Aerodynamic Design of Axial Flow Compressors and Fans
Professor Chetankumar Surreshbhai Mistry
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur
Lecture - 6
Tutorial

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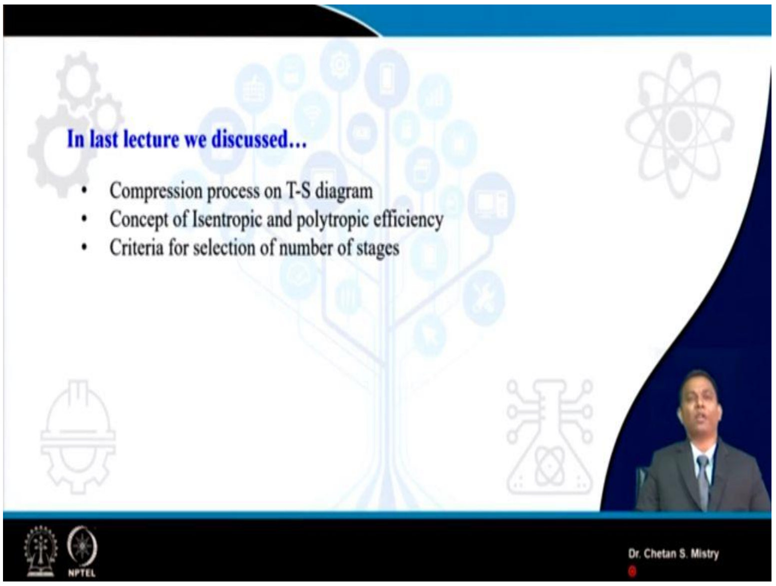

Hello and welcome to tutorial session for module one. This is what we have discussed in our last lecture. We have represented our compression process on T-S diagram. Then we have discussed about different concept of isentropic efficiency and polytropic efficiency. And slowly we have realized this is what is a fundamental that is what will be giving us idea in sense of selection of number of stages. So, let us try to understand that by using a numerical.

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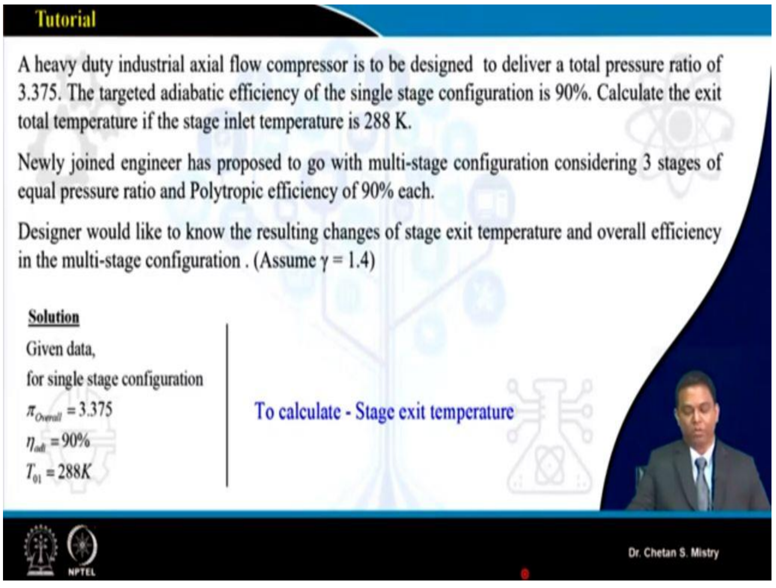

So, take this as a numerical. What it says? A heavy duty industrial axial flow compressor is to be designed to deliver a total pressure ratio of 3.375. The targeted efficiency for single stage configuration is 90%; so, for that particular pressure ratio, your targeted efficiency is 90%. Calculate the total temperature if the stage inlet temperature is 288 K. Now, newly joined engineer has proposed to go with multistage configuration calculation considering three stages of equal pressure ratio and polytropic efficiency of 90%, okay. So, designer would like to know what will be the resulting change of stage exit temperature and overall efficiency.

It says assume your γ =1.4. So, let us try to understand, say… it says we are having already having our axial flow compressor, that's what is generating your pressure ratio of say… 3.375; and it says adiabatic efficiency is roughly 90%. Now, new engineers like you or already experienced engineer like you, he has proposed let us try to do calculation with what number of stages we have made that compressor; and assume this polytropic efficiency to be 90%, okay. So, let us start with the solution. What it says? We are having our given data that's what is for single stage configuration, overall pressure ratio is 3.375.

Our adiabatic efficiency is 90% and total temperature at the entry is 288 K. And we are looking for our stage exit temperature.

