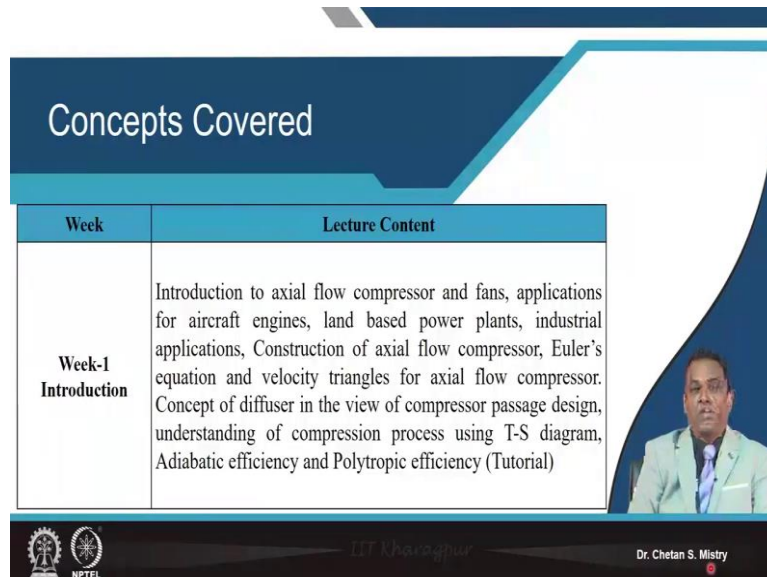


Aerodynamic Design of Axial Flow Compressors and Fans
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Lecture 7
Stage Configurations and Parameters

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Week	Lecture Content
Week-1 Introduction	Introduction to axial flow compressor and fans, applications for aircraft engines, land based power plants, industrial applications, Construction of axial flow compressor, Euler's equation and velocity triangles for axial flow compressor. Concept of diffuser in the view of compressor passage design, understanding of compression process using T-S diagram, Adiabatic efficiency and Polytropic efficiency (Tutorial)

Hello, and welcome to lecture-7. So, in last week, we were discussing about say... introduction to axial flow compressor. We have discussed about say... various application those are related to say... aircraft, land base power plant; we have discussed about the industrial applications. Then we were started discussing about the construction of axial flow compressor.

Now, with this detail, we move forward by using our fundamental equation, that's what is called Euler's equation. We have discussed about various velocity components and what are the uses, then, most importantly, we have correlated our diffuser or diffusing section or say... diffuser, that's what is related with our flow passage within say... axial flow compressor blades, okay.

Then, we have started discussing about this process of compression on T-S diagram, we have discussed very important parameter called adiabatic efficiency and polytropic efficiency. And we have realized, like adiabatic efficiency, that's what is related with the overall stage of the compressor and polytropic efficiency, that's what is say individual stage.

Now, all these things what we have learned, that is what will be making the base for us in order to move forward. Now, again and again when we will be moving ahead, we will be using Euler's equation, we will be using velocity triangle; and this velocity triangle, that's what is

most important thing in order to realize what is happening within your axial flow compressor. Now, in order to understand what all are your adiabatic and polytropic efficiency, we have taken one tutorial session and I had given you the assignment part, just have quickly look for that.

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Assignment

Industrial axial flow compressor is to be designed to deliver a total pressure ratio of 7.36. It is proposed to have three stages with individual pressure ratios of 1.6, 2.0 and 2.3 respectively. The polytropic efficiencies of the first, second and third stages are 80%, 75% and 79% respectively. Assuming $\gamma = 1.4$

(1) Calculate the overall total-to-total efficiency of the resulting stage.
 (2) What should be the polytropic efficiency of the last stage in order to have an overall efficiency of 78%?
 Consider first 2 stages as case-1 (PR- 1.6, 2.0 Efficiency as 80%,75%) ?

From the given data,
 $\pi_1 = 1.6, \eta_1 = 0.8$
 $\pi_2 = 2.0, \eta_2 = 0.75$
 $\pi_3 = 2.3, \eta_3 = 0.79$
 Case-1 $\eta_{all\ stage} = ?$
 Case-2 $\eta_{all\ stage} = 78\%, \eta_{i-3} = ?$

Hint Case-1

Given- polytropic efficiency, Total pressure ratio

↓

Calculate stage Temperature ratio

↓

Calculate stage efficiency

Hint Case-2


Given- Overall efficiency, Total pressure ratio of 2 stages

↓

Calculate stage Temperature ratio of stage-3

↓

Calculate polytropic efficiency



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So, this was the problem; it says my industrial axial flow compressor which delivers the pressure ratio of 7.36. It is proposed that the individual stage they are having different pressures here, say... it is 1.6, 2, 2.3, respectively, and their polytropic efficiencies are 80%, 75% and 79%. So, you can realize what numerical we have solved in last session they are having same pressure ratio and same efficiency - polytropic efficiency. Here in this case, we are having polytropic efficiency and pressure ratios to be different. That is what will be giving you some different flavor.

