Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 57
Design of Transonic Compressor (Contd.)

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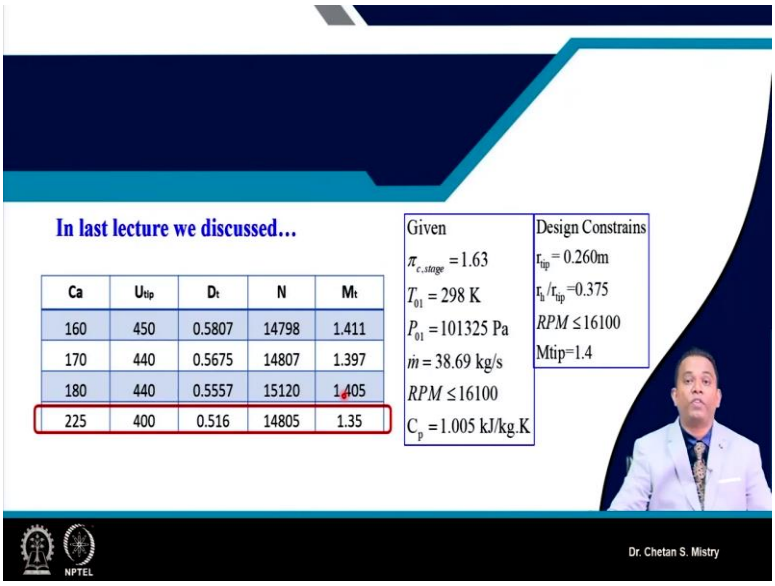

In last lecture we discussed…

Ca	U_{tip}	D_t	N	M_t
160	450	0.5807	14798	1.411
170	440	0.5675	14807	1.397
180	440	0.5557	15120	1.405
225	400	0.516	14805	1.35

Given

$\pi_{c,stage} = 1.63$

$T_{01} = 298$ K

$P_{01} = 101325$ Pa

$\dot{m} = 38.69$ kg/s

$RPM \leq 16100$

$C_p = 1.005$ kJ/kg.K

Design Constrains

$r_{tip} = 0.260$m

$r_h / r_{tip} = 0.375$

$RPM \leq 16100$

Mtip=1.4

Welcome to lecture 57. In last lecture, we were discussing about the design of transonic compressor, which was having say expected pressure ratio of 1.63, we were having constraints with tip diameter, hub to tip ratio at the entry is given to us. Speed, it was limited by 16,100. And we started doing design with initial consideration of assuming certain parameters. And finally, we have come up with the solution, say, our axial velocity to be 225 m/s, our peripheral speed to be 400 m/s, tip diameter, that's what was coming 0.516, that's what is less than what we are supposed to be. And that is the reason why we have accepted with.

Rotational speed, it was coming 14,805; and tip Mach number which was constrained saying 1.4, in spite of that, for our assumed values of axial velocity, and peripherals speed, that's what was coming 1.35. And that's what we have accepted with. Then we were discussing about the flow track configurations.

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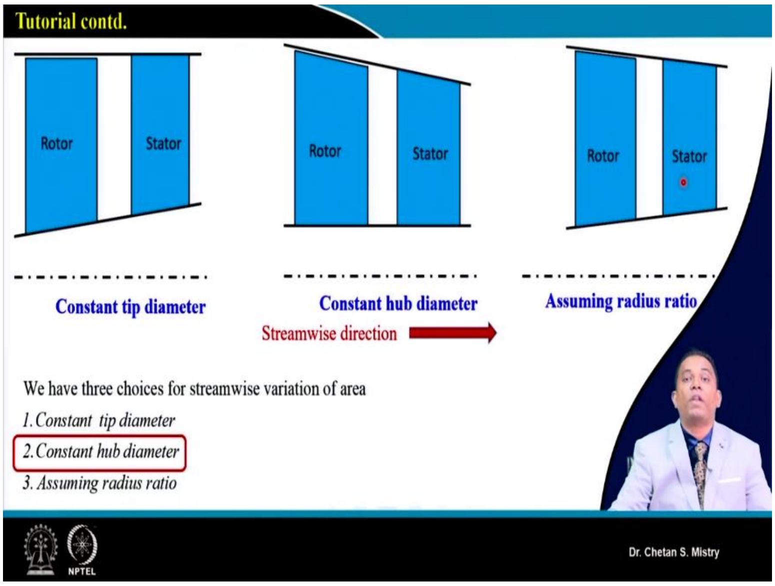

Tutorial contd.

We have three choices for streamwise variation of area

1. *Constant tip diameter*
2. *Constant hub diameter*
3. *Assuming radius ratio*

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So, if you recall, this is what were…we were discussing in last lecture, where we have configured our say flow track for axial flow compressor. First, that's what is with say constant tip diameter configuration. In that, our tip diameter, that's what will be remained constant. And as we know, our expected pressure ratio, that's what is in high range 1.63, that's what is higher in that range.

That means we will be having our exit configuration or exit dimensions to be different. Because we are planning to have satisfaction of continuity equation. If we consider our pressure ratio to be higher, that means that's what will be giving our density also to be higher. And in order to maintain our axial velocity to be constant, our exit area, that's what was decreasing. And if we configure our requirement as say constant tip diameter, then my hub diameter that's what will be changing compared to my entry condition and my exit condition. There is second configuration that is also possible in which we will be maintaining our hub diameter to be constant.

Now, when we say our hub diameter to be constant, in order to satisfy the continuity equation, we will be having our flow passage, flow track, that's what will be looking like this. In which by tip diameter that's what will be going to be decreased towards the exit of my stage. There is one more possibility in which we will be assuming some radius ratio, or maybe we will be configuring saying like maybe constant mean diameter kind of configuration, then we will be having our passage to be converging passage like this. Now, today, we will be discussing in detail design using say constant hub diameter configuration.

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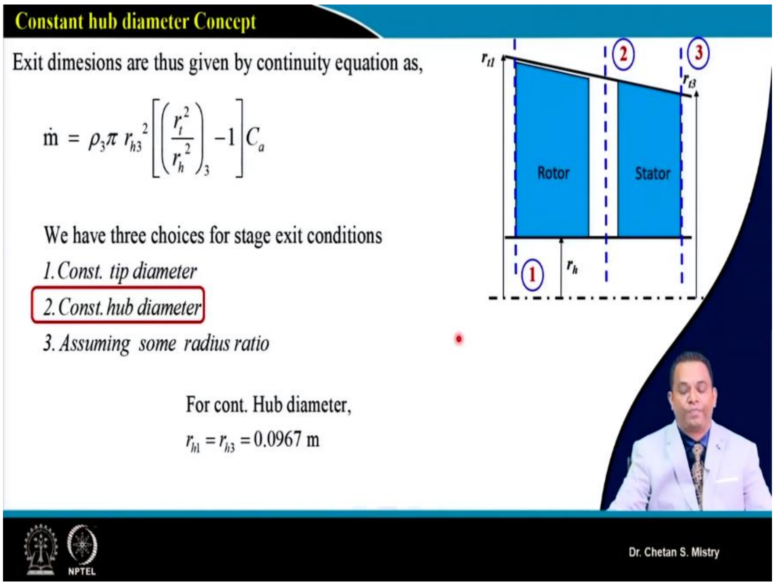

Constant hub diameter Concept

Exit dimesions are thus given by continuity equation as,

$$\dot{m} = \rho_3 \pi r_{h3}^2 \left[\left(\frac{r_t^2}{r_h^2}\right)_3 - 1\right] C_a$$

We have three choices for stage exit conditions

1. Const. tip diameter
2. Const. hub diameter
3. Assuming some radius ratio

For cont. Hub diameter,

$r_{h1} = r_{h3} = 0.0967$ m

So, in order to move with constant hub diameter configuration, very first thing that's what we need to find, that is with the diameter ratio. So, what we know? We have our continuity equation, based on my mass flow rate equation that's what we can write down saying like $density \times area \times axial\ velocity$, this is what we are configuring at the exit.

That is the reason why this is what is say

$$\dot{m} = \rho_3 \pi r_{h3}^2 \left[\left(\frac{r_t^2}{r_h^2}\right)_3 - 1\right] C_a$$

Now, as we have discussed, we have done our calculation for hub diameter and tip diameter earlier. For this configuration, we are assuming our hub diameter to be constant that means, my r_{1h} and r_{3h}, this is what will be remaining constant, and that's what is say 0.0967 m.

$$For\ constant\ hub\ diameter,$$

$$r_{h1} = r_{h3} = 0.0967\ m$$

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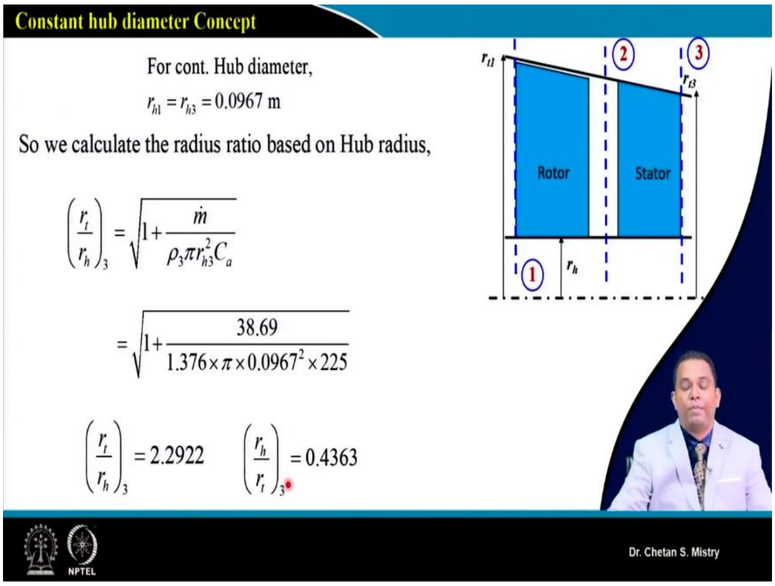

Constant hub diameter Concept

For cont. Hub diameter,

$r_{h1} = r_{h3} = 0.0967$ m

So we calculate the radius ratio based on Hub radius,

$$\left(\frac{r_t}{r_h}\right)_3 = \sqrt{1+\frac{\dot{m}}{\rho_3 \pi r_{h3}^2 C_a}}$$

$$= \sqrt{1+\frac{38.69}{1.376 \times \pi \times 0.0967^2 \times 225}}$$

$$\left(\frac{r_t}{r_h}\right)_3 = 2.2922 \qquad \left(\frac{r_h}{r_t}\right)_3 = 0.4363$$

Now, once we know this hub diameter, based on my continuity equation we can calculate what will be my say ratio of hub to tip, and that’s what is coming 0.4363.

So, we calculate the radius ratio based on Hub radius,

$$\left(\frac{r_t}{r_h}\right)_3 = \sqrt{1+\frac{\dot{m}}{\rho_3 \pi r_{h3}^2 C_a}}$$

$$= \sqrt{1+\frac{38.69}{1.376 \times \pi \times 0.0967^2 \times 225}}$$

$$\left(\frac{r_t}{r_h}\right)_3 = 2.2922$$

$$\left(\frac{r_h}{r_t}\right)_3 = 0.4363$$

Now, here in this case once we understand the dimensions for hub diameter, based on this ratio we can calculate what will be my tip diameter.

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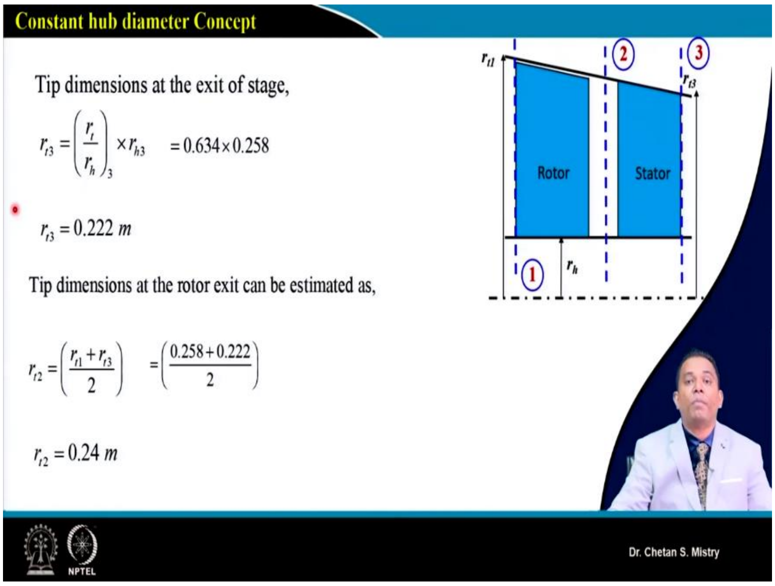

Constant hub diameter Concept

Tip dimensions at the exit of stage,

$$r_{t3} = \left(\frac{r_t}{r_h}\right)_3 \times r_{h3} \quad = 0.634 \times 0.258$$

$$r_{t3} = 0.222\ m$$

Tip dimensions at the rotor exit can be estimated as,

$$r_{t2} = \left(\frac{r_{t1} + r_{t3}}{2}\right) \quad = \left(\frac{0.258 + 0.222}{2}\right)$$

$$r_{t2} = 0.24\ m$$

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And that's what is giving me my tip diameter to be 0.22 m. So, if we draw our passage, we can say, this is what is my hub diameter and this is what is my tip diameter, okay.

