Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 59
Design of Transonic Compressor (Contd.)

Hello, and welcome to lecture 59. We are discussing about design of transonic compressor.

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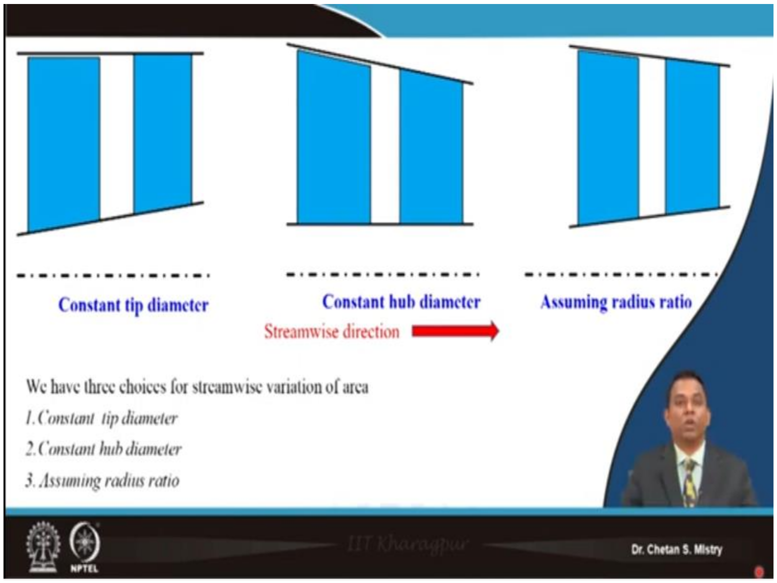

So, in last few lectures we were discussing about design of transonic compressor with different configuration in sense of flow tracks. So, we were discussing as transonic compressors are mainly high pressure compressors and per stage pressure ratio, that's what is very high. And, that is the reason why the flow, that's what will be coming out at the exit that will be having higher pressure that means its density also will be higher and in order to satisfy the continuity equation, assuming our axial velocity to be constant my exit area will be coming to be lower.

And, for that we have discussed about three different configurations; one, we have discussed as say constant tip diameter, constant hub diameter, and third approach we have discussed about assuming some radius ratio. Now, what all design methodology we have adopted for transonic compressor with two configurations; constant hub diameter and constant tip diameter, where we have discussed about the fundamental design approach.

At the same time, we have discussed like as per the requirement when people they are doing their transonic fan design, that's what is having somewhat different approach, and that's what is the most recent approach. So, we should explore that possibility in this subject, specially with the interest. So, let us move towards that part.

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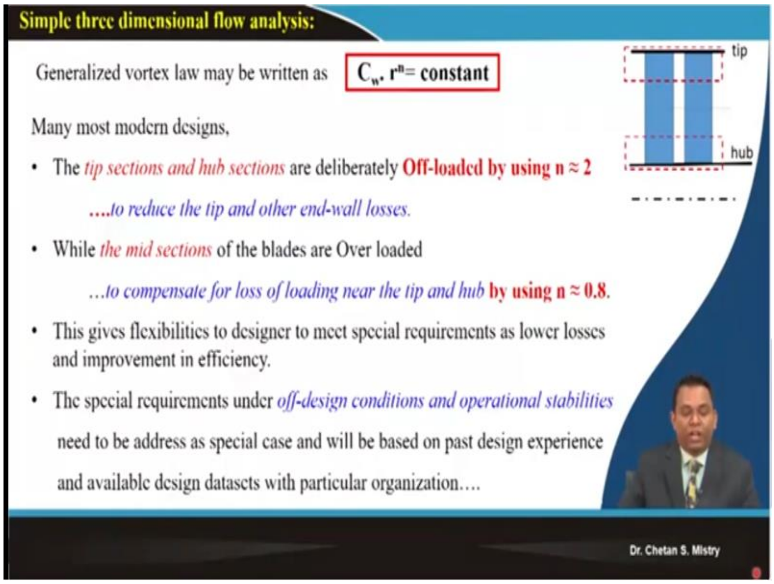

So, what we have realized, if we consider we are having say vortex law, that's what it $C_w \cdot r^n = constant$. So, for most modern design, people, they are changing their load distribution at the mid span, they are changing the load distribution at the tip and near the hub region.

So, what exactly they are doing, say near tip and hub, the design it is say off-loaded design where these exponents, that's what has been taken as $n = 2$. We can say, this is what is helping in order to reduce the endwall losses, mainly, say near the tip region, that's what is reducing your tip clearance losses, at the same time at the hub or near the hub, that's what will be reducing hub corner losses. Now, at the mid span, in order to compensate what load we will be losing near hub and tip, that's what will be getting compensated at the mid span region where this exponent, that's what has been selected as $n = 0.8$.

Now this, this is what is giving the flexibility in order to reduce the losses and that's what will be improving the efficiency. One other criteria, that's what is a wider operating range and in order to operate our compressor or in order to have wider operating range under off design condition, some design strategies need to be modified. And, that's what is depending on say kind of experience the designer has and what all are the strategies they are been decided by engine manufacturing company. So, for particular organizations, they are having certain rules and they are following certain laws in order to design those transonic compressors and fans.

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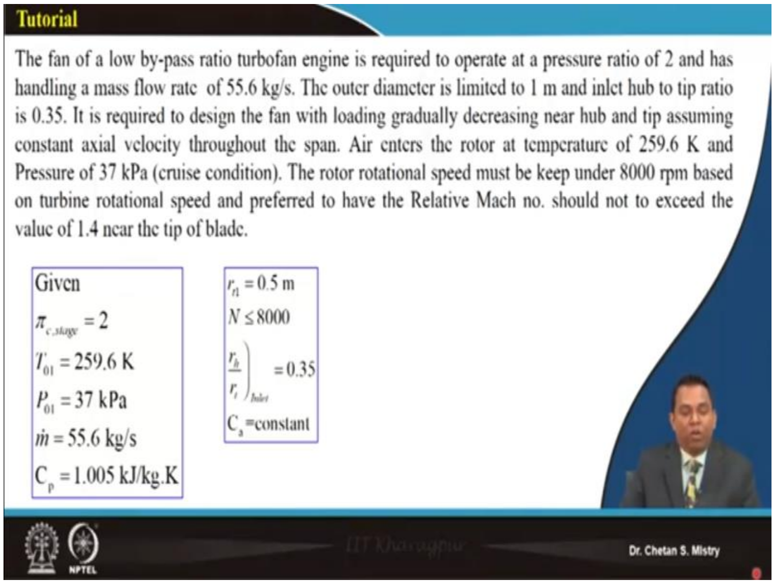

So, what we have learned from that let us try to take one of the numerical. What it says? The fan of low bypass ratio turbofan engine is required to operate at the pressure ratio of 2 and has mass handling capacity that's what is 55.6 kg/s. The outer diameter is limited to 1 m and inlet hub to tip ratio is 0.35. So, we say, these are the constraints; say we should not exceed our casing diameter beyond 1, and at the entry we are having hub to tip ratio to be 0.35.

It is required to design the fan with loading gradually decreased near the hub and tip region assuming constant axial velocity throughout the span. Air enters the rotor at a temperature of 259.6 K and pressure of 37 kPa. This is what is we can say low bypass ratio engine, that's what is mainly been used for say fighter aircrafts, where we are doing design. So, many times the designer, they are preferred to go with the cruise condition.

So, this is what is a cruise condition we can say. The rotor rotational speed must be kept under 8000 rpm based on the turbine rotational speed and preferred to have Mach number should not exceed by 1.4 near the tip region of the blade. So, in overall if you look at, what we are planning to do is a design of say axial flow fan for low bypass ratio engine, that's what is having say compression ratio in the range of 2, that's what is slightly on the higher side. We can say entry pressure and temperature they have been defined under the cruise condition, mass flow rate is given that's what is 55.6 kg/s.

And, what it says? My casing diameter should not exceed by 1 m, rotational speed also is constrained by 8000 rpm. And, it says my axial velocity that need to be constant. So, we must realize, actually by changing our whirl distribution or load distribution, we are violating the law of radial equilibrium. So, as we have discussed earlier also for many times, especially for axial flow fan design, this violation that's what has been valid and people, they have done design and those designs, those fans are working fine, absolutely fine as on today also. And that's what we are opting here.

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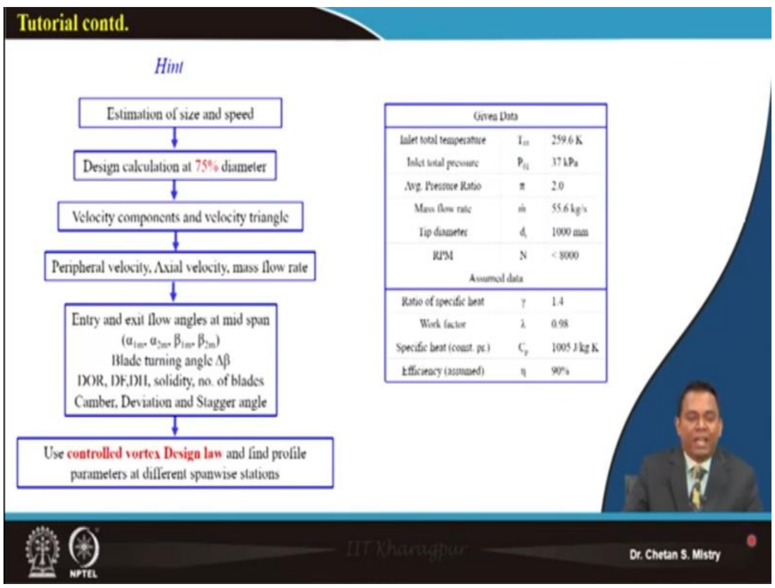

So, let us try to have solution for this. So, what all we know is my size and speed, that's what we need to estimate, okay. We need to do our design at 75% of span because this compressor we can say that's what is a transonic compressor or we can say this is what is transonic fan. Now, we are looking for different velocity components and velocity triangle in order to determine the components like axial velocity, peripheral speed, relative velocity, absolute velocity, different flow angles, all those things, those need to be calculated at different stations.

So, initially we will be doing our calculation at the mid station and then after we will be applying our law. Here in this case, we are not opting for say straight free vortex design or straight force vortex design or maybe fundamental design. So, that is why it is named as say controlled vortex design. So, let us try to understand what exactly is the meaning of that?

