Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 55
Design of Transonic Compressor (Contd.)

Hello, and welcome to lecture 55, we are discussing about the Design of Transonic Compressor.

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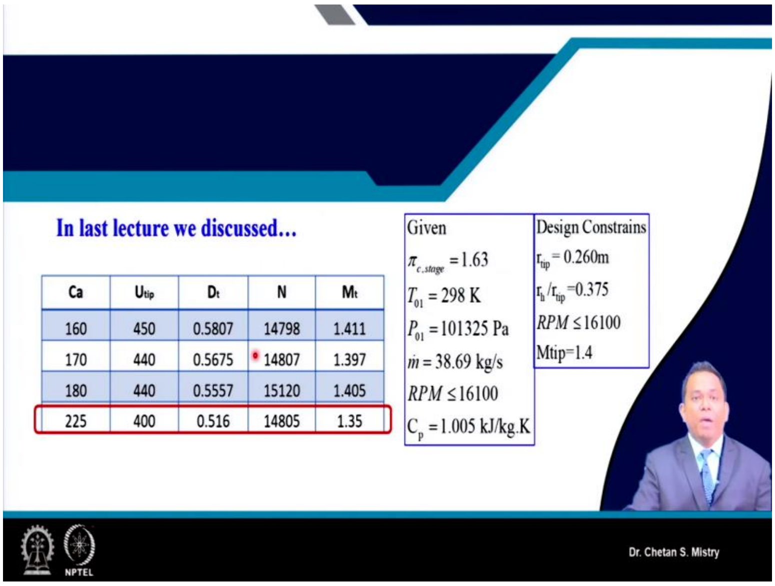

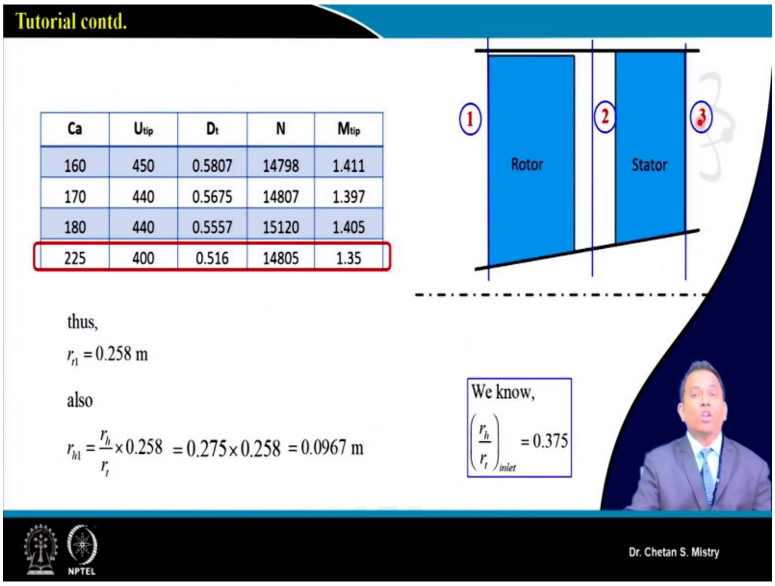

In last lecture, we have discussed about say what data that's what is given to us for the design. It says we are looking for pressure ratio of 1.63, mass flow rate of 38.69 kg/s and we have constraints with say tip diameter, hub to tip radius ratio at the entry that is also fixed, and it says our rotational speed should not exceed by 16,100. In last lecture, we were discussing about different approaches we can address the design, one of the approaches we have decided with it was assuming the axial velocity and assuming say peripheral speed.

And with different combination we have come up with the solution, it says, for current design, we will be taking our axial velocity to be say 225 m/s. Peripheral speed to be 400 m/s, that's what is giving us say tip diameter of 0.516 m. Our rotational speed will be 14,805 rpm, and our tip Mach number, that's what is coming within the range of 1.4 and that number is coming 1.35.

C_a	U_{tip}	D_t	N	M_t
160	450	0.5807	14798	1.411
170	440	0.5675	14807	1.397
180	440	0.5557	15120	1.405
225	400	0.516	14805	1.35

So, now, this is what all we were discussing in sense of entry condition. And, once we are decided with our entry diameter at the tip, we have done our calculation that what will be the radius at the entry near the hub station. And that's what was coming 0.0967 m.

$$thus,$$

$$r_{t1} = 0.258\, m$$

$$Also,$$

$$r_{h1} = \frac{r_h}{r_t} \times 0.258 = 0.275 \times 0.258 = 0.0967\, m$$

Now, the next step, that's what is we are looking for calculating different geometrical parameter for the stage. That's what is say my rotor and stator as in combination.