Now, once we have done this calculation for hub and tip diameter, we are looking for say what will be the dimension at the mid station, because at mid station if we configure at station 2, my r_{2h} that's what will be say constant, and my r_{2t} that means, my tip dimension at station 2, that's what we will be taking as a average value, and this is what is coming as say 0.24 m.

$$Tip\ dimensions\ at\ the\ exit\ of\ stage,$$

$$r_{t3} = \left(\frac{r_t}{r_h}\right)_3 \times r_{h3} = 0.634 \times 0.258$$

$$r_{t3} = 0.222\ m$$

$$Tip\ dimensions\ at\ the\ rotor\ exit\ can\ be\ estimated\ as,$$

$$r_{t2} = \left(\frac{r_{t1} + r_{t3}}{2}\right) = \left(\frac{0.258 + 0.222}{2}\right)$$

$$r_{t2} = 0.24\ m$$

Now, we can say we are having all the dimensions at entry, at mid station and the exit.

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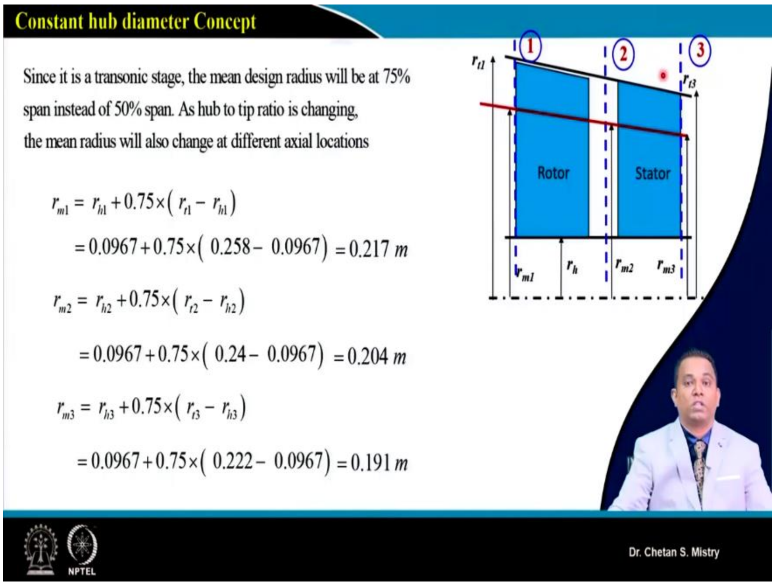

Constant hub diameter Concept

Since it is a transonic stage, the mean design radius will be at 75% span instead of 50% span. As hub to tip ratio is changing, the mean radius will also change at different axial locations

$$r_{m1} = r_{h1} + 0.75\times\left(r_{t1} - r_{h1}\right)$$
$$= 0.0967 + 0.75\times\left(0.258 - 0.0967\right) = 0.217\ m$$
$$r_{m2} = r_{h2} + 0.75\times\left(r_{t2} - r_{h2}\right)$$
$$= 0.0967 + 0.75\times\left(0.24 - 0.0967\right) = 0.204\ m$$
$$r_{m3} = r_{h3} + 0.75\times\left(r_{t3} - r_{h3}\right)$$
$$= 0.0967 + 0.75\times\left(0.222 - 0.0967\right) = 0.191\ m$$

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Now, if this is what is known to us, what we know is we are planning to do our calculation; since, this is what is high pressure ratio configuration, and we know this is what is a transonic compressor that means, we need to do calculation for our mid station. And, as we have discussed when we will be having say subsonic compressor, we are considering our mid station at 50% of span, and when we are configuring or when we are considering our compressor as transonic compressor, all calculation that's what we are doing at 75% of span.

So, we can say this is what is our mid station. So, at the entry, we need to have value of radius, at mid station also we are looking for radius, and at the exit also we are looking for radius because that's what will be giving me the streamline pattern at 75% span, okay.

So, if we configure in that direction, then we can say my mean radius at the entry (r_{m1}) that's what is given by $r_{h1} + 0.75 \times (r_{t1} - r_{h1})$, we can say this is coming say 0.217 m.

$$r_{m1} = r_{h1} + 0.75 \times (r_{t1} - r_{h1})$$

$$= 0.0967 + 0.75 \times (0.258 - 0.0967) = 0.217\ m$$

Same way, at mid station also at location 2, we can calculate that's what is coming 0.204 m; and at the exit, that's what is coming 0.191 m.

$$r_{m2} = r_{h2} + 0.75 \times (r_{t2} - r_{h2})$$

$$= 0.0967 + 0.75 \times (0.24 - 0.0967) = 0.204\ m$$

$$r_{m3} = r_{h3} + 0.75 \times (r_{t3} - r_{h3})$$

$$= 0.0967 + 0.75 \times (0.222 - 0.0967) = 0.191\ m$$

Now, this is what is must in sense of doing our calculation, we need to be very careful in considering and calculating this mean radius, okay. So, it is advisable based on what dimensions we are getting for hub and tip, just draw a diagram and based on that you just try to calculate this radius, okay.

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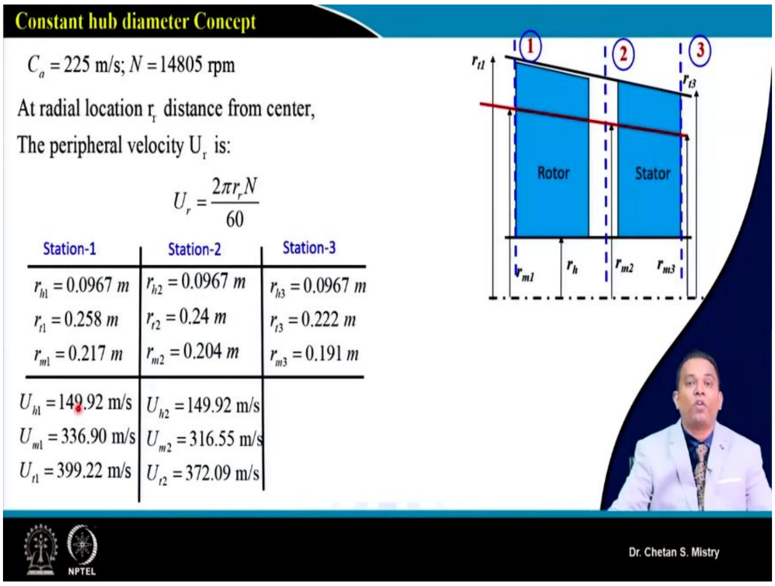

Constant hub diameter Concept

$C_a = 225$ m/s; $N = 14805$ rpm

At radial location r_r distance from center,

The peripheral velocity U_r is:

$$U_r = \frac{2\pi r_r N}{60}$$

Station-1	Station-2	Station-3
$r_{h1} = 0.0967\ m$	$r_{h2} = 0.0967\ m$	$r_{h3} = 0.0967\ m$
$r_{t1} = 0.258\ m$	$r_{t2} = 0.24\ m$	$r_{t3} = 0.222\ m$
$r_{m1} = 0.217\ m$	$r_{m2} = 0.204\ m$	$r_{m3} = 0.191\ m$
$U_{h1} = 149.92$ m/s	$U_{h2} = 149.92$ m/s	
$U_{m1} = 336.90$ m/s	$U_{m2} = 316.55$ m/s	
$U_{t1} = 399.22$ m/s	$U_{t2} = 372.09$ m/s	

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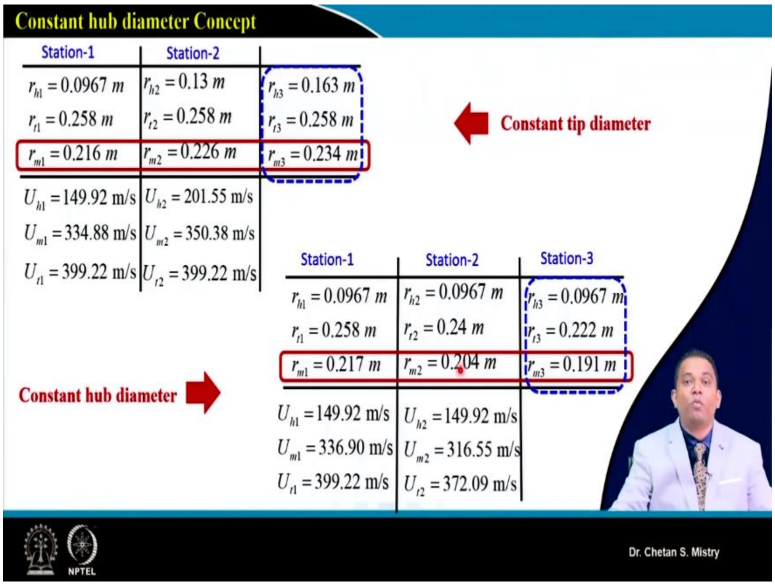

Constant hub diameter Concept

Station-1	Station-2	
$r_{h1} = 0.0967\ m$	$r_{h2} = 0.13\ m$	$r_{h3} = 0.163\ m$
$r_{t1} = 0.258\ m$	$r_{t2} = 0.258\ m$	$r_{t3} = 0.258\ m$
$r_{m1} = 0.216\ m$	$r_{m2} = 0.226\ m$	$r_{m3} = 0.234\ m$
$U_{h1} = 149.92$ m/s	$U_{h2} = 201.55$ m/s	
$U_{m1} = 334.88$ m/s	$U_{m2} = 350.38$ m/s	
$U_{t1} = 399.22$ m/s	$U_{t2} = 399.22$ m/s	

Constant tip diameter

Station-1	Station-2	Station-3
$r_{h1} = 0.0967\ m$	$r_{h2} = 0.0967\ m$	$r_{h3} = 0.0967\ m$
$r_{t1} = 0.258\ m$	$r_{t2} = 0.24\ m$	$r_{t3} = 0.222\ m$
$r_{m1} = 0.217\ m$	$r_{m2} = 0.204\ m$	$r_{m3} = 0.191\ m$
$U_{h1} = 149.92$ m/s	$U_{h2} = 149.92$ m/s	
$U_{m1} = 336.90$ m/s	$U_{m2} = 316.55$ m/s	
$U_{t1} = 399.22$ m/s	$U_{t2} = 372.09$ m/s	

Constant hub diameter

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Now, this is what we can say in sense of the configuration what we have decided with. With our axial velocity of 225 m/s, and our peripheral speed that's what we are writing as say $\frac{2\pi r_r N}{60}$. Here, our rotational speed we have considered as 14,805 rpm.

$$C_a = 225\, m/s\,; \qquad N = 14805\, rpm$$

So, this is what will be giving me dimensions at different stations, and we need to be very careful here at hub, at mid station and at the tip station, at say location 1, we will be having U that's what will be different.

Same way, for mid station also, we will be having these dimensions to be different. And, this is what is my station 3, basically that's what is the exit of stator. So that's what where we are not considering any peripheral speed configuration. So, we need to be very careful in sense of doing our calculation for peripheral speeds.

Station-1	Station-2	Station-3
$r_{h1} = 0.0967\, m$ $r_{t1} = 0.258\, m$ $r_{m1} = 0.217\, m$	$r_{h2} = 0.0967\, m$ $r_{t2} = 0.24\, m$ $r_{m2} = 0.204\, m$	$r_{h3} = 0.0967\, m$ $r_{t3} = 0.222\, m$ $r_{m3} = 0.191\, m$
$U_{h1} = 149.92\, m/s$ $U_{m1} = 336.90\, m/s$ $U_{t1} = 399.22\, m/s$	$U_{h2} = 149.92\, m/s$ $U_{m2} = 316.55\, m/s$ $U_{t2} = 372.09\, m/s$	

Let us try to look at what all are the changes. Suppose if we consider, this is what is with our constant tip diameter for which we have done our calculation, we have done our design. And, these are the station 1, 2 and 3 for constant hub diameter. Let us try to compare what exactly will be the change, because this change, that's what will be reflecting in our design. So, if we configure, this is what is my main station calculation. It says my entry mean station, that's what is 0.216. At mid station it is 0.226, and say at the exit we will be at 0.234. So, we can say this is what is giving upward direction kind of configuration for the streamlines.