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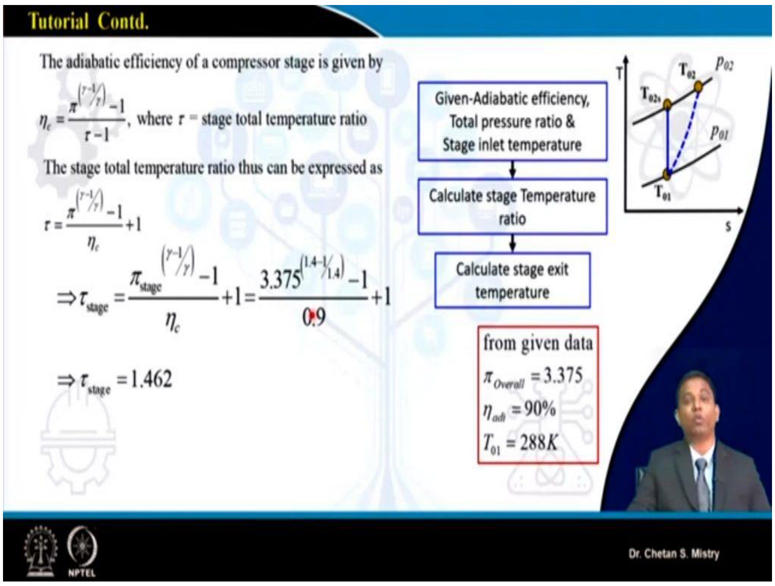

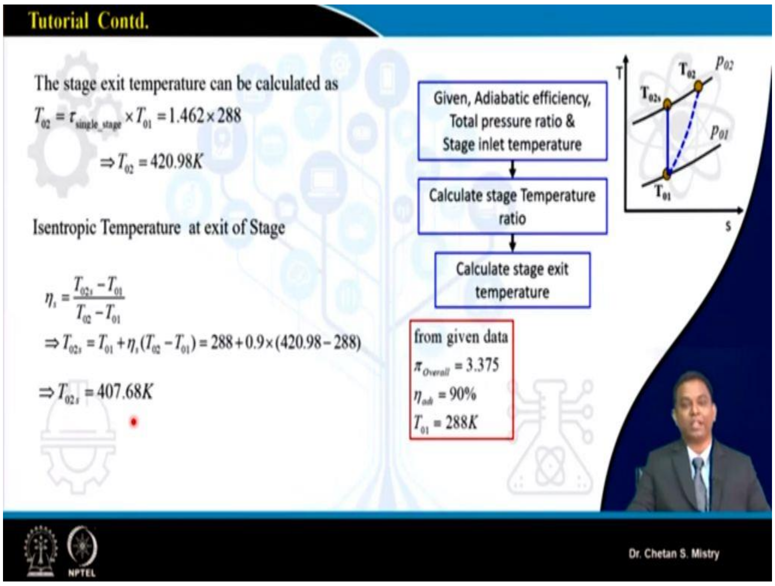

So, let us see. Suppose if I consider this is what is representing my overall compression ratio of 3.375; so that process we are representing on T-S diagram. So, it will be having my entry temperature T_{01} and my exit temperature as say… T_{02}. This (T_{02s}) is what is representing my isentropic exit temperature from the compressor. So, how do we move forward? Let say, what all we know we are having our adiabatic efficiency; for the stage it is known to us. We know what is our total pressure ratio; we know what is our entry temperature; so, based on that we can calculate our temperature ratio.

And from that we can try to calculate what is our exit temperatures. Say, what is my T_{02} and what will be my T_{02s}. So, with this given data, let us move ahead. What we know from our fundamentals? Our adiabatic efficiency for the compressor that's what is represented by my pressure ratio divided by my temperature ratio, okay.

$$\eta_c = \frac{\pi^{\left(\frac{\gamma-1}{\gamma}\right)} - 1}{\tau - 1}$$

Here in this case, we can write down our, say… stage total temperature ratio in sense of my pressure ratio and efficiency. Because what I know? I know what is my, say… pressure ratio; so, let me put these numbers.