So, what you need to do is, you need to calculate your overall efficiency from the resulting stage and then it says what needs to be your polytropic efficiency of the last stage in order to have your overall efficiency to be 78%. Assuming, say first two stages are as it is means that pressure ratio and polytropic efficiencies are same. So, I had given you the hint in order to solve the numerical. What it says, in order to solve your first case, we are having our overall pressure ratio.

So, based on that we can calculate our temperature ratio for individual stage and we can calculate what will be our efficiency - overall efficiency, okay. Same way second numerical or second case, it says we are given with overall efficiency, we are given with pressure ratio of

two stages and their polytropic efficiencies. Now, for the same configuration you need to calculate the efficiency of the third stage, okay. And for that it says you are about to achieve your overall efficiency of 78%. So, let us move ahead.

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Assignment

Solution-1 From the given data,
 $\pi_1 = 1.6, \eta_1 = 0.8$
 $\pi_2 = 2.0, \eta_2 = 0.75$
 $\pi_3 = 2.3, \eta_3 = 0.79$
 Case-1 $\eta_{all\ stage} = ?$

All 3-stages combined
 $\tau_{all\ stage} = \tau_1 \times \tau_2 \times \tau_3 = 1.179 \times 1.29 \times 1.319 = 2.006$
 $\pi_{all\ stage} = \pi_1 \times \pi_2 \times \pi_3 = 1.6 \times 2 \times 2.3 = 7.36$
 $\Rightarrow \eta_{all\ stage} = \frac{\pi_{all\ stage}^{\frac{\gamma-1}{\gamma}} - 1}{\tau_{all\ stage} - 1} = \frac{7.36^{\frac{\gamma-1}{\gamma}} - 1}{2.006 - 1} = 0.764$
 $\Rightarrow \eta_{all\ stage} = 76.40\%$

Stage temperature ratios for individual stages
 $\tau_1 = \frac{(1.6^{\frac{1.4}{1.4}} - 1)}{0.8} + 1 = 1.179$
 $\tau_2 = \frac{(2.0^{\frac{1.4}{1.4}} - 1)}{0.75} + 1 = 1.29$
 $\tau_3 = \frac{(2.3^{\frac{1.4}{1.4}} - 1)}{0.79} + 1 = 1.319$

It can be observed that the total-to-total efficiency of the resulting stage is 76.4%.

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Suppose if I consider, these all are my given data. So, I say pressure ratio is 1.6, my polytropic efficiency of first stage is 0.8, my pressure ratio of second stage is 2, polytropic efficiency is 75%, pressure ratio for the third stage is 2.3, and my polytropic efficiency is 0.79. Now, as we discussed, we can calculate what will be our stage temperature ratio for the individual stage using our fundamental equation. So, it says these are my temperature ratios for stage 1, stage 2 and stage 3.

$$\tau_1 = \frac{1.6^{\frac{1.4-1}{1.4}} - 1}{0.8} + 1 = 1.179$$

$$\tau_2 = \frac{2^{\frac{1.4-1}{1.4}} - 1}{0.75} + 1 = 1.29$$

$$\tau_3 = \frac{2.3^{\frac{1.4-1}{1.4}} - 1}{0.79} + 1 = 1.319$$

Based on that we can calculate our overall temperature ratio for all the three stages, we know what is our total pressure ratio for all the three stages. And based on that, if I will be putting, I can calculate my overall efficiency including all the stages. So, that's what is nothing but in sense of overall pressure ratio, and overall temperature ratio.

All 3 – stages combined

$$\tau_{all_stage} = \tau_1 \times \tau_2 \times \tau_3 = 1.179 \times 1.29 \times 1.319 = 2.006$$

$$\pi_{all_stage} = \pi_1 \times \pi_2 \times \pi_3 = 1.6 \times 2 \times 2.3 = 7.36$$

$$\eta_{all_stage} = \frac{\pi_{all_stage}^{\frac{\gamma-1}{\gamma}} - 1}{\tau_{all_stage} - 1} = \frac{7.36^{\frac{\gamma-1}{\gamma}} - 1}{2.006 - 1} = 0.764$$

$$\eta_{all_stage} = 76.4\%$$

So, if I am putting these numbers, it says my overall efficiency, that's what is coming 76.40%. So, this is what is giving you idea, like what all will be the impact of, you know, different pressure ratio and different polytropic efficiencies.

