

Aerodynamics Design of Axial Flow Compressors & Fans
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Lecture 56
Design of Transonic Compressor (Contd.)

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In last lecture we discussed...

Ca	U _{tip}	D _t	N	M _t
160	450	0.5807	14798	1.411
170	440	0.5675	14807	1.397
180	440	0.5557	15120	1.405
225	400	0.516	14805	1.35

Given

$\pi_{c,stage} = 1.63$

$T_{01} = 298 \text{ K}$

$P_{01} = 101325 \text{ Pa}$

$\dot{m} = 38.69 \text{ kg/s}$

$RPM \leq 16100$

$C_p = 1.005 \text{ kJ/kg.K}$



Design Constrains

$r_{tip} = 0.260 \text{ m}$

$r_h/r_{tip} = 0.375$

$RPM \leq 16100$

$M_{tip} = 1.4$

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Welcome to lecture 56, we are discussing design of transonic compressor. So, this design that's what is mainly been focusing for the pressure ratio of the order of 1.63. Mass flow rate, that's what is fixed with 38.69 kg/s and we have our design constraints. For that in initial stage, we have assumed certain axial velocity numbers, say peripheral speed based on that we have done our calculation for entry. We have done our calculation for say exit of the stage. In between the stage that means, at the exit of my rotor and at the entry of stator, we have done the calculation for the radius. And, at mid-station we started doing the calculation for say different parameters.

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Tutorial contd.

Design sheet 75% at desired total rise

Span	7.75% Span
r_1 (m)	0.217
r_{1r}	0.84
f_{1m}	0.228
r_{1mean} (m)	0.222
f_{1m}	0.234
Mass flow (kg/sec)	38.89
U_1 (m/s)	336.80
U_2 (m/s)	350.38
C_d (m/s)	225
α_1 (deg)	0
T_{01} (K)	298
P_{01} (Pa)	101325
ΔP_{01} (Pa)	40530
P_{02} (Pa)	141855
ΔT_{01} (K)	48.89
C_{u1} (m/s)	0
C_{u2} (m/s)	140.50
β_1 (deg)	36.28
β_2 (deg)	43.01
β_2' (deg)	13.25
α_2 (deg)	31.88
C_1 (m/s)	225.00
C_2 (m/s)	265.28
V_1 (m/s)	405.12
V_2 (m/s)	307.70
oh No.	0.78
σ	0.32
DOR	0.80
Chord (m)	0.090
ecc cis (calculated)	1.58
SP	0.32
m (slope factor)	0.324
deviation angle, δ (deg)	4.42
Corrected deviation, δ_{cor} (deg)	4.92
Corrected Camber, δ_{cam} (deg)	21.17
Corrected Stagger, k_{stg} (deg)	48.88

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So, when we are discussing about say, design at the mid-station, we have assumed our pressure ratio to be expected pressure ratio at the mid station, that is 75% of span. We have done our calculation for all flow parameters, flow angles. And, in order to have say calculation for the number of blades, we have assumed our diffusion factor to be 0.32 at mid-station and that's what was helping us for the calculation for say solidity. Based on the aspect ratio, we have done our calculation for the chord. And, let us move towards the next calculation. So, as we know once, we have done our calculation for the mid-station, we are interested in what is happening at the hub-station, what is happening at the tip-station.

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Tutorial contd.

Calculations at hub:

Let's take total pressure rise at hub is to be **40530 Pa** which corresponds to a total pressure ratio of **1.4**

Given

$P_{01} = 101325 \text{ Pa}$

$T_{01} = 298 \text{ K}$

Total temperature rise at hub can be calculated from

$$\Delta T_{0h} = \left[\left(\frac{P_{01} + \Delta P_{0,h}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \times \frac{T_{01}}{\eta_p} = \left[\left(\frac{101325 + 40530}{101325} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \times \frac{298}{0.93}$$

$$\Delta T_{0h} = 32.33 \text{ K}$$

Hence hub will be designed for a total temperature rise of 32.33 K

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So, let us move with the calculation at the hub-station. Now, here in this case, we have considered our fundamental design approach that means we need to assume certain pressure rise expected near the hub, okay. So, say for the sake of understanding, sake of simplicity, we are assuming our pressure ratio expected at the hub to be 1.4, that means that's what is giving me my total pressure rise expected to be 40530 Pa. So, do not get confused from where this number that's what is coming, that's what is approximately we are assuming our pressure ratio of 1.4.

If that's what is the case, at the hub-station we can do our calculation for say total pressure rise. So, here in this case at the entry my pressure P_{01} , that's what is known to us, this is what is say my total pressure what we are expecting, that's what is 40530, we are assuming our efficiency to be 93%, that's what is giving me my ΔT_0 at the hub to be 32.33 K. Now, this is what is very much helpful for us in order to do the calculation for different velocity components.

Let's take total pressure rise at hub is to be 40530 Pa

which corresponds to a total pressure ratio of 1.4

Total temperature rise at hub can be calculated from

$$\begin{aligned}\Delta T_{0h} &= \left[\left(\frac{P_{01} + \Delta P_{0,h}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \times \frac{T_{01}}{\eta_p} \\ &= \left[\left(\frac{101325 + 40530}{101325} \right)^{\frac{1.4-1}{1.4}} - 1 \right] \times \frac{298}{0.93}\end{aligned}$$

$$\Delta T_{0h} = 32.33 \text{ K}$$

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Tutorial contd.

Balancing Aerodynamic and Thermodynamic work at hub

$$C_p \Delta T_{0,h} = \lambda \omega (r_{h2} C_{wh2} - r_{h1} C_{wh1}) \quad (\because \text{hub radius is changing})$$

where $\lambda = 0.98$

As $C_{wh1} = 0$ (Flow is axial at inlet)

$$C_{wh2} = \frac{C_p \Delta T_{0h}}{\lambda U_{h2}} = \frac{1.005 \times 10^3 \times 32.33}{0.98 \times 201.55} = 164.52 \text{ m/s}$$

Given

$$\Delta T_{0h} = 32.33 \text{ K}$$

$$U_{h2} = 201.55 \text{ m/s}$$

$$C_p = 1.005 \text{ kJ/kg K}$$

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So, what we will be doing say in line to what we have done at the mid-station, we will be assuming our aerodynamic work and thermodynamic work to be same at the hub-station, if you are putting that as a case, my C_{w1} , as we have discussed, it is axial entry, that is the reason why C_{w1} , that's what is equal to 0 and if we are putting these numbers, it says $\frac{C_p \Delta T_0}{\lambda U_{h2}}$ at exit, that's what is giving say whirl component it is as say 164.52 m/s. Now, once at exit station, this is what is known to us, it will make our calculation easy.

Balancing Aerodynamic and Thermodynamic work at hub,

$$C_p \Delta T_{0,h} = \lambda \omega (r_{h2} C_{wh2} - r_{h1} C_{wh1})$$

$$\text{where } \lambda = 0.98$$

$$\text{As } C_{wh1} = 0 \text{ (Flow is axial at inlet)}$$

$$C_{wh2} = \frac{C_p \Delta T_{0h}}{\lambda U_{h2}} = \frac{1.005 \times 10^3 \times 32.33}{0.98 \times 201.55} = 164.52 \text{ m/s}$$

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Tutorial contd.

From inlet velocity triangle at hub,
 $\alpha_{h1} = 0^\circ$ (Axial Entry)

also,
 $\beta_{h1} = \tan^{-1}\left(\frac{U_{h1}}{C_a}\right) = \tan^{-1}\left(\frac{149.92}{225}\right)$
Hence, $\beta_{h1} = 33.68^\circ$

From exit velocity triangle,
 $\beta_{h2} = \tan^{-1}\left(\frac{U_{h2} - C_{wh2}}{C_a}\right) = \tan^{-1}\left(\frac{201.55 - 164.52}{225}\right)$
 $\beta_{h2} = 9.35^\circ$

Given
 $C_a = 225 \text{ m/s}$
 $U_{h1} = 149.92 \text{ m/s}$
 $U_{h2} = 201.55 \text{ m/s}$
 $C_{wh2} = 164.52 \text{ m/s}$

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So, let us move with what velocity triangle we will be having. So, this is what is representing my velocity triangle at the entry of the rotor, this is what is the velocity triangle at the exit of the rotor. So, based on our understanding, we are having our axial entry that means α_1 that's what is equal to 0. β_1 , we can calculate based on U/C_a , that's what is coming 33.68°. Same way, β_2 that is at the exit station, we can do calculation that's what is coming 9.35°.

From inlet velocity triangle,

$$\alpha_{1h} = 0^\circ \text{ (Axial Entry)}$$

also,

$$\beta_{h1} = \tan^{-1}\left(\frac{U_{h1}}{C_a}\right) = \tan^{-1}\left(\frac{149.92}{225}\right)$$

$$\text{Hence, } \beta_{h1} = 33.68^\circ$$

From exit velocity triangle,

$$\beta_{h2} = \tan^{-1}\left(\frac{U_{h2} - C_{wh2}}{C_a}\right) = \tan^{-1}\left(\frac{201.55 - 164.52}{225}\right)$$

$$\beta_{h2} = 9.35^\circ$$

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Tutorial contd.