Here, these are the dimensions at hub, tip, at mean station. Suppose say, if you are comparing these two numbers, we can see here at mid station, we are having radius, that's what is same at say mid station, we are having that's what is say 0.204, you can compare this value; it is 0.226, for constant tip. And for say constant hub configuration, it is 0.204. At the exit, also my mean radius, that's what is decreasing. So, we can say our streamline direction, or if we draw the line, that's what is connecting all these points, that's what is showing say the flow to be moving towards the hub direction. So, that's what is the imaginary line, but we can realise, we can understand that part.

Now what is happening? If we are configuring here, we can say, my radius at the hub, at the exit, that's what is lower. At mid station, that's what is also we are having lower value. And at say, our tip station, there also we are having this value coming to the lower. Now, this is what all numbers we are looking at, that's what is basically reflecting on our peripheral speed.

And we know, our peripheral speed, that's what is a function of my radius. If we are having the change in this peripheral speed, accordingly, we will be having change in flow velocity components, we also will be having change in flow angles.

And, if we are having this configuration, that's what is different. We can say what all say observation parameters like diffusion factor, degree of reaction, de Haller's factor, my deflection angle, my different flow angles, those all, that's what will be getting changed, or that's what we will be having different values. There is a purpose specifically, one lecture, that's what is dedicated for this particular design, because that's what will be giving us the feeling of suppose say, if you will be configuring this kind of design, okay. So, let us move with what all we are discussing.

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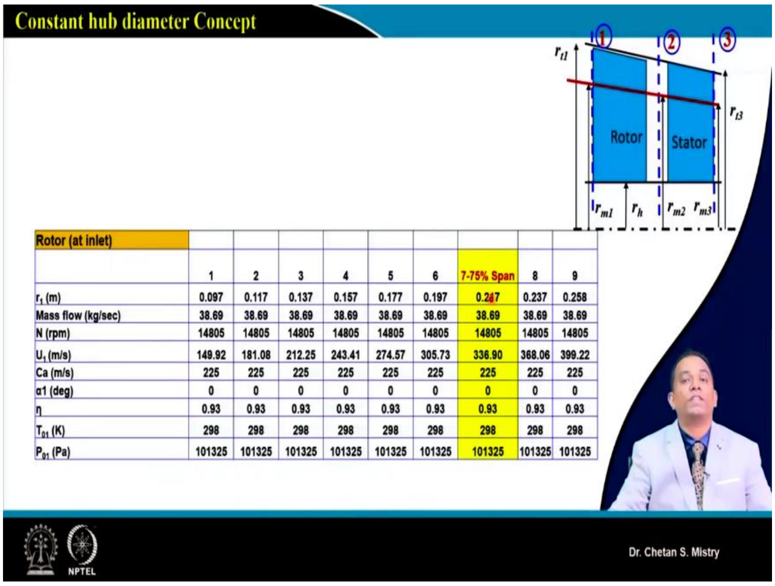

Constant hub diameter Concept

Rotor (at inlet)									
	1	2	3	4	5	6	7-75% Span	8	9
r_1 (m)	0.097	0.117	0.137	0.157	0.177	0.197	0.2[illegible]7	0.237	0.258
Mass flow (kg/sec)	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69
N (rpm)	14805	14805	14805	14805	14805	14805	14805	14805	14805
U_1 (m/s)	149.92	181.08	212.25	243.41	274.57	305.73	336.90	368.06	399.22
Ca (m/s)	225	225	225	225	225	225	225	225	225
α1 (deg)	0	0	0	0	0	0	0	0	0
η	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
T_{01} (K)	298	298	298	298	298	298	298	298	298
P_{01} (Pa)	101325	101325	101325	101325	101325	101325	101325	101325	101325

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Now, here in this case, once we know what is our entry station, that's what we will be dividing into say nine station. So, if we configure my radius at the hub, that's what is 0.097. And this is what is at the tip, that's what is say 0.258. Be careful, we are always looking for r_1. We are looking for r_2, and we are looking for r_3. If you recall, this is what is we are looking for the calculation of U.

So, make a habit of writing this r_1, and r_2 separately; even r_3 also separately, when you are doing your design for stator. Otherwise, there are chances that you will be making mistake which radius to be selected, that's what is always very important. So, this is what is at say 75% span, that's what is coming 0.217.

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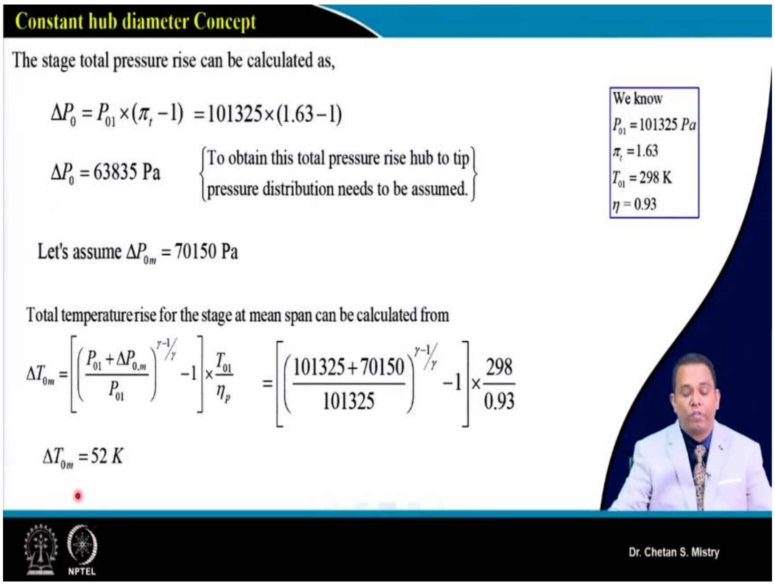

Constant hub diameter Concept

The stage total pressure rise can be calculated as,

$$\Delta P_0 = P_{01} \times (\pi_t - 1) = 101325 \times (1.63 - 1)$$

$$\Delta P_0 = 63835 \text{ Pa} \quad \begin{Bmatrix} \text{To obtain this total pressure rise hub to tip} \\ \text{pressure distribution needs to be assumed.} \end{Bmatrix}$$

We know
$P_{01} = 101325\ Pa$
$\pi_t = 1.63$
$T_{01} = 298$ K
$\eta = 0.93$

Let's assume $\Delta P_{0m} = 70150$ Pa

Total temperature rise for the stage at mean span can be calculated from

$$\Delta T_{0m} = \left[\left(\frac{P_{01} + \Delta P_{0,m}}{P_{01}} \right)^{\gamma-1/\gamma} - 1 \right] \times \frac{T_{01}}{\eta_p} = \left[\left(\frac{101325 + 70150}{101325} \right)^{\gamma-1/\gamma} - 1 \right] \times \frac{298}{0.93}$$

$$\Delta T_{0m} = 52\ K$$

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Now, here, this is what is our design requirements. So, we will be initiating our design at the mid station. Now, if you recall, when we were discussing constant tip diameter configuration, we have initially assumed our total pressure, say ΔP_0 or say pressure ratio at the mid station, that's what we have initially assumed.

And then after in later case, if you recall, we have changed our loading in order to set the loading both towards the tip as well as towards the hub region, in order to receive expected pressure rise. So, here also we can move with say assuming our pressure ratio to be 1.63. And later on, we can modify the total pressure ratio or expected pressure ratio as per the design.

But in order to reduce the iteration, we are not doing like that, let us assume our ΔP_0 at the mid station to be on the higher side. From initial only we will be taking this as say 70,150 Pa. Now, this is what we are assuming at say ΔP_0 at the mid station. Now, once this ΔP_0, that's what is known to us, we can calculate what will be our temperature rise…total temperature rise at the mid station.

And, this is what we are writing in terms of let us say polytropic efficiency, my entry temperature and ΔP_0, and total pressure at the entry and that's what is coming say 52 K.

$$The\ stage\ total\ pressure\ rise\ can\ be\ calculated\ as,$$

$$\Delta P_0 = P_{01} \times (\pi_t - 1)$$

$$\Delta P_0 = 101325 \times (1.63 - 1)$$

$$\Delta P_0 = 63835\ Pa$$

$$Let's\ assume\ \Delta P_{0m} = 70150\ Pa$$

$$Total\ temperature\ rise\ for\ the\ stage\ at\ mean\ span\ can\ be\ calculated\ from$$

$$\Delta T_{0m} = \left[\left(\frac{P_{01} + \Delta P_{0,m}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \times \frac{T_{01}}{\eta_p}$$

$$= \left[\left(\frac{101325 + 70150}{101325}\right)^{\frac{1.4-1}{1.4}} - 1\right] \times \frac{298}{0.93}$$

$$\Delta T_{0m} = 52\ K$$

Now, basically this ΔP_0, that's what is very important because in order to do the calculation at mid station, we are looking for velocity components, we are looking for flow angles.

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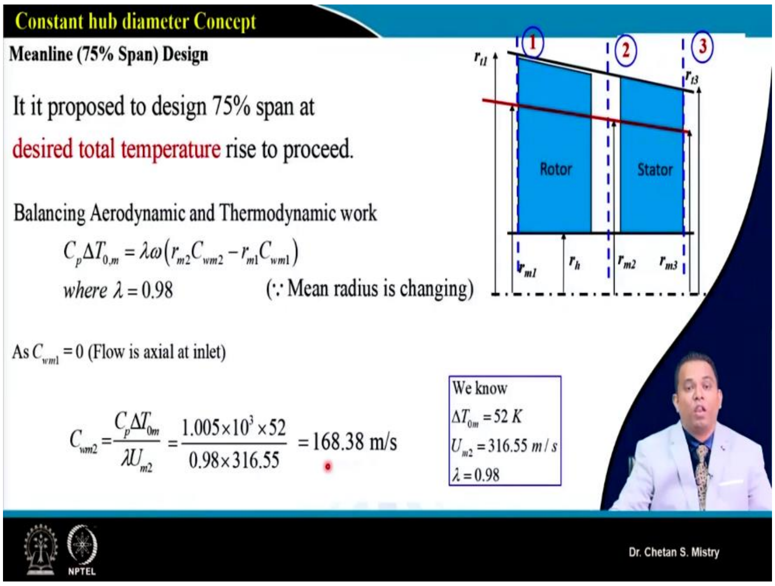

Constant hub diameter Concept

Meanline (75% Span) Design

It it proposed to design 75% span at desired total temperature rise to proceed.

Balancing Aerodynamic and Thermodynamic work

$$C_p \Delta T_{0,m} = \lambda \omega \left(r_{m2} C_{wm2} - r_{m1} C_{wm1}\right)$$

$where\ \lambda = 0.98$ (∵ Mean radius is changing)

As $C_{wm1} = 0$ (Flow is axial at inlet)

$$C_{wm2} = \frac{C_p \Delta T_{0m}}{\lambda U_{m2}} = \frac{1.005 \times 10^3 \times 52}{0.98 \times 316.55} = 168.38\ \text{m/s}$$

We know
$\Delta T_{0m} = 52\ K$
$U_{m2} = 316.55\ m/s$
$\lambda = 0.98$

So, what we will be doing? We will be assuming our aerodynamic work and thermodynamic work, that's what is to be same. Or we can say, here, we are writing $C_p \Delta T_0$, that's what is say, $\lambda \cdot \omega$. Here, we need to be very careful, say this radius, say when we were doing our design for subsonic compressor, that time we were writing that as say, $U_2 C_{w2} - U_1 C_{w1}$. But here in this case, say there U1 and U2 that's what our radius that's what was coming to be say same. And, that is the reason why we are putting $U_2 C_{w2} - U_1 C_{w1}$. But here in this case, our radius that's what is different.