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Assignment

Solution-2 From the given data,
 $\pi_1 = 1.6, \eta_1 = 0.8$
 $\pi_2 = 2.0, \eta_2 = 0.75$
 $\pi_3 = 2.3$
 Case-2 $\eta_{all_stage} = 78\%, \eta_{1-3} = ?$

The overall pressure ratio is constant as
 $\pi_{all_stage} = \pi_1 \times \pi_2 \times \pi_3 = 7.36$

Hence, required overall temperature ratio

$$\tau_{overall} = \frac{(\pi_{all_stage}^{\frac{\gamma-1}{\gamma}} - 1)}{\eta_{req}} + 1 = \frac{(7.36^{\frac{1.4-1}{1.4}} - 1)}{0.78} + 1$$

$$\Rightarrow \tau_{overall} = 1.985$$

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Very interesting case, that's what is with say... case two. For that case, what is given, say... we are given all these numbers, pressure ratio for stage 3 that's what is given to us, that's what is say 2.3. Now, what we know, my overall pressure ratio that's what is remained same, it is 7.36. So, based on that, I can calculate what is my overall temperature ratio, okay. So, my overall temperature ratio that's what is coming 1.985, okay.

$$\tau_{overall} = \frac{\pi_{all_stage}^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{req}} + 1 = \frac{7.36^{\frac{1.4-1}{1.4}} - 1}{0.78} + 1$$

$$\tau_{overall} = 1.985$$

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Assignment

We know,

$$\tau_{overall} = (\tau_1 \times \tau_2) \times \tau_{3_new}$$

The temperature ratio of first two stages will be same as before.
It is the last stage whose temperature ratio changes

$$\Rightarrow \tau_{3_new} = \frac{\tau_{overall}}{\tau_1 \times \tau_2} = \frac{1.985}{1.52} = 1.306$$

In order to limit the temperature ratio to 1.306, while giving the same pressure ratio the required efficiency of last stage can be calculated by

$$\eta_{3_new} = \frac{\pi_3^{\frac{\gamma-1}{\gamma}} - 1}{\tau_{3_new} - 1} = \frac{2.3^{\frac{1.4-1}{1.4}} - 1}{1.306 - 1}$$

$$\Rightarrow \eta_{3_new} = 0.878 = 87.80\%$$

The last stage should have polytropic efficiency of 87.8%.


From the given data,


$\pi_1 = 1.6, \eta_1 = 0.8$

$\pi_2 = 2.0, \eta_2 = 0.75$

$\pi_3 = 2.3$

Case-2 $\eta_{all\ stage} = 78\%, \eta_{1-3} = ?$





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Now, what we know my overall temperature ratio that I can write down, what is my temperature ratio of stage 1, temperature ratio of stage 2, and what is my new temperature ratio; because now I am looking for my efficiency - overall efficiency to be some number and for that, we do not know what is our polytropic efficiency. So, what we will be doing? We will be calculating our overall temperature ratio based on this relation. It says my temperature ratio for the new stage that's what is coming or the third stage it is coming 1.306.

$$\tau_{overall} = (\tau_1 \times \tau_2) \times \tau_{3_new}$$

$$\therefore \tau_{3_new} = \frac{\tau_{overall}}{\tau_1 \times \tau_2} = \frac{1.985}{1.52} = 1.306$$

Now, if we know this temperature ratio; now, I can calculate, I can use my relation for isentropic efficiency, that's what is in sense of my pressure ratio and my temperature ratio. If I will be putting this number, it says my efficiency it is coming 87.80.

$$\eta_{3_new} = \frac{\pi_3^{\frac{\gamma-1}{\gamma}} - 1}{\tau_{3_new} - 1} = \frac{2.3^{\frac{1.4-1}{1.4}} - 1}{1.306 - 1}$$

$$\therefore \eta_{3_new} = 0.878 = 87.80\%$$

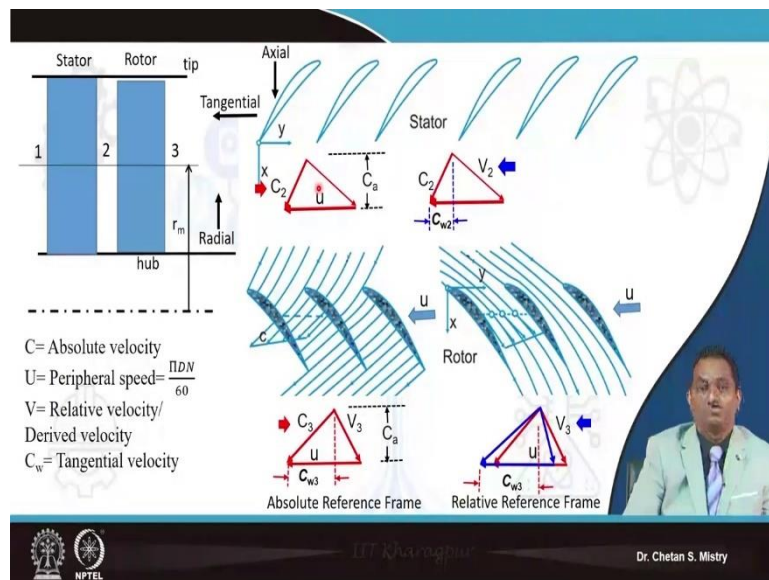
Now, you can understand when we are looking for our efficiency to be on higher side; say...when we are looking for our efficiency to be increased from 76.8 to 78%, one of the