So, my pressure ratio we are putting 3.375; we are having our compressor efficiency to be 90%. And if we are putting that, that is what will be giving me my stage temperature ratio as 1.462, okay.

$$\tau = \frac{\pi^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} + 1$$

$$\tau = \frac{3.375^{\frac{1.4-1}{1.4}} - 1}{0.9} + 1$$

$$\tau = 1.462$$

Now, with this temperature ratio, we will try to calculate what will be our exit temperature. So, let me write down here. We can write down; my temperature ratio is given by T_{02}/T_{01}. Since my T_{01} is known to me, so I will be putting that; it is 288; that is what will be giving me what is my actual temperature T_{02} from the exit of my stage. It comes 420.98 K.

$$\frac{T_{02}}{T_{01}} = \tau_{single_{stage}}$$

$$T_{02} = \tau_{single_{stage}} \times T_{01} = 1.462 \times 288$$

$$\Rightarrow T_{02} = 420.98\, K$$

Now, let us see we are also interested in what is our exit temperature, if we are considering our compression process to be isentropic. So, from our fundamental equation I can write down efficiency as T_{02s} minus T_{01} divided by T_{02} minus T_{01}.

$$\eta_s = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

This T_{01}, it is 288 K; that's what is known to us. This T_{02} that is what we have calculated, it is 420.98. If we put these numbers here, it says my isentropic temperature coming from my stage, it is 407.68 K. So, that is how we are calculated if we are considering our overall pressure ratio is of 3.375.

$$\Rightarrow T_{02s} = T_{01} + \eta_s(T_{02} - T_{01}) = 288 + 0.9 \times (420.98 - 288)$$

$$\Rightarrow T_{02s} = 407.68\, K$$

This (T_{02}) will be my exit absolute temperature and this (T_{02s}) is what will be I say… exit isentropic temperature, okay. Now, this is what is a first requirement; so, I can this is what is our solution one.

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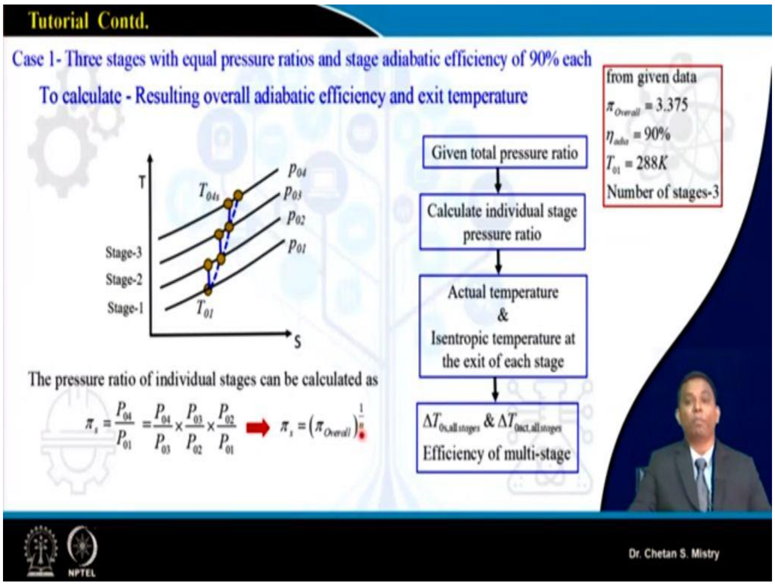

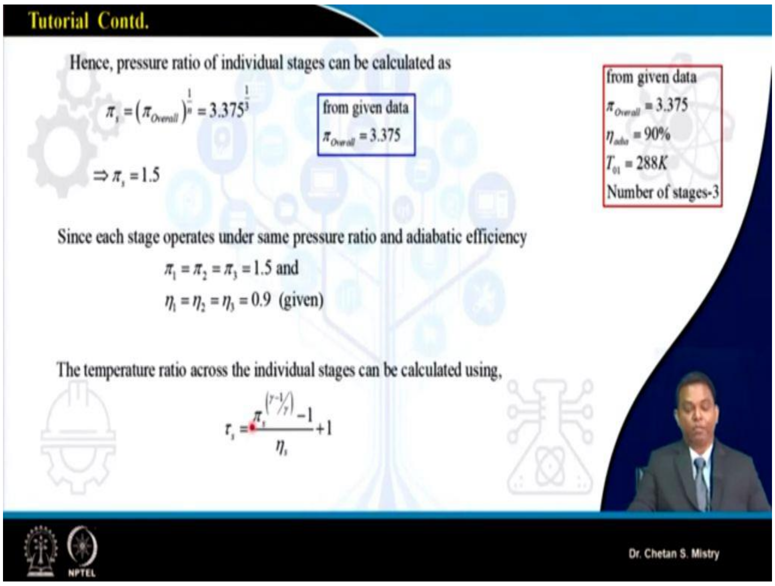

Now, what it says; like as per the proposal, we will be dividing our compression process using three stages. And it says they are having same pressure ratio and same polytropic efficiency; it say 90%. So, this is what is being represented on our T-S diagram; I can say, this is what is my entry temperature, and this will be my exit temperature. So, how do we proceed further? Here we are looking for calculation of our adiabatic efficiency considering multistage configuration. So, what we will be doing? We know our overall pressure ratio.