So, what is given to us? It say like our pressure ratio per stage, that's what we are expecting. That means, that's what we will be taking as a base for our future calculation. So, what will be our future calculation or what we are looking for is we need to do our calculation at the station 3 because per stage pressure rise that's what is given to us, okay.

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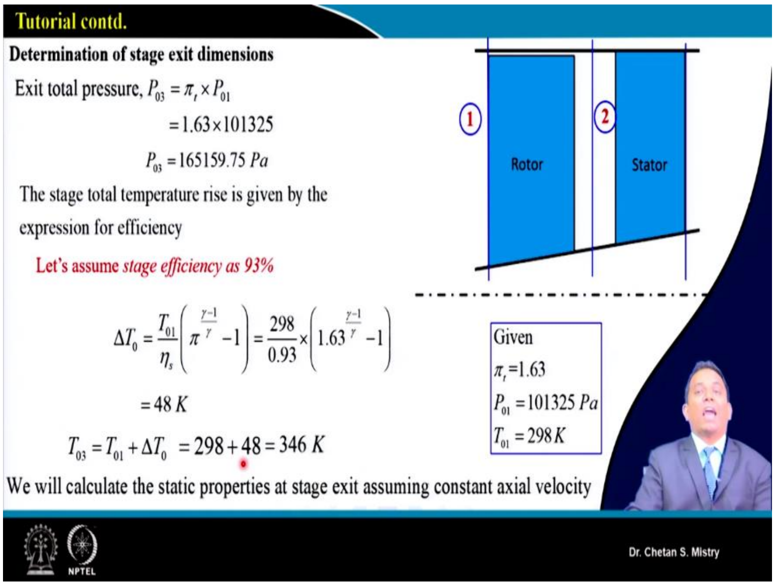

So, now in order to do that calculation, let us see; what it says, our pressure ratio that's what is given it is 1.63. Our inlet pressure that's what is 101.325 kPa. So, based on that we can do our calculation for what will be our exit pressure…exit total pressure. Now, from our fundamentals, what we know for per stage configuration, we need to correlate with the rise of temperature. This rise of temperature that's what will be helping us in order to calculate what will be my total temperature at the exit.

Now, once the total temperature at the exit is known to us, we can do our calculation for static temperature. We also can do our calculation for say our static pressure and that's what is required for doing calculation for what will be my exit density. Now, once the exit density, that's what is known to us, we can do our calculation what will be my exit area for this particular stage and that is how we will be following with.

$$Determination\ of\ stage\ exit\ dimensions$$

$$Exit\ total\ pressure, P_{03} = \pi_t \times P_{01}$$

$$= 1.63 \times 101325$$

$$P_{03} = 165159.75\ Pa$$

Here, in this case, what we know? This total temperature rise, ΔT_0 that's what we are writing in sense of entry temperature, my efficiency; here, this is what is stage efficiency, you can consider this as a polytropic efficiency because we are considering only one stage. And this is my pressure ratio. So, now in order to do that calculation, we are assuming this efficiency to be 93%. This is what is slightly on the higher side, we are going aggressively in that direction. That is the reason why we have assumed that let us say suppose it is say 93%, okay. Now, once we are knowing this ΔT_0, we can do our calculation in sense of what will be my T_{03}. So, this T_{03}, that is coming 346 K. Now, this is what is known to us.

$$Let's\ assume\ stage\ efficiency\ as\ 93\%$$

$$\Delta T_0 = \frac{T_{01}}{\eta_s}\left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right) = \frac{298}{0.93}\left(1.63^{\frac{1.4-1}{1.4}} - 1\right)$$

$$= 48\ K$$

$$T_{03} = T_{01} + \Delta T_0 = 298 + 48 = 346\ K$$

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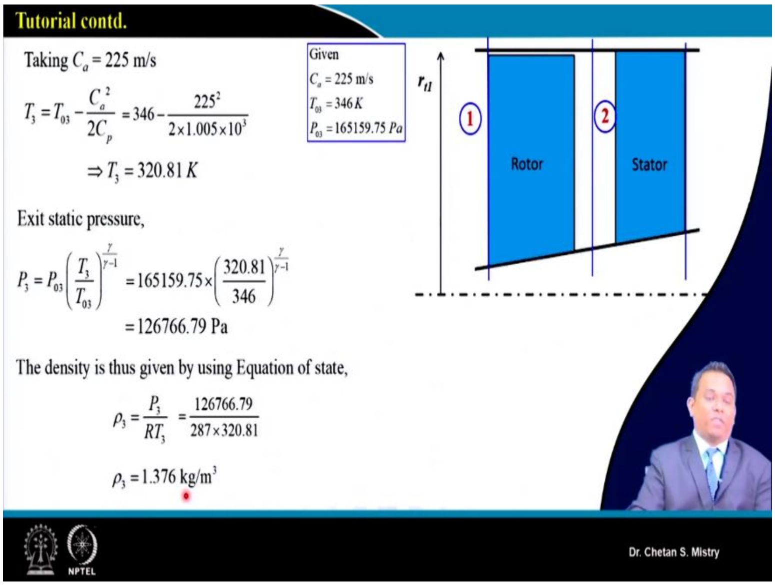