possibility for us to increase the polytropic efficiency. And that is the reason why... when we are doing our design, we are taking care of efficiency of individual stage, okay. That is what say, we are taking care of our polytropic efficiency.

And again let me repeat, say... what I was discussing, whenever you are having conversation with the sales person or see maybe those who are supplying you say... compressor or maybe your engine, when they are claiming some numbers in sense of efficiency be careful, as and when required just inquire and check with whether they are talking of overall efficiency of the compressor or polytropic efficiency of the compressor.

So, when we will move ahead with our design part that time we will be taking care of polytropic efficiencies. So, all stages let us say... like for your engine, you will be having your fan... suppose high bypass ratio engine or low bypass ratio engine for that you will be having your fan, you will be having your LP spool, you will be having say... HP spool that means LP compressor, HP compressor maybe if it is a three spool configuration, you... we will be having intermediate pressure compressor; and for all of them this polytropic efficiencies we need to assume and there is a systematic way to use these numbers. So, we will be discussing in detail how we will be deciding these numbers.

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So, with this let us move ahead. Say, this is what is representing... this is what all we have discussed in sense of our velocity triangle for axial flow compressor. And, it says like this is what is say my stator, I will be having my rotor, okay. And if I will be taking my mid-section,

I will put my stations, say... station 1, station 2 and station 3, that's what is representing my condition at stator inlet, stator exit, rotor entry and say... rotor exit, okay.

Now, you know, like we have discussed we will be having three components. That's what is say... one, we say it is my axial component, that's what is, you know, in the direction of my flow. That's what is responsible for my fluid to flow through. So, you know, like when I say, I am writing my mass flow rate, I used to say my mass flow rate is given by density into area into my axial velocity. Next component that is what we say, it is my tangential component, and we have realized using our Euler's equation, this tangential component that's what is responsible for doing your compression work, we are working on our working fluid, okay.

We are having third component that's what is say perpendicular to this, that's what is representing my radial direction, that means that's what is along my span. Then we have realized how my flow that is what will be coming out from my stator, okay; how I will be having my whirl component at the entry of my rotor, what will be my whirl component at the exit of my rotor. Then we have realized in order to have your, you know, work done or the pressure rise requirement to be higher, we need to go with increase of our whirl component at the exit of my rotor, okay. So, this is what all we were discussing.

Then we were discussing about the change in velocity components. So, here if you look at, this is what is representing my relative velocity component. If you look at. at the entry of my rotor, my relative velocity component that's what is higher, at the exit of my rotor, my relative velocity component that's what is coming to be lower, okay. So, when we say axial flow compressor or say any compression device, for that compression device, your relative velocity through the rotor that will be decreasing, that's what is representing the diffusion work that is what has happened, okay.

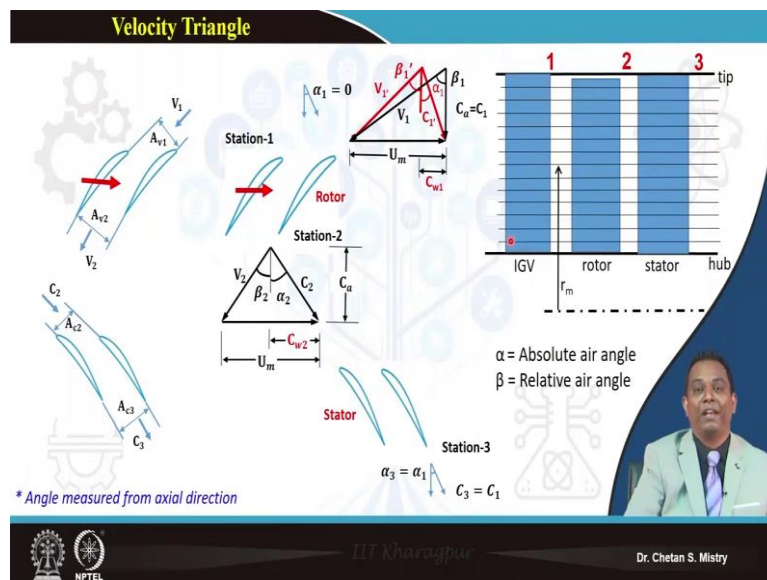
Now, if you consider, say... your component of absolute velocity, say... at the entry of my rotor my absolute velocity component and my exit of my rotor, say...absolute velocity component, it is clearly been seen like my absolute velocity at the exit of my rotor will be higher compared to my entry case, okay. So, this is what all will be giving us idea about what all are the velocity components for axial flow compressor.