And, that is the reason why we are writing as say $r_{m2} \cdot \omega_{m2} - r_{m1} \cdot \omega_{m1}$, and we will be putting this is what is say $\lambda\omega(rC_{w2} - rC_{w1})$. Be careful about when we are doing our calculation for say supersonic or say transonic compressor, okay. We are assuming our entry condition to be axial, and that is the reason my whirl component we are assuming to be 0 m/s. And that's what will be giving me my whirl velocity at the exit of my rotor, and that's what is coming 168.38 m/s.

$$Balancing\ Aerodynamic\ and\ Thermodynamic\ work,$$

$$C_p \Delta T_{0,m} = \lambda\omega(r_{m2} C_{wm2} - r_{m1} C_{wm1})$$

$$where\ \lambda = 0.98$$

$$As\ C_{wm1} = 0\ (flow\ is\ axial\ at\ inlet)$$

$$C_{wm2} = \frac{C_p \Delta T_{0m}}{\lambda U_{m2}} = \frac{1.005 \times 10^3 \times 52}{0.98 \times 316.55} = 168.38\ m/s$$

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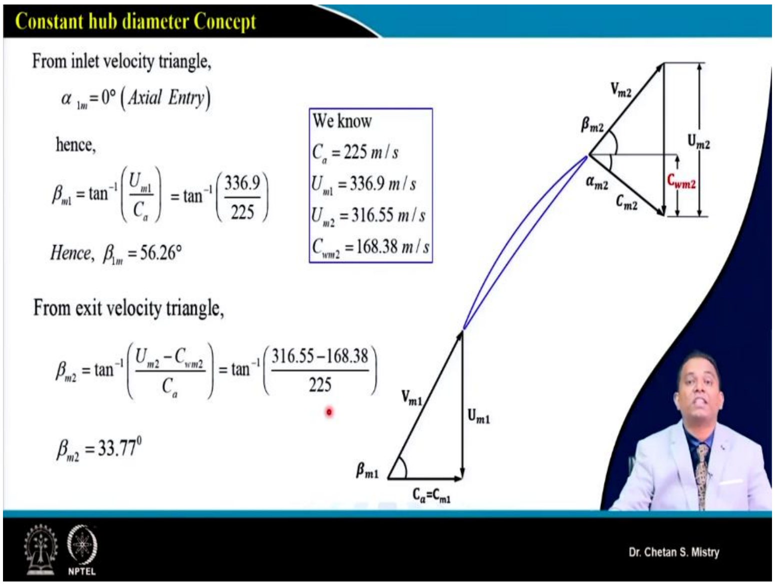

Constant hub diameter Concept

From inlet velocity triangle,

$\alpha_{1m} = 0° \ (Axial\ Entry)$

hence,

$\beta_{m1} = \tan^{-1}\left(\frac{U_{m1}}{C_a}\right) = \tan^{-1}\left(\frac{336.9}{225}\right)$

$Hence,\ \beta_{1m} = 56.26°$

We know
$C_a = 225\ m/s$
$U_{m1} = 336.9\ m/s$
$U_{m2} = 316.55\ m/s$
$C_{wm2} = 168.38\ m/s$

From exit velocity triangle,

$\beta_{m2} = \tan^{-1}\left(\frac{U_{m2} - C_{wm2}}{C_a}\right) = \tan^{-1}\left(\frac{316.55 - 168.38}{225}\right)$

$\beta_{m2} = 33.77^0$

Now, this is what we realise. Now, α_1, that's what is known to us, my peripheral speed is known to me, axial velocity is also known to us. For that, we can do our calculation for say angles β_1, β_2. We can calculate what will be our absolute velocity at the entry, what will be our absolute velocity at the exit, my relative velocity at the entry, and relative velocity at the exit. So, this we are writing in sense, if we configure, this is what is say my entry velocity triangle. Since this is what is my axial entry, so $\alpha_1 = 0$. So, from trigonometry, we can say, tan β, that's what is given by U/C_a. And, that's what is coming 56.26°.

$$From\ inlet\ velocity\ triangle,$$

$$\alpha_{1m} = 0° \ (Axial\ Entry)$$

$$Hence,$$

$$\beta_{m1} = \tan^{-1}\left(\frac{U_{m1}}{C_a}\right) = \tan^{-1}\left(\frac{336.9}{225}\right)$$

$$Hence,\ \ \beta_{1m} = 56.26°$$

Now, same way at the exit, we can write down say tan β, that's what is $\frac{U-C_{w2}}{C_a}$, and that's what is giving my β_2 at the mid station, at the exit of rotor as 33.77°.

$$From\ exit\ velocity\ triangle,$$

$$\beta_{m2} = \tan^{-1}\left(\frac{U_{m2} - C_{wm2}}{C_a}\right) = \tan^{-1}\left(\frac{316.55 - 168.38}{225}\right)$$

$$\beta_{m2} = 33.77°$$

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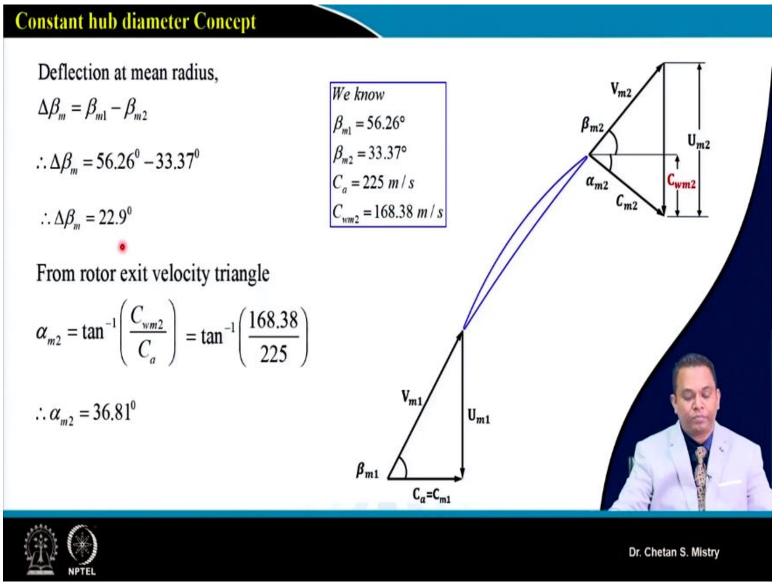

Constant hub diameter Concept

Deflection at mean radius,

$$\Delta\beta_m = \beta_{m1} - \beta_{m2}$$

$$\therefore \Delta\beta_m = 56.26^0 - 33.37^0$$

$$\therefore \Delta\beta_m = 22.9^0$$

From rotor exit velocity triangle

$$\alpha_{m2} = \tan^{-1}\left(\frac{C_{wm2}}{C_a}\right) = \tan^{-1}\left(\frac{168.38}{225}\right)$$

$$\therefore \alpha_{m2} = 36.81^0$$

We know

$\beta_{m1} = 56.26°$

$\beta_{m2} = 33.37°$

$C_a = 225\ m/s$

$C_{wm2} = 168.38\ m/s$

Now, once we are calculating our β_1 and β_2, we can calculate what will be our $\Delta\beta$ and that's what is coming 22.9°, okay.

$$Deflection\ at\ mean\ radius,$$

$$\Delta\beta_m = \beta_{m1} - \beta_{m2}$$

$$\therefore \Delta\beta_m = 56.26° - 33.37°$$

$$\therefore \Delta\beta_m = 22.9°$$

Same way, we can do our calculation for α at the exit, so α_2 at the exit, that's what is given by C_{w2}/U_a. These numbers are unknown to us, we can calculate our α_2, that's what is coming 36.81°.

$$From\ rotor\ exit\ velocity\ triangle,$$

$$\alpha_{m2} = \tan^{-1}\left(\frac{C_{wm2}}{C_a}\right) = \tan^{-1}\left(\frac{168.38}{225}\right)$$

$$\therefore \alpha_{m2} = 36.81°$$

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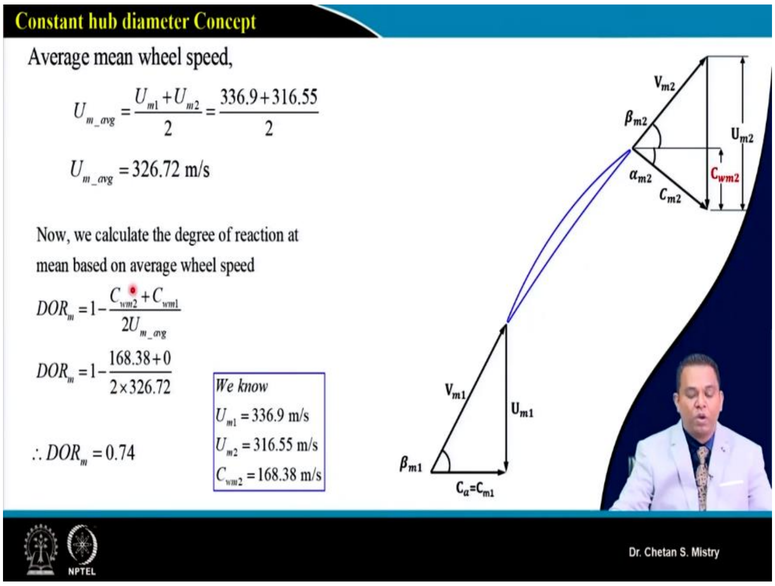

Constant hub diameter Concept

Average mean wheel speed,

$$U_{m\_avg} = \frac{U_{m1}+U_{m2}}{2} = \frac{336.9+316.55}{2}$$

$$U_{m\_avg} = 326.72\ \text{m/s}$$

Now, we calculate the degree of reaction at mean based on average wheel speed

$$DOR_m = 1 - \frac{C_{wm2}+C_{wm1}}{2U_{m\_avg}}$$

$$DOR_m = 1 - \frac{168.38+0}{2\times 326.72}$$

$$\therefore DOR_m = 0.74$$

We know

$U_{m1} = 336.9$ m/s

$U_{m2} = 316.55$ m/s

$C_{wm2} = 168.38$ m/s

Now, what we are looking for is observation for our degree of reaction at the mid station. And as we have discussed, this degree of reaction that's what we are calculating as say $1 - \frac{C_{w2}+C_{w1}}{2U_{m\_avg}}$. Here, in order to avoid the confusion, we will be taking say mean or we can say average peripherals speed, and that’s what is say 326.72 m/s. That’s what is giving degree of reaction as say 0.74.

$$Average\ mean\ wheel\ speed,$$

$$U_{m\_avg} = \frac{U_{m1} + U_{m2}}{2} = \frac{336.9 + 316.55}{2}$$

$$U_{m\_avg} = 326.72 m/s$$

$$Now, we\ calculate\ the\ degree\ of\ reaction\ at\ mean\ based\ on\ average\ wheel\ speed,$$

$$DOR_m = 1 - \frac{C_{wm2} + C_{wm1}}{2U_{m\_avg}}$$

$$DOR_m = 1 - \frac{168.38 + 0}{2 \times 326.72}$$

$$\therefore DOR_m = 0.74$$

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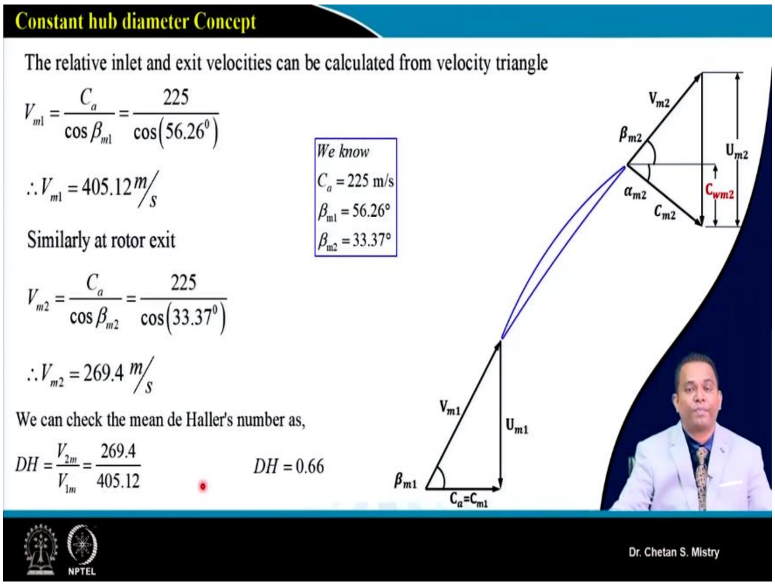

Constant hub diameter Concept

The relative inlet and exit velocities can be calculated from velocity triangle

$$V_{m1} = \frac{C_a}{\cos\beta_{m1}} = \frac{225}{\cos\left(56.26^0\right)}$$

$$\therefore V_{m1} = 405.12 m/s$$

Similarly at rotor exit

$$V_{m2} = \frac{C_a}{\cos\beta_{m2}} = \frac{225}{\cos\left(33.37^0\right)}$$

$$\therefore V_{m2} = 269.4\ m/s$$

We can check the mean de Haller's number as,

$$DH = \frac{V_{2m}}{V_{1m}} = \frac{269.4}{405.12} \qquad DH = 0.66$$

We know
$C_a = 225$ m/s
$\beta_{m1} = 56.26°$
$\beta_{m2} = 33.37°$

Now, next component, that’s what is of our interest, that’s what is our relative velocity at the entry, and relative velocity at the exit of the rotor.