Deflection at hub,
 $\Delta\beta_h = \beta_{h1} - \beta_{h2}$
 $\therefore \Delta\beta_h = 33.68^\circ - 9.35^\circ$
 $\therefore \Delta\beta_h = 24.33^\circ$

From rotor exit velocity triangle
 $\alpha_{h2} = \tan^{-1}\left(\frac{C_{wh2}}{C_a}\right) = \tan^{-1}\left(\frac{164.52}{225}\right)$
 $\therefore \alpha_{h2} = 36.17^\circ$

Given
 $\beta_{h1} = 33.68^\circ$
 $\beta_{h2} = 9.35^\circ$
 $C_{wh2} = 164.52 \text{ m/s}$
 $C_a = 225 \text{ m/s}$

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Now, here in this case, if this is the case, we can have our $\Delta\beta$, that's what is coming 24.33° , okay. So, with this $\Delta\beta$, we will be moving forward with say calculation of α_2 , we will be calculating what will be our relative velocities, okay. So, based on velocity triangle, we can say $\tan \alpha_2$, that's what is C_{w2}/C_a and this is what is coming as say 36.17° .

Deflection at hub,

$$\Delta\beta_h = \beta_{h1} - \beta_{h2}$$

$$\therefore \Delta\beta_h = 33.68^\circ - 9.35^\circ$$

$$\therefore \Delta\beta_h = 24.33^\circ$$

From rotor exit velocity triangle,

$$\alpha_{h2} = \tan^{-1}\left(\frac{C_{wh2}}{C_a}\right) = \tan^{-1}\left(\frac{164.52}{225}\right)$$

$$\therefore \alpha_{h2} = 36.17^\circ$$

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Tutorial contd.


Average wheel speed at hub,

$$U_{h_avg} = \frac{U_{h1} + U_{h2}}{2} = \frac{149.92 + 201.55}{2}$$
$$U_{h_avg} = 175.74 \text{ m/s}$$

Now, we calculate the degree of reaction at hub,

$$DOR_h = 1 - \frac{C_{wh2} + C_{wh1}}{2U_{h_avg}}$$
$$DOR_h = 1 - \frac{164.52 + 0}{2 \times 175.74}$$
$$\therefore DOR_h = 0.53$$

Given

$$U_{h1} = 149.92 \text{ m/s}$$
$$U_{h2} = 201.55 \text{ m/s}$$
$$C_{wh2} = 164.52 \text{ m/s}$$


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Now, in order to calculate our degree of reaction as we have discussed, we will be taking say peripheral speed to be average peripheral speed. So, at hub at the entry we have U_{h1} , at the exit we have U_{h2} , if you are putting that as average it is coming 175.74 m/s . And if we are putting that in the equation for degree of reaction, the degree of reaction at the hub that's what is coming 0.53 , okay.

Average wheel speed at hub,

$$U_{h_avg} = \frac{U_{h1} + U_{h2}}{2} = \frac{149.92 + 201.55}{2}$$

$$U_{h_avg} = 175.74 \text{ m/s}$$

Now, we calculate the degree of reaction at hub,

$$DOR_h = 1 - \frac{C_{wh2} + C_{wh1}}{2U_{h_avg}}$$

$$DOR_h = 1 - \frac{164.52 + 0}{2 \times 175.74}$$

$$\therefore DOR_h = 0.53$$

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Tutorial contd.

The relative inlet and exit velocities can be calculated from velocity triangle

$$V_{h1} = \frac{C_a}{\cos \beta_{h1}} = \frac{225}{\cos(33.68^\circ)}$$

$$\therefore V_{h1} = 270.37 \text{ m/s}$$

Similarly at rotor exit

$$V_{h2} = \frac{C_a}{\cos \beta_{h2}} = \frac{225}{\cos(9.35^\circ)}$$

$$\therefore V_{h2} = 228.03 \text{ m/s}$$

de Haller's no at hub, $DH = \frac{V_{h2}}{V_{h1}} = \frac{228.03}{270.37}$

$$DH = 0.84$$

Given

- $\beta_{h1} = 33.68^\circ$
- $\beta_{h2} = 9.35^\circ$
- $C_a = 225 \text{ m/s}$

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Now, in order to do the calculation for relative velocity components, based on velocity triangle we can say, \cos component of β_1 that's what is given by C_a/V_h and that's what is giving me relative velocity component at the hub at the entry it is 270.32 m/s . Similarly, at the exit we can do this calculation it says 228.03 m/s . We can say the number we need to be careful, we need to keep on eye with say V_1 and V_2 . You can say, this is what is coming as say 228.03 , that's what is indicating the diffusion that's what is happening. And, that is why if you are calculating our de Haller's factor, that's what is coming 0.84 . So, this is what is in the range of what we are deciding with 0.72 .

The relative inlet and exit velocities can be calculated from velocity triangle,

$$V_{h1} = \frac{C_a}{\cos \beta_{h1}} = \frac{225}{\cos(33.68^\circ)}$$

$$\therefore V_{h1} = 270.37 \text{ m/s}$$

Similarly at rotor exit,

$$V_{h2} = \frac{C_a}{\cos \beta_{h2}} = \frac{225}{\cos(9.35^\circ)}$$

$$\therefore V_{h2} = 228.03 \text{ m/s}$$

de – Haller's number at hub,

$$DH = \frac{V_{h2}}{V_{h1}} = \frac{228.03}{270.37}$$

$$DH = 0.84$$

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Tutorial contd.

The number of blades has already been determined from mean line calculations as 29.

Pitch/spacing at hub is calculated as,

$$s_h = \frac{2\pi r_h}{Z} = \frac{2\pi \times 0.113}{29}$$

$$s_h = 0.025 \text{ m}$$


where, $r_h = \frac{r_{h1} + r_{h2}}{2}$
 $r_h = \frac{0.0967 + 0.130}{2} = 0.113 \text{ m}$

Solidity at hub is calculated based on mean chord and hub pitch,

$$\sigma_h = \frac{c}{s_h} = \frac{0.09}{0.025}$$

$$\sigma_h = 3.665$$

Given
 $c = 0.09 \text{ m}$
 $Z = 29$



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Now, in order to do the calculation for say our diffusion factor at the hub-station, we need to have some parameters. So, since we have already assumed our diffusion factor at the mid-station to be 0.32. And based on that we have done our calculation for number of blades. So, for the same number of blade, we can calculate what will be my pitch, okay. And, that pitch is coming 0.025 m. And, based on that pitch and chord say here we are assuming our chord to be constant, okay.

The number of blades has already been determined from meanline calculation as 29

Pitch/spacing at hub is calculated as,

$$s_h = \frac{2\pi r_h}{Z}$$

$$\text{where, } r_h = \frac{r_{h1} + r_{h2}}{2} = \frac{0.0967 + 0.13}{2} = 0.113 \text{ m}$$

$$= \frac{2\pi \times 0.113}{29}$$

$$s_h = 0.025 \text{ m}$$

So, there are many design where people they are assuming this chord to be different, okay. So in such kind of design, that's what we will be discussing as and when required. So, at this moment, we are assuming our chord to be constant from hub to tip. And that is the reason why it is giving me solidity as 3.665.

Solidity at hub is calculated based on mean chord and hub pitch,

$$\sigma_h = \frac{c}{s_h} = \frac{0.09}{0.025}$$

$$\sigma_h = 3.665$$

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Tutorial contd.

Thus the Diffusion factor at hub is given by

$$(DF)_{h,rotor} = 1 - \frac{\cos \beta_{1h}}{\cos \beta_{2h}} + \frac{\cos \beta_{1h}}{2 \times \sigma_h} (\tan \beta_{1h} - \tan \beta_{2h})$$

$$= 1 - \frac{\cos(33.68^\circ)}{\cos(9.35^\circ)} + \frac{\cos(33.68^\circ)}{2 \times 3.665} (\tan 33.68^\circ - \tan 9.35^\circ)$$

$(DF)_{h,rotor} = 0.21$

Given

- $\beta_{1h} = 33.68^\circ$
- $\beta_{2h} = 9.35^\circ$
- $C_{w02} = 164.52 \text{ m/s}$
- $\sigma_h = 3.665$

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Once, the solidity it is known to us, we can do our calculation for diffusion factor at the hub and that's what is coming 0.21, okay.