So, let me repeat again, we are having our axial flow component, that's what is say axial velocity, which is responsible for my fluid to flow. I am having my absolute velocity component with which my flow that will be entering inside the stator. I will be having my

relative velocity component, that is what will be entering to my rotor and will be coming out at the exit, that's what we say... relative reference frame components. We are having say... whirl component, that's what is responsible for your tangential force, and that's what is responsible for your work done, okay. So, now we are clear with this.

One more parameter that's what is your peripheral speed, that is nothing but my rotational speed is correlated with that. We will see today what is the use of this peripheral speed. So peripheral speed it is mainly been written as say $\frac{\pi DN}{60}$, where D is the diameter or the location where we are doing our calculation, and N, that's what is nothing but the rotational speed of my wheel, okay. So, I am sure this is what is what all we have discussed in earlier class also. Now, let us try to understand how we will be using that for our actual case.

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So, here again if you look at, this is what is representing my rotor, this is what is representing my stator; if I will be taking my midsection, let me put here, okay. Now, you know, like in order to have your configuration here, it says I am having my rotor at that mid-section I will be putting this component, okay. I will be putting that... I will be showing that as say my airfoil.

So, my flow, that will be entering inside the rotor with relative velocity of V_1 . Here if you look at, this is what is representing my area at the entry of the rotor, this is what is representing the area at the exit of my rotor and it will be leaving with velocity V_2 . Do not forget what passage we are making for our rotor or even for stator that will be having diffusing passage, okay. And, that's what is it is reflected at the entry my area is lower at the exit my area that's what is coming to be higher, okay.

Now, let us talk in sense of say stator, what is happening? Say, this is what is happening in the stator. I say, my flow that is what will be entering with my absolute velocity, C_2 ; and it will be coming out with the absolute velocity, C_3 , okay. Now, with this background, I am sure you can make the velocity triangle because we already have discussed about this part, okay.

So, let us take your pen and paper and try to plot the velocity triangle for this with this background, okay. Now, let me put this airfoil; this airfoil that's what is representing my rotor and this is what is say... we are having rotational direction and as we have discussed our rotational direction that is what will be mainly from if you consider, it is from my suction side to the pressure side, okay.

Now, you know, like this is what is. Now if we consider, say my flow that's what is entering in axial direction means my entry it is axial. If that's what is your case, you can say this is what is representing my axial entry and I am having my absolute velocity that's what is to be same as my axial velocity. Now what we know, this is what is my rotational direction. So, if I will be putting, that's what is representing my peripheral speed, U ; this is what is at the mid-section that is the reason I have written that as say... m , okay.

Now, as per our velocity triangle law what all we have discussed, I will be joining that with say relative velocity component, okay, and this angle now what I am putting, let me put this angle as say... β . Now this β angle, that's what is defined as say relative air angle. Sometimes people used to say this as say blade angle, okay, sometimes they used to say relative blade angle. So, do not get confused with this part. So, the component of my angle for rotor that is what we are representing using β . That's what is a function, that's what is co relating your relative velocity component.

Now, at the exit, what will happen? My flow that will be coming out with relative velocity, V_2 , okay. I know what is my rotational speed, U ; so, I will be putting that as a rotational speed, okay. And if I will be joining that, that is what will be giving me my absolute velocity component. Now, let me show you, here if you look at, I will be putting my angle; so, you know, careful when we are talking about this axial flow machine we are measuring angle with reference to axial direction, okay. Do not get confused; universally people they are measuring these angles in reference with the axial direction, okay.

So, this angles what we are measuring, as we discussed for relative velocity component, that's what is I am representing as β_2 , it is nothing but my blade outlet angle or relative blade angle

at the outlet, this is what is representing my absolute angle at the exit, okay. Now, what happens? This flow of velocity that's what is say C_2 with that velocity my flow that will be entering inside my stator, okay. So, let me put this as a stator, okay.