We can now do our calculation for static temperature. Now, again here in this case, we can assume our exit to be axial one. So, we can say my absolute velocity at the exit, that's what is suppose say C_3, this is equal to my axial velocity. And we are assuming our axial velocity to be constant for this configuration. So, if we are putting that as a number, it says we will be calculating our static temperature at the exit or at the station 3. So, this station 3 it is coming as say 320.81 K.

$$Taking\ C_a = 225\ m/s$$

$$T_3 = T_{03} - \frac{C_a^2}{2C_p} = 346 - \frac{225^2}{2 \times 1.005 \times 10^3}$$

$$\Rightarrow T_3 = 320.81\ K$$

Now, based on our isentropic relation, we can do our calculation for say static pressure P_3. So, if we are putting this relation and if we are putting our static temperature and total temperature, we will be getting our pressure…static pressure at the exit as say 126.76 kPa, okay.

$$Exit\ static\ pressure,$$

$$P_3 = P_{03}\left(\frac{T_3}{T_{03}}\right)^{\frac{\gamma}{\gamma-1}} = 165159.75 \times \left(\frac{320.81}{346}\right)^{\frac{1.4}{1.4-1}}$$

$$= 126766.79\ Pa$$

Once, this is what is known to us, we can do our calculation for exit density and that exit density, that's what is given by P_3/RT_3. If we are putting these numbers in sense of say static temperature and static pressure, it says my density is coming 1.376 kg/m^3. Now, this density, that's what is very important for us.

$$The\ density\ is\ thus\ given\ by\ using\ equation\ of\ state,$$

$$\rho_3 = \frac{p_3}{RT_3} = \frac{126766.79}{287 \times 320.81}$$

$$\rho_3 = 1.376\,\frac{kg}{m^3}$$

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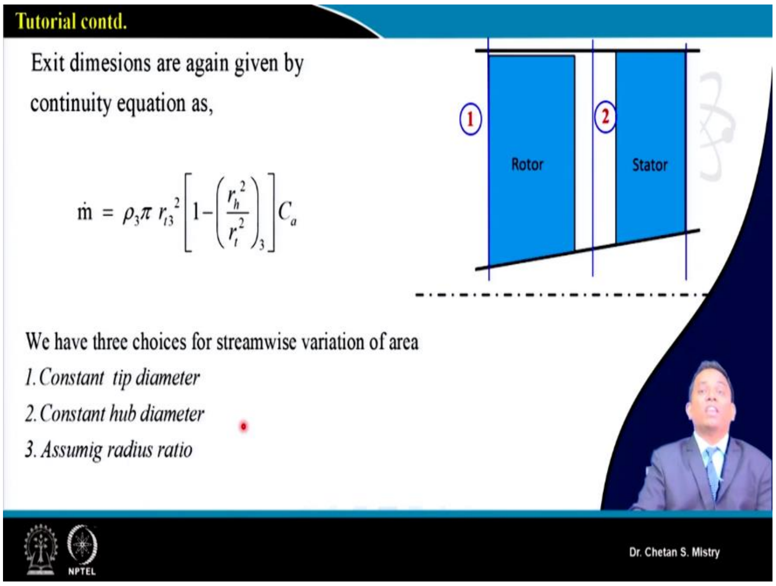

Now, if we are considering, we need to have our continuity to be satisfied both at the entry as well as at the exit station. So, when we say at the exit station, my mass flow rate equation I can write down, it is

$$\dot{m} = \rho_3 \pi r_{t3}^2 \left[1 - \left(\frac{r_h^2}{r_t^2}\right)_3\right] C_a$$

Now, here let me put this case as a special case. And, if you recall when we were discussing say design of axial flow compressor at the initial stage. We were discussing about the flow track design where we were discussing about say different type of configuration which are possible. We have discussed about constant tip diameter, constant hub diameter, constant mean diameter kind of configuration.

So, for say single stage transonic compressor, what we know is by entry area and exit area, these two will be different. Now, in order to do the calculation for our exit dimension we need to fix with one of the radius. When I say, we need to fix one of the radius, based on our equation we can calculate what will be the radius of other station, okay.