Thus the Diffusion factor at hub is given by,

$$(DF)_{h,rotor} = 1 - \frac{\cos \beta_{1h}}{\cos \beta_{2h}} + \frac{\cos \beta_{1h}}{2 \times \sigma_h} (\tan \beta_{1h} - \tan \beta_{2h})$$

$$= 1 - \frac{\cos(33.68^\circ)}{\cos(9.35^\circ)} + \frac{\cos(33.68^\circ)}{2 \times 3.665} (\tan 33.68^\circ - \tan 9.35^\circ)$$

$$(DF)_{h,rotor} = 0.21$$

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Tutorial contd.
Calculations at tip:

The total pressure rise at tip is prescribed to be **67000 Pa** which corresponds to a total pressure ratio of **1.66**


total temperature rise at tip can be calculated from

$$\Delta T_{0t} = \left[\left(\frac{P_{01} + \Delta P_{0,t}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \times \frac{T_{01}}{\eta_p} = \left[\left(\frac{101325 + 67000}{101325} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \times \frac{298}{0.93}$$

Given
 $P_{01} = 101325 \text{ Pa}$
 $T_{01} = 298 \text{ K}$

$$\Delta T_{0t} = 50.01 \text{ K}$$

Hence tip will be designed for a total temperature rise of 50.01 K



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Now, in order to understand, like in order to do the calculation at the tip; so, here as we know, this is what is our fundamental design approach, in which we need to assume our ΔP_0 distribution. So, at the tip, we are assuming suppose say our pressure ratio to be say 1.66 near the tip region, and that's what is giving say my pressure is expected to be 67,000 Pa, okay. If that's what is the case, we can do our calculation of say my ΔT_0 at the tip-station, and this is what is coming 50.01 K, okay.

The total pressure rise at tip is prescribed to be 67000 Pa

which corresponds to a total pressure ratio of 1.66

Total temperature rise at tip can be calculated from

$$\begin{aligned} \Delta T_{0t} &= \left[\left(\frac{P_{01} + \Delta P_{0,t}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \times \frac{T_{01}}{\eta_p} \\ &= \left[\left(\frac{101325 + 67000}{101325} \right)^{\frac{1.4-1}{1.4}} - 1 \right] \times \frac{298}{0.93} \end{aligned}$$

$$\Delta T_{0t} = 50.01 \text{ K}$$

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Tutorial contd.

Balancing Aerodynamic and Thermodynamic work at tip,

Given
 $\Delta T_{0t} = 50.01 \text{ K}$
 $U_{t2} = 399.22 \text{ m/s}$
 $C_p = 1.005 \text{ kJ/kg K}$

$$C_p \Delta T_{0,t} = \lambda \omega (r_{t2} C_{wt2} - r_{t1} C_{wt1}) \quad (\because \text{hub radius is changing})$$

where $\lambda = 0.98$

$$C_{wt2} = \frac{C_p \Delta T_{0t}}{\lambda U_{t2}} = \frac{1.005 \times 10^3 \times 50.01}{0.98 \times 399.22} = 128.46 \text{ m/s} \quad (C_{wt1} = 0, \text{ Flow is axial at inlet})$$


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Now, in line to what all calculation we have done at mid-station and at the hub, we can do our calculation by comparing aerodynamic and thermodynamic work. So, here in this case, C_w that's what is at tip-station, C_{w2} at the tip-station, that's what we can calculate by comparing our aerodynamic and thermodynamic work. So, if you will be putting that number, it says my whirl component at tip at the exit, that's what is coming as say 128.46 m/s , okay.

Balancing Aerodynamic and Thermodynamic work at tip,

$$C_p \Delta T_{0,t} = \lambda \omega (r_{t2} C_{wt2} - r_{t1} C_{wt1})$$

$$\text{where } \lambda = 0.98$$

As $C_{wt1} = 0$ (Flow is axial at inlet)

$$C_{wt2} = \frac{C_p \Delta T_{0t}}{\lambda U_{t2}} = \frac{1.005 \times 10^3 \times 50.01}{0.98 \times 399.22} = 128.46 \text{ m/s}$$

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Tutorial contd.

From inlet velocity triangle at tip,

$$\alpha_{t1} = 0^\circ \text{ (Axial Entry)}$$

also,

$$\beta_{t1} = \tan^{-1} \left(\frac{U_{t1}}{C_a} \right) = \tan^{-1} \left(\frac{399.22}{225} \right)$$

Hence, $\beta_{t1} = 60.59^\circ$

From exit velocity triangle,

$$\beta_{t2} = \tan^{-1} \left(\frac{U_{t2} - C_{wt2}}{C_a} \right) = \tan^{-1} \left(\frac{399.22 - 128.46}{225} \right)$$

$$\beta_{t2} = 50.27^\circ$$

Given

$C_a = 225 \text{ m/s}$

$U_{t1} = 399.22 \text{ m/s}$

$U_{t2} = 399.22 \text{ m/s}$

$C_{wt2} = 128.46 \text{ m/s}$

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Now, based on this, we can do our calculation for different velocity components near the tip region. So, α_1 that's what we are assuming to be 0, we are considering axial entry throughout the span, okay. And, based on our velocity triangle we can calculate what will be our β_1 at the tip and what will be my β_2 at the tip. If you are considering β_1 at the tip it is coming 60.59° and β at the tip, that's what is at the exit it is coming 50.27° .

From inlet velocity triangle at tip,

$$\alpha_{t1} = 0^\circ \text{ (Axial Entry)}$$

also,

$$\beta_{t1} = \tan^{-1} \left(\frac{U_{t1}}{C_a} \right) = \tan^{-1} \left(\frac{399.22}{225} \right)$$

Hence, $\beta_{t1} = 60.59^\circ$

From exit velocity triangle,

$$\beta_{t2} = \tan^{-1} \left(\frac{U_{t2} - C_{wt2}}{C_a} \right) = \tan^{-1} \left(\frac{399.22 - 128.46}{225} \right)$$

$$\beta_{t2} = 50.27^\circ$$

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Tutorial contd.

Deflection at tip,
 $\Delta\beta_t = \beta_{t1} - \beta_{t2}$
 $\therefore \Delta\beta_t = 60.59^\circ - 50.27^\circ$
 $\therefore \Delta\beta_t = 10.32^\circ$

From rotor exit velocity triangle
 $\alpha_{t2} = \tan^{-1}\left(\frac{C_{wt2}}{C_a}\right) = \tan^{-1}\left(\frac{128.46}{225}\right)$
 $\therefore \alpha_{t2} = 29.72^\circ$

Given
 $\beta_{t1} = 60.59^\circ$
 $\beta_{t2} = 50.27^\circ$
 $C_{wt2} = 128.46 \text{ m/s}$
 $C_a = 225 \text{ m/s}$

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Now, this is what we will be giving us our $\Delta\beta$ to be in the range of say 10° . So, this is what is 10.32° and based on that we can make our blades, okay. Here, in this case from the exit velocity triangle, we can do our calculation for α_2 , this α_2 , that's what is will be helpful for us in order to do calculation for this data, okay.

Deflection at tip,

$$\Delta\beta_t = \beta_{t1} - \beta_{t2}$$

$$\therefore \Delta\beta_t = 60.59^\circ - 50.27^\circ$$

$$\therefore \Delta\beta_t = 10.32^\circ$$

From rotor exit velocity triangle,

$$\alpha_{t2} = \tan^{-1}\left(\frac{C_{wt2}}{C_a}\right) = \tan^{-1}\left(\frac{128.46}{225}\right)$$

$$\therefore \alpha_{t2} = 29.72^\circ$$

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Tutorial contd.


Average wheel speed at tip,

$$U_{t_avg} = 399.22 \text{ m/s} \quad (\because \text{tip is constant from inlet to outlet})$$

Given

$$U_t = 399.22 \text{ m/s}$$
$$C_{wt2} = 128.46 \text{ m/s}$$

Now, we calculate the degree of reaction at tip,

$$DOR_t = 1 - \frac{C_{wt2} + C_{wt1}}{2U_{t_avg}}$$
$$DOR_t = 1 - \frac{128.46}{2 \times 399.22}$$
$$\therefore DOR_t = 0.84$$


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Now, based on all this calculation, we are looking for say degree of reaction at the tip-station, again, we will be doing our peripheral speed averaging and this averaged value, that's what is coming 399.22 m/s and that's what will be giving degree of reaction to be 0.84, okay. Now, all these numbers what we are discussing in sense of degree of reaction, you must realize what is the meaning of this degree of reaction, that's what is varying all the way from up to tip, okay.

Average wheel speed at tip,

$$U_{t_avg} = 399.22 \text{ m/s}$$

Now, we calculate the degree of reaction at tip,

$$DOR_t = 1 - \frac{C_{wt2} + C_{wt1}}{2U_{t_avg}}$$

$$DOR_t = 1 - \frac{128.46}{2 \times 399.22}$$

$$\therefore DOR_t = 0.84$$

(Refer Slide Time: 11:37)

Tutorial contd.