Then, it says we need to calculate what is our individual pressure ratio; because we are considering three stage configuration. Once we are calculating our individual pressure ratio, we can do our calculation for my actual temperature and we can do our calculation for isentropic temperature. Once this is what is known to you, you know it says, like designer is interested in what is the change in temperature, and what is the change in efficiency. So, we will be comparing what solution what one we have done with; and next what… what we are going now, okay.

So, with this understanding what all we know? Say, for this three number of stages, I can write down my pressure ratio as P_{04}/P_{01}, okay. It says for all the stages my pressure ratio is same; so, this P_{04}/P_{01}, that is what I am representing as

$$\pi_s = \frac{P_{04}}{P_{01}} = \frac{P_{04}}{P_{03}} \times \frac{P_{03}}{P_{02}} \times \frac{P_{02}}{P_{01}}$$

What I have done? I have multiplied and divide with this pressure ratios, or these pressures; P_{03} I am multiplied here and divided here; same way multiplied here and divided here. Remember one thing, this compression ratio is not the addition.

We have common understanding, what it says? We are talking about say… compression ratio of 20; then we are adding these compression ratios; remember that's what is a wrong idea, okay. So, basically my compression ratio that is what has been represented in this form. What it says? This is what I can write down, my overall pressure ratio or my stage pressure ratio, that is what has been represented in sense of say,

$$\pi_s = (\pi_{Overall})^{\frac{1}{n}}$$

Let us put here, what it says? My total pressure ratio that's what is given to me, it is say 3.375. I can write down what I am looking for is say, three number of stages.

$$\pi_s = (\pi_{Overall})^{\frac{1}{n}} = 3.375^{\frac{1}{3}}$$

$$\Rightarrow \pi_s = 1.5$$

Since my pressure ratio is same for all the stages; that is the reason I am writing that as, say… $3.375^{\frac{1}{3}}$. Be careful when we are doing our calculation, what instructions that's what is given, you follow those instructions. When we are designing our axial flow compressor, it may be possible that for few stages, you will be having your pressure ratio to be different. For a few stages, you will be having your polytropic efficiency to be different. We will be discussing how this pressure ratio and how this polytropic efficiencies we are considering for particular stages, okay.

For this numerical we have considered our pressure ratio to be same for all the stages; we have considered our polytropic efficiency to be same. So, that is what it says my pressure ratio for individual stage that's what is coming 1.5. So, now I can write down this is what is my pressure ratio for individual stage; it is coming 1.5. And this is what is my efficiency; that's what is given, it is a 90%.

$$\pi_1 = \pi_2 = \pi_3 = 1.5 \; and$$

$$\eta_1 = \eta_2 = \eta_3 = 0.9 \; (given)$$

Now, with this background, you know, we can use our fundamental understanding of say temperature ratio. This temperature ratio that is what we are representing now in sense of stage, so be careful! Here this is what is written in sense of particular stage, okay.

$$\tau_s = \frac{\pi_s^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s} + 1$$

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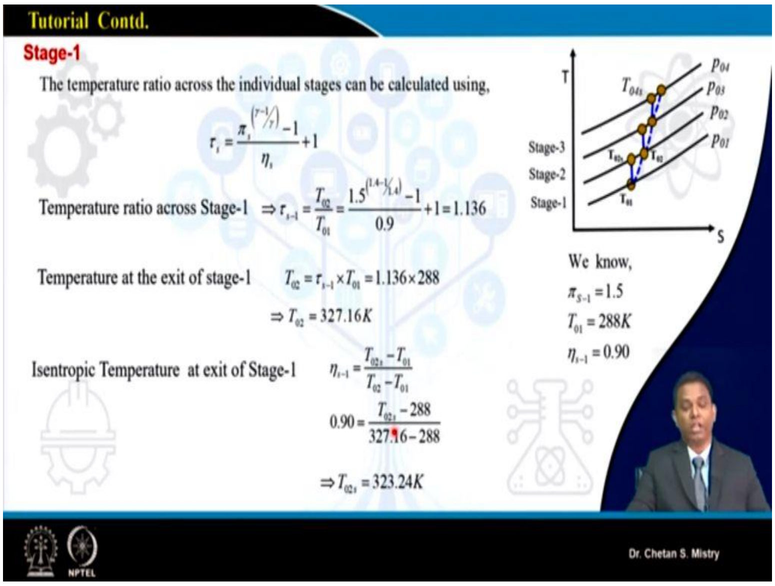