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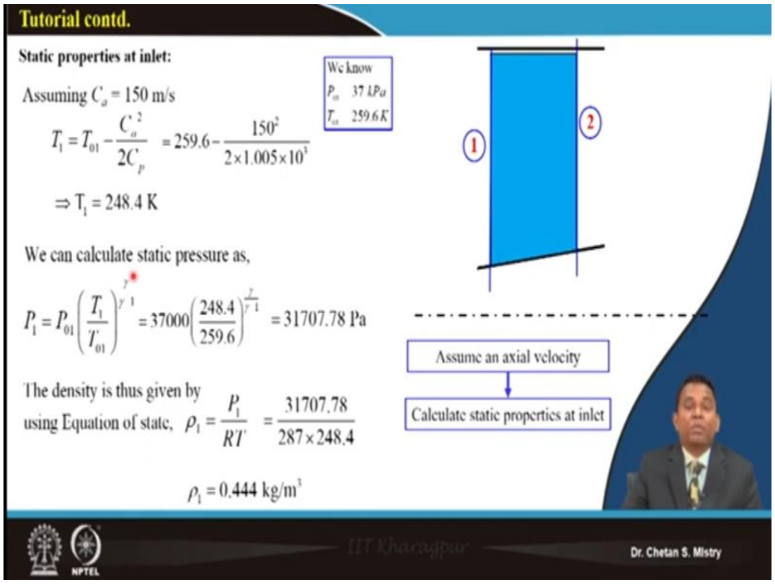

Now, as we know at the entry condition, we need to have our entry dimensions. Now, in order to have those dimensions we must know what will be our entry density. At the same time, we are looking for say axial velocity as well as peripherals speed also need to be known, in order to complete our velocity triangle.

So, what we will be doing? Let us say for this case, we are assuming our axial velocity, suppose say 150 m/s. And, nothing it is mentioned, so, we will straightway say at that particular station or…say…we can say our entry that's what is say axial entry that means my C_a and C_1, they both are same. If you are putting that, that's what is giving me my entry temperature as say 248.4 K.

$$Static\ properties\ at\ inlet,$$

$$Assuming\ C_a = 150\ m/s$$

$$T_1 = T_0 - \frac{C_a^2}{2C_p} = 259.6 - \frac{150^2}{2 \times 1.005 \times 10^3}$$

$$\Rightarrow T_1 = 248.4\ K$$

Now, we know, what is our entry pressure? What is our entry temperature? So, based on that we can calculate our static entry pressure and that's what is coming, say it is 31.70 kPa.

$$We\ can\ calculate\ static\ pressure\ as,$$

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 37000\left(\frac{248.4}{259.6}\right)^{\frac{1.4}{1.4-1}} = 31707.78\ Pa$$

Now, these things that's what is already been discussed in so many cases, that is the reason we will not be taking much time in explanation. Here, we can say, we can calculate our density based on our static pressure and static temperature, that's what is coming to be 0.44 kg/m^3.

$$The\ density\ is\ thus\ given\ by\ using\ Equation\ of\ state,$$

$$\rho_1 = \frac{P_1}{RT} = \frac{31707.78}{287 \times 248.4}$$

$$\rho_1 = 0.444\ kg/m^3$$

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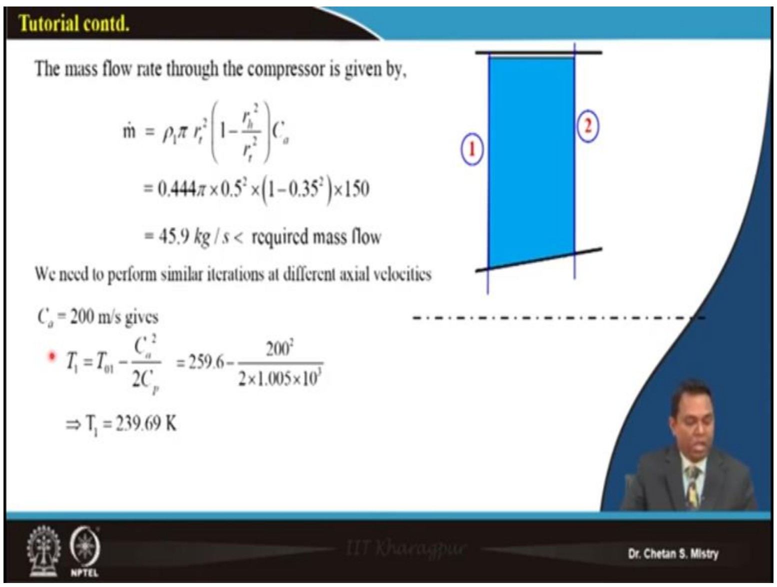

Now, once this is what is known to us, we know our continuity, we can write down that is nothing but it is in sense of say my $density \times area \times axial\ velocity$. Here, in this case at the entry, my hub to tip ratio that's what is given, it is 0.35. So, based on that if you are assuming our tip dimension to be 0.5, it says my mass flow rate, that's what is coming 45.9 kg/s.

$$The\ mass\ flow\ rate\ through\ the\ compressor\ is\ given\ by,$$

$$\dot{m} = \rho_1 \pi r_t^2 \left(1 - \frac{r_h^2}{r_t^2}\right) C_a$$

$$= 0.444\pi \times 0.5^2(1 - 0.35^2) \times 150$$

$$= 45.9\ {}^{kg}/_{s} < required\ mass\ flow$$

Now, this is what is less than what mass flow rate we are expecting. So, we need to realize if my entry mass flow rate, that's what is low, that's what will be reflecting in sense of my thrust generation capacity for particular engine.

And, that is the reason why we can say, this is not acceptable mass flow rate at this moment. That means, what axial velocity we have assume that's what is say on a lower side. So, let us try to take our axial velocity to be say 200 m/s. If we are taking that to be 200 m/s, we will be having our temperature…static temperature, it is 239.69 K.

$$We\ need\ to\ perform\ similar\ iterations\ at\ different\ axial\ velocities$$

$$Assuming\ C_a = 150\ m/s$$

$$T_1 = T_0 - \frac{C_a^2}{2C_p} = 259.6 - \frac{200^2}{2 \times 1.005 \times 10^3}$$

$$\Rightarrow T_1 = 239.69\ K$$

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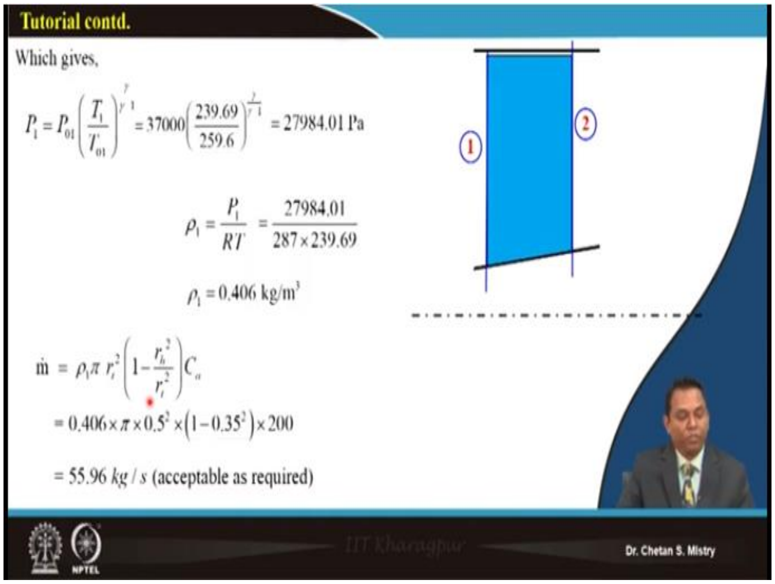

We can determine our pressure…static pressure, that's what is coming say 27.98 kPa, and we can calculate our density. This density is coming 0.406 kg/m^3.

$$Which\ gives,$$

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 37000\left(\frac{239.69}{259.6}\right)^{\frac{1.4}{1.4-1}} = 27984.01\ Pa$$

$$\rho_1 = \frac{P_1}{RT} = \frac{27984.01}{287\times 239.69}$$

$$\rho_1 = 0.406\ kg/m^3$$

Now, if you will be putting this in our mass flow rate equation, that says this is what is coming as say 55.96 kg/s. So, we can say this is what is in acceptable range.

$$\dot{m} = \rho_1 \pi r_t^2\left(1-\frac{r_h^2}{r_t^2}\right)C_a$$

$$= 0.406\pi\times 0.5^2(1-0.35^2)\times 200$$

$$= 55.96\ {}^{kg}/_{s}\ (acceptable\ as\ required)$$

Now, this is what is very important when we are doing our assumption for axial velocity. We have discussed in earlier numerical also, sometimes we need to assume with the axial velocity, many times axial velocity may be known to you, maybe flow coefficient is known to you, all those design strategies we have already discussed.

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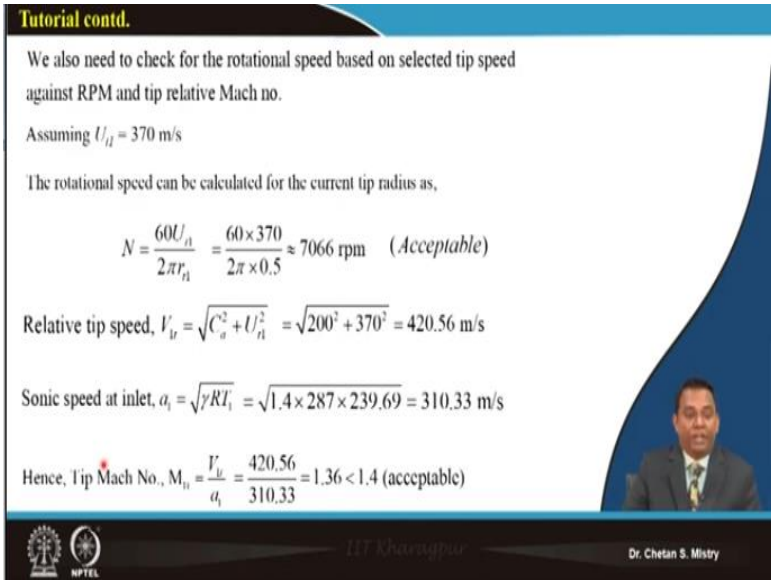

Now, based on that, we need to define with what need to be my peripheral speed, okay. So, in order to have that peripheral speed to be calculated, let us assume say my tip peripheral speed as say 370 m/s. If we are taking that my rotational speed is coming 7066 rpm. You can say that's what is say near to my 8000 rpm.

$$Assuming\ U_{t1} = 370\frac{m}{s}$$

$$The\ rotational\ speed\ can\ be\ calculated\ for\ the\ current\ tip\ radius\ as,$$

$$N = \frac{60U_{t1}}{2\pi r_{t1}} = \frac{60\times370}{2\pi\times0.5} \approx 7066\ rpm\ (Acceptable)$$

So, at this moment, let us…let us put that as a number. We need to check with our relative Mach number at the tip. And, in order to do that calculation, we need to calculate our relative velocity. If you are calculating our relative velocity, that's what is coming 420.56 m/s. Our sonic speed that is 310.33 m/s.

$$Relative\ tip\ speed, V_{1t} = \sqrt{C_a^2 + U_{t1}^2} = \sqrt{200^2 + 370^2} = 420.56\frac{m}{s}$$

$$Sonic\ speed\ at\ inlet, a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 239.69} = 310.33\ m/s$$

$$Hence, Tip\ Mach\ number, M_{1t} = \frac{V_{1t}}{a_1} = \frac{420.56}{310.33} = 1.36 < 1.4\ (acceptable)$$

So, we can say, the tip Mach number, that's what is coming 1.36. We can say, that's what is less than 1.4. So, this number that's what is acceptable. So, we can say, now we are fixing with our axial velocity, we are fixing with our peripheral speed. So, you know, like many times, based on certain assumptions, you can go forward with. It says, it should not exceed by 8000 rpm. You have that flexibility, you can assume the speed to be say 7500 rpm, and you can do calculation, there is nothing wrong in that. So, this is what is based on few of our iteration and that is the reason why we are straight way putting our peripheral speed as say 370 m/s.