So, there are three different possibilities we have; one, that’s what is say you can go with constant tip diameter configuration, we can go with say constant hub diameter configuration and there is a possibility we can assume say some radius ratio at the exit, okay. So, the kind of design what we are looking for based on that we need to go with this kind of configuration. So, we need to be very careful in sense of understanding here.

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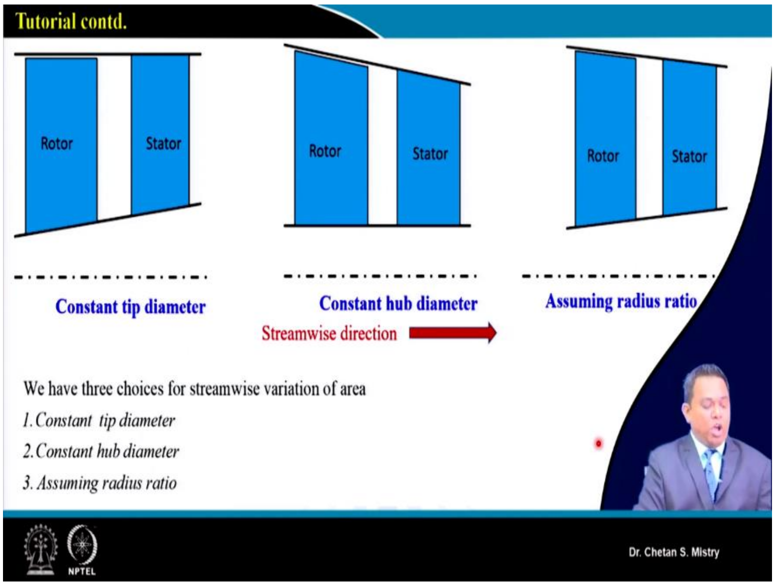

So, let us try to look at what all we are looking for. So, when I say we are having constant tip diameter configuration; so, this is what will be my tip that's what is constant. So, along the streamwise direction, what will happen, my hub radius that's what is going to be increased. We know what all are the benefits of configuring having say constant tip diameter kind of configuration, okay. Same way, suppose if you are considering constant hub diameter kind of configuration, then at the exit, my area that's what will be lower. And if that's what is lower, you can say my tip diameter, that's what is going to be reduced.

So, do not get confused here, this is what is very important for us to realize to understand, okay. Now, this is what is one of the configuration where we are assuming our say radius ratio. Maybe you can assume same radius ratio as your entry condition, that is also possible. You can assume my exit radius ratio to be different, even it is possible you can assume your mean diameter or you can say my mean radius that's what is same, okay.

If that's what is your case, you can do your calculation for hub diameter, you can do your calculation for tip diameter. So, be careful about all these aspects. So, if you recall, when we were discussing about Pratt and Whitney engine that time we were discussing about say different kinds of flow tracks, okay. So, initially you will be assuming with some configuration, maybe later on as per the requirement, you may need to modify your design, okay.

Suppose say, you are doing your design for say transonic kind of configuration and that is to in multistage; suppose say, we are considering say low bypass ratio engine, where you may be having more than one axial flow fan on LP spool. So, under that configuration, based on downstream requirement; when I say downstream requirement that means, what connecting duct we will be having for LP spool and HP spool that need to be done in an aggressive way. And, maybe as per your requirement, you may need to go with some modification in sense of designing say different configurations, okay.

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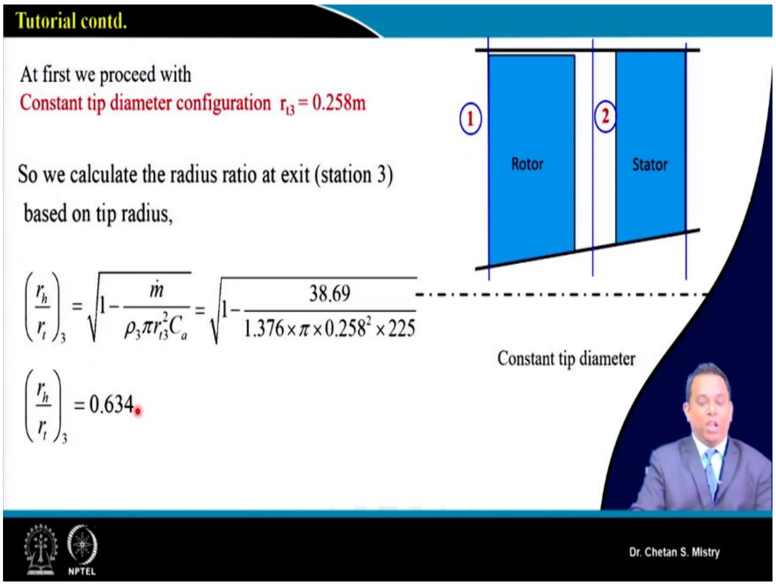

Now, here in this case, what we will be doing? We will be going with say constant tip diameter configuration. So, you know do not get confused at this moment, we will be discussing the same design with two different approaches, we will be discussing this design with constant tip, diameter configuration, we will be discussing our design with constant hub diameter configuration also. So, that's what will be giving you the idea like how exactly you will move forward with.