So, if I am considering my Stage-1, so just look at here, this is what is my Stage-1, where I am having my entry temperature as say T_{01}; my exit actual temperature that's what is say T_{02}, and my isentropic temperature at the exit is T_{02s}. So, what we are looking for is we want to calculate what is my T_{02} and what is my T_{02s}, okay? So, based on that I can write down my temperature ratio for Stage-1 is say T_{02}/T_{01}. That's what is 1.5 to the power this ratio; that's what is giving me my temperature ratio as say 1.136, okay.

$$Temperature\ ratio\ across\ Stage - 1,$$

$$\tau_{s-1} = \frac{T_{02}}{T_{01}} = \frac{1.5^{\left(\frac{1.4-1}{1.4}\right)} - 1}{0.9} + 1 = 1.136$$

$$Temperature\ at\ the\ exit\ of\ Stage - 1,$$

$$T_{02} = \tau_{s-1} \times T_{01} = 1.136 \times 288 = 327.16\ K$$

Now, for Stage-1, I know what is my entry temperature. So, based on that I can calculate what is my actual temperature at the exit of my Stage-1, okay. So, if I am putting this number, that's what is giving me my actual temperature from the exit of my Stage-1 as say 327.16 K. Now, in order to calculate what is my isentropic temperature at the exit of my Stage-1, I will be using my fundamental equation. What it says? It is my isentropic enthalpy rise divided by my actual enthalpy rise, and that is what if I will be putting my numbers; so T_{02s}, that's what is known or known to me in this equation. I say, this T_{01} and T_{02} are the known parameters.

$$Isentropic\ Temperature\ at\ exit\ of\ Stage - 1,$$

$$\eta_{s-1} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$$0.9 = \frac{T_{02s} - 288}{327.16 - 288}$$

$$\Rightarrow T_{02s} = 323.24\ K$$

So, if I am putting that; that is what it says my isentropic temperature at the exit of my Stage-1, it is 323.24. Now, you know, like this is what is giving you indication, okay. Just look at these numbers, this is what is coming to be higher, this is what is in line to what we have discussed; my T_{02}, that's what is coming to be higher compared to my T_{02s}.

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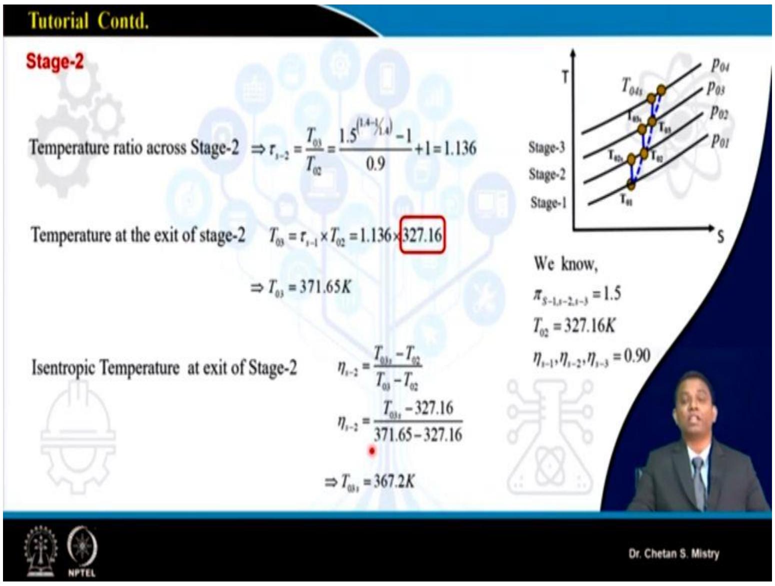

Similar to this, I can go for a calculation for my Stage-2. Now for Stage-2, I can write down my temperature ratio, that temperature ratio is nothing but T_{03} by T_{02}. If I am putting these numbers, because my pressure ratio is known to me and my efficiency, that's what is given to me. I can calculate what is my temperature at the exit of my Stage-2, okay. If I am putting this number from earliest stage, we have done our calculation for T_{02}; that's what is say… 327.16. And if I am putting this number, it says my temperature at the exit of my Stage-2 is 371.65 K.