Now, at the same way at the exit, I will be putting my angle that's what is $\alpha_1 = \alpha_3$. Let me put it here in way, say... we are assuming our axial velocity to be constant throughout the stage, okay; for multistage also, we are assuming our axial velocity to be constant. At this moment, this is what is my assumption as we go ahead, I will say my axial velocity will not remains constant. But, you know, in order to simplify our problem, we are initially assuming our axial velocity to be constant. So, you can see here, this axial velocity (at entry of rotor) and this axial velocity (at exit of rotor) their magnitudes are same.

Now, let me put here, this is what is my C_{w2} component that is nothing but my whirl component, okay, that is the component, it is you know, component of my absolute velocity. And as we have discussed, this is what is responsible for your compression work to be done, okay. So, this is what is very important. If you recall, when I was discussing about the T-S diagram, that time I was talking to you, say... when people they are doing design, at the entry absolute velocity and the exit of my stage absolute velocity, that is what they are assuming to be constant or they are assuming to be same. This is what is for ease of making an ease of designing the components or the stages, okay.

So, when we are designing multistage axial flow compressor, that time in order to, you know, make repetitive stages designers they are preferring to have this absolute velocity at the entry of the stage and absolute velocity at the exit of stage, that's what is to be same. So, it says my α_3 and α_1 they are same, it says my absolute velocity C_3 and C_1 they both will be same, okay.

Now, suppose if I consider, I will try to enter my flow not axially, let my flow to enter at some angle. So, that is what we are doing. Suppose if I consider, my flow that's what is entering at some angle α_1 . We are measuring, again I am repeating, we are measuring our angle with reference to axial direction.

So, this is what is representing my absolute flow angle at the entry. If that's what is your case, my peripheral speed suppose if I am taking to be same, okay. Under that case, I will be having my relative velocity component, that's what is coming, V_1' . Now, if you compare these two relative velocities, you will realize, at the entry my relative velocity that's what is going to be lower, okay. Now, here in this case, the angle what we are measuring with axial direction, that

is what we are writing as β_1' . That is also my blade angle, or you can say air flow angle, or relative blade angle or relative air angle, okay.

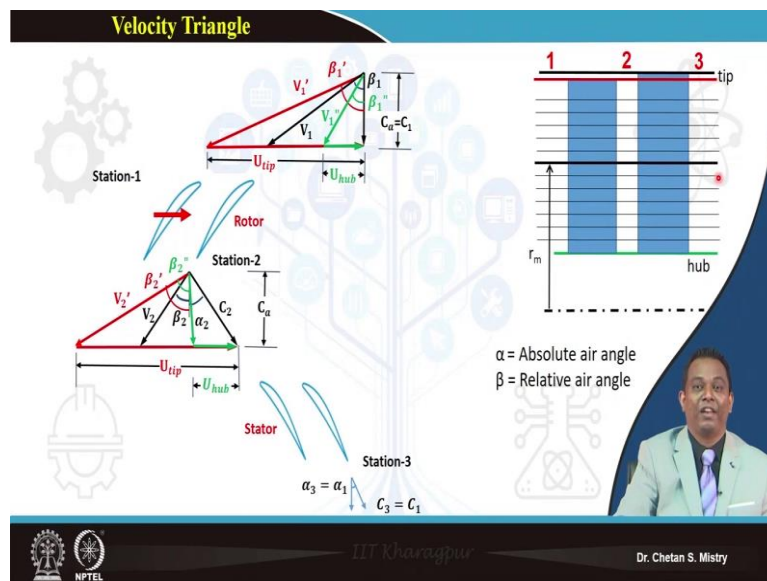
So, you can see if I am using my flow to be entered at say not in axial direction, it will be entering at some angle, my β_1 angle, that's what is going to be reduce. My relative velocity component that's what is going to be reduce, okay. And in order to achieve that part, we need to incorporate a component in this stage, okay.

So, let me introduce that component, this is what is defined as say... inlet guide vane, okay. So, what it will be doing? Here if you look at, my flow that will be entering at some angle α_1 , that's what is contributing some whirl component, okay. So here, I will be having entry swirl component as C_{w1} , okay. So, when I am incorporating this, my flow to be entered in the stage at some angle, basically I am reducing my relative velocity and I am introducing my whirl component, okay.

So, we will be discussing all these in detail very soon. Why we are incorporating inlet guide vane? If you recall, in introduction session I was talking, saying like on both the sides of Pacific people they are having different thought process for incorporating the inlet guide vanes. Some of them, they say it is of no use, some of them, they are preferring to use the inlet guide vane, okay. So, we will be discussing all these in detail.

So, now you can understand this is what is representing my velocity triangle at midsection. Now, what we realize say... my rotor, stator, my inlet guide vane all these components they are made up of number of stations. So, you know, I will be dividing my span - this is what is called span or the height of the blade, I will be dividing into number of stations.