Yes, this is what is kind of spoon feeding at this moment to all of you, but later on you will not get confused how exactly you will be doing your design. And, that is the reason we will be discussing our fundamental design approach with two different kind of configuration. So, if you go through overall course design this is what will be giving you whole idea of design of any kind of axial flow compressor or any kind of axial flow fan. You will not be having any confusion. So, do not jump for week ,straight away from week 1 to week 4, week 5 saying this is known to us. It may possible that some of the important discussion what all we have done in different weeks, that's what is very important, that's what is making the base for our future designs, okay.

So, here in this case, let us see we will be taking our tip radius to be 0.258 m. If that's what is your case, we can say, we will be calculating what will be our radius ratio. You can see, my radius ratio that's what is coming 0.634, okay. So, you know, like this is what is giving you idea suppose say you will be doing your design for some kind of radius ratio assumption, maybe this will help you in sense of what numbers need to be selected with.

$$At\ first\ we\ proceed\ with,$$

$$Constant\ tip\ dimaeter\ configuration\ r_{t3} = 0.258\ m$$

$$So, we\ can\ calculate\ the\ radius\ ratio\ at\ exit\ (station\ 3) based\ on\ tip\ radius,$$

$$\left(\frac{r_h}{r_t}\right)_3 = \sqrt{1 - \frac{\dot{m}}{\rho_3 \pi r_{t3}^2 C_a}} = \sqrt{1 - \frac{38.69}{1.376 \times \pi \times 0.258^2 \times 225}}$$

$$\left(\frac{r_h}{r_t}\right)_3 = 0.634$$

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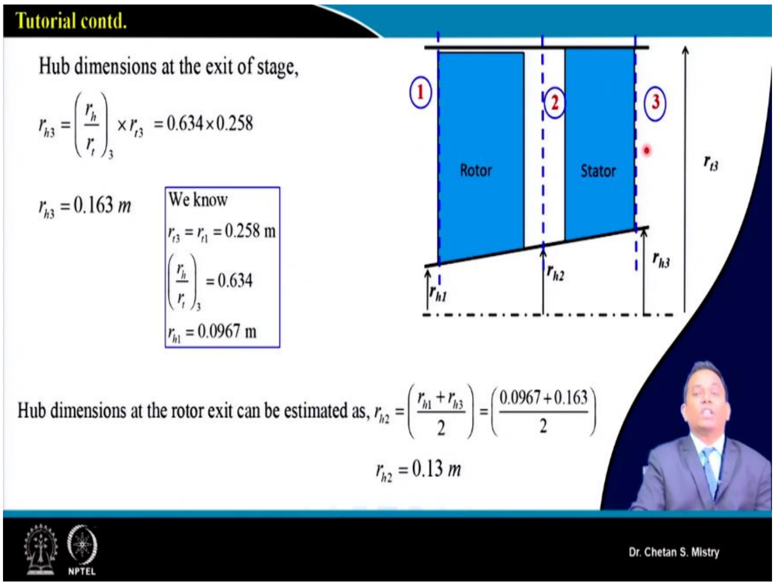

Now, once this is what is known to us, we can do our calculation for what will be our hub radius and my hub radius, that's what is coming 0.163 m, okay.

$$Hub\ dimensions\ at\ the\ exit\ of\ stage,$$

$$r_{h3} = \left(\frac{r_h}{r_t}\right)_3 \times r_{t3} = 0.634 \times 0.258 = 0.163\ m$$

So, now you can say at station 1, I know what is my tip radius, what is my hub radius. At station 3, I know what will be my hub radius and what will be my tip radius. Now here, in this case, I am locating my station 2 in between stator and rotor somewhere, okay. So, if this is your case, you can do your calculation for hub radius, safely; maybe you can take the average of these two at the hub, that's what is giving me my hub radius equals to 0.13 m, my tip radius that's what we have assumed to be constant.

$$Hub\ dimensions\ at\ the\ rotor\ exit\ can\ be\ estimated\ as,$$

$$r_{h2} = \left(\frac{r_{h1} + r_{h3}}{2}\right)$$

$$r_{h2} = \frac{0.0967 + 0.163}{2}$$

$$r_{h2} = 0.13\ m$$

Now, here in this case, there are different possibilities when you are doing your design, okay. So, initially you are doing your preliminary design. So, this calculation, that's what will be helping you. Very soon the question will come, Sir, what will be the gap…axial gap between rotor and stator. Let me tell you for transonic kind of configuration, people, they are safely assuming the axial gap, that's what is varying say 20% to 35% of the rotor chord, okay.