$$Temperature\ ratio\ across\ Stage-2,$$

$$\tau_{s-2} = \frac{T_{03}}{T_{02}} = \frac{1.5^{\left(\frac{1.4-1}{1.4}\right)} - 1}{0.9} + 1 = 1.136$$

$$Temperature\ at\ the\ exit\ of\ Stage-2,$$

$$T_{03} = \tau_{s-2} \times T_{02} = 1.136 \times 327.16 = 371.65\ K$$

$$Isentropic\ Temperature\ at\ exit\ of\ Stage-2,$$

$$\eta_{s-2} = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}}$$

$$0.9 = \frac{T_{03s} - 327.16}{371.65 - 327.16}$$

$$\Rightarrow T_{02s} = 367.2\ K$$

In line to what we have done calculation for our isentropic temperature at the exit of the stage, here for exit of my Stage-2, I can write down this equation for, say… my polytropic efficiency for particular stage and I am able to calculate here or we are calculating here, this is what is say my T_{03s}; that is nothing but my isentropic temperature at the exit of my Stage-2. Now, with this understanding you can easily calculate what will be my temperature at the exit of my Stage-3, okay.

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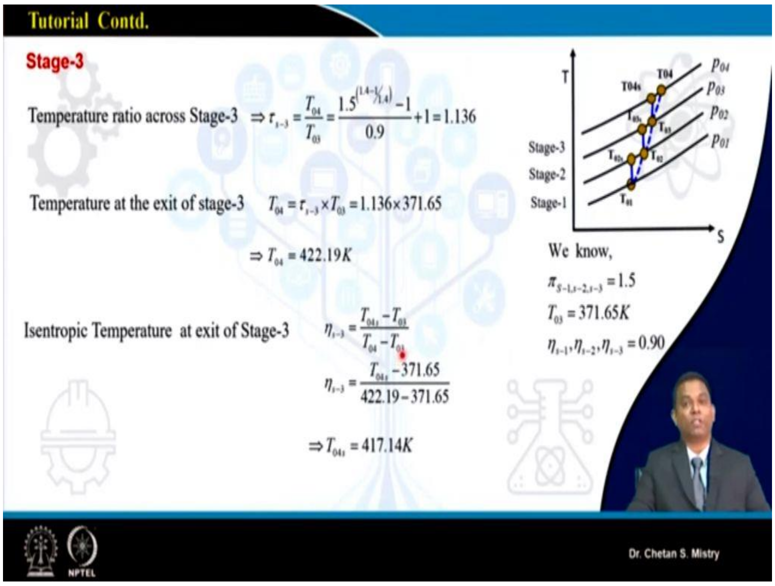

So, here what we are doing? We are having our temperature ratio; that is what we are representing in sense of T_{04}/T_{03}. We know our pressure ratio, we know our efficiency; based on that we can calculate what is our T_{04}. So, this T_{04} is nothing but this is what is my exit temperature from Stage-3; or for this case, this is what is my temperature - total exit temperature from all the stages. Now, in line to what we have discussed, here I can do my calculation, what will be my isentropic temperature at the exit of my Stage-3? So, this temperature that's what is coming T_{04s}; that's what is 417.14 K, okay.

$$Temperature\ ratio\ across\ Stage-3,$$

$$\tau_{s-3} = \frac{T_{04}}{T_{03}} = \frac{1.5^{\left(\frac{1.4-1}{1.4}\right)} - 1}{0.9} + 1 = 1.136$$

$$Temperature\ at\ the\ exit\ of\ Stage-3,$$

$$T_{04} = \tau_{s-3} \times T_{03} = 1.136 \times 371.65 = 422.19\ K$$

$$Isentropic\ Temperature\ at\ exit\ of\ Stage-2,$$

$$\eta_{s-2} = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}}$$

$$0.9 = \frac{T_{03s} - 371.65}{422.19 - 371.65}$$

$$\Rightarrow T_{02s} = 417.14\,K$$

Now, this all temperatures, they are known to us. So, that is what will be helping us in sense of doing our calculation for the efficiency - overall efficiency, when I am going with multistage configuration, okay.

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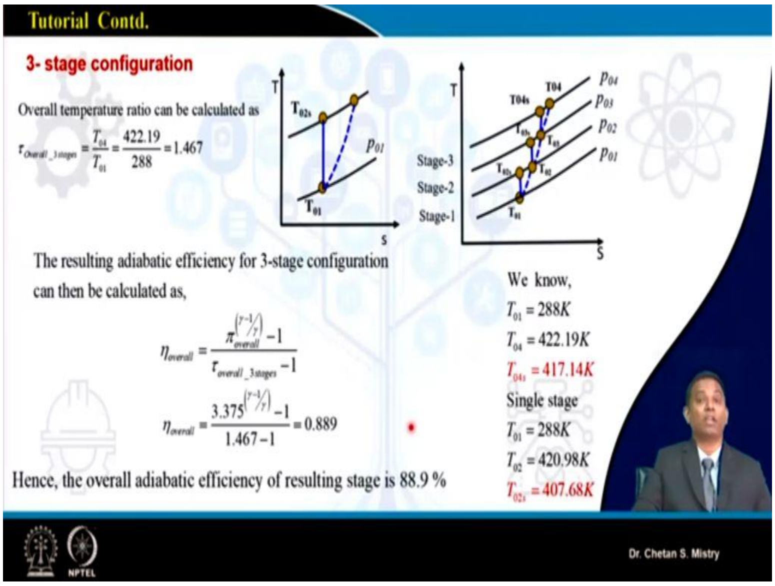