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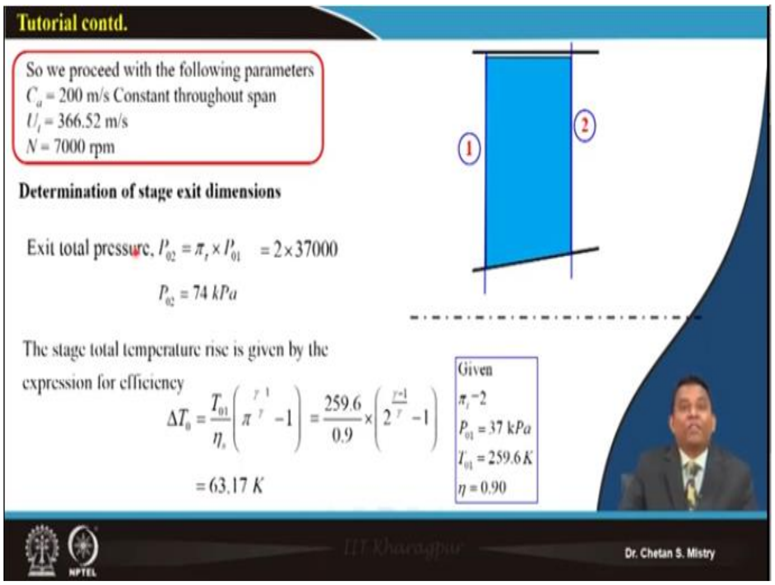

So, based on that, we can say, we are finalizing our axial velocity, that's what is 200 m/s and it is given that's what is constant throughout my span. And, our peripheral speed we are considering 366.52 m/s, rotational speed we are considering 7000 rpm. If this is what is known to us, so, at the entry condition, we can say, we are having our parameters known for making of a velocity triangle. Now, this is what is a fan that's what need to be designed, so, if we consider, this is what is my entry condition, we can say that as a station 1 and this is what is my exit condition we can say that as say station 2.

Now, here in this case, in order to calculate my exit dimensions, we know what is our pressure ratio. So, our pressure ratio given, that's what is say 2 and based on that we can calculate what will be our total pressure at the exit. Now, this is what we will be using in order to calculate our ΔT_0. It is nothing but my total temperature rise across my rotor and that's what is coming 63.17 K.

$$Determination\ of\ stage\ exit\ dimensions$$

$$Exit\ total\ pressure, P_{02} = \pi_t \times P_{01} = 2 \times 37000$$

$$P_{02} = 74\ kPa$$

$$The\ stage\ total\ temperature\ rise\ is\ given\ by\ the\ expression\ for\ efficiency$$

$$\Delta T_0 = \frac{T_{01}}{\eta_s}\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right) = \frac{259.6}{0.9} \times \left(2^{\frac{1.4-1}{1.4}} - 1\right)$$

$$= 63.17\ K$$

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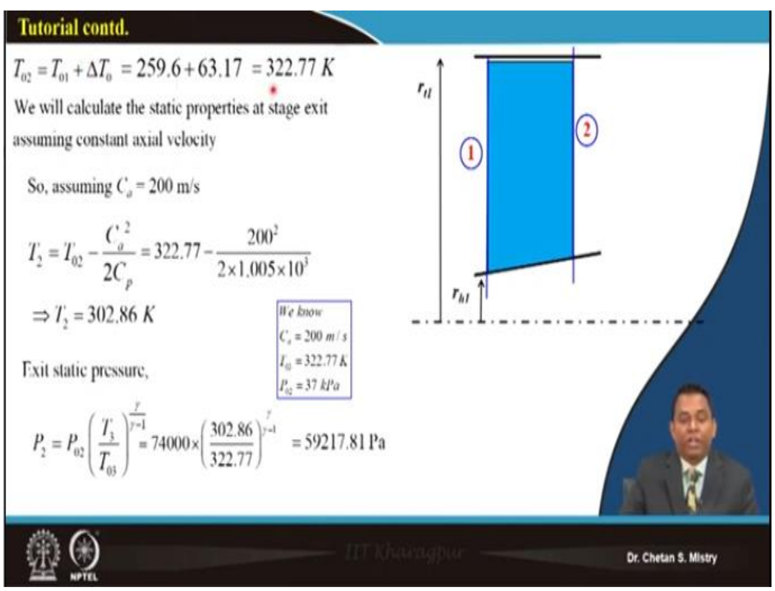

Now, from the known parameter, if this is what is known to us, we can calculate what will be our total temperature at the exit. Since my T_{01} it is known to us, ΔT_0, that is also known, so, we can say, T_{02}, that's what is coming 322 .77 K.

$$T_{02} = T_{01} + \Delta T_0 = 259.6 + 63.17 = 322.77\ K$$

Now, what it says? Like, we have considered or it is known say axial velocity, that's what is say 200 m/s. So, we will be putting and taking that as same. We are considering say assuming say exit to be axial in for the sake of simplicity, we can say. So, for that, we will be calculating our exit temperature and that's what is coming 302.86 K. Now, once this is what is known to us, we can calculate what will be our static pressure at the exit, and that's what is coming 59.21 kPa.

$$So, assuming\ C_a = 200\ {}^{m}/_{s}$$

$$T_2 = T_{02} - \frac{C_a^2}{2C_p} = 322.77 - \frac{200^2}{2 \times 1.005 \times 10^3}$$

$$\Rightarrow T_2 = 302.86\ K$$

$Exit\ static\ pressure,$

$$P_2 = P_{02}\left(\frac{T_3}{T_{03}}\right)^{\frac{\gamma}{\gamma-1}} = 74000 \times \left(\frac{302.86}{322.77}\right)^{\frac{1.4}{1.4-1}} = 59217.81\ Pa$$

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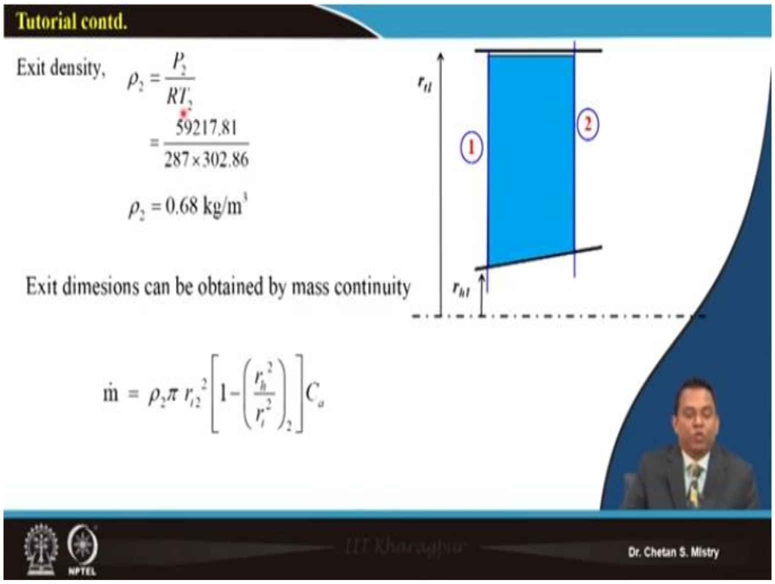

Now, once the static pressure and static temperature, they are known to us, we can calculate our static density and that's what is say P_2/RT_2 that is coming as say 0.68 kg/m^3.

$$Exit\ density,$$

$$\rho_2 = \frac{P_2}{RT_2}$$

$$= \frac{59217.81}{287 \times 302.86}$$

$$\rho_2 = 0.68\ {}^{kg}/_{m^3}$$

Now, in order to calculate our exit dimension, we will be taking help of our continuity equation. So, from this continuity equation at the exit system, if we are writing that's what is nothing $\rho_2 \times A_2 \times axial\ velocity$. So, this A_2 that's what we are representing in sense of hub to tip ratio at the outlet.

$$Exit\ dimensions\ can\ be\ obtained\ by\ mass\ continuity$$

$$\dot{m} = \rho_2 \pi r_{t2}^2 \left[1 - \left(\frac{r_h^2}{r_t^2}\right)_2\right] C_a$$

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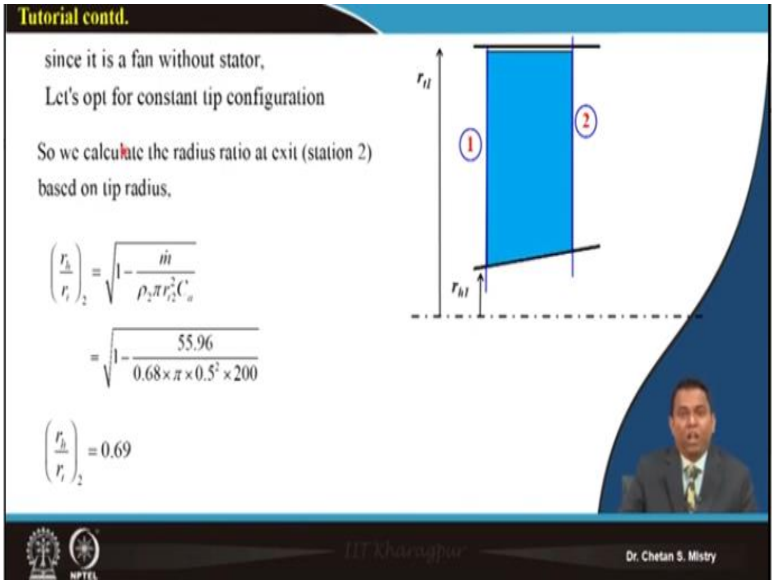

So, if we are considering this as a case, we will be first calculating what will be my hub to tip ratio at the outlet. Now, this is what is a fan, that's what we are designing to be on say safe side or we can say, we can straightway assume constant tip kind of configuration. There is nothing wrong, you can move with even constant hub also, even you can go with say constant mean diameter configuration also. But, we can understand at the entry condition, it is most preferred to go with say constant tip diameter.