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Now, let us see what is the use of that. Now, here if you look at, this is what is representing my midsection, okay. And, as I told we are making this in number of stages, so let me put this as say... this is what is my tip section, okay. Let me put it here, say... at tip section what is happening? You can see my radius, my radius, that's what is going to increase when I am moving towards the tip. What is the meaning of that? We know our peripheral speed we are writing as $\frac{\pi DN}{60}$. Now, that D is nothing but the diameter, so up till now what we were discussing, that's what was only up to mid-section.

Now, when I am going for the tip, I know my tip diameter that is what is going to increase. If that's what is your case, you can see I am having my peripheral speed for this rotor, that's what is to be large. If I consider my flow is entering axially only, so this is what is my axial component. If we connect this, that's what is representing my relative velocity component. And, you can see my angle, okay. Here, what is my angle at the entry of my rotor that's what is coming to be larger compared to what we have seen at the midsection, okay.

Now, let me take, say... at the exit of rotor what is happening? So, here in this case, this is what is say my tip section and this is what is representing my peripheral speed, okay. And, if I will be connecting that, you will see, my relative velocity that's what is coming out from my rotor that also will be larger near the tip region, okay.

Now, this angle β_2' also if you are calculating that number is coming large, okay. So, what it says, you know, like my work done we have discussed that's what is mainly been depend on

my peripheral speed; when I say peripheral speed, that's what is changing, basically that's what is changing my $\Delta\beta$ that is nothing but my flow deflection angle.

So, near the tip region you will be having your $\Delta\beta$ to be coming to be large, okay. Now, suppose if I consider the next station, suppose if I am considering this is what is say my hub; what we know, my hub diameter that's what is coming to be lower. Let me put in this case, say... this is what is representing my hub peripheral speed, okay.

Now, if I will be connecting, surprisingly you will see my relative velocity component that's what is coming to be lower, okay; and if I look at my angle, my angle also is coming to be lower, okay. In line to that what will happen at the exit? Let me put this my U_{hub} that's what I am representing by the small component. It is $\left(\frac{\pi D_{hub} N}{60}\right)$, and this is what is representing my blade angle.

Now, this is what is making your fundamental understanding for blade. Now, people used to say or your... the blades what you have observed for the compressor, these blades are twisted blade, okay. So, it is not that people they are making that blade twist, okay. This is what is geometry that's what is insisting you to make that blade to be twisted blade. Why it is twisted? You can see, my angle, that's what is varying from β_1'' (near hub) at the entry to β_1' at the tip. Same way at the exit, I will be having my variation of angle β_2'' , to β_2' at the tip region, okay. Now, you can see here, what we have done? We have divided whole our airfoil in number of stations.

Now, when I say number of stations, I will be having radius at each station, okay. Now at the each station, I will be having my peripheral speed. If I am considering my axial velocity to be constant, you can make number of velocity triangles, you can make calculation of your variation of β_1 , you can make variation of your - variation of say angle β_2 , okay. So, this is what is representing how my angles that's what is varying from hub to tip, okay. Similar thing, that is what you will be getting at stator also, okay. So, this is what is very fundamental, and that's what is required by you people in order to move forward, okay. And, I am sure you will get in detail what all that's what is happening here.

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Using Euler's equation work done is given by (for $C_a = \text{const}$)

Specific workdone $W = U(C_{w2} - C_{w1})$
 $W = U C_a (\tan \alpha_2 - \tan \alpha_1)$

From geometry,

$\frac{U_1}{C_{a1}} = \tan \alpha_1 + \tan \beta_1$
 $\frac{U_2}{C_{a2}} = \tan \alpha_2 + \tan \beta_2$

Idea about
 Low speed and/or low PR
 $r_1 = r_2$
 High speed and/or high PR
 configuration $r_1 \neq r_2$

If there is no axial acceleration / deceleration,
 $C_{a1} = C_{a2}$ So across radius
 $U_1 / C_{a1} = U_2 / C_{a2}$

Aerodynamic work
 $W = U C_a (\tan \beta_1 - \tan \beta_2) \text{ kJ / kg of air}$

Thermodynamic work
 $W = \dot{m} C_p (T_{02} - T_{01}) \text{ kW}$

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Now, let me try to explain what all we have learned from your basic Euler's equation. When I say my Euler's equation; specific work done, that is what we are representing as say $U(C_{w2} - C_{w1})$. Now, if I am using this velocity triangle, then you can say my C_{w2} , that's what is I am representing as say $C_a \tan(\alpha_2)$ and C_{w1} that's what is say $C_a \tan(\alpha_1)$, okay.