So, from trigonometry, if we will be taking say cosine, say $\cos\beta_1$, that’s what is say C_a/V_1, that’s what is given say V_1 as say 405.12 m/s.

$$The\ relative\ inlet\ and\ exit\ velocities\ can\ be\ calculated\ from\ velocity\ triangle,$$

$$V_{m1} = \frac{C_a}{\cos \beta_{m1}} = \frac{225}{\cos(56.26°)}$$

$$\therefore V_{m1} = 405.12\ m/s$$

Now, same way we can do our calculation for relative velocity, at the exit and that's what is coming as say 269.4.

$$Similarly\ at\ rotor\ exit,$$

$$V_{m2} = \frac{C_a}{\cos \beta_{m2}} = \frac{225}{\cos(33.37°)}$$

$$\therefore V_{m2} = 269.4\ m/s$$

You can say, here we are having effective kind of diffusion, that's what is happening. My relative velocity at the entry is higher, my relative velocity at the exit, that's what is going to be lower. And that's what we are calculating as say de Haller's factor. And that's what is coming say 0.66.

$$We\ can\ check\ the\ mean\ de-Haller's\ number\ as,$$

$$DH = \frac{V_{2m}}{V_{1m}} = \frac{269.4}{405.12}$$

$$DH = 0.66$$

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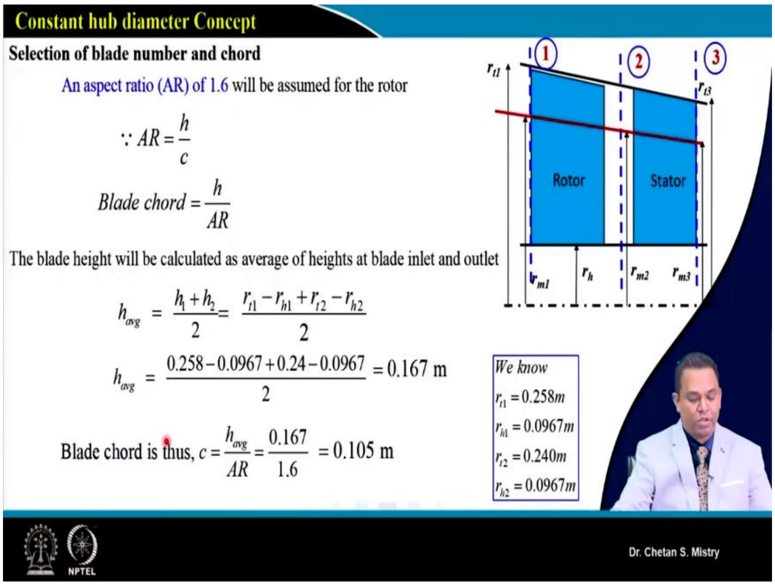

Constant hub diameter Concept

Selection of blade number and chord

An aspect ratio (AR) of 1.6 will be assumed for the rotor

$$\because AR = \frac{h}{c}$$

$$Blade\ chord = \frac{h}{AR}$$

The blade height will be calculated as average of heights at blade inlet and outlet

$$h_{avg} = \frac{h_1 + h_2}{2} = \frac{r_{t1} - r_{h1} + r_{t2} - r_{h2}}{2}$$

$$h_{avg} = \frac{0.258 - 0.0967 + 0.24 - 0.0967}{2} = 0.167\ \text{m}$$

$$\text{Blade chord is thus, } c = \frac{h_{avg}}{AR} = \frac{0.167}{1.6} = 0.105\ \text{m}$$

We know

$r_{t1} = 0.258m$

$r_{h1} = 0.0967m$

$r_{t2} = 0.240m$

$r_{h2} = 0.0967m$

Now, next important parameter for us, that's what will be to decide with the number of blades, and what will be the chord of my blade? So, as we have discussed our aspect ratio in earlier design, that's what was coming say 1.6. So, in order to compare these two designs, in that sense, we will not be changing this aspect ratio, let us take same aspect ratio here for constant tip diameter configuration. So, aspect ratio, I am taking as say 1.6.

Now, in order to calculate the height of my blade or the span of my blade, that's what we are doing as say $\frac{h_1+h_2}{2}$. So, h_1 and h_2, that's what is given by $r_t - r_h$ at entry, and $r_t - r_h$ at the exit. And, this is what is coming to be 0.167 m. Based on that, we will be calculating our chord and that's what is coming as say 0.105 m.

$$An\ aspect\ ratio\ (AR)\ of\ 1.6\ will\ be\ assumed\ for\ the\ rotor$$

$$\because AR = \frac{h}{c}$$

$$c = \frac{h}{AR}$$

$$The\ blade\ height\ will\ be\ calculated\ as\ average\ of\ heights\ at\ blade\ inlet\ and\ outlet$$

$$h_{avg} = \frac{h_1 + h_2}{2} = \frac{r_{t1} - r_{h1} + r_{t2} - r_{h2}}{2}$$

$$h_{avg} = \frac{0.258 - 0.0967 + 0.24 - 0.0967}{2} = 0.167\ m$$

$$Blade\ chord, c = \frac{h_{avg}}{AR} = \frac{0.167}{1.6} = 0.105\ m$$

Now, if we compare this number with our earlier calculation, with say constant tip diameter configuration, this chord was coming 0.09 m meter. So, you can say, here our chord that's what is coming to be larger, okay. So, the changes, that's what will start reflecting now.

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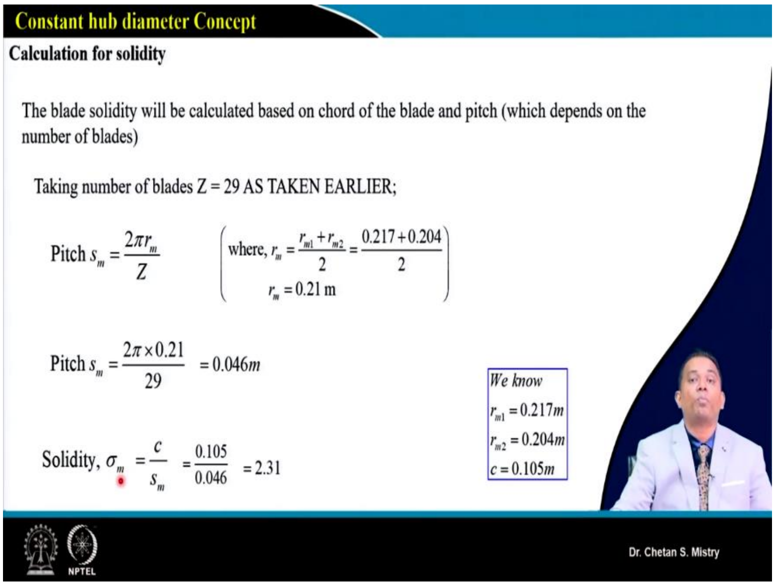

Constant hub diameter Concept

Calculation for solidity

The blade solidity will be calculated based on chord of the blade and pitch (which depends on the number of blades)

Taking number of blades Z = 29 AS TAKEN EARLIER;

$$\text{Pitch } s_m = \frac{2\pi r_m}{Z} \qquad \left(\text{where, } r_m = \frac{r_{m1}+r_{m2}}{2} = \frac{0.217+0.204}{2} \quad r_m = 0.21\text{ m} \right)$$

$$\text{Pitch } s_m = \frac{2\pi \times 0.21}{29} = 0.046m$$

$$\text{Solidity, } \sigma_m = \frac{c}{s_m} = \frac{0.105}{0.046} = 2.31$$

We know
$r_{m1} = 0.217m$
$r_{m2} = 0.204m$
$c = 0.105m$

Dr. Chetan S. Mistry

Same way, in earlier design, we have assumed our number of blades for the rotor as say 29. So, we will not be changing those number of blades, okay. So, number of blades, that’s what we have assumed to be 29.

Same we will be putting here, that’s what will be giving us the pitch to be 0.046 m. Now, based on pitch and chord, we can calculate our solidity, and that’s what is coming as say 2.31.

$$Taking\ number\ of\ blades\ Z = 29\ (as\ taken\ earlier)$$

$$Pitch, s_m = \frac{2\pi r_m}{Z}$$

$$where, r_m = \frac{r_{m1} + r_{m2}}{2} = \frac{0.217 + 0.204}{2} = 0.21\ m$$

$$Pitch, s_m = \frac{2\pi \times 0.21}{29} = 0.046\ m$$

$$Solidity, \sigma_m = \frac{c}{s_m} = \frac{0.105}{0.046} = 2.31$$

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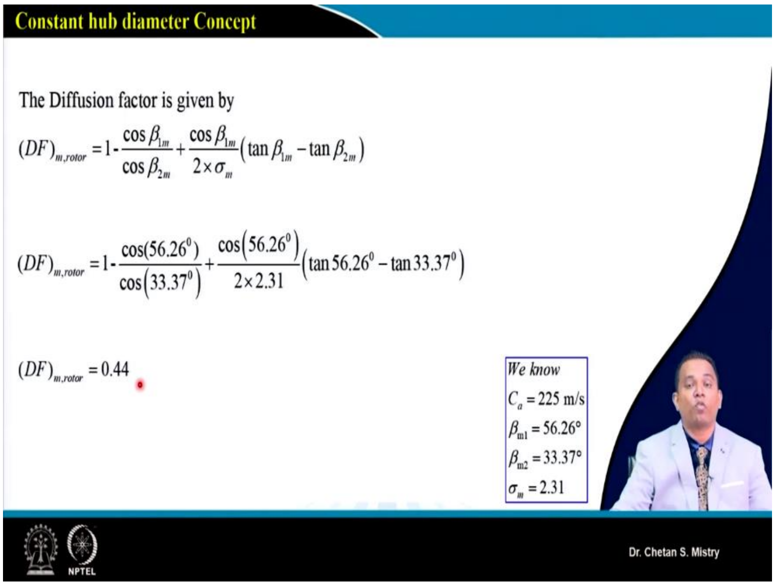

Constant hub diameter Concept

The Diffusion factor is given by

$$(DF)_{m,rotor} = 1 - \frac{\cos\beta_{1m}}{\cos\beta_{2m}} + \frac{\cos\beta_{1m}}{2\times\sigma_m}\left(\tan\beta_{1m} - \tan\beta_{2m}\right)$$

$$(DF)_{m,rotor} = 1 - \frac{\cos(56.26^0)}{\cos(33.37^0)} + \frac{\cos(56.26^0)}{2\times 2.31}\left(\tan 56.26^0 - \tan 33.37^0\right)$$

$$(DF)_{m,rotor} = 0.44$$

We know

$C_a = 225$ m/s

$\beta_{m1} = 56.26°$

$\beta_{m2} = 33.37°$

$\sigma_m = 2.31$

Dr. Chetan S. Mistry

Once, we know what is our solidity, my immediate parameter that need to be calculated, that’s what is say my diffusion factor. So, the diffusion factor of this rotor, that’s what we can calculate based on beta 1, beta 2 and based on my what is the solidity, so if you are writing this, this is what is coming as 0.44.

The Diffusion factor is given by,

$$(DF)_{m,rotor} = 1 - \frac{\cos\beta_{m1}}{\cos\beta_{m2}} + \frac{\cos\beta_{m1}}{2\times\sigma_m}(\tan\beta_{m1} - \tan\beta_{m2})$$

$$= 1 - \frac{\cos(56.26°)}{\cos(33.37°)} + \frac{\cos(56.26°)}{2\times 2.31}(\tan 56.26° - \tan 33.37°)$$

$$(DF)_{m,rotor} = 0.44$$

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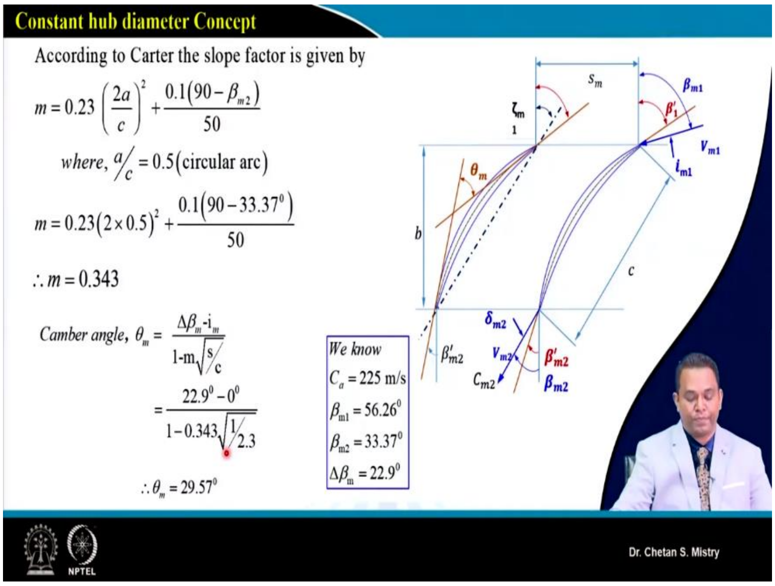

Constant hub diameter Concept

According to Carter the slope factor is given by

$$m = 0.23\left(\frac{2a}{c}\right)^2 + \frac{0.1(90-\beta_{m2})}{50}$$

$$where,\ a/c = 0.5(\text{circular arc})$$

$$m = 0.23(2\times0.5)^2 + \frac{0.1(90-33.37^0)}{50}$$

$$\therefore m = 0.343$$

$$Camber\ angle,\ \theta_m = \frac{\Delta\beta_m - i_m}{1-m\sqrt{s/c}}$$

$$= \frac{22.9^0 - 0^0}{1-0.343\sqrt{1/2.3}}$$

$$\therefore \theta_m = 29.57^0$$

We know

$C_a = 225$ m/s

$\beta_{m1} = 56.26^0$

$\beta_{m2} = 33.37^0$

$\Delta\beta_m = 22.9^0$

Dr. Chetan S. Mistry

Now, in order to calculate my blade parameters, very first parameter, which we need to calculate is say slope factor or say Carter’s parameter. And, that’s what we are writing here, we are considering our say camber line, that’s what is say circular arc camber line, and that is the reason my a/c that’s what we are taking as say 0.5, and my parameter or m factor that’s what is coming as say 0.343.