The relative inlet and exit velocities can be calculated from velocity triangle

$$V_{t1} = \frac{C_a}{\cos \beta_{t1}} = \frac{225}{\cos(60.59^\circ)}$$

$$\therefore V_{t1} = 458.26 \text{ m/s}$$

Similarly at rotor exit

$$V_{t2} = \frac{C_a}{\cos \beta_{t2}} = \frac{225}{\cos(50.27^\circ)}$$

$$\therefore V_{t2} = 352.05 \text{ m/s}$$

de Haller's number at tip, $DH = \frac{V_{t2}}{V_{t1}} = \frac{352.05}{458.26}$

$$DH = 0.77$$

Given
 $\beta_{t1} = 60.59^\circ$
 $\beta_{t2} = 50.27^\circ$
 $C_a = 225 \text{ m/s}$

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Now, based on what all calculation we have done for our blade angle, relative inflow blade angle, relative outlet angle, we can calculate what will be our relative velocity at the entry, what will be our relative velocity at the exit and based on that we can do our calculation for de-Haller's factor.

So, from velocity triangle, we can write down our relative velocity at the entry, that's what is $C_a / \cos \beta_1$ and that's what is coming 458.26 m/s. You can see the range, if we compare what is happening at the hub and what is happening at the tip. Same way, at the exit we can do calculation it says this is what is coming as say 352.05 m/s. Based on this relative velocity, we can do calculation for our de Haller's factor and that's what is coming 0.77, it says that needs to be more than 0.72, so we can say this is what is coming in line to what we are expecting.

The relative inlet and exit velocities can be calculated from velocity triangle,

$$V_{t1} = \frac{C_a}{\cos \beta_{t1}} = \frac{225}{\cos(60.59^\circ)}$$

$$\therefore V_{t1} = 458.26 \text{ m/s}$$

Similarly at rotor exit,

$$V_{t2} = \frac{C_a}{\cos \beta_{t2}} = \frac{225}{\cos(50.27^\circ)}$$

$$\therefore V_{t2} = 352.05 \text{ m/s}$$

de – Haller's number at tip,

$$DH = \frac{V_{t2}}{V_{t1}} = \frac{352.05}{458.26}$$

$$DH = 0.77$$

(Refer Slide Time: 12:47)

Tutorial contd.

Pitch/spacing at tip is calculated as,

$$s_t = \frac{2\pi r_t}{Z} = \frac{2\pi \times 0.258}{29}$$


$$s_t = 0.056 \text{ m}$$

Solidity at tip is calculated based on mean chord and tip pitch,

$$\sigma_t = \frac{c}{s_t} = \frac{0.09}{0.056}$$

$$\sigma_t = 1.612$$

Given
 $r_t = 0.258 \text{ m}$
 $Z = 29$
 $c = 0.09$



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Now, say at the tip, we have already assumed with our number of blades, that's what it say 29. So, we can do our calculation for the solidity at the tip-station. Here, let me say it is chord that's what we are asking to be constant that's what is coming 0.09 m and we can do calculation for the solidity this is what is coming 1.61.

Pitch/spacing at tip is calculated as,

$$s_t = \frac{2\pi r_t}{Z}$$

$$= \frac{2\pi \times 0.258}{29}$$

$$s_t = 0.056 \text{ m}$$

Solidity at tip is calculated based on mean chord and tip pitch,

$$\sigma_t = \frac{c}{s_t} = \frac{0.09}{0.056} = 1.612$$


(Refer Slide Time: 13:17)

Tutorial contd.


Thus the Diffusion factor at tip is given by

$$(DF)_{t,rotor} = 1 - \frac{\cos \beta_{1t}}{\cos \beta_{2t}} + \frac{\cos \beta_{1t}}{2 \times \sigma_t} (\tan \beta_{1t} - \tan \beta_{2t})$$
$$= 1 - \frac{\cos(60.59^\circ)}{\cos(50.27^\circ)} + \frac{\cos(60.59^\circ)}{2 \times 1.612} (\tan 60.59^\circ - \tan 50.27^\circ)$$
$$(DF)_{t,rotor} = 0.32$$

Given

$$\beta_{1t} = 60.59^\circ$$
$$\beta_{2t} = 50.27^\circ$$
$$\sigma_t = 1.612$$


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So, based on that, we can do our calculation for the diffusion factor. For diffusion factor at the tip this is what is coming as say 0.32.

Thus the Diffusion factor at tip is given by,

$$(DF)_{t,rotor} = 1 - \frac{\cos \beta_{1t}}{\cos \beta_{2t}} + \frac{\cos \beta_{1t}}{2 \times \sigma_t} (\tan \beta_{1t} - \tan \beta_{2t})$$
$$= 1 - \frac{\cos(60.59^\circ)}{\cos(50.27^\circ)} + \frac{\cos(60.59^\circ)}{2 \times 1.612} (\tan 60.59^\circ - \tan 50.27^\circ)$$

$$(DF)_{t,rotor} = 0.32$$

(Refer Slide Time: 13:31)

Tutorial contd.

	1-hub	7.75% Span	8-tip
Rotor			
r_1 (m)	0.697	0.217	0.258
r_{1h}	0.37	0.84	1.00
r_2 (m)	0.130	0.226	0.258
r_{2mean} (m)	0.113	0.222	0.258
r_3 (m)	0.163	0.234	0.258
Mass flow (kg/sec)	38.69	38.69	38.69
U_1 (m/s)	148.92	336.60	399.22
U_2 (m/s)	201.55	350.38	399.22
C_a (m/s)	225	225	225
α_1 (deg)	0	0	0
T_1 (K)	298	298	298
P_0 (Pa)	101325	101325	101325
ΔP_0 (Pa)	40530	63835	67000
P_{01} (Pa)	141855	165160	168325
M_1 (K)	32.33	46.99	56.01
C_{u1} (m/s)	0	0	0
C_{u2} (m/s)	164.52	140.50	128.48
β_1 (deg)	33.88	36.28	60.59
β_2 (deg)	9.55	43.21	59.27
β_3 (deg)	24.33	13.25	10.32
α_2 (deg)	36.17	31.98	29.72
C_1 (m/s)	225.00	225.00	225.00
C_2 (m/s)	278.72	285.26	259.09
V_1 (m/s)	270.37	405.12	458.28
V_2 (m/s)	228.03	307.70	352.05
Oh No.	0.84	0.76	0.77
λ	2.60	0.99	-2.00
DDR	0.53	0.80	0.84
Chord (m)	0.690	0.690	0.690
arc cis (calculated)	3.08	1.58	1.35
DR	0.31	0.32	0.32
m (slope factor)	0.391	0.324	0.309
deviation angle, δ (deg)	5.74	4.42	3.97
Corrected deviation, δ_{cor} (deg)	8.24	6.92	6.47
Corrected Camber, θ_{cor} (deg)	30.57	21.17	18.79
Corrected stagger, $C_{stagger}$ (deg)	16.39	46.48	53.28

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Now, after doing all this calculation, as we know, we can make our excel sheet program that's what will be helpful for us in order to check...cross checked with what all calculation we have done at hub-station, mid-station and at the tip-station. Now, here you can see, this is what is giving us the variation of ΔP_0 all the way at the hub, mid-station and at the tip-station.

Now, you can divide say from hub to span, you can divide into some number of station. Same way, you can divide from say 75% span to tip span as say some number of stations, okay. And based on your design program, you can do all your calculation part, but here in this case, we need to assume this ΔP_0 . So, this ΔP_0 assumption that's what need to be very careful with catch on eye for what is my $\Delta\beta$. It should have particular trend, this is what all we have discussed when we have done design for our subsonic compressor.

So, in line to that only we need to assume our ΔP_0 in such a way that $\Delta\beta$ will be coming in line. Second observed parameter, that's what is in sense of my de-Haller's factor. Same way, we can catch on eye with say diffusion factor, okay. Even we can check with our camber angle variation. So, this is what is a calculation we can say at three different stations.

(Refer Slide Time: 15:10)

Tutorial contd.


Stator design:

Assuming absolute velocity at the exit of rotor is same at entry velocity of stator

$C_{m3} = C_{m2}$
 $\alpha_{m3} = \alpha_{m2} = 31.98^\circ$
Also, axial exit is expected,
Hence, $\alpha_{m4} = 0^\circ$

Turning of the flow,
 $\Delta\alpha_m = \alpha_{m3} - \alpha_{m4}$
 $\therefore \Delta\alpha_m = 31.98^\circ - 0^\circ$
 $\therefore \Delta\alpha_m = 31.98^\circ$

Given
 $\alpha_{m1} = 31.98^\circ$



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Now, let me put the point here. So, after doing this design, we will be looking for the design for the stator. So, for stator design at the mid-station, what we are assuming is say what is my absolute velocity at the exit of rotor, that's what is equal to absolute velocity at the entry of my stator, okay. And here, in this case, my $\alpha_2 = \alpha_3$, this what we have calculated is say 31.98° . Now at the exit, we are expecting the exit to be say axial one and this is what is say in line to what we say assuming $\alpha_4 = 0$.

Stator design:

Assuming absolute velocity at the exit of rotor is same at entry velocity of stator

$$C_{m3} = C_{m2}$$

$$\alpha_{m3} = \alpha_{m2} = 31.98^\circ$$

Also, axial exit is expected,

$$\text{Hence, } \alpha_{m4} = 0^\circ$$

(Refer Slide Time: 15:58)

Tutorial contd.