Now, this is what is representing my specific work done in terms of α_1 and α_2 . If you use your pen and paper and if you try to derive the relation, you will get this equation $\frac{U_1}{C_{a1}}$, that's what is equal to $\tan \alpha_1 + \tan \beta_1$; $\frac{U_2}{C_{a2}}$, that is what is $\tan \alpha_2 + \tan \beta_2$, okay. Now, here in this case intentionally I have written $\frac{U_1}{C_{a1}}$ and $\frac{U_2}{C_{a2}}$, okay.

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Courtesy Pratt & Whitney

Dr. Chetan S. Mistry

Using Euler's equation work done is given by (for $C_a = \text{const}$)

Specific workdone $W = U(C_{w2} - C_{w1})$
 $W = UC_a(\tan \alpha_2 - \tan \alpha_1)$

From geometry,

$$\frac{U_1}{C_{a1}} = \tan \alpha_1 + \tan \beta_1$$

$$\frac{U_2}{C_{a2}} = \tan \alpha_2 + \tan \beta_2$$

Idea about
Low speed and/or low PR
 $r_1 = r_2$
High speed and/or high PR
configuration $r_1 \neq r_2$

If there is no axial acceleration / deceleration,
 $C_{a1} = C_{a2}$ So across radius
 $U_1/C_{a1} = U_2/C_{a2}$

Aerodynamic work
 $W = UC_a(\tan \beta_1 - \tan \beta_2) \text{ kJ / kg of air}$

Thermodynamic work
 $W = \dot{m} C_p (T_{02} - T_{01}) \text{ kW}$

Dr. Chetan S. Mistry

Suppose, if I consider here if you look at, this is what is representing one of my stage, okay. For the stage my area - entry area and exit area, that's what is coming to be same, okay. Both the area that's what is coming to be same. So, at any station, my radius will remain constant, okay. So, that is the reason why you can write down $U_1 = U_2$, but here if you look at, this is what is representing my fan; and for fan if you look at, you know, my entry area and exit area, that's what is coming to be different. Why?

Because I will be having the rise of pressure. When I say rise of pressure, at the exit you will be having rise in density and according to our continuity, my continuity at entry and exit must be satisfied. So, in order to accommodate the rise of your density, my area that's what will be coming to be lower, okay.

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Courtesy Pratt & Whitney

Dr. Chetan S. Mistry

Using Euler's equation work done is given by (for $C_a = \text{const}$)

Specific workdone $W = U(C_{w2} - C_{w1})$
 $W = UC_a(\tan \alpha_2 - \tan \alpha_1)$

From geometry,

$$\frac{U_1}{C_{a1}} = \tan \alpha_1 + \tan \beta_1$$

$$\frac{U_2}{C_{a2}} = \tan \alpha_2 + \tan \beta_2$$

Idea about
 Low speed and/or low PR $r_1 = r_2$
 High speed and/or high PR configuration $r_1 \neq r_2$

If there is no axial acceleration / deceleration,
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 $U_1/C_{a1} = U_2/C_{a2}$

Aerodynamic work

$$W = UC_a(\tan \beta_1 - \tan \beta_2) \text{ kJ / kg of air}$$

Thermodynamic work

$$W = \dot{m} C_p (T_{02} - T_{01}) \text{ kW}$$

Same way if I consider, this is what is representing say my HP spool or HP compressor. Now, for this HP compressor if you look at, this is what is having systematic shape, which we can say, we can assume that as say constant hub diameter configuration. Again, you are having the change in your radius, okay; and that is the reason why, let me go back.

That is the reason why we are having this to be you know, for high speed or high-pressure ratio configuration, my r_1 and r_2 they will not be same, okay. And, if we are assuming our flow to be axial one, it is not accelerating, or decelerating, we can say my $\frac{U_1}{C_{a1}} = \frac{U_2}{C_{a2}}$. And that is what will be giving me very important equation, that's what is called aerodynamic work.

We can represent that as say, $U C_a(\tan \beta_1 - \tan \beta_2) \text{ kJ / kg}$ of air, okay. I can write down $\frac{W}{\dot{m}}$, that is what will be representing my specific work done; do not get confused with this term, okay. So, you can say my aerodynamic work that's what is a function of my peripheral speed. That's what is a function of my axial speed and my angles β_1 and β_2 .

We will be discussing in detail in next lecture, what all is a use of this equation. And this is what all we have learned from our basic understanding. It says this is what is say my thermodynamic work. That's what is $\dot{m} C_p \Delta T_0$, that's what is representing in sense of kilowatt, okay. So, here we are stopping with. This is what is giving us all idea what all we have learned in sense of our velocity triangle. And what we have learned about Euler's equation and it says application for the calculation of work done. Thank you!