Here, the expected pressure ratio is in the range of 2 that is also giving one of the indication for selection of constant tip diameter configuration. So, if we are considering constant tip diameter configuration, we will be getting our hub to tip ratio at the outlet. And that's what is coming as say 0.69.

So, we calculate the radius ratio at exit (station 2) based on tip radius,

$$\left(\frac{r_h}{r_t}\right)_2 = \sqrt{1-\frac{\dot{m}}{\rho_2 \pi r_{t2}^2 C_a}}$$

$$= \sqrt{1-\frac{55.96}{0.68\times\pi\times0.5^2\times200}} = 0.69$$

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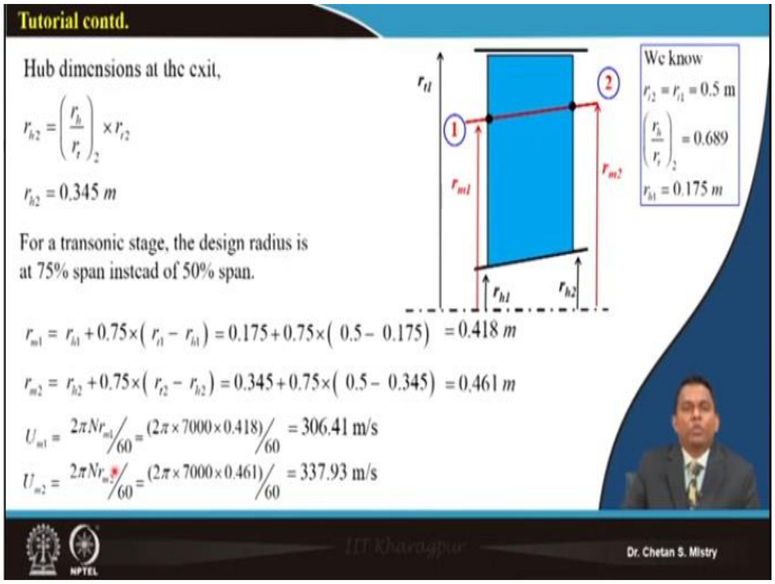

Now, here in this case, once the hub to tip ratio at the outlet station, that's what is known, we can calculate what will be the hub diameter or hub radius at the exit, and this is what is coming as say 0.345 m.

$$Hub\ dimensions\ at\ the\ exit,$$

$$r_{h2} = \left(\frac{r_h}{r_t}\right)_2 \times r_{t2}$$

$$r_{h2} = 0.345\ m$$

Now, in order to do our calculation at 75% span, we need to have our mid station. So, if we draw our inlet area and outlet area based on the radius, we will be getting our 75% span like this. So, we need to calculate what will be our r_{1m} and what will be our r_{2m} . So, that's what has been calculated based on 75% span. So, my mean radius at the entry it is coming 0.418 and at the exit, that's what is coming 0.461.

$$r_{m1} = r_{h1} + 0.75 \times (r_{t1} - r_{h1}) = 0.175 + 0.75 \times (0.5 - 0.175) = 0.418\ m$$

$$r_{m2} = r_{h2} + 0.75 \times (r_{t2} - r_{h2}) = 0.345 + 0.75 \times (0.5 - 0.345) = 0.461\ m$$

Now, we need to be very careful here, since my peripheral speed it is a function of my diameter and that is the reason why my entry peripheral speed, that's what need to be calculated based on entry radius and that's what is coming 306.41 m/s. Same way, for outlet it is coming 337.93 m/s.

$$U_{m1} = \frac{2\pi N r_{m1}}{60} = \frac{2\pi \times 7000 \times 0.418}{60} = 306.41\, {}^{m}/_{s}$$

$$U_{m2} = \frac{2\pi N r_{m2}}{60} = \frac{2\pi \times 7000 \times 0.461}{60} = 337.93\, {}^{m}/_{s}$$

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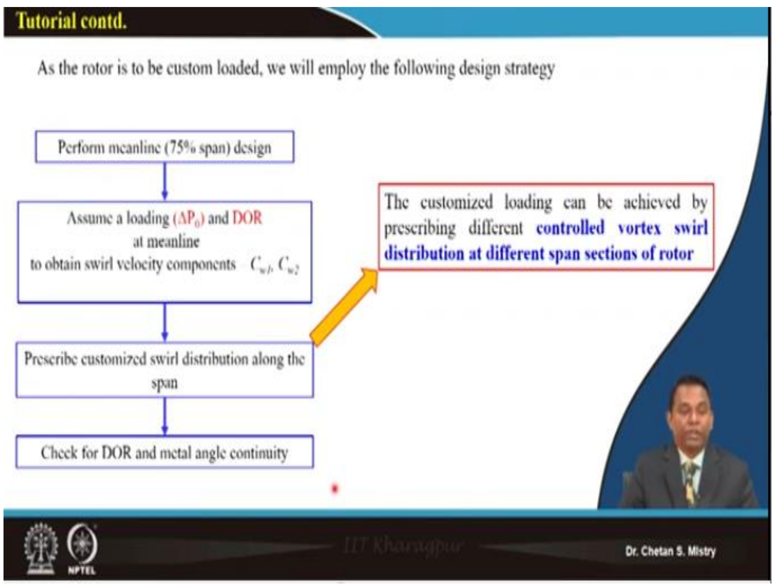

Now, very important configuration or very important discussion, that's what is coming here. What is our case, say we want to do our design, that's what will be based on say controlled vortex kind of configuration. Say, it is been asked like we need to low load say rotor at hub region and near the tip region. In order to lower that configuration, we need to change certain parameters. We can say my loading or work done capacity that's what is a function of my say whirl component. So, if we will be able to manage my entry whirl component and exit whirl component, we will be able to reduce the loading near that region.

Now, in order to do that configuration, we need to have value of ΔP_0 at the mid station. At the same time, we need to have certain known parameter, that's what is suppose say degree of reaction. Now, here in this case, my degree of reaction, that's what is not given. So, what we will be doing? We will be assuming our degree of reaction at 75% span.

Based on our fundamental understanding, we can calculate what will be our entry whirl component and exit whirl component. And, that is where we will be applying our Controlled Vortex law. So, let us move, how exactly we will be checking with? Now, here there is nothing mentioned in sense of what need to be the number of say my whirl component or my loading component.

That is the reason why we need to keep on eye for parameter say degree of reaction. We can understand, if we are reducing our load near the hub region, there may be possibility that my degree of reaction will be going low, it may be going 0 or sometimes it may go negative, and we know consequences for that. And, as the reason why we need to keep on eye for that degree of reaction near that hub region. Same way, we will be taking care of my blade angle variation, we can say metal angles, β_1 *and* $\beta_2$variation or we can say $\Delta\beta$ variation that need to be smooth, okay.

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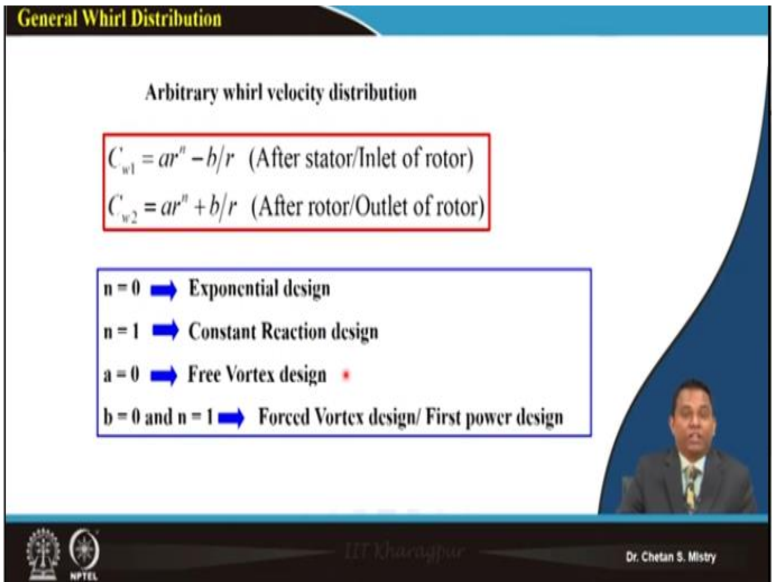

So, for that, let us see what can be done here? So, when we say, we need to assume our whirl component at the entry and whirl component at the exit, that's what we will be calculating based on assuming degree of reaction. Now, throughout the span we need to vary our whirl component at the entry as well as exit. So, what we have learned in week three, we have certain possibility, it says we can assume arbitrary whirl velocity distribution. Say, we will be taking $C_{w1} = ar^n - {}^{b}/_{r}$, we can consider C_{w2} as say $ar^n + {}^{b}/_{r}$. It like be at the entry you can consider this as a negative sign, at the exit you can take that as a positive sign.

$$C_{w1} = ar^n - {}^{b}/_{r}\,(After\ stator/Inlet\ of\ rotor)$$

$$C_{w2} = ar^n + {}^{b}/_{r}\,(After\ rotor/Outlet\ of\ rotor)$$

Now, this exponent we already have discussed when we are configuring $n = 0$, we can say it is an exponential design. If you are considering $n = 1$, that's what is my constant reaction design. If you are considering $a = 0$ that is nothing but it is a free vortex design. If you are considering $b = 0\ and\ n = 1$, that's what will be first power design or forced vortex design. Now, there are ample of possibilities here. So, it is designer's choice, how he or she will be deciding the distribution of whirl component at the entry and the exit.

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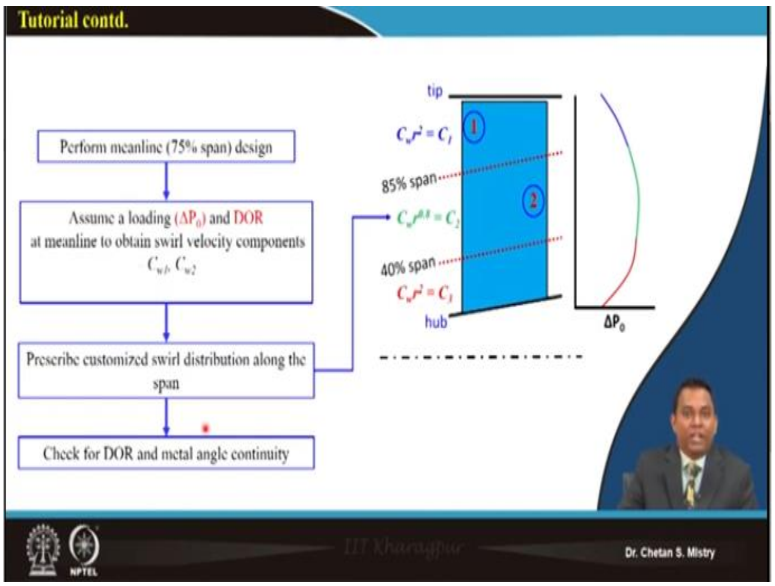

What we will be doing here? Say, in order to simplify and in order to reduce our calculation and for understanding, we will be configuring our say span of the blade into three different stations. We will be considering my first station, that's what is from hub to 40% of span then from 45 to 85% span and above 85% span.