Rotor			
	1-hub	7.75% Span	8-tip
r ₁ (m)	0.697	0.217	0.258
r ₂ (m)	0.27	0.84	1.00
r ₃ (m)	0.130	0.226	0.258
r _{mean} (m)	0.113	0.222	0.258
r _{fm} (m)	0.163	0.234	0.258
Mass flow (kg/sec)	38.69	38.69	38.69
U ₁ (m/s)	148.92	336.60	399.22
U ₂ (m/s)	201.55	350.38	399.22
C _a (m/s)	225	225	225
α ₁ (deg)	0	0	0
T ₀₁ (K)	298	298	298
P ₀₁ (Pa)	101325	101325	101325
ΔP _r (Pa)	40530	63835	67000
P ₀₂ (Pa)	141855	165160	168325
ΔT ₀ (K)	32.33	46.99	50.01
C _{u1} (m/s)	0	0	0
C _{u2} (m/s)	164.52	140.50	128.48
β ₁ (deg)	33.88	36.28	60.59
β ₂ (deg)	9.55	43.21	59.27
β ₃ (deg)	24.33	13.25	10.32
α ₂ (deg)	36.17	31.98	29.72
C ₁ (m/s)	225.00	225.00	225.00
C ₂ (m/s)	278.73	265.26	259.09
V ₁ (m/s)	270.37	405.12	458.28
V ₂ (m/s)	228.03	307.70	352.05
St No.	0.84	0.76	0.77
λ	2.60	0.99	-2.00
DDR	0.53	0.60	0.84
Chord (m)	0.090	0.090	0.090
arc cis (calculated)	3.08	1.58	1.35
CF	0.21	0.32	0.32
m (slope factor)	0.391	0.324	0.300
deviation angle, δ (deg)	5.74	4.42	3.97
Corrected deviation, δ _{cor} (deg)	8.24	6.92	6.47
Corrected Camber, θ _{cam} (deg)	30.57	21.17	18.79
Corrected stagger, C _{stg} (deg)	16.39	46.68	53.28

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Now here, if we look at what is happening in sense of our axial velocity at the exit of the rotor, this is what is coming in the range of subsonic flow, okay. So, that's what is giving us indication that our stator, that will be subsonic stator. So, based on design, so this is what all we have discussed when we were discussing about the transonic compressor. So, there are many compressor in which you will be having your absolute velocity coming out from the rotor, that's what is on higher side. And under that condition, you need to go with your stator to be supersonic stator, okay. So, here in this case, these numbers, that's what is coming to be in slightly lower range in subsonic range or high subsonic range, we can safely go with say subsonic kind of stator.

(Refer Slide Time: 16:56)

Tutorial contd.

Stator design:

Assuming absolute velocity at the exit of rotor is same at entry velocity of stator

Given
 $\alpha_{m3} = 31.98^\circ$

$C_{m3} = C_{a2}$
 $\alpha_{m3} = \alpha_{m2} = 31.98^\circ$
 Also, axial exit is expected,
 Hence, $\alpha_{m1} = 0^\circ$

Turning of the flow,

$\Delta\alpha_m = \alpha_{m3} - \alpha_{m1}$
 $\therefore \Delta\alpha_m = 31.98^\circ - 0^\circ$
 $\therefore \Delta\alpha_m = 31.98^\circ$

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And, that is the reason why if you look at this airfoil that's what they say subsonic airfoil, we can say, we can go with say C4 kind of airfoil for this, okay. Initially, you can go with C4 airfoil, later on as per your requirement, you can modify the shape, you can morph your airfoil as per the expectation. Here, in this case, my $\Delta\alpha$, that's what is coming to be say 31.98.

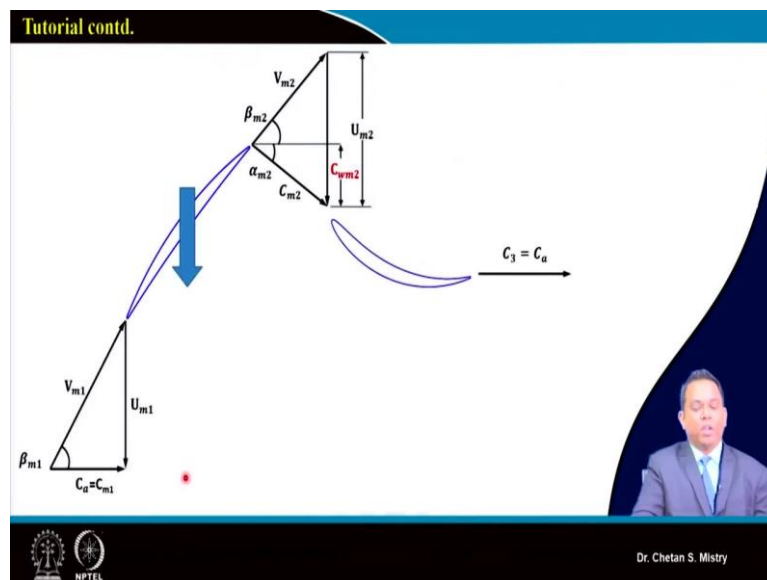
Turning of the flow,

$$\Delta\alpha_m = \alpha_{m3} - \alpha_{m4}$$

$$\therefore \Delta\alpha_m = 31.98^\circ - 0^\circ$$

$$\therefore \Delta\alpha_m = 31.98^\circ$$

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Now, this is what is the kind of configuration we are discussing at this moment.

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Tutorial contd.

An aspect ratio (AR) of 1.4 will be assumed for the stator

$$\therefore AR = \frac{h}{c}$$

$$\text{chord of blade} = \frac{h}{AR}$$

The blade height will be approximated as average of heights at rotor exit and stator exit

$$h = \frac{r_{t2} - r_{h2} + r_{t3} - r_{h3}}{2}$$

$$h = \frac{0.258 - 0.13 + 0.258 - 0.163}{2} = 0.111 \text{ m}$$

Blade chord is thus, $c = \frac{h}{AR} = \frac{0.111}{1.4} = 0.08 \text{ m}$

We know

- $r_{t2} = r_{t3} = 0.258 \text{ m}$
- $r_{h2} = 0.13 \text{ m}$
- $r_{h3} = 0.163 \text{ m}$

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Now, next step, that's what is for the design of stator. Here also we are looking for what will be the height and what needs to be the chord, even we need to have assumption or we need to have say number of blades for the stator. So, for the sake of understanding, here also, let us assume say aspect ratio to be 1.4, okay, this 1.4 that's what is coming with certain assumption in order to catch on eye for the chord of our stator, okay. So, here we have assumed this aspect ratio to be 1.4 and as we have discussed, we need to calculate our height based on entry height and exit height and this is what is my average height.

So, this is what is in line to what we have done for our say rotor, okay. Same way, you can calculate your average height for the stator and this is what is coming 0.11 m. Now, this chord calculation, we can say this is what is coming as say 0.08 m.

An aspect ratio (AR) of 1.4 will be assumed for the stator

$$\therefore AR = \frac{h}{c}$$

$$\text{chord of blade} = \frac{h}{AR}$$

The blade height will be approximated as average of heights at rotor exit and stator exit

$$h = \frac{r_{t2} - r_{h2} + r_{t3} - r_{h3}}{2}$$

$$h = \frac{0.258 - 0.13 + 0.258 - 0.163}{2} = 0.111m$$

$$\text{Blade chord is thus, } c = \frac{h}{AR} = \frac{0.111}{1.4} = 0.08 \text{ m}$$

So, we can say this is what is in line to what we are expecting. Suppose, again we need to be very careful this number, if this is what is coming to be lower under that condition maybe you need to modify your assumption for aspect ratio, okay. So, certain assumptions when we are making, we need to be very careful in that aspects, okay.

(Refer Slide Time: 19:15)

Tutorial contd.

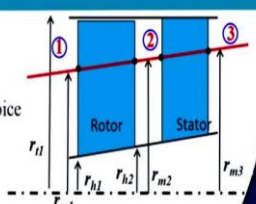
In order to limit the DF, value of $\sigma_m = 2.0$ for stator seems to be a good choice

$$\text{Since, } \sigma_m = \frac{c}{s_m}$$

$$s_m = \frac{c}{\sigma_m} = \frac{2\pi r_m}{Z}$$

(where, $r_m = \frac{r_{m2} + r_{m3}}{2} = \frac{0.226 + 0.234}{2}$)
 $r_m = 0.23 \text{ m}$

Thus, no. of stator blades, $Z = 36$



The diagram shows a cross-section of a rotor and stator. The rotor is a blue rectangle with a red line passing through its center. The stator is a blue rectangle to the right of the rotor. Radii are labeled as r_{l1} , r_{h1} , r_{h2} , r_{m2} , and r_{m3} . Three points are numbered 1, 2, and 3 along the top edge of the rotor and stator.