Now, once we know what is our parameter m, we can calculate our camber angle. That’s what is $\Delta\beta - i$, and as a function of say my m parameter and solidity. So, if you are doing this calculation, my camber angle at the mid station, that’s what is coming 29.57°.

$$According\ to\ Carter, the\ slope\ factor\ is\ given\ by$$

$$m = 0.23\left(\frac{2a}{c}\right)^2 + \frac{0.1(90-\beta_{m2})}{50}$$

$$where, \frac{a}{c} = 0.5(circular\ arc)$$

$$m = 0.23(2\times0.5)^2 + \frac{0.1(90-33.37°)}{50}$$

$$\therefore m = 0.343$$

$$Camber\ angle, \theta_m = \frac{\Delta\beta_m - i_m}{1-m\sqrt{S/c}}$$

$$= \frac{22.9° - 0°}{1 - 0.343\sqrt{1/_{2.3}}}$$

$$\therefore \theta_m = 29.57°$$

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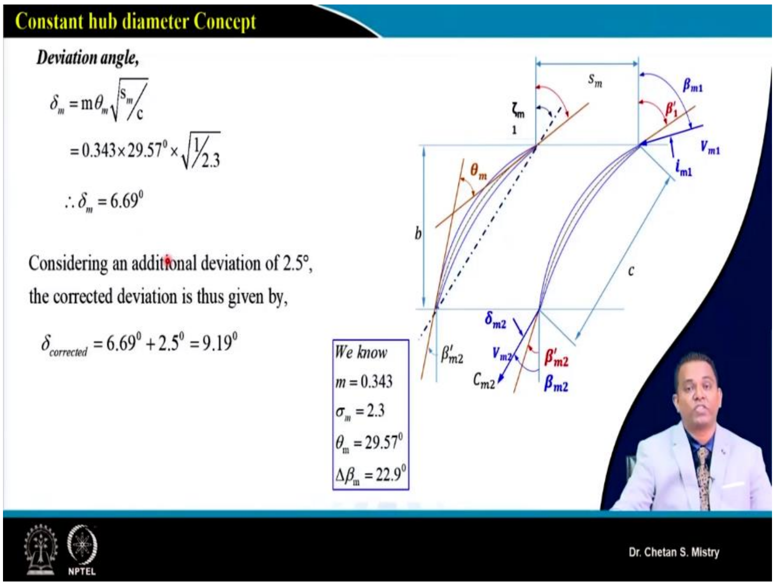

Constant hub diameter Concept

Deviation angle,

$$\delta_m = m\theta_m\sqrt{s_m/c}$$

$$= 0.343 \times 29.57^0 \times \sqrt{1/2.3}$$

$$\therefore \delta_m = 6.69^0$$

Considering an additional deviation of 2.5°, the corrected deviation is thus given by,

$$\delta_{corrected} = 6.69^0 + 2.5^0 = 9.19^0$$

We know

$m = 0.343$

$\sigma_m = 2.3$

$\theta_m = 29.57^0$

$\Delta\beta_m = 22.9^0$

Dr. Chetan S. Mistry

Same way, we can use our Carter's relation for calculation of deviation angle, and that's what is coming as say 6.69. Similar to what all we have discussed earlier for almost all design based on your computational study or based on your past experience, you can do the correction for your deviation angle. Same procedure we will be following here also.

Deviation angle,

$$\delta_m = m\theta_m\sqrt{s_m/_{c}}$$

$$= 0.343 \times 29.57°\sqrt{1/_{2.3}}$$

$$\therefore \delta_m = 6.69°$$

Considering an additional deviation of 2.5°, *the corrected deviation is thus given by,*

$$\delta_{corrected} = 6.69° + 2.5° = 9.19°$$

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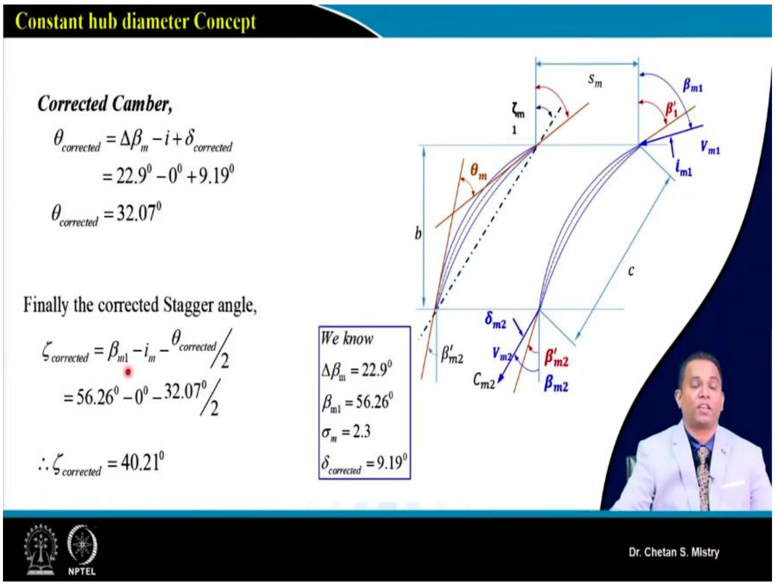

Constant hub diameter Concept

Corrected Camber,

$$\theta_{corrected} = \Delta\beta_m - i + \delta_{corrected}$$
$$= 22.9^0 - 0^0 + 9.19^0$$
$$\theta_{corrected} = 32.07^0$$

Finally the corrected Stagger angle,

$$\zeta_{corrected} = \beta_{m1} - i_m - \theta_{corrected}/2$$
$$= 56.26^0 - 0^0 - 32.07^0/2$$
$$\therefore \zeta_{corrected} = 40.21^0$$

We know
$\Delta\beta_m = 22.9^0$
$\beta_{m1} = 56.26^0$
$\sigma_m = 2.3$
$\delta_{corrected} = 9.19^0$

Dr. Chetan S. Mistry

And, we can do our camber angle correction. And, we can do our calculation for say Stagger angle correction. So, these are the angles what we are getting in sense of 32.07, that's what is my corrected camber angle, and 40.21, that's what is my corrected Stagger angle.

$$Corrected\ Camber,$$

$$\theta_{corrected} = \Delta\beta_m - i + \delta_{corrected}$$

$$= 22.9° - 0° + 9.19°$$

$$\theta_{corrected} = 32.07°$$

$$Finally\ the\ corrected\ Stagger\ angle,$$

$$\zeta_{corrected} = \beta_{m1} - i_m - {}^{\theta_{corrected}}/_{2}$$

$$= 56.26° - 0° - {}^{32.07°}/_{2}$$

$$\therefore \zeta_{corrected} = 40.21°$$

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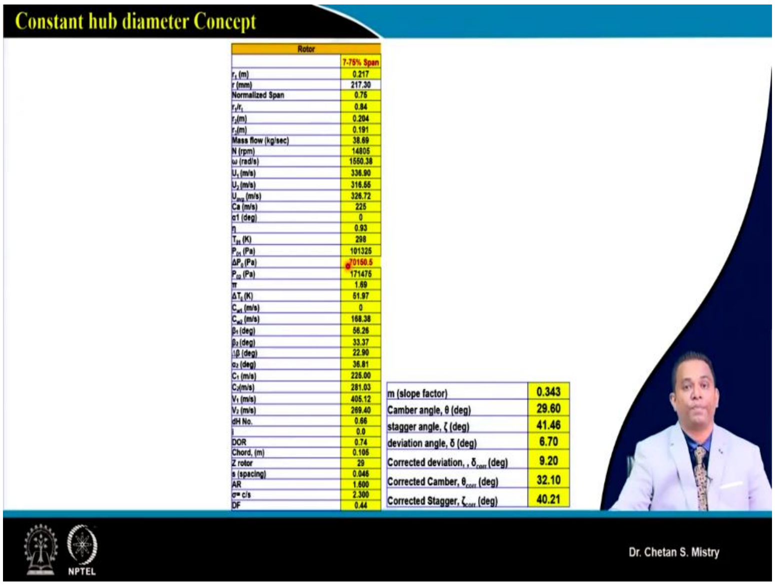

Constant hub diameter Concept

Rotor	
	7-75% Span
r_1 (m)	0.217
r (mm)	217.30
Normalized Span	0.76
r_1/r_1	0.84
r_2(m)	0.204
r_3(m)	0.191
Mass flow (kg/sec)	38.69
N (rpm)	14806
ω (rad/s)	1550.38
U_1 (m/s)	336.90
U_2 (m/s)	316.55
U_{avg} (m/s)	326.72
Ca (m/s)	225
α1 (deg)	0
η	0.93
T_{01} (K)	298
P_{01} (Pa)	101325
ΔP_0 (Pa)	70150.5
P_{02} (Pa)	171475
π	1.69
ΔT_0 (K)	51.97
C_{w1} (m/s)	0
C_{w2} (m/s)	168.38
β_1 (deg)	56.26
β_2 (deg)	33.37
$\Delta\beta$ (deg)	22.90
α_2 (deg)	36.81
C_1 (m/s)	225.00
C_2(m/s)	281.03
V_1 (m/s)	405.12
V_2 (m/s)	269.40
dH No.	0.66
i	0.0
DOR	0.74
Chord, (m)	0.106
Z rotor	29
s (spacing)	0.046
AR	1.600
σ= c/s	2.300
DF	0.44

m (slope factor)	0.343
Camber angle, θ (deg)	29.60
stagger angle, ζ (deg)	41.46
deviation angle, δ (deg)	6.70
Corrected deviation, , δ_{corr} (deg)	9.20
Corrected Camber, θ_{corr} (deg)	32.10
Corrected Stagger, ζ_{corr} (deg)	40.21

Now, in overall if we look at, this is what is my station. So, if we configure here, as I was saying, we need to be very careful in sense of my radius. So, this is what is my radius r_1, and this is what is my radius r_2, this is what is my radius r_3. Because that's what will be giving me U_1 and U_2 to be different, okay. So, based on that this is what is a table, that's what we have prepared, assuming our ΔP_0 that to be here, okay.

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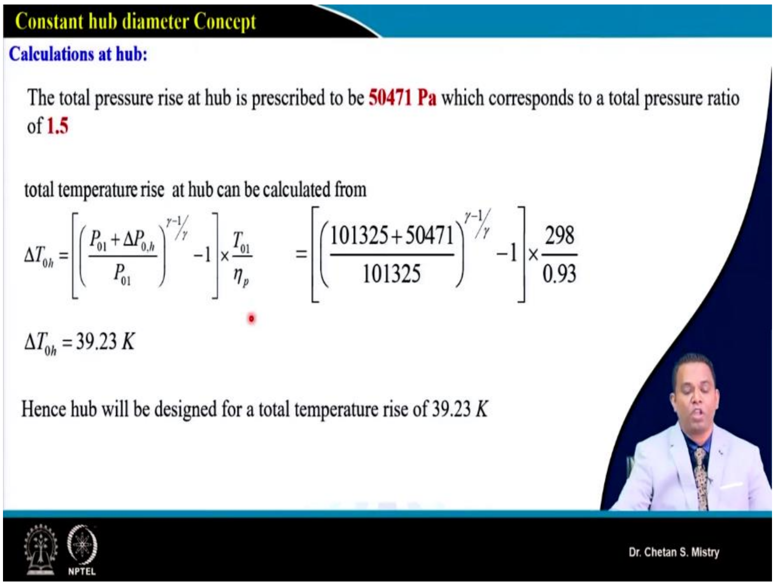

Constant hub diameter Concept

Calculations at hub:

The total pressure rise at hub is prescribed to be **50471 Pa** which corresponds to a total pressure ratio of **1.5**

total temperature rise at hub can be calculated from

$$\Delta T_{0h} = \left[\left(\frac{P_{01}+\Delta P_{0,h}}{P_{01}}\right)^{\gamma-1/\gamma} - 1\right] \times \frac{T_{01}}{\eta_p} \quad = \left[\left(\frac{101325+50471}{101325}\right)^{\gamma-1/\gamma} - 1\right] \times \frac{298}{0.93}$$

$$\Delta T_{0h} = 39.23\ K$$

Hence hub will be designed for a total temperature rise of 39.23 K

Now, once this is what is we have done at the mid station. Next step for us it is to calculate at hub station as well as at the tip station. So, in order to do calculation at the hub station, we are following the approach, that's what is say our fundamental design approach, we need to assume ΔP_0 near the hub station.