Now, what it says? We need to load our hub and tip in order to have proper distribution or in order to have lower loading in that particular region. Now, in order to have that kind of configuration, we will be assuming say from hub to say 40% of span, my distribution will be $C_w \cdot r^2 = constant$, in say mid span region or say from 40% to 85% span, we will be taking $C_w \cdot r^{0.8} = constant$ and from 85% to tip, we will be taking $C_w \cdot r^2 = constant$. Now, this is what is very important, we need to realize that part. So, you can say, this is what we are looking for in sense of our distribution, okay.

We are expecting our total pressure rise of 2 or we are expecting our pressure ratio of the 2. So, it says, in this particular region we will be loading on higher side; near the hub region and near the tip region, we will be loading slightly lower. This is what is very interesting and you know, this is what is now the design concept which people they are adopting these days.

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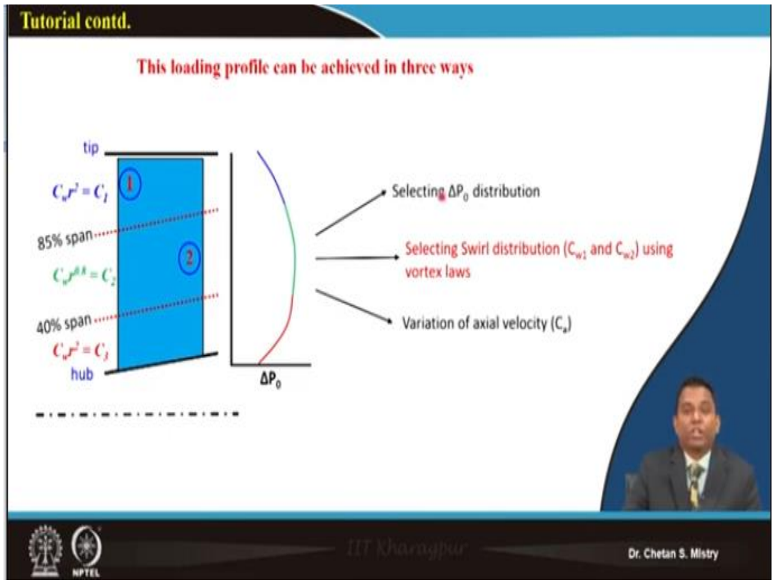

So, if you are configuring this, there are other chances also. The question may come in your mind, Sir, in order to change the C_w, we have different possibilities with us. One that's what I have discussed that maybe you can assume say, some $C_w \cdot r^n = constant$, that's what you can assume. Second possibility, that's what is you can take fundamental design approach where you will be having your ΔP_0 distribution, but this is what we have discussed for many approaches. So, at this moment we will not opting for this configuration.

Now, there is one more possibility, many designers, they are changing the axial velocity from say hub to certain span location and then after from say that particular span to higher span, they are configuring higher axial velocity; and, towards the tip region, they are configuring lower axial velocity. So, that is also possible.

But here in this case, what it says? My axial velocity, that’s what is constant. So, we will not be exploring that possibility. What we will be exploring is having the whirl distribution in the range of $C_w \cdot r^2 = constant$ from say hub to 40% span; and, from 85% to tip region, we will be taking $C_w \cdot r^2 = constant$. And, at the mean stream region, between say 40% to 85% span, we will be taking $C_w \cdot r^{0.8} = constant$.

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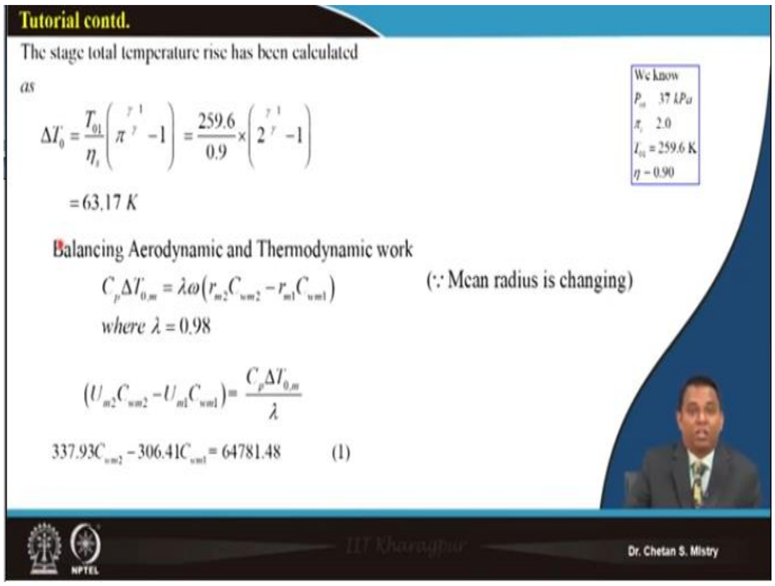

Now, the thing is once this is what is been decided with us, the question will come we need to have our whirl velocity component at the entry and our whirl velocity component at the exit, then only we will be having that law to be applicable with. So, what we will be doing? Suppose say, if we are considering, we are having our aerodynamic work and we are having our thermodynamic work; so, we can say, aerodynamic work that’s what is giving us the whirl velocity distribution say like C_{w1} component and C_{w2} component, so, what we will be doing?

Say, we will be calculating our ΔT_0 at that mid station, that’s what is coming say 63.17 K. If we are comparing our aerodynamic work and thermodynamic work assuming work done factor to be 0.98, we will be getting some distribution of say my whirl components C_{w2} and C_{w1}. So, this is what I am writing as say equation 1, okay. So, in this case, we are not knowing what is our C_{w1} and what is our C_{w2}, okay.

$$The\ stage\ total\ temperature\ rise\ has\ been\ calculated\ as,$$

$$\Delta T_0 = \frac{T_{01}}{\eta_s}\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$= \frac{259.6}{0.9} \times \left(2^{\frac{1.4-1}{1.4}} - 1\right)$$

$$\Delta T_0 = 63.17\ K$$

$$Balancing\ Aerodynamic\ and\ Thermodynamic\ work,$$

$$C_p \Delta T_{0,m} = \lambda\omega(r_{m2}C_{wm2} - r_{m1}C_{wm1})$$

$$(\because\ Mean\ radius\ is\ changing)$$

$$where\ \lambda = 0.98$$

$$(U_{m2}C_{wm2} - U_{m1}C_{wm1}) = \frac{C_p \Delta T_{0,m}}{\lambda}$$

$$337.93C_{wm2} - 306.41C_{wm1} = 64781.48 \qquad (1)$$

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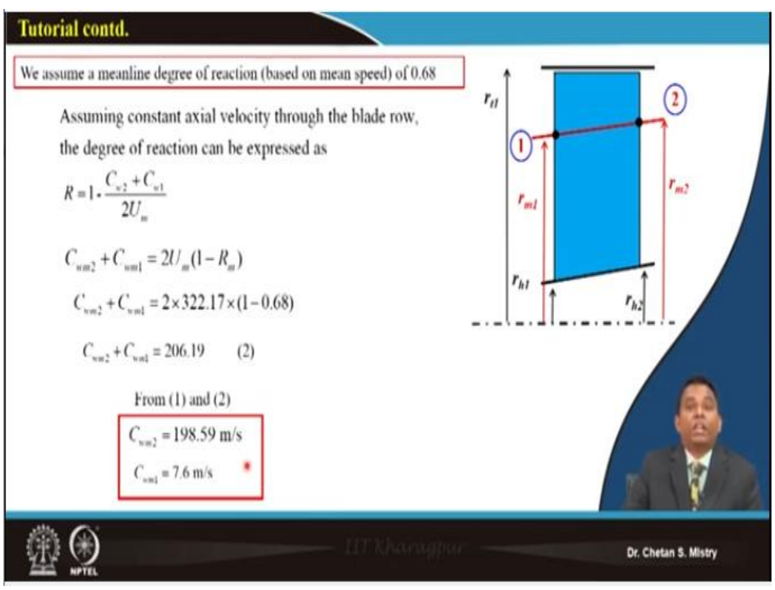

Now, say second configuration, what we will be doing is we will be assuming our degree of reaction. So, this is what is a little tricky in that sense. If suppose say, if I am assuming my degree of reaction to be lower at 75% span, there may be chances that near the hub region, my degree of reaction will go even low, it may go to 0.

So, in order to avoid such kind of situation, let us assume, say degree of reaction to be 0.68. So, what we know? Our degree of reaction, that's what we are correlating in sense of whirl components at the entry and exit and mean peripheral speed. So, that's what will be giving us the equation in the form of C_{w1} and C_{w2}.

$$We\ assume\ a\ meanline\ degree\ of\ reaction\ (based\ on\ mean\ speed) of\ 0.68$$

$$Assuming\ constant\ axial\ velocity\ through\ the\ blade\ row,$$

$$the\ degree\ of\ reaction\ can\ be\ expressed\ as$$

$$R = 1 - \frac{C_{w2} + C_{w1}}{2U_m}$$

$$C_{wm2} + C_{wm1} = 2U_m(1 - R_m)$$

$$C_{wm2} + C_{wm1} = 2 \times 322.17 \times (1 - 0.68)$$

$$C_{wm2} + C_{wm1} = 206.19 \qquad (2)$$

$$From\ (1)\ and\ (2)$$

$$C_{wm2} = 198.59\ {}^{m}/_{s}$$

$$C_{wm1} = 7.6\ {}^{m}/_{s}$$

Now, we are having two equations; one, that's what is from our say aerodynamic and thermodynamic work comparison. And second, that's what is based on our degree of reaction correlation. So, that's what will be giving me C_{w1} and C_{w2} distribution at the mid span, okay. So, this is what we need, to learn this is what we need to understand, okay.

Now, we have discussed some of the numerical in week 3 for say calculation by using different laws. So, if required, just go through repeat those calculation, go through those slides again, that's what will give you brush up of your knowledge. Again, you will be coming to the momentum for this numerical.