So, here, this is what we are representing in sense of my multi stage configuration. So, for multi stage configuration, I can write down my overall stage temperature ratio as T_{04}/T_{01}, okay. This T_{04}, that's what is my say… actual temperature that's what is coming from my multi stage configuration. This T_{01} that's what is 288 K, it is given, okay. So, this is what will be giving me what is my overall temperature ratio when I am considering a multistage configuration. Now, based on that from our fundamental understanding, we can do our calculation for overall efficiency with multistage configuration.

$$\textit{Overall temperature ratio can be calculated as}$$

$$\tau_{overall\_3stages} = \frac{T_{04}}{T_{01}} = \frac{422.19}{288} = 1.467$$

The resulting adabatic efficiency for 3 stage configuration can then be calculated as,

$$\eta_{overall} = \frac{\pi_{overall}^{\frac{\gamma-1}{\gamma}} - 1}{\tau_{overall\_3stages} - 1}$$

$$\eta_{overall} = \frac{3.375^{\frac{1.4-1}{1.4}} - 1}{1.467 - 1} = 0.889$$

So, I will be putting my overall pressure ratio that's what is known to us, okay. So, let me put that number, it says my overall pressure ratio is 3.375, okay; and my temperature ratio for all the three stages that's what is 1.467. And let me calculate that efficiency; that's what is coming 88.9%. Now, what we were discussing in our class? As I told, like people they are giving efficiency as a number, okay. What they are saying? My compressor is having efficiency of 90%. So, I told, you just check with whether that efficiency what they are talking is per stage efficiency, or say its overall efficiency.

Just look at, though we have considered our per stage efficiency 90%; my overall efficiency is coming at 88.9%. So, do not get confused with this number game.

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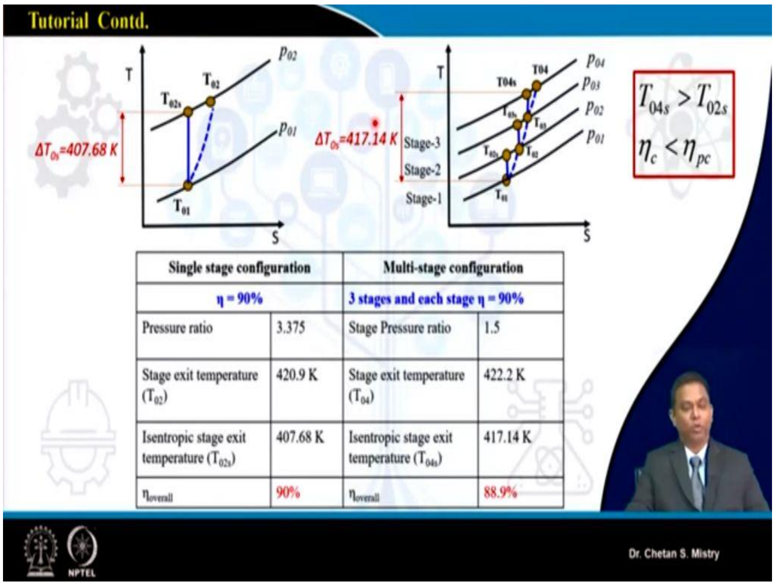

Let us try to understand what exactly is happening and why we are having this difference in number. So, here if you look at, this is what is a calculation for my single stage configuration, where my pressure ratio is 3.375. We are having our exit temperature what we have calculated, it is 420.9 K. My isentropic exit temperature what we have calculated, that's what is 407.68 K; and you know, our overall efficiency, that's what was given - it is 90%, okay. Now, if I do my calculation for my ΔT_{0s} that's what is coming here. This is what is coming in sense of my - what is my temperature.

Here if you look at, when I am going with say… my say… multi-stage configuration; it says my pressure ratio is coming 1.5. Same way if I am calculating my isentropic temperature at the exit of my stage, it is coming 417.14; and my overall efficiency is coming 88.9. So, here if you look at, this ΔT_0 when I am considering single stage, and when I am considering my multi-stage; that's what is giving me high temperature at the exit of my stage, okay. So, with this understanding, I am sure you maybe able to solve the further numericals or you maybe able to understand what all can be done if you are doing your design; because this is what is initial starting of your design concept.