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Now, once this is what is known to us, next parameter that's what will be coming is for the selection of say number of blades for the stator, okay. There are different approaches for say assuming say number of blades for the stator. Here in this case, what we will be saying in order to limit our diffusion factor, say we have assumed our diffusion factor for the rotor, diffusion factor for the stator also can be assumed, but do not go into the detail we will be doing say assumption for say solidity. Let us assume the solidity to be 2. You may be having questions or why 2, you can go with say lower number, you can go even with higher number, okay. Why we are assuming this solidity to be 2? Let us see.

So here, based on solidity to be 2, we can do our calculation for say number of blades and this number of blades that's what is coming say 36, okay. So, you know, many times as we have discussed earlier, mechanical engineers that's what they will be putting, they will be insisting to have say maybe less number of blades. Again, this less number of blades and more number

of blade it is a hot discussion for the design, okay. When we are looking for say design in which we are expecting our say weight of the engine to be lower it is but obvious it says you go with say lower number of blades both for rotor and stator.

But when we say we are assuming our number of blades to be say lower it may be possible that it will not give what performance we are expecting because my flow passage will be, you know, it is diffusing passage. And, when we are having less number of blades, it will be possible that we will be having say special kind of configuration where we will be having say diffusion action will be lower.

There are many chances in which you will be having the flow separation that's what will be happening towards the trailing edge, okay. So, those all things that's what will be coming when we will be doing the detailed design. At this moment this is what we are doing as a preliminary design, so we are taking our number of stators to be 36.

In order to limit the DF, value of $\sigma_m = 2.0$ for stator seems to be a good choice

$$\text{Since, } \sigma_m = \frac{c}{s_m}$$

$$s_m = \frac{c}{\sigma_m} = \frac{2\pi r_m}{Z}$$

$$\left(\text{where, } r_m = \frac{r_{m2} + r_{m3}}{2} = \frac{0.226 + 0.234}{2} = 0.23 \text{ m} \right)$$

Thus, number of stator blades, $Z = 36$

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Tutorial contd.

According to Carter the slope factor is given by

$$m = 0.23 \left(\frac{2a}{c} \right)^2 + \frac{0.1(90 - \alpha_{m1})}{50}$$

where, $a/c = 0.5$ (circular arc)

$$m = 0.23(2 \times 0.5)^2 + \frac{0.1(90 - 0^\circ)}{50}$$

$$\therefore m = 0.41$$

Camber angle, $\theta_m = \frac{\Delta\alpha_m - i_m}{1 - m\sqrt{s/c}}$

$$= \frac{31.98^\circ - 0^\circ}{1 - 0.32\sqrt{1/1.87}}$$

$$\therefore \theta_m = 45.05^\circ$$

We know

$\alpha_{m4} = 0^\circ$
 $\Delta\alpha_m = 31.98^\circ$
 $s/c = \frac{1}{\sigma} = \frac{1}{2}$

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Now, once this is what is done, we can say we are calculating our say 'm' factor that's what is my slope factor. So, my slope factor that's what we are calculating based on α . So, this is what is representing my cascade, okay. So, we will be taking this as a parameter. Here, in this case when we are calculating our camber angle, again at the mid-station even for stator we are assuming our incidence angle to be 0, okay. So, if I am taking that to be 0, at mid-station we can do our calculation for camber angle and that's what is coming 45° .

According to Carter the slope factor is given by

$$m = 0.23 \left(\frac{2a}{c} \right)^2 + \frac{0.1(90 - \alpha_{m4})}{50}$$

where, $a/c = 0.5$ (circular arc)

$$m = 0.23(2 \times 0.5)^2 + \frac{0.1(90 - 0^\circ)}{50}$$

$$\therefore m = 0.41$$

$$\text{Camber angle, } \theta_m = \frac{\Delta\alpha_m - i_m}{1 - 0.32\sqrt{1/1.87}}$$

$$= \frac{31.98^\circ - 0^\circ}{1 - 0.32\sqrt{1/1.87}} = 45.05^\circ$$

(Refer Slide Time: 22:25)

Tutorial contd.

Deviation angle,

$$\delta_m = m\theta_m \sqrt{s_m/c}$$

$$= 0.41 \times 45.05^\circ \sqrt{1/2}$$

$$\therefore \delta_m = 13.09^\circ$$

Considering an additional deviation of 2.2°, the corrected deviation is thus given by,

$$\delta_{corrected} = 13.09^\circ + 2.2^\circ = 15.29^\circ$$

Corrected Camber,

$$\theta_{corrected} = \Delta\alpha_m - i + \delta_{corrected}$$

$$= 31.98^\circ - 0^\circ + 15.29^\circ$$

$$\theta_{corrected} = 47.25^\circ$$

We know

$$\Delta\alpha_m = 31.98^\circ$$

$$s/c = \frac{1}{\sigma} = \frac{1}{2}$$

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Now, we can do our calculation based on Carter's relation for the deviation angle and this deviation angle that's what is coming say 13.09. Maybe again we will be putting this as a note, we can put additional deviation as per your requirement, this is basically been done based on computational study, okay. So, we need to be very careful what numbers need to be selected for say corrected deviation, okay. So, this is what will be giving me corrected deviation. Same way, based on this corrected deviation, we need to calculate what will be our corrected camber and this camber that's what we are using in our design program for say making of airfoils, we need to be very careful about this.

Deviation angle,

$$\delta_m = m\theta_m \sqrt{s_m/c}$$

$$= 0.41 \times 45.05^\circ \sqrt{1/2}$$

$$\therefore \delta_m = 13.09^\circ$$

Considering an additional deviation of 2.2°, the corrected deviation is thus given by,

$$\delta_{corrected} = 13.09^\circ + 2.2^\circ = 15.29^\circ$$

Corrected Camber,

$$\theta_{corrected} = \Delta\alpha_m - i + \delta_{corrected}$$

$$= 31.98^\circ - 0^\circ + 15.29^\circ = 47.25^\circ$$

(Refer Slide Time: 23:17)

Tutorial contd.

Finally the corrected Stagger angle,

$$\zeta_{corrected} = \alpha_{m3} - i_m - \theta_{corrected} / 2$$

$$= 31.98^\circ - 0^\circ - 47.25^\circ / 2$$

$$\therefore \zeta_{corrected} = 8.34^\circ$$

We know

$\beta_{m1} = 56.10^\circ$

$\theta_{corrected} = 19.59^\circ$

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Now, based on that we need to set our blades that's what is called say setting angle or what we say as a stagger angle. So, this stagger angle for blade that's what is coming to be say 8.3 at the mid-station, okay.

Finally the corrected stagger angle,

$$\zeta_{corrected} = \alpha_{m3} - i_m - \theta_{corrected} / 2$$

$$= 31.98^\circ - 0^\circ - 47.25^\circ / 2$$

$$\therefore \zeta_{corrected} = 8.34^\circ$$

(Refer Slide Time: 23:37)

Tutorial contd.

The transonic stage is designed at 75% span rather than 50%.
The average span loading needs to be equivalent to required overall loading.

Thus loading distribution needs to be revised as per the following reasons.

Loading 75% span to desired pressure rise results in-

- *Extremely high loading in 75% span to tip. This invites shock losses due to high blade camber and relative speed.*
- *The extreme loading can be avoided by loading the rest of the span (below 75%) to high pressure rise. This results in very uneven loading distribution throughout the span.*

A possible solution is to load the 75% span at a higher loading than required loading.
This ensures moderately loaded tip and hub.

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Now, so this is what will be giving us idea about how do we do design for the stator. You must be realizing there is nothing more to do for the stator design, because we are assuming our absolute velocity at the entry to be same as exit velocity...exit absolute velocity from the rotor and what we are expecting at the exit maybe axial or maybe when we are doing multistage configuration, it may be possible that we need to enter the flow to the next stage at certain angle. So, that's what will be giving me what will be my α_4 , okay. So, the stator design that's what we can say, we can do simply by understanding in sense with certain assumptions.

Now, here very important point that's what we need to discuss is in sense of selection of what we say pressure distribution or what performance we are expecting. Say for 50% span, we were assuming what pressure ratio we are expecting? We are assuming the same number. Similar approach we have done here for say our transonic design, we have assumed our design pressure ratio at 75% of span.

Now, what we need to do? We need to vary our pressure ratio...expected pressure ratio at different station based on certain assumptions. So, for this case, we have assumed our pressure ratio expected near the hub, it is 1.4 and pressure ratio expected near the tip as say 1.66. If we are doing this kind of configuration, what will happen? We will be having say distribution of the pressure that's what we will be varying in a different way.

Suppose say, if we are varying our pressure ratio to be on higher side from 75% span towards the tip then it will be very highly loaded near the tip region and that's what will be subjected

to give very high camber angle and maybe relative velocity also will be coming to be on higher side, this will lead to increase the losses.