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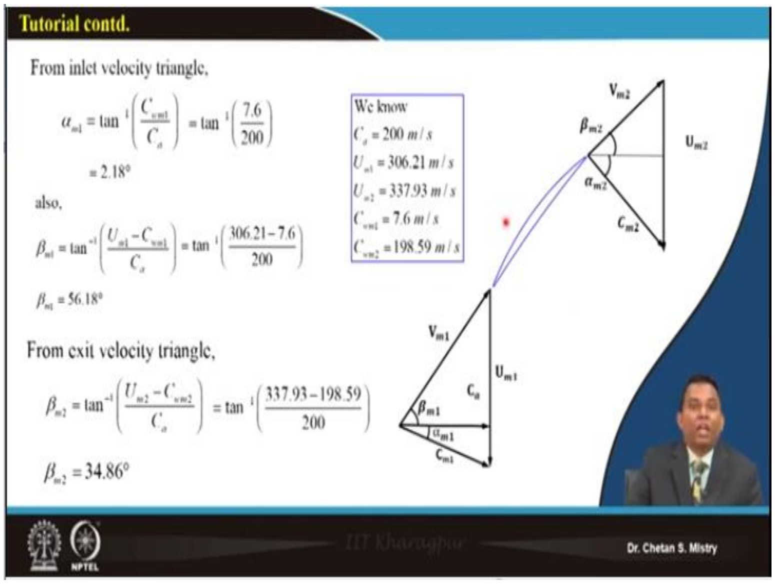

Now, once at the mid station, we know what is a whirl component at the entry and exit, we can make our velocity triangle. And, from that velocity triangle, we can calculate what is our flow angle at the entry that's what is coming 2.18. My β_1 angle, that's what is coming 56.18 and my β_2 angle, that is coming 34.86.

From inlet velocity triangle,

$$\alpha_{m1} = \tan^{-1}\left(\frac{C_{wm1}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{7.6}{200}\right)$$

$$= 2.18°$$

Also,

$$\beta_{m1} = \tan^{-1}\left(\frac{U_{m2} - C_{wm1}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{306.21 - 7.6}{200}\right)$$

$$\beta_{m1} = 56.18°$$

$$From\ exit\ velocity\ triangle,$$

$$\beta_{m2} = \tan^{-1}\left(\frac{U_{m2} - C_{wm2}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{337.93 - 198.59}{200}\right)$$

$$\beta_{m2} = 34.86°$$

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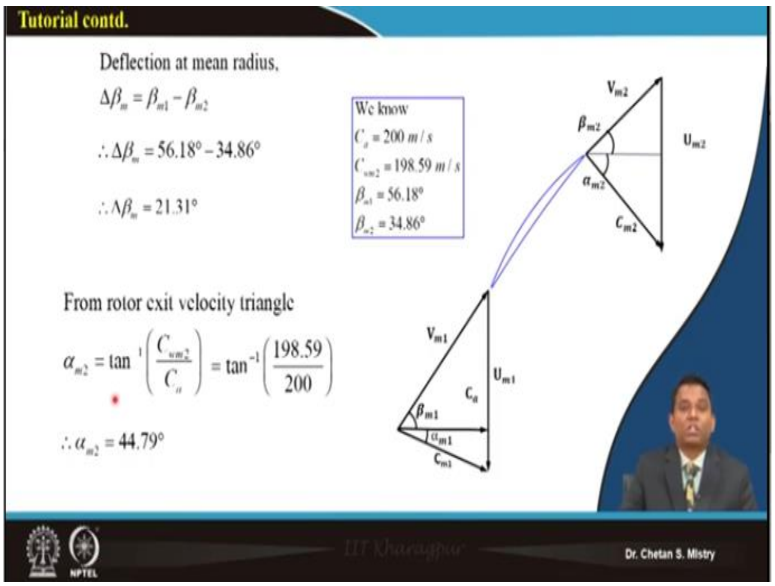

Now, this β_1 and β_2, they are known to us, we can calculate what is our $\Delta\beta$ and this $\Delta\beta$ is coming 21.31°.

$$Deflection\ at\ mean\ radius,$$

$$\Delta\beta = \beta_{m1} - \beta_{m2}$$

$$\therefore \Delta\beta_m = 56.18° - 34.86°$$

$$\therefore \Delta\beta_m = 21.31°$$

Same way, we can calculate what will be our α_2. This α_2, that's what is coming 44.79°.

$$From\ rotor\ exit\ velocity\ triangle,$$

$$\alpha_{m2} = \tan^{-1}\left(\frac{C_{wm2}}{C_a}\right)$$

$$= \tan^{-1}(\frac{198.59}{200})$$

$$\therefore \alpha_{m2} = 44.79°$$

Now, we are interested in checking with some of the parameters. So, in order to have those calculations, we will be calculating what is our relative velocity at the entry and what will be our relative velocity at the outlet. So, if you are using our triangle law and from that it says, my relative velocity at the entry is coming 359.33 m/s and relative velocity at the exit is say 243.74 and that's what is giving de-Haller's factor of 0.68.

$$The\ relative\ inlet\ and\ exit\ velocities\ can\ be\ calculated\ from\ velocity\ triangle,$$

$$V_{m1} = \frac{C_a}{\cos\beta_{m1}} = \frac{200}{\cos(56.18°)}$$

$$\therefore V_{m1} = 359.33\ m/s$$

$$Similarly\ at\ rotor\ exit,$$

$$V_{m2} = \frac{C_a}{\cos\beta_{m2}} = \frac{200}{\cos(34.86°)}$$

$$\therefore V_{m2} = 243.74\ m/s$$

$$We\ can\ check\ the\ mean\ de - Haller's\ number\ as,$$

$$DH = \frac{V_{m2}}{V_{m1}} = \frac{243.74}{359.33}$$

$$DH = 0.68$$

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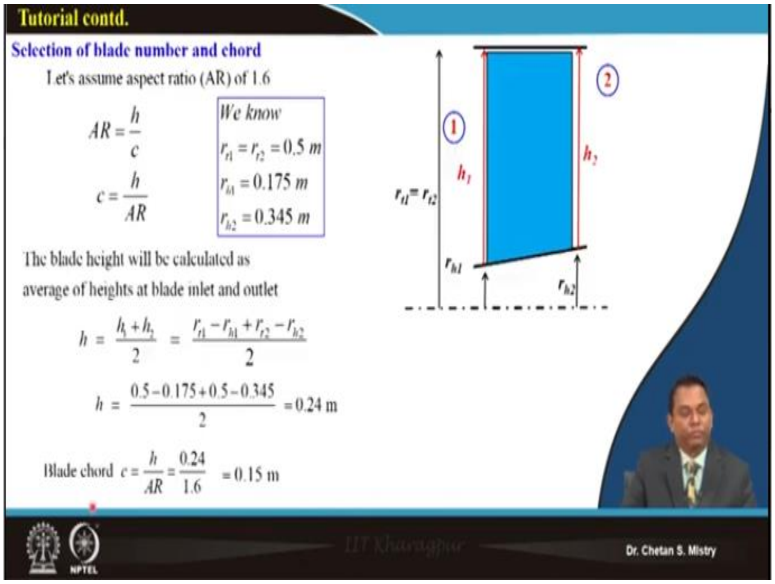

Now, next important parameter for us it is to select or to calculate the number of blades and chord. So, what we will be doing, in line two what all we have done; here also, we will be assuming our aspect ratio. Let us say for this blade aspect ratio is say 1.6. If we are putting that aspect ratio as 1.6, we are able to calculate our blade chord and that's what is coming as say 0.15 m.

$$Let's\ assume\ aspect\ ratio\ (AR)\ of\ 1.6\ for\ the\ rotor$$

$$\because AR = \frac{h}{c}$$

$$c = \frac{h}{AR}$$

$$The\ blade\ height\ will\ be\ calculated\ as\ average\ of\ heights\ at\ blade\ inlet\ and\ outlet$$

$$h = \frac{h_1 + h_2}{2} = \frac{r_{t1} - r_{h1} + r_{t2} - r_{h2}}{2}$$

$$h = \frac{0.5 - 0.175 + 0.5 - 0.345}{2} = 0.24\ m$$

$$Blade\ chord, c = \frac{h}{AR} = \frac{0.24}{1.6} = 0.15\ m$$

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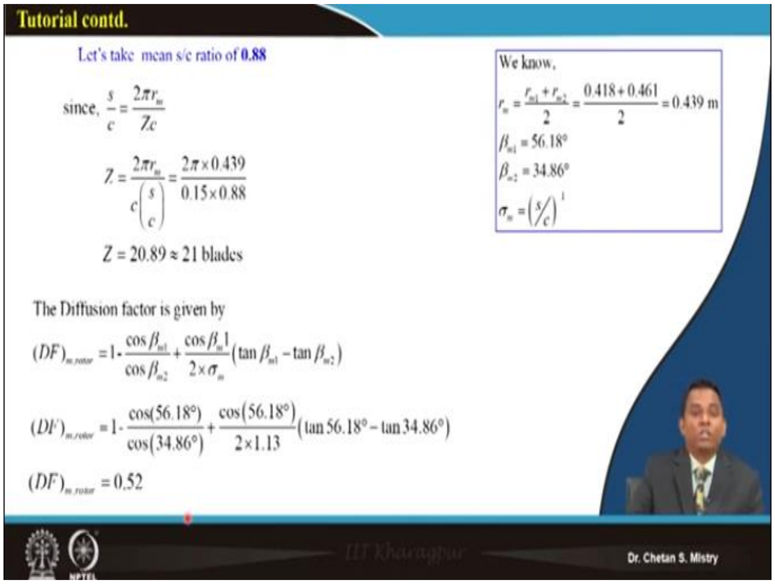

Now, once this is what is known, our next calculation that's what is to calculate the number of blades. So, in earlier numerical what all we have solved, there we have assumed our diffusion factor and based on that we use to calculate our number of blades. Let us take different approach in order to understand and realize this calculation in a different way. We can assume, suppose say mean say, s/c ratio as say 0.88. If you are considering that as say 0.88, we can calculate our number of blade and that's what is coming 20.89; so, safely we can consider that as say 21 number of blades.

$$Since, \frac{s}{c}=\frac{2\pi r_m}{Zc}$$

$$Z=\frac{2\pi r_m}{c\left(\frac{s}{c}\right)}=\frac{2\pi\times0.439}{0.15\times0.88}$$

$$Z=20.89\approx21\ blades$$

Here, you may have arguments saying sir, why we are not assuming diffusion factor? Just understand one thing, we are having our keep on I parameters, like say de-Haller's factor, degree of reaction, diffusion factor. So, one of the parameter, we already have assumed that's what is our degree of reaction. So, it is preferred not to go with assumption of second parameter also, that's what may give you very aggressive design. So, in order to avoid that kind of situation, we are assuming our say s/c ratio or pitch to chord ratio as 0.88, if you are putting that as a number diffusion factor is coming 0.52.

