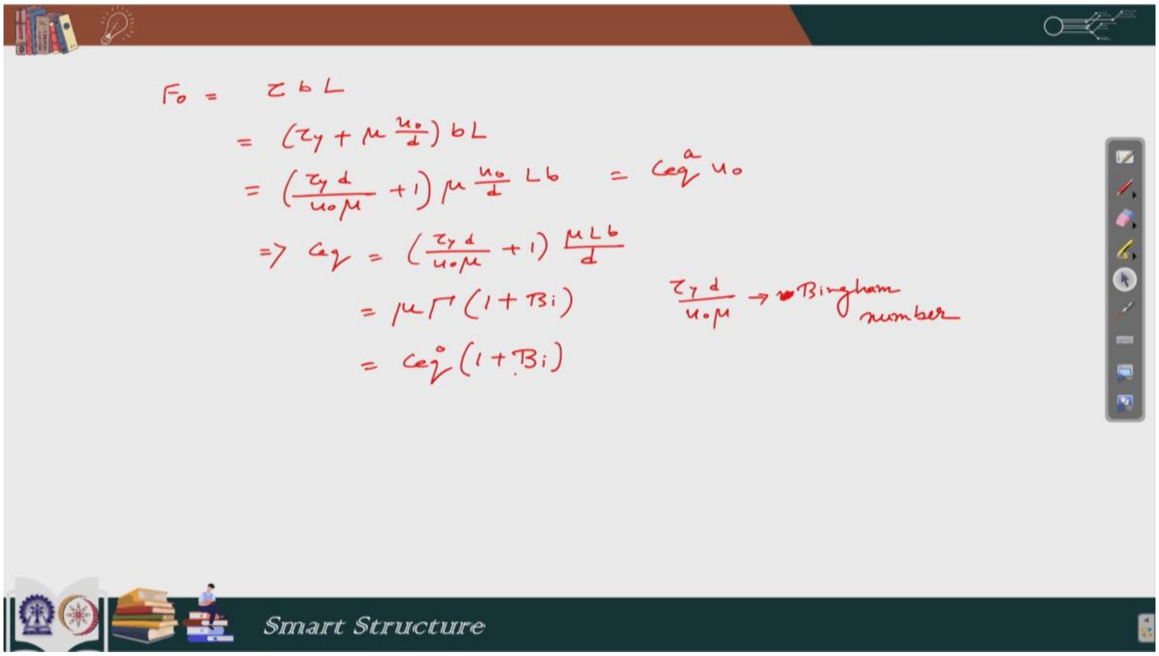

So, these are the two cases that we have described so far - one is the flow mode, another is the shear mode and under each of these modes we have non-zero applied field and zero applied field. And we have seen that in flow mode there are several regions that comes out because of the nonzero applied field. In shear mode, we do not have that shear mode is somewhat more simplified in terms of analysis.

So, with that I would like to conclude this lecture here.

Thank you.