Now, it depends, this is what is a proprietary thing. So, many engine design manufacturing companies, they are not putting that in open literature but safely you can assume that. Anyway, you will be having your computational tool with you with which you can verify your performance and later on suppose if you are having the facility for testing, then you can do say parametric study by changing the axial spacing between rotor and stator, okay.

So, this is also one of the possibilities, we can say, we are observing this as r_2, okay. Sometimes what people, they are doing? Say, this radius, that's what is known to us, this entry radius, that's what is known to us. Based on calculation of angle θ by that we also you can do that design, okay. And if you recall, when I was discussing about the track design, we say, we are looking for our flow passage to be smooth passage, okay. So, it may be possible that you may need to play with certain parameters, maybe you will be changing your hub diameter here and there, may be at the entry maybe at the exit, maybe you will be changing your tip diameters slightly, okay.

So, that's what is all designer's choice and as per your requirement. So, in order to simplify whole our design process, what we are doing? We are safely assuming at station 2 my hub diameter that's what is averaged, okay. So, now you can say, we know what is our radius at entry, what is our radius at the exit, and what is our radius at say station 3, okay, that's what is my exit.

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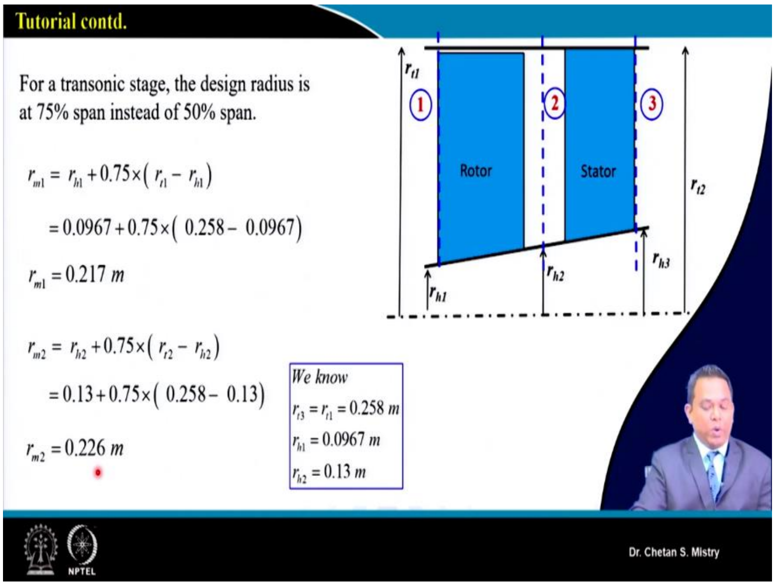

Now, once this is what is known to us, very first step, as we have discussed, is we need to start doing calculation at say mid station and for transonic compressor our mid station we are taking at 75% of span, that's what is a thumb rule, okay. If, that's what is your case, we can say at the entry we can calculate what will be my mean radius. So, that's what is given by say this is what is my $r_{h1} + 0.75 \times (r_{t1} - r_{h1})$, that's what will be giving me what will be my mean radius at the entry.

$$r_{m1} = r_{h1} + 0.75 \times (r_{t1} - r_{h1})$$

$$= 0.0967 + 0.75 \times (0.258 - 0.0967)$$

$$r_{m1} = 0.217\ m$$

Be careful, as we have discussed, here in this case, my mid station, that's what will be coming somewhere here.