$$The\ Diffusion\ factor\ is\ given\ by,$$

$$(DF)_{m,rotor} = 1 - \frac{\cos\beta_{m1}}{\cos\beta_{m2}} + \frac{\cos\beta_{m1}}{2 \times \sigma_m}(\tan\beta_{m1} - \tan\beta_{m2})$$

$$(DF)_{m,rotor} = 1 - \frac{\cos(56.18°)}{\cos(34.86°)} + \frac{\cos(56.18°)}{2 \times 1.13}(\tan 56.18° - \tan 34.86°)$$

$$(DF)_{m,rotor} = 0.52$$

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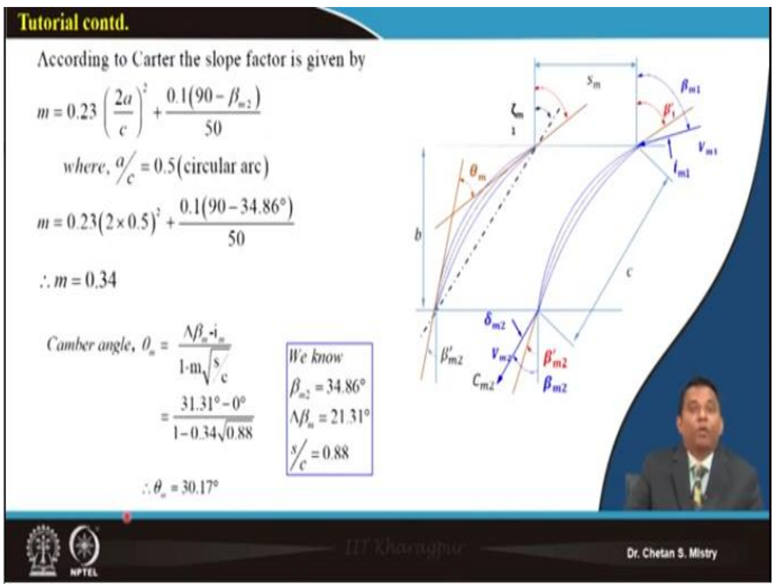

Now, we need to calculate our blade parameter or say blade airfoil or airfoil geometrical parameter that's what will be based on Carter rule. So, we will be calculating our slope factor. Here, this blade also we are considering as a circular Camber line that is why a/c is 0.5, that's what is giving me 'm' equal to 0.34. My Camber angle, that's what is coming 30.17. Here also, we are assuming our incidence angle at the mid station to be say 0.

$$According\ to\ Carter, the\ slope\ factor\ is\ given\ by$$

$$m = 0.23\left(\frac{2a}{c}\right)^2 + \frac{0.1(90 - \beta_{m2})}{50}$$

$$where, \frac{a}{c} = 0.5 (circular\ arc)$$

$$m = 0.23(2 \times 0.5)^2 + \frac{0.1(90 - 34.86°)}{50}$$

$$\therefore m = 0.34$$

$$Camber\ angle, \theta_m = \frac{\Delta\beta_m - i_m}{1 - m\sqrt{S/c}}$$

$$= \frac{31.31° - 0°}{1 - 0.34\sqrt{0.88}}$$

$$\therefore \theta_m = 30.17°$$

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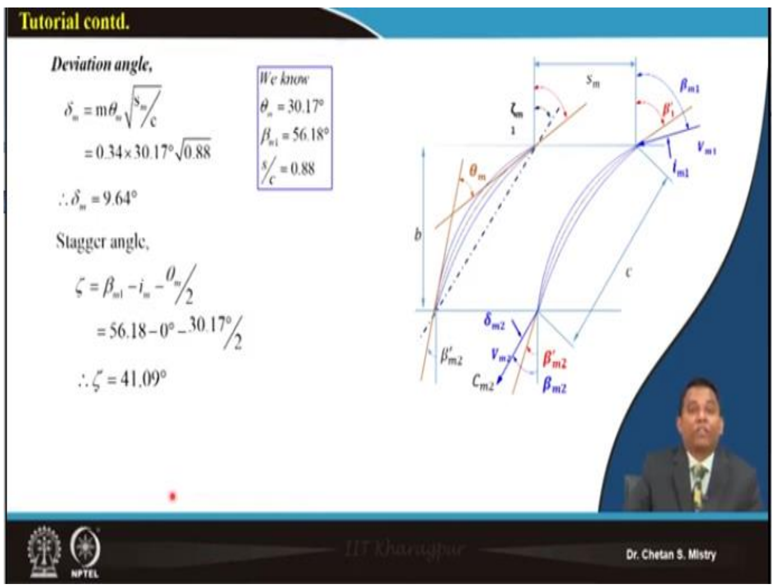

We can calculate what is our deviation angle and what will be our stagger angle. So, this calculation what we are discussing those are the cascade parameter, the methodology opted for the calculation, that's what will remain same, okay. And, mean line calculation almost if you look at, say that's what is coming as a similar kind of procedure. And, if you look at the textbooks or if you are referring the books, people, they are mostly they are discussing about the mean line calculation.

And then after they are putting say same way we can do our calculation for tip and hub; but we can understand, in order to do systematic design, very particular design, we need to have more number of station that's what will be giving us our blade to be in our hand as per our expected performance, okay.

$$\textit{Deviation angle,}$$

$$\delta_m = m\theta_m\sqrt{^{s_m}/_c}$$

$$= 0.34 \times 30.17°\sqrt{0.88}$$

$$\therefore \delta_m = 9.64°$$

$$\textit{Stagger angle,}$$

$$\zeta = \beta_{m1} - i_m - \theta_m/2$$

$$= 56.18° - 0° - 30.17°/2$$

$$\therefore \zeta = 41.09°$$

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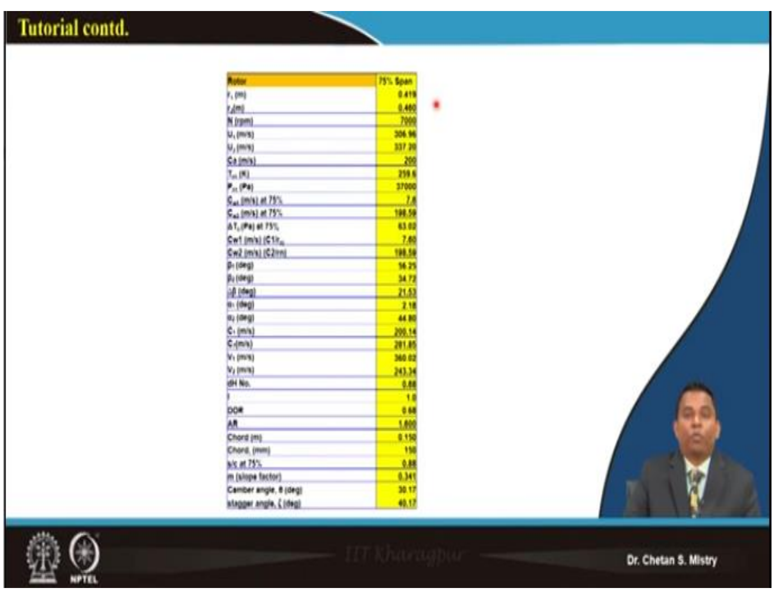

Now, this is what we are doing our calculation at say 75% of span. So, these all are the parameters we are calculating.

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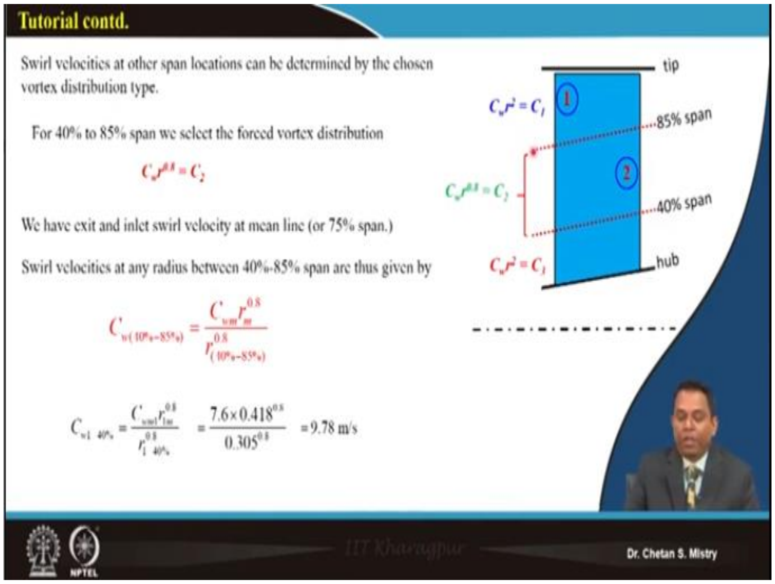

Now, very next step, that's what is coming that's what is to distribute our whirl components. So here in this case, if you look at from say 40% to 85% of span, we are considering our $C_w \cdot r^{0.8} = C_2$ that's what is coming say constant, it is say C_2 we are putting, okay. Same way, any radius in between these two, that's what we can calculate, because we have done our calculation at the mid station 75% span. So, we will be taking that as a constant and that constant we will be applying between 40% span to 85% span. So, this is what we can calculate in sense of C_{w1}, that's what is coming 9.78 m/s.

$$For\ 40\%\ to\ 85\%\ span\ we\ select\ the\ forced\ vortex\ distribution$$

$$C_w \cdot r^{0.8} = C_2$$

$$We\ have\ exit\ and\ inlet\ swirl\ velocity\ at\ mean\ line\ (or\ 75\%\ span)$$

$$Swirl\ velocities\ at\ any\ radius\ between\ 40\% - 85\%\ span\ are\ thus\ given\ by$$

$$C_{w(40\%-85\%)} = \frac{C_{wm} r_m^{0.8}}{r^{0.8}_{40\%-85\%}}$$

$$C_{w1\_40\%} = \frac{C_{wm1} r_{1m}^{0.8}}{r^{0.8}_{1\ 40\%}} = \frac{7.6 \times 0.418^{0.8}}{0.305^{0.8}} = 9.78\ m/s$$

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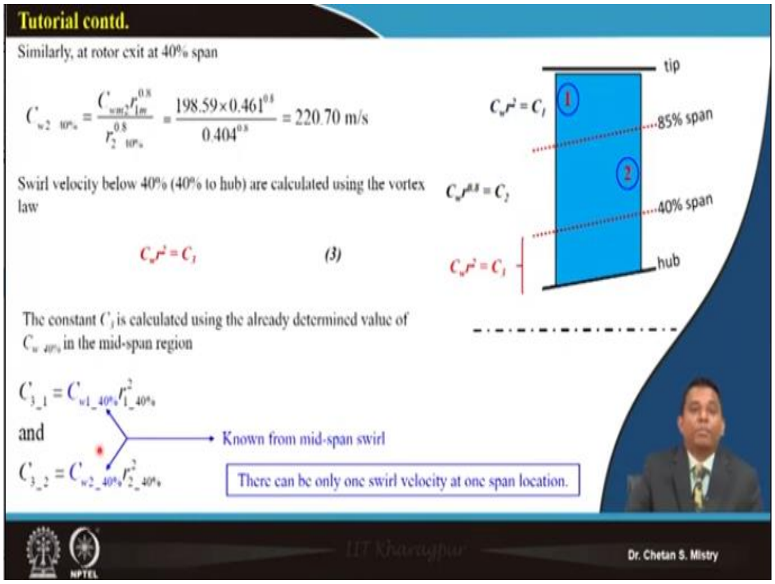

Same way, my C_{w2}, that's what is coming 220.70 m/s at 40% span.

$$Similarly, at\ rotor\ exit\ at\ 40\%\ span,$$

$$C_{w2\_40\%} = \frac{C_{wm2} r_{2m}^{0.8}}{r_{2\ 40\%}^{0.8}}$$

$$= \frac{198.59 \times 0.461^{0.8}}{0.404^{0.8}} = 220.70\ {}^{m}/_{s}$$

Now, say below 40% span, we are looking for $C_w \cdot r^2 = constant$.

$$Swirl\ velocity\ below\ 40\%\ (40\%\ to\ hub) are\ calculated\ using\ the\ vortex\ law$$

$$C_w \cdot r^2 = C_3 \qquad (3)$$

So, our requirement it is to calculate the C_w or we can say, that's what is say C_3, the constant need to be calculated. So, here we must realize at particular span, suppose say 40% span, we have opted for $C_w \cdot r^{0.8} = constant$.