Because initially only we will be starting with our assumption of polytropic efficiency. We are interested in overall efficiency, but when we are doing our design, we are designing individual stage. That means we are more interested, what we say, in sense of our polytropic efficiency.

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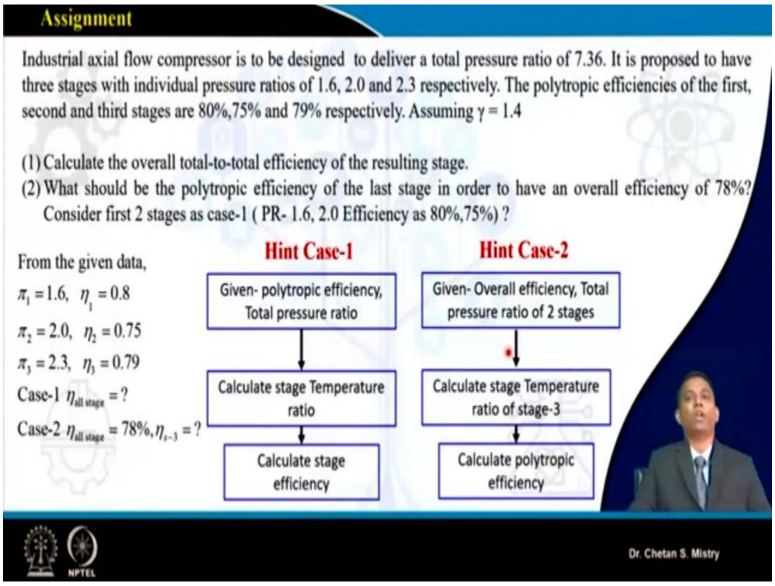

Now, in order to build your confidence, let us see, you do one calculation at your own in homework; this is what is your assignment problem. What it says, like industrial axial flow compressor is to be designed to deliver your total pressure ratio of 7.36. It is proposed to have three stage with individual pressure ratio of 1.6, 2 and 2.3. Just look at here, my individual pressure ratio is not same; they are given different, do not get confused with this, okay. You have learnt these fundamentals in the class; so, you will be able to understand if I am having say… different pressure ratio what needs to be done.

The polytropic efficiency for first, second and third stage that's what is say… 80%, 75% and 79%. Assume your γ to be 1.4. Calculate your overall total-to-total efficiency for resulting stage. What should be the polytropic efficiency of your last stage in order to achieve your overall efficiency of 78%, okay? Consider first two stages, say… the pressure ratio of 1.6 and

2; and their efficiency as say… 80% and 75%, okay. So, I am sure you are able to solve this numerical, okay. What it says? You are given data with pressure ratio of 1.6 and efficiency 80%.

For second stage, it is having pressure ratio of 2 and efficiency as 75%. For Stage-3, it is having pressure ratio of 2.3 and efficiency as 0.79, okay. You are looking for overall efficiency and for stage or say, for second case, you are looking for your overall efficiency to be 78%. What need to be the efficiency of my third stage, okay? So, this is what I am sure you are able to do this part. Let me try to give you a hint; that is what will be helping you in solving this numerical, okay. Let me give you the hint for case-1.

$$Given\ data,$$

$$\pi_1 = 1.6, \qquad \eta_1 = 0.8$$

$$\pi_2 = 2.0, \qquad \eta_2 = 0.75$$

$$\pi_3 = 2.3, \qquad \eta_1 = 0.79$$

$$Case - 1\ \eta_{all_{stage}} = ?$$

$$Case - 2\ \eta_{all_{stage}} = 78\%, \qquad \eta_{s-3} = ?$$

What it says? It is given with the polytropic efficiency and total pressure ratio for all the stages. So, you are able to calculate what will be your temperature rise or temperature at the exit of all the stages, okay. And that is what will be helping you in sense of calculating your overall stage efficiency; this is what is doable, okay. Now, in order to solve your second case what it says? You are having your overall efficiency that’s what is given to you. And total pressure ratio for two stages that’s what is given to you. So, you know at the entry of your third stage, you know what is your temperature and what is your pressure, okay.

So, based on that, for particular efficiency requirements, you can have your stage temperature ratio for stage-3 you can calculate; and based on your understanding, you can put your polytropic relation, and you can calculate your polytropic efficiency. So, this is what is the end of our first module. That’s what is say, introduction. Now, we will be moving ahead with our next stage from the next lecture. Thank you. Thank you very much!