Though, we have taken our tip radius to be constant, my mean radius at the entry, mean my mean radius in between stator and rotor or at the station 2 and at the station 3 that's what will be different. And, that is the reason why we are calculating at say station 2, this is what can be calculated by this way. What we know, we know what is our hub diameter at station 2, 75% span, that's what is giving me my mean radius at station 2 it is coming say 0.226, okay.

$$r_{m2} = r_{h2} + 0.75 \times (r_{t2} - r_{h2})$$

$$= 0.13 + 0.75 \times (0.258 - 0.13)$$

$$r_{m2} = 0.226\, m$$

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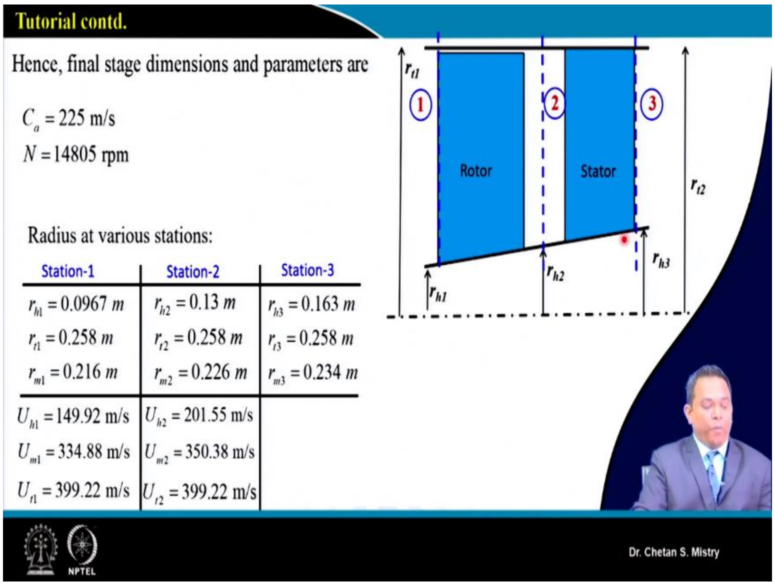

Same way, if we are having our exit condition; so, we can do that calculation. So here, you can see, we are putting different radius, this is what you can do as a calculation at the entry station, this is what is known to us, okay. This mean radius, that's what we have calculated. Same way at station 2 we can do our calculation for mid station.

At station 3 also, you can do this calculation. So now, here if you look at, you can clearly see my mean radius that's what is not constant. You can say this is what is increasing in the nature. So, you will be having your mid station that's what will be coming like this, it depends, okay. And, that's what will help us in sense of doing our further design.

$$Hence, final\ stage\ dimensions\ and\ parameters\ are$$

$$C_a = 225 \frac{m}{s}$$

$$N = 14805\ rpm$$

Station-1	Station-2	Station-3
$r_{h1} = 0.0967\ m$	$r_{h2} = 0.13\ m$	$r_{h3} = 0.163\ m$
$r_{t1} = 0.258\ m$	$r_{t2} = 0.258\ m$	$r_{t3} = 0.258\ m$
$r_{m1} = 0.216\ m$	$r_{m2} = 0.226\ m$	$r_{m3} = 0.234\ m$
$U_{h1} = 149.92\ m/s$	$U_{h2} = 201.55\ m/s$	
$U_{m1} = 334.88\ m/s$	$U_{m2} = 350.38\ m/s$	
$U_{t1} = 399.22\ m/s$	$U_{t2} = 399.22\ m/s$	

Now, since my radius, that's what is different, we need to realize my peripheral speed at the entry and my peripheral speed at the exit of my rotor, those also will be different, okay. So, you can see here, at the entry my peripheral speed that's what is at the hub is 149.92 and at the exit that's what is say 201.55.

Same way, at the tip, that's what is coming nearly say 400 and this is what is because that's what is we are assuming to be constant, okay. So, I am sure, this is what will be giving us idea how do we do our calculation at say entry, how do we do our calculation at the exit and how do we do our calculation at the mid station and we need to assume certain approach for the design.

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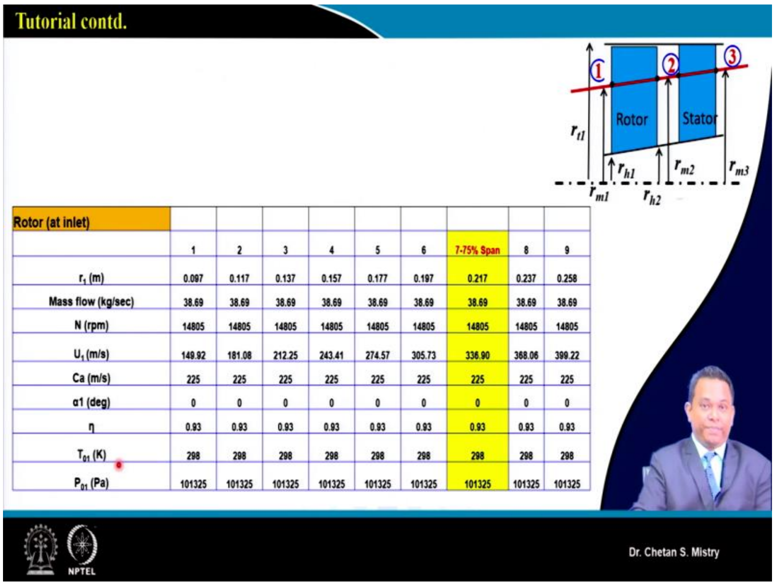