So, at one station we cannot have say two different whirl velocity components. So, what we will be doing? Say, at 40% span, we will be applying the same law, okay, because this is what is known to us at 40% span. So, that's what will be helping us in order to calculate C_3. Same way, we can opt for say C_1 also, near the tip region.

$$\textit{The constant } C_3 \textit{ is calculated using the already}$$

$$\textit{determined value of } C_{w\,40\%} \textit{ in the mid span region}$$

$$C_{3\_1} = C_{w1\_40\%} r_{1\_40\%}^2$$

and,

$$C_{3\_2} = C_{w2\_40\%} r_{2\_40\%}^2$$

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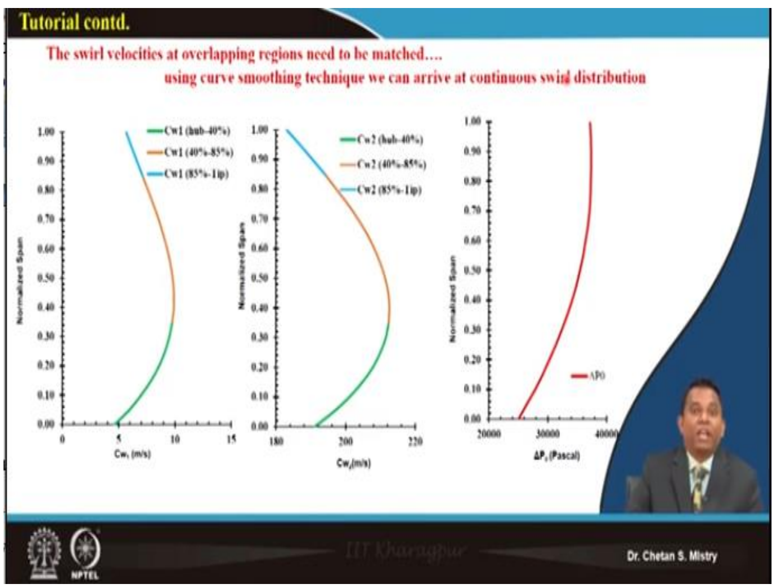

Now, when we are configuring our design say, that's what we have discussed here. So, there may be chances near this 40% span as well as near the 75% span we may be getting our distribution of whirl component in a zigzag way. So, we need to have smoothening of the curve in that particular region. And, that's what has been done. So, here if you look at, this is what is representing the distribution of say C_{w1} at entry from say 0 to 40% span, from 40% to 85% span and from 85% span to tip region. So, this is what is representing the distribution of C_{w1}.

In line to that, this is what is representing the distribution of C_{w2} and based on that if we will be putting our eye and if you calculate our ΔP_0, this is what is representing the loading. Now, here in this case, by changing the value of C_{w1} or C_{w2}, even we are able to modify the tip loading, but at this moment this is what is serving a purpose in sense of expected pressure ratio to be 2. So, this is what has been calculated and this is what has been assumed with. So, you can say this is what is representing C_{w1}, this is what is representing C_{w2}.

So, we can understand, we are having our excel sheet program, for that excel sheet program we will be doing our calculation from 40% span to 85% span. And, then after later on that same sheet we will be using for different whirl distribution. And, at the junction point it may require we need to modify our distribution, okay.

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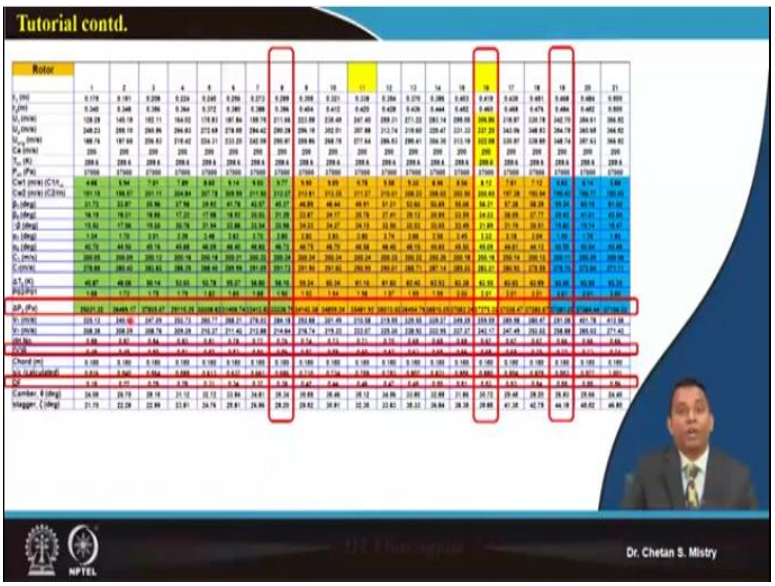

Now, let us look at this, this is what is representing the excel sheet with having say smoothing of the curve. So, if we look at, this is what is representing very first calculation that's what we have done at 75% span. Same way, this is what is representing the calculation at 85% span and this is what is representing our calculation at 40% of span, okay.

Now, we are putting our eye in order to understand say what is the variation of ΔP_0, because that is also very important for us. Same way, we will be putting our eye for say degree of reaction. So, if we look at, my degree of reaction at the hub, that's what is coming 0.48 and near the tip, that's what is coming 0.74. Now, diffusion factor, that's what is varying all the way from 0.19 to 0.56 and we are having our variation of Camber angle and Stagger angle accordingly. So, we need to check with what is happening with all this parameter in line to what all we have discussed up till now.

So, this sheet what you are observing, that's what has come after many iterations, okay. So likewise, when you are doing your design, you need to be very careful, it is not that you are sitting for an hour and you will get the design. That's what is required a lot of parameters to be observed. And, those observed parameter again and again we are putting with the highlighting point that's what is say ΔP_0, $\Delta\beta$, degree of reaction, diffusion factor, de Haller's factor, Camber angle, all those parameters you need to keep on eye.

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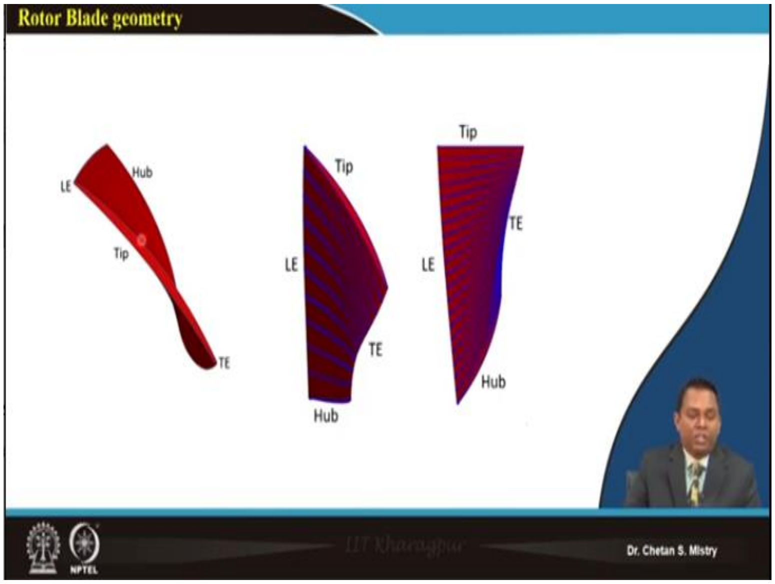

Now, based on what all calculation we have done, this is what is a blade that's what is coming. So, here if you look at, this is what is representing my curve at the hub and this is what is at the tip region. And, in this case, if you look at, this is what is my hub region and this is what is the tip region. And, in three-dimensional way, if we look at, this is what will be the shape of our blade.

So, now if you look at this blade, you must realize what all blades we are looking for high bypass ratio fan or say low bypass ratio fan, that's what is having three-dimensional funny shape. And, just realize that shape it has come by design. Just understand that thing it is not because it looks nice, but this is what has come by design. Here in this case, we are not discussing anything about the variation of chord, we are assuming our chord length to be same throughout the span. It may be possible, designer may modify the chord length near the hub region, maybe at the mid station and even at the tip station, as per the requirement.

So, now once you are making this blade ready with, use your computational tool in order to simulate this blade. Once you are simulating your blade, just observe certain parameters what we are looking at in sense of say increasing the losses or decreasing of losses or how my flow that's what is behaving on suction surface? How my flow that's what is behaving on pressure surface? Based on that certain modifications you need to do. And, after doing those iteration, you will come up with the final design. That's what it says this is what is working fine under design condition.

It may be possible when you are doing your design, that's what is given best performance expected pressure rise even efficiency at a design point, but when you are putting that under off design condition, suppose say 80% mass flow rate or 75% mass flow rate, you will find that's what is not giving wider operating range in sense of my stall margin. Under that condition, you may need to modify your blades, you may need to modify your airfoils. If possible, you may need to modify your dimensions of the blade.

So, this is what is all we are discussing in sense of transonic compressor. So, in overall this week, if we configure week 10, that week 10 we have dedicated for design of transonic compressor. And, in that we have discussed detailed design of transonic compressor, that's what will build the confidence in you for designing the future axial flow compressor or future axial flow fan which are of transonic nature.

So, here we are stopping with. I am sure, you will get the confidence, it is preferred that you just make your own design sheet, do this calculation what all design we have done, you can go with some other configuration also, say configuration like maybe you can go with the free vortex, you can go with say different loading and as we have discussed, you can go with change of axial velocity. And that's what we will be giving you the feeling of real design. Thank you. Thank you very much.