So, the meaning is, you know, we need to assume certain amount of say pressure distribution from 75% span towards the tip. Same way, from downside of mid-span towards the hub also we need to assume our ΔP_0 in such a way that we will be having uniform distribution of the pressure or we will be having distribution that's what will be even distribution.

If this is not the case, then it may lead to give some kind of zigzag kind of configuration for our blade angle, okay. Now, in order to get rid of this difficulty, maybe what we need to do is at 75% span, other than what we are expecting in sense of pressure ratio, suppose say for this compressor expected pressure ratio is 1.63, maybe you can assume say at 75% span this pressure ratio to be on higher side, okay.

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Tutorial contd.
Rotor design sheet with increased loading at 75% span.

Rotor	1-hub	2	3	4	5	6	7	8	9	100% Span	8	6-90
r_1 (m)	0.007	0.117	0.137	0.167	0.177	0.197	0.217	0.237	0.257	0.277	0.297	0.317
r_2 (m)	0.07	0.48	0.53	0.61	0.69	0.76	0.84	0.92	1.00	1.08	1.16	1.24
$r_{1,mean}$ (m)	0.130	0.146	0.162	0.178	0.194	0.210	0.226	0.242	0.258	0.274	0.290	0.306
$r_{2,mean}$ (m)	0.113	0.131	0.149	0.168	0.186	0.204	0.222	0.240	0.258	0.276	0.294	0.312
$\beta_{1,mean}$ (deg)	0.163	0.176	0.187	0.199	0.211	0.222	0.234	0.246	0.258	0.269	0.280	0.291
Mean flow (kg/sec)	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69
U_1 (m/s)	149.92	181.08	212.26	243.41	274.57	305.72	336.90	368.05	399.22	430.39	461.55	492.72
U_2 (m/s)	291.56	326.38	361.18	395.97	430.77	465.56	500.35	535.14	569.93	604.72	639.51	674.30
C_u (mm)	226	226	226	226	226	226	226	226	226	226	226	226
β_1 (deg)	0	0	0	0	0	0	0	0	0	0	0	0
β_2 (deg)	298	298	298	298	298	298	298	298	298	298	298	298
P_{01} (Pa)	161926	161926	161926	161926	161926	161926	161926	161926	161926	161926	161926	161926
ΔP_0 (Pa)	63943	63943	63943	63943	63943	63943	63943	63943	63943	63943	63943	63943
P_{02} (Pa)	161926	161926	161926	161926	161926	161926	161926	161926	161926	161926	161926	161926
ΔT_0 (K)	43.11	48.37	49.61	49.84	48.06	49.28	50.48	51.67	52.82	53.95	55.07	56.18
C_{u1} (mm)	0	0	0	0	0	0	0	0	0	0	0	0
C_{u2} (mm)	219.38	201.00	186.23	174.07	163.88	155.21	147.75	141.24	135.93	131.24	127.04	123.24
β_1 (deg)	33.88	38.83	43.33	47.26	50.87	53.86	56.28	58.24	60.00	61.60	63.08	64.48
β_2 (deg)	-4.83	6.43	18.10	28.36	37.13	44.51	50.51	55.12	58.48	60.68	62.68	64.48
β_1 (deg)	46.29	47.46	47.49	47.08	46.39	45.42	44.24	42.84	41.24	39.51	37.68	35.74
β_2 (deg)	46.27	47.74	49.51	51.73	54.07	56.39	58.59	60.59	62.42	64.04	65.44	66.74
C_{u1} (mm)	226.00	226.00	226.00	226.00	226.00	226.00	226.00	226.00	226.00	226.00	226.00	226.00
C_{u2} (mm)	214.24	201.71	186.27	174.47	163.36	153.34	144.17	135.66	127.66	120.00	112.66	105.66
V_1 (m/s)	270.37	268.82	269.51	271.47	274.99	279.60	285.12	291.54	298.86	306.08	313.20	320.22
V_2 (m/s)	226.70	226.42	224.18	217.00	203.37	183.22	157.60	127.60	95.40	62.80	30.80	0.00
β_1 (deg)	0.83	0.78	0.76	0.75	0.74	0.74	0.74	0.74	0.74	0.74	0.74	0.74
β_2 (deg)	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0	0.9	0.8	0.7
DOE	0.34	0.51	0.60	0.68	0.72	0.75	0.76	0.76	0.76	0.76	0.76	0.76
Chord (m)	0.790	0.790	0.790	0.790	0.790	0.790	0.790	0.790	0.790	0.790	0.790	0.790
inc. angle (deg)	1.72	1.76	1.81	1.81	1.78	1.76	1.76	1.76	1.76	1.76	1.76	1.76
DP	0.26	0.30	0.33	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34
pt. slope factor	0.819	0.829	0.838	0.841	0.841	0.841	0.841	0.841	0.841	0.841	0.841	0.841
deviation angle, δ (deg)	10.17	8.82	7.82	6.48	5.08	4.53	4.48	4.13	3.90	3.68	3.46	3.24
Corrected deviation, δ_{cor} (deg)	12.87	11.32	10.03	8.95	8.08	7.43	6.99	6.63	6.40	6.20	6.00	5.80
Corrected Camber, δ_{cor} (deg)	48.98	41.97	36.76	30.88	26.56	23.48	21.21	19.69	18.74	18.10	17.60	17.10
Corrected Stagger, L_{cor} (mm)	7.28	16.10	23.94	30.72	36.51	41.44	45.66	49.28	52.38	55.00	57.20	59.00

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And, this is what we have done here. So, here in this case, if you look at, say earlier at 75% span, we have assumed our ΔP_0 to be slightly same as what we are expecting in sense of design, this is what has been modified here, okay. So, here you can see, this is what has been modified. So, once you are having your design excel sheet with you, this is what is the flexibility what you need to keep on eye with.

So, what we are expecting in sense of pressure rise of 1.63, this is what is a total pressure rise we are expected to be, okay. And, based on modified version, with at mid-span we have modified our ΔP_0 this is what is representing the distribution of ΔP_0 . So, when we have done

design for subsonic compressor that time also we have discussed the same approach. So, in line to that this is what we are discussing here, okay.

Now, when we are doing this, we need to keep on eye as we discussed in sense of my $\Delta\beta$. So, this is what is representing my $\Delta\beta$, that's what is varying from 38 to 11, okay. Same way, we are putting our eye for degree of reaction, since this is what is having higher rotational speed and my diameter is also slightly on the higher side near the hub region and as the reason why degree of reaction is not a problem. But maybe when we are discussing about the low speed configuration, this is what was a problem. So, accordingly we were modifying our ΔP_0 at the hub. This is what is in a reasonable range, we can say 0.38 and at the tip this is what is coming 0.83.

Now, diffusion factor also that's what is now modified earlier it was 0.22 and at the mid-station this is what was coming as say 0.32, that's what we have assumed. Now based on this, this is what has been modified as say 0.34. We have considered our number of blades to be same, we have considered our chord also to be same and that's what will be giving us say corrected camber, corrected stagger, and, you know, our corrected deviation angle. So, this is what is a modification that's what we have done.

So, you know, like when we are discussing, doing design using fundamental approach, there are infinite number of solutions which are possible, okay. Again, as a designer, it is you to decide which configuration you will be moving with. Now, many times what happens people they are realizing, if I will be assuming my ΔP_0 such that, you know, you will be setting some trend and based on that you will be assuming this number, No. It is preferred, you just check with how my angles are varying, how my velocity components are varying and based on that only you assume these numbers, be careful about this aspect, okay! So, maybe when you are using your excel sheet, on side by you can keep on eye putting say $\Delta\beta$ is one of the parameter and that's what will be helping you in sense of understanding what need to be my ΔP_0 .

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Tutorial contd.

Stator design sheet as per updated rotor loading

Stator									
r_3 (m)	0.163	0.175	0.187	0.199	0.211	0.222	0.234	0.246	0.258
α_3 (deg)= α_2 (deg)	44.27	41.78	39.61	37.73	36.07	34.60	33.29	32.12	31.14
α_4 (deg)=0	0	0	0	0	0	0	0	0	0
$\Delta\alpha$ (deg)	44.27	41.78	39.61	37.73	36.07	34.60	33.29	32.12	31.14
Chord (m)	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Z stator	36.00	36.00	36.00	36.00	36.00	36.00	36.00	36.00	36.00
s_1 (Spacing) (m)	0.026	0.028	0.030	0.033	0.035	0.038	0.040	0.043	0.045
l	2.00	1.50	1.00	0.50	0.00	-0.50	-1.00	-1.50	-2.00
$\sigma = c/s$	3.13	2.86	2.63	2.43	2.27	2.12	1.99	1.88	1.78
C_1 (m/s)	314.24	301.71	292.07	284.47	278.36	273.34	269.17	265.66	262.87
C_4 (m/s)	225	225	225	225	225	225	225	225	225
dH	0.72	0.75	0.77	0.79	0.81	0.82	0.84	0.85	0.86
DOR_stator	0.624	0.493	0.402	0.335	0.285	0.246	0.215	0.190	0.170
m (slope factor)	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
Corrected deviation, δ_{corr} (deg)	14.96	15.10	15.27	15.47	15.70	15.96	16.24	16.55	16.92
Corrected Camber, θ_{corr} (deg)	57.23	55.37	53.88	52.70	51.77	51.06	50.53	50.17	50.06
Corrected Stagger, ζ_{corr} (deg)	13.66	12.59	11.67	10.88	10.18	9.57	9.03	8.53	8.11

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Now, we will not be discussing about the stator design at the hub, we will not be discussing stator design at the tip, because this is what is known to us in sense of my α_3 , that's what is equal to α_2 and α_4 (we are assuming to be 0). We have done our calculation for number of stator blades that's what is say 36. And, again here in this case, incidence angle at the hub we are taking as $+2^\circ$ and say we are assuming our incidence angle at the tip that's what is say -2° , okay and this is what is all calculation at the hub-station. Now, based on all this design for say rotor and stator, now the term that's what is coming is to make the blade, okay.