And we know, hub is critical in sense of the having flow turning angle to be large, and it may be possible that my degree of reaction may be going lower in that region. So, it may possible, like we have discussed earlier also, some of the portion near the hub region for this compressor may act like a turbine, if we will not be configuring, or if we are not designing properly.

And that's what will lead to losses, maybe in sense of rise of pressure, or maybe in sense of, say my operating range. And that is the reason why we need to select these number iteratively, okay. And in order to have that flexibility, we are having now our Excel sheet program with which we can do the modification what we are looking for.

Say for design; here, in this case, we have assumed near the hub region, going aggressively, we have taken this pressure ratio expected near the hub to be 1.5. And that's what is giving me my pressure rise near the hub to be 50,471 Pa. Now, once we have resumed this ΔP_0, we can do our calculation for ΔT_0 at the hub station. And that's what is coming 39.23 K.

$$Let's take\ total\ pressure\ rise\ at\ hub\ is\ to\ be\ 50471\ Pa$$

$$which\ corresponds\ to\ a\ total\ pressure\ ratio\ of\ 1.5$$

$$Total\ temperature\ rise\ at\ hub\ can\ be\ calculated\ from$$

$$\Delta T_{0h} = \left[\left(\frac{P_{01} + \Delta P_{0,h}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \times \frac{T_{01}}{\eta_p}$$

$$= \left[\left(\frac{101325 + 50471}{101325}\right)^{\frac{1.4-1}{1.4}} - 1\right] \times \frac{298}{0.93}$$

$$\Delta T_{0h} = 39.23\ K$$

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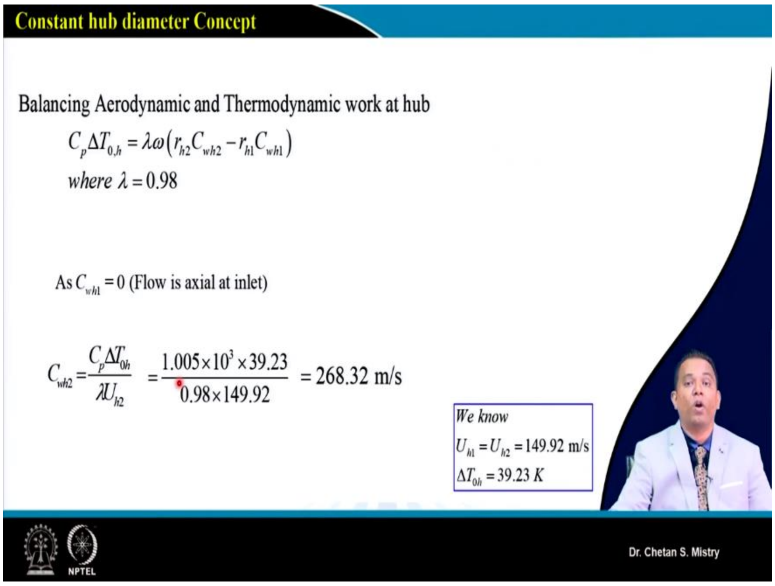

Constant hub diameter Concept

Balancing Aerodynamic and Thermodynamic work at hub

$$C_p\Delta T_{0,h} = \lambda\omega\left(r_{h2}C_{wh2} - r_{h1}C_{wh1}\right)$$

$$where\ \lambda = 0.98$$

As $C_{wh1} = 0$ (Flow is axial at inlet)

$$C_{wh2} = \frac{C_p\Delta T_{0h}}{\lambda U_{h2}} = \frac{1.005\times10^3\times39.23}{0.98\times149.92} = 268.32\ \text{m/s}$$

We know
$U_{h1} = U_{h2} = 149.92$ m/s
$\Delta T_{0h} = 39.23\ K$

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Now, if we compare our aerodynamic work and thermodynamic work, we will be getting our whirl component. Be careful, here also what we are discussing, in sense of my, you know, r_2 into C_{w2}. Be careful about this design calculation. Do not make any mistake here, that's what is giving me my whirl component at the hub to be 268.32 m/s.

$$Balancing\ Aerodynamic\ and\ Thermodynamic\ work\ at\ hub,$$

$$C_p\Delta T_{0,h} = \lambda\omega(r_{h2}C_{wh2} - r_{h1}C_{wh1})$$

$$where\ \lambda = 0.98$$

$$As\ C_{wh1} = 0\ (Flow\ is\ axial\ at\ inlet)$$

$$C_{wh2} = \frac{C_p\Delta T_{0h}}{\lambda U_{h2}} = \frac{1.005\times10^3\times39.23}{0.98\times149.92} = 268.32\ m/s$$

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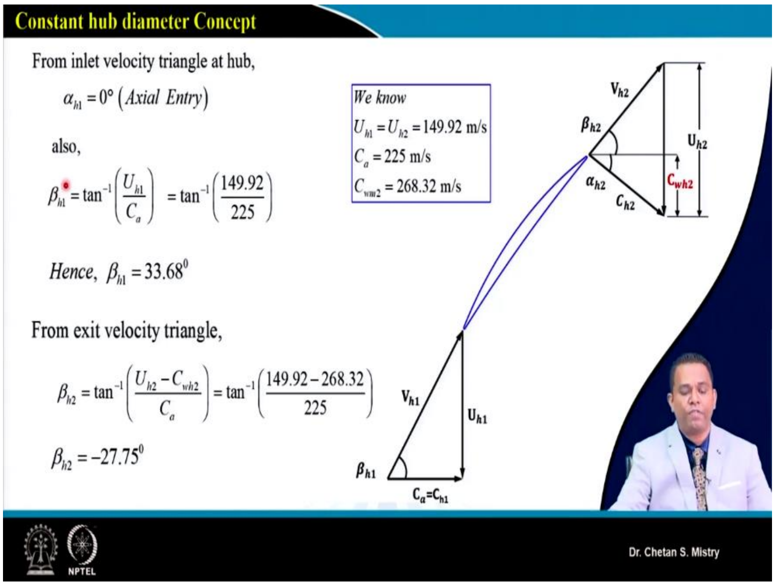

Constant hub diameter Concept

From inlet velocity triangle at hub,

$\alpha_{h1} = 0° \ (Axial\ Entry)$

also,

$\beta_{h1} = \tan^{-1}\left(\frac{U_{h1}}{C_a}\right) = \tan^{-1}\left(\frac{149.92}{225}\right)$

$Hence,\ \beta_{h1} = 33.68^0$

From exit velocity triangle,

$\beta_{h2} = \tan^{-1}\left(\frac{U_{h2} - C_{wh2}}{C_a}\right) = \tan^{-1}\left(\frac{149.92 - 268.32}{225}\right)$

$\beta_{h2} = -27.75^0$

We know

$U_{h1} = U_{h2} = 149.92$ m/s

$C_a = 225$ m/s

$C_{wm2} = 268.32$ m/s

Now, once we know what is our axial velocity, our peripheral speed, one of the whirl component based on our velocity triangle at the hub station, this is what is at the hub station, okay. If you are putting this, that's what will be giving me my tan β_1 as say U/C_a, if you are putting these numbers, that's what will be giving me my β_1 to be 33.68°. Same way, we can do our calculation at the exit of the rotor, and that's what is coming $-27.75°$.

$$From\ inlet\ velocity\ triangle,$$

$$\alpha_{h1} = 0° \ (Axial\ Entry)$$

$$also,$$

$$\beta_{h1} = \tan^{-1}\left(\frac{U_{h1}}{C_a}\right) = \tan^{-1}\left(\frac{149.92}{225}\right)$$

$$Hence,\ \ \beta_{h1} = 33.68°$$

$$From\ exit\ velocity\ triangle,$$

$$\beta_{h2} = \tan^{-1}\left(\frac{U_{h2} - C_{wh2}}{C_a}\right) = \tan^{-1}\left(\frac{149.92 - 268.32}{225}\right)$$

$$\beta_{h2} = -27.75°$$

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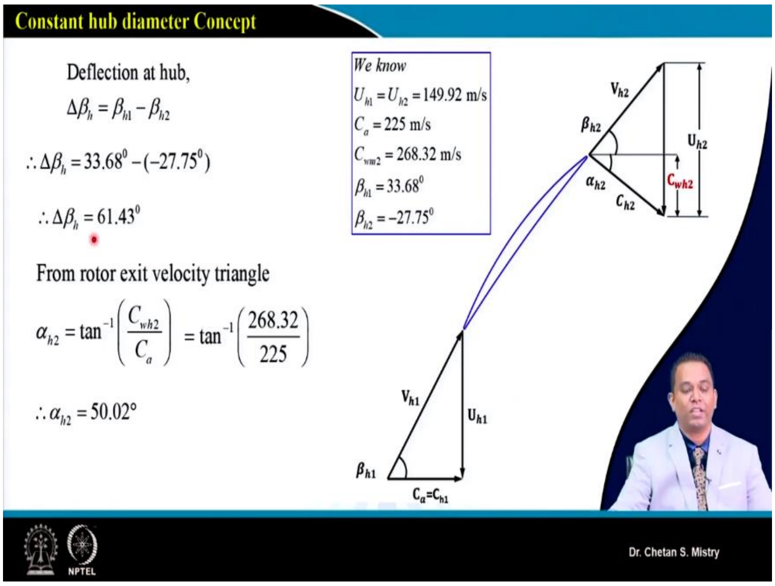

Constant hub diameter Concept

Deflection at hub,

$$\Delta\beta_h = \beta_{h1} - \beta_{h2}$$

$$\therefore \Delta\beta_h = 33.68^0 - (-27.75^0)$$

$$\therefore \Delta\beta_h = 61.43^0$$

From rotor exit velocity triangle

$$\alpha_{h2} = \tan^{-1}\left(\frac{C_{wh2}}{C_a}\right) = \tan^{-1}\left(\frac{268.32}{225}\right)$$

$$\therefore \alpha_{h2} = 50.02^\circ$$

We know

$U_{h1} = U_{h2} = 149.92$ m/s

$C_a = 225$ m/s

$C_{wm2} = 268.32$ m/s

$\beta_{h1} = 33.68^0$

$\beta_{h2} = -27.75^0$

Dr. Chetan S. Mistry

Now, here in this case, my $\Delta\beta$ as we were discussing near the hub, that's what is coming to be large, this is what is say 61.43°, okay. Now, we can do our calculation for what will be the α_2 at the hub station, and that's what is coming 50.02°.

Deflection at hub,

$$\Delta\beta_h = \beta_{h1} - \beta_{h2}$$

$$\therefore \Delta\beta_h = 33.68^\circ - (-27.75^\circ)$$

$$\therefore \Delta\beta_h = 61.43^\circ$$

From rotor exit velocity triangle,

$$\alpha_{h2} = \tan^{-1}\left(\frac{C_{wh2}}{C_a}\right) = \tan^{-1}\left(\frac{268.32}{225}\right)$$

$$\therefore \alpha_{h2} = 50.02^\circ$$

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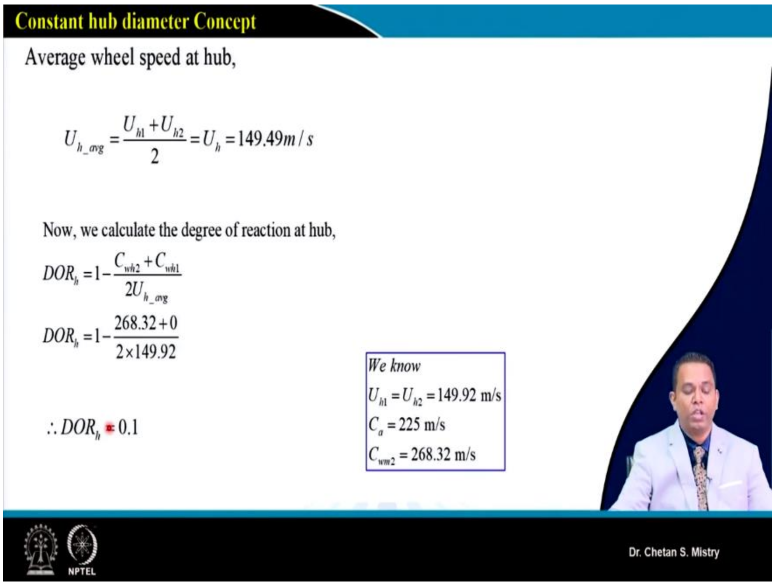

Constant hub diameter Concept

Average wheel speed at hub,

$$U_{h\_avg} = \frac{U_{h1} + U_{h2}}{2} = U_h = 149.49 m/s$$

Now, we calculate the degree of reaction at hub,

$$DOR_h = 1 - \frac{C_{wh2} + C_{wh1}}{2U_{h\_avg}}$$

$$DOR_h = 1 - \frac{268.32 + 0}{2 \times 149.92}$$

$$\therefore DOR_h = 0.1$$

We know

$U_{h1} = U_{h2} = 149.92$ m/s

$C_a = 225$ m/s

$C_{wm2} = 268.32$ m/s

Dr. Chetan S. Mistry

Now, in order to calculate our degree of reaction, we can do the calculation for degree of reaction. And if you look at my degree of reaction at the hub, that's what is coming 0.1.

$$Average\ wheel\ speed\ at\ hub,$$

$$U_{h\_avg} = \frac{U_{h1} + U_{h2}}{2} = U_h = 149.49\ m/s$$

$$Now, we\ calculate\ the\ degree\ of\ reaction\ at\ hub,$$

$$DOR_h = 1 - \frac{C_{wh2} + C_{wh1}}{2U_{h\_avg}}$$

$$DOR_h = 1 - \frac{268.32 + 0}{2 \times 149.92}$$

$$\therefore DOR_h = 0.1$$

So, we need to be very careful. And that's what is a flexibility we are getting when you are using Excel sheet program, if you are doing pen, paper calculation, it requires say number of iteration and it may possible you will come up with some wrong solution.

So, it is advisable you use your Excel sheet program and based on that you modify your ΔP_0, keep an eye for your degree of reaction, keep an eye for your $\Delta\beta$, keep on your eye for say your deviation angle, keep an eye for your camber angle. Be careful, this is what is very important because there are more chances to come with say some problem near the hub region, okay. And, that too, this is what is we are configuring constant hub diameter, okay.

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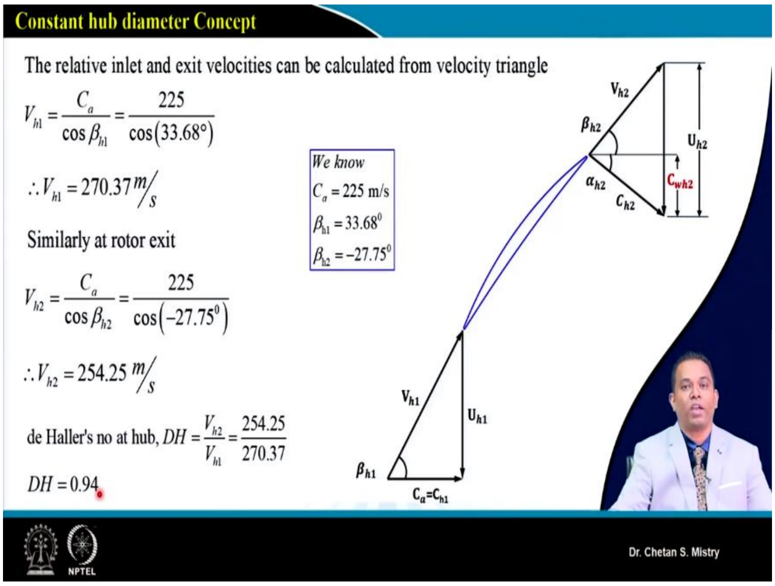

Constant hub diameter Concept

The relative inlet and exit velocities can be calculated from velocity triangle

$$V_{h1} = \frac{C_a}{\cos\beta_{h1}} = \frac{225}{\cos(33.68^\circ)}$$

$$\therefore V_{h1} = 270.37\, {}^{m}\!/_{s}$$

Similarly at rotor exit

$$V_{h2} = \frac{C_a}{\cos\beta_{h2}} = \frac{225}{\cos\left(-27.75^0\right)}$$

$$\therefore V_{h2} = 254.25\, {}^{m}\!/_{s}$$

de Haller's no at hub, $DH = \frac{V_{h2}}{V_{h1}} = \frac{254.25}{270.37}$

$DH = 0.94$

We know
$C_a = 225$ m/s
$\beta_{h1} = 33.68^0$
$\beta_{h2} = -27.75^0$

Now, once we have done this calculation, we can have our relative velocity components at the hub station. So here, if you calculate V_1, that's what is coming 270.37 m/s. Same way, my V_2 at the exit, that's what is coming 254.25 m/s. And if we look at say de-Haller's factor, that's what is coming 0.94, okay.

$$The\ relative\ inlet\ and\ exit\ velocities\ can\ be\ calculated\ from\ velocity\ triangle,$$

$$V_{h1} = \frac{C_a}{\cos\beta_{h1}} = \frac{225}{\cos(33.68°)}$$

$$\therefore V_{h1} = 270.37\ m/s$$

$$Similarly\ at\ rotor\ exit,$$

$$V_{h2} = \frac{C_a}{\cos\beta_{h2}} = \frac{225}{\cos(-27.75°)}$$

$$\therefore V_{h2} = 254.25\ m/s$$

$$de - Haller's\ number\ at\ hub,$$

$$DH = \frac{V_{h2}}{V_{h1}} = \frac{254.25}{270.37}$$

$$DH = 0.94$$

So, this is what is also indicating like we are having diffusion, that's what is happening near the hub region, but you can compare these velocity components, that's what is showing like my diffusion that's what is not happened as per our expectation in that sense, okay.

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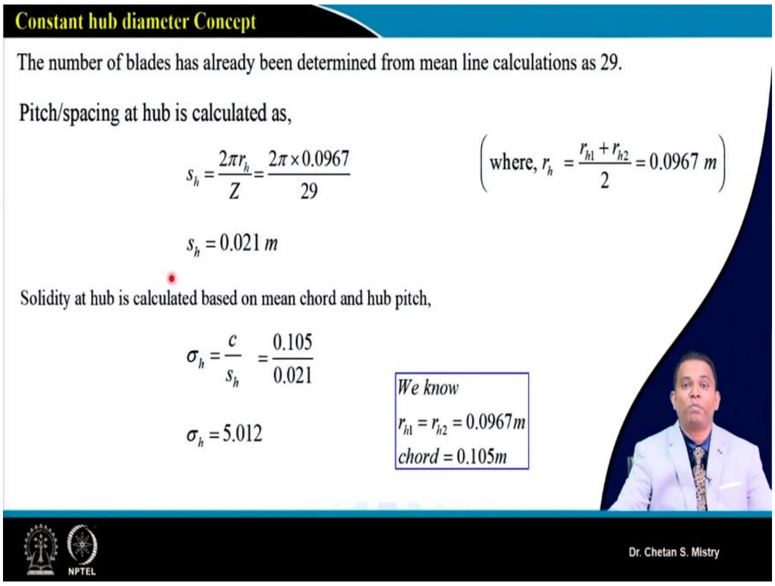

Constant hub diameter Concept

The number of blades has already been determined from mean line calculations as 29.

Pitch/spacing at hub is calculated as,

$$s_h = \frac{2\pi r_h}{Z} = \frac{2\pi \times 0.0967}{29}$$

$$\left(\text{where, } r_h = \frac{r_{h1} + r_{h2}}{2} = 0.0967\ m\right)$$

$$s_h = 0.021\ m$$

Solidity at hub is calculated based on mean chord and hub pitch,

$$\sigma_h = \frac{c}{s_h} = \frac{0.105}{0.021}$$

$$\sigma_h = 5.012$$

We know

$r_{h1} = r_{h2} = 0.0967m$

$chord = 0.105m$

Dr. Chetan S. Mistry

Now, here for say number of blades as we have assumed that's what will be remaining same. So, we will be taking say number of blades to be 29, we can do our calculation for pitch. This also many times we are committing mistake, we are considering pitch to be same, but we must realise, my pitch, it is a function of radius, okay. So, we can do our calculation for solidity and that's what is coming 5.012, okay.

The number of blades has already been determined from meanline calculation as 29

Pitch/spacing at hub is calculated as,

$$s_h = \frac{2\pi r_h}{Z}$$

$$where, r_h = \frac{r_{h1} + r_{h2}}{2} = 0.0967\ m$$

$$= \frac{2\pi \times 0.0967}{29}$$

$$s_h = 0.021\ m$$

$$Solidity\ at\ hub\ is\ calculated\ based\ on\ mean\ chord\ and\ hub\ pitch,$$

$$\sigma_h = \frac{c}{s_h} = \frac{0.105}{0.021}$$

$$\sigma_h = 5.012$$

Now, here in this case, let me put a comment say all designs what all we are discussed or we will be discussing where we are configuring say our chord to be constant from hub to tip. But in order to manage the flow particularly say, when we say near the hub region, many times designers they have preferred to change the chord length. If you are changing our chord length, accordingly we will be having change in our camber angle.

So, there are many iterations people they are doing in order to manage the flow properly near the hub region. So, they will be having variation of chord from say hub to tip. For many industrial designs, maybe industrial compressor as well as for industrial fans, this kind of configuration you can see. Now, at the same time, suppose if we consider, say we are having say exhaust fans, for those exhaust fans, many times for design, they are taking, say chord at the tip region to be large. So, those all are different design philosophies.

And as I told, like with what all numbers you are playing with, and ultimately what all you will be getting in sense of your final shape of the blade, just go with your computational tool, and try to achieve the expected performance. Maybe at a design point, maybe at off design point, just verify with that part. And if that’s what is coming as per your expectation, that’s what will be your design, okay. There are certain rules which we need to keep on eye, but at the same time, superficially there is no such fixed design rule for say compressor as well as for fans, okay.

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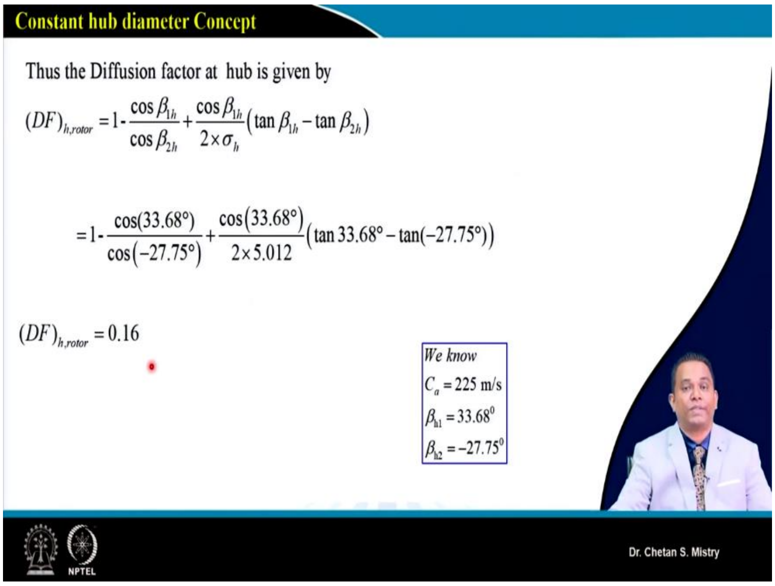

Constant hub diameter Concept

Thus the Diffusion factor at hub is given by

$$(DF)_{h,rotor} = 1 - \frac{\cos\beta_{1h}}{\cos\beta_{2h}} + \frac{\cos\beta_{1h}}{2\times\sigma_h}\left(\tan\beta_{1h} - \tan\beta_{2h}\right)$$

$$= 1 - \frac{\cos(33.68°)}{\cos(-27.75°)} + \frac{\cos(33.68°)}{2\times 5.012}\left(\tan 33.68° - \tan(-27.75°)\right)$$

$$(DF)_{h,rotor} = 0.16$$

We know
$C_a = 225$ m/s
$\beta_{h1} = 33.68^0$
$\beta_{h2} = -27.75^0$

Dr. Chetan S. Mistry

Now, here we will be doing our calculation for the diffusion factor. And as we know our solidity, we can say diffusion factor at the hub, that’s what is coming 0.16.

$$Thus\ the\ Diffusion\ factor\ at\ hub\ is\ given\ by,$$

$$(DF)_{h,rotor} = 1 - \frac{\cos\beta_{1h}}{\cos\beta_{2h}} + \frac{\cos\beta_{1h}}{2\times\sigma_h}(\tan\beta_{1h} - \tan\beta_{2h})$$

$$= 1 - \frac{\cos(33.68°)}{\cos(-27.75°)} + \frac{\cos(33.68°)}{2\times 5.012}(\tan 33.68° - \tan(-27.75°))$$

$$(DF)_{h,rotor} = 0.16$$

So, here, this is what is the discussion about design method using constant hub diameter, and we have done our calculation for say the radius at hub, mid, and tip station, at the exit of the stage, and at the mid of this stage. Then we have started doing calculation for say mid station, we have shown the calculation at the hub station, we will see how the calculation that’s what will be proceed further for the tip station.

And based on that, we will be coming up with the final design sheet, that’s what we will be discussing in the next lecture. Thank you very much for your kind attention. We will be discussing further comparison, we will be discussing about design calculation, and the comparison of both the methods in next lecture. Thank you. Thank you very much!