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Tutorial contd.

Rotor Geometry with Constant tip configuration

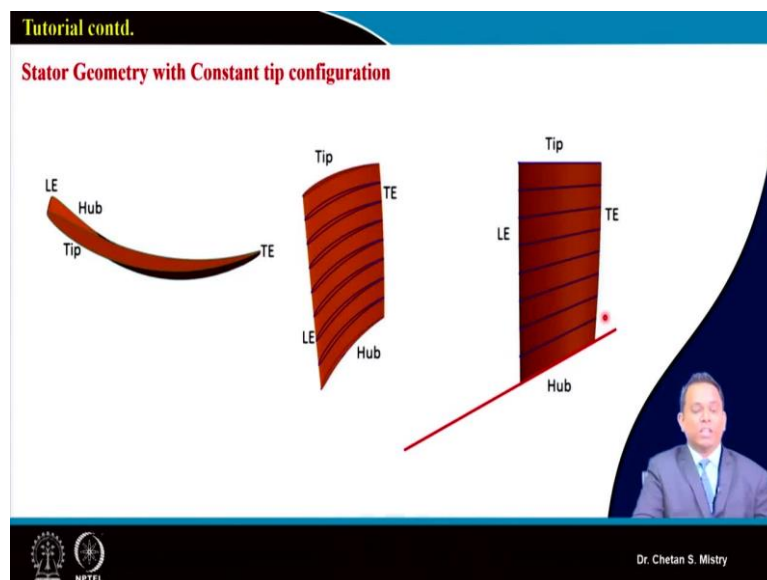
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So, here if you look at, this is what is representing my rotor blade, okay. And, as we have discussed, we have discussed about the development of DCA airfoil in last week, you are expert

enough to make the program for that. This is what is representing my distribution of the thickness in a circular arc, okay. And, these are the locations what we are discussing at hub, at tip, okay.

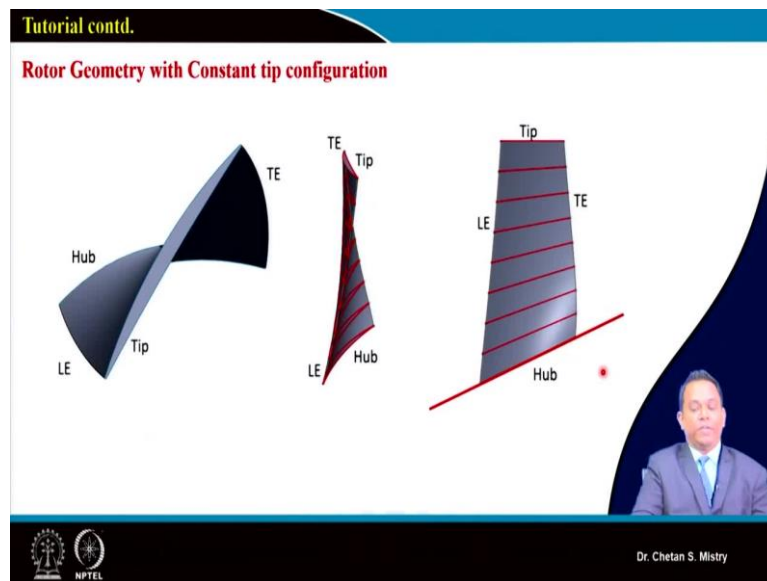
And, if you look at this leading edge what we are discussing, that's what is slightly rounded, trailing edge is also is slightly rounded. So, you make your design program at one station, maybe same logic you can apply, okay. Here, in this case my whole blade, that's what is or say all airfoils they are been stacked about CG. So, this is what is representing my rotor blade. So, you can see, this is what is constant tip configuration, this is what is representing in sense of what is happening at the hub, okay.

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Now, once this is what is, you can have your stator geometry and as we have discussed our stator, that's what a subsonic stator, so we will be using our C4 airfoil. So, here this is what is representing my curvature at hub and tip and this is also been stacked about CG. As per your requirement you can modify in a later case maybe you can stack about leading edge, maybe you can stack about say trailing edge, okay. So, this is what is my blade, and this is what is representing my placement of the blade.

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Now, very interesting point, here you need to observe for both rotor as well as for stator. What needs to be observed? You know, Like what all airfoils we have discussed, that's what program we have discussed, this is what is making 2D airfoil, that's what is having x and y coordinate and that too it is in one plane, okay.

Now, here in this case, as we have discussed, because of change of my density, because of change of my pressure ratio, my exit dimension will be different. So, under that condition, my airfoil here at the hub-station is not 2D airfoil. Suppose if I say, 2D airfoil that's what it say, I will be having my hub radius to be same.

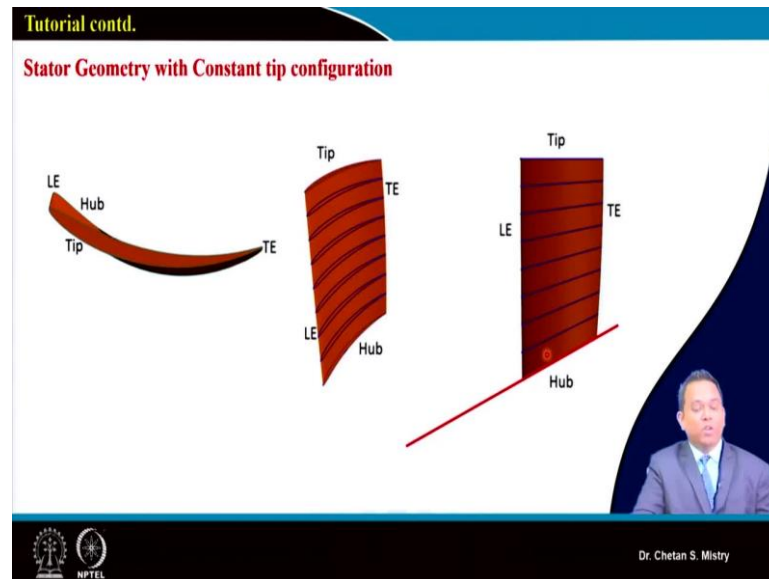
Now, based on what is my inclination, I need to modify. So, this airfoil is no more to the airfoil, this is what is 3D airfoil, okay. So, what all cascade data that's what is available with us may not work straightaway here, okay. For transonic kind of airfoil, those data may not work straightaway. Now, there your computational study that will be helping.

Here in this case, what will happen? Maybe you can have your chord, you need to modify that's what is one of the way, okay. Based on your inclination, you can modify your chord, this is what is say with modified chord or maybe your expected chord you can change, okay. This is what is one of the possibility. Later on, maybe you need to modify your thickness, okay. So, this is what is very important when we are discussing transonic compressor.

For say subsonic compressor, there is no problem; straight way what all cascade data, that's what is available with you, you can use and that is the reason why most of the time when we are doing our design for transonic stage, we are using or we are making use of computational

tools only, okay. So, most of the designers, most of the engine design companies, they people, they are taking help of their own code for doing the simulation or maybe they are using commercial tools which are available, okay.

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Now, in line to this, here also if you look at for subsonic case also, say this stator is subsonic say in the case. Here, in this case, here we are having this hub that's what is inclined hub, this is tip, that's what is say constant tip. So, this airfoils also you need to modify, okay. So, you need to be very careful about selection of these airfoils and how you will be incorporating these airfoils when you are making your rotor, when you are making your stator, okay.

So here, we will be stopping with. We are interested in doing design for transonic compressor. So, very first approach we have selected that's what is constant tip diameter configuration and we have done all our calculation at the mid-station, we have done calculation at hub-station, we have done calculation at the tip-station. Based on that we have changed our aerodynamic loading for fundamental design approach and we have come up with these blades.

So, this same design you can try with free vortex concept also having say constant tip diameter configuration. Now, in next lecture we will be discussing about constant hub diameter configuration, that also will be very interesting! Thank you, thank you very much for your kind consideration. See you again in the next lecture. Thank you.