Rotor (at inlet)									
	1	2	3	4	5	6	7-75% Span	8	9
r_1 (m)	0.097	0.117	0.137	0.157	0.177	0.197	0.217	0.237	0.258
Mass flow (kg/sec)	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69	38.69
N (rpm)	14805	14805	14805	14805	14805	14805	14805	14805	14805
U_1 (m/s)	149.92	181.08	212.25	243.41	274.57	305.73	336.90	368.06	399.22
Ca (m/s)	225	225	225	225	225	225	225	225	225
α1 (deg)	0	0	0	0	0	0	0	0	0
η	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
T_{01} (K)	298	298	298	298	298	298	298	298	298
P_{01} (Pa)	101325	101325	101325	101325	101325	101325	101325	101325	101325

Now, once this is known to us, what we need to do is my entry location. So here, this is what is representing one of the streamline at 75% span, okay. And, these are the points which we have calculated with, okay. So, that's what is giving us say my tip dimension, my hub dimension and mean dimension. Now, here we need to be very careful, this my entry radius what we are discussing that's what is say my r_{1h} and r_{1t}, okay. So, at the entry, that is the reason it is specifically we are doing our calculation. So, we are writing this as say r_1. Do not get confused in the later stage.

So, that is the reason we are saying like this is what is varying from 0.097 to 0.258 and this is what we are doing calculation since we are assuming our entry to be axial one. So, α_1 we are taking to be 0 and these are the pressures and temperatures. So, this is what will be giving me what will be my entry design or entry parameters.

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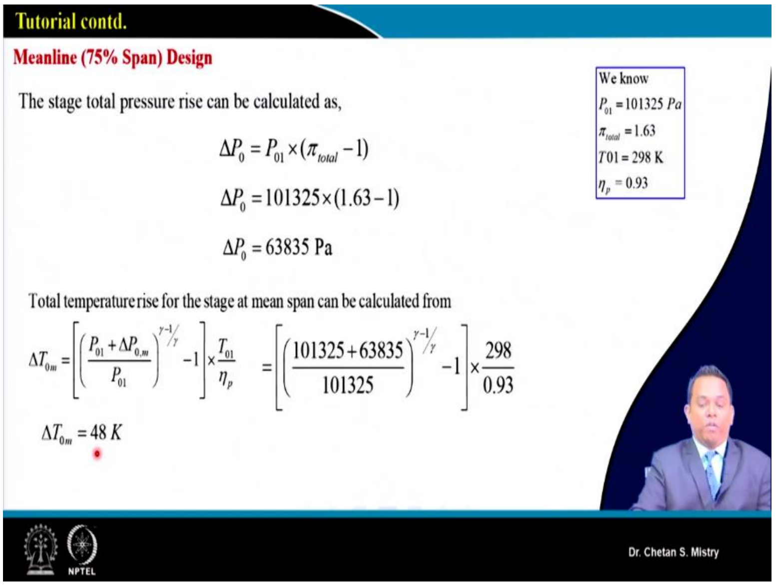

Now, what we are looking for is we are looking for doing our calculation at that say exit station, okay. Now, in order to do that calculation, what we are looking for, here in this case, we are assuming our fundamental design approach, okay. So, in order to do our calculation at the mid station, in order to do calculation for different velocity components, in order to do calculation for different flow angles, in order to do calculation for say other parameters like diffusion factor, de-Haller's factor, degree of reaction, different flow angles. We need to do the calculation at the mid station.

So, for the sake of simplicity at the initial stage, what we are doing? At that station, at 75% span, we are taking our pressure ratio expected that's what is 1.63, okay. So, it says like across my 75% span, we are expecting our ΔP_0, that's what is coming as say $P_{01} \times (\pi_{total} - 1)$ is what is coming as ΔP_0.

$$\textit{The stage total pressure rise can be calculated as,}$$

$$\Delta P_0 = P_{01} \times (\pi_{total} - 1)$$

$$\Delta P_0 = 101325 \times (1.63 - 1)$$

$$\Delta P_0 = 63835\ Pa$$

So, you can say at 75% span, we are looking for this pressure rise, okay. If this is what is your case, based on our understanding, we can do our calculation for ΔT_0. So, when we are putting this, it says this is what is my ΔP_0 at mid station, entry temperature, that's what is known to us and efficiency is known to us. So, we can do our calculation for ΔT_0. So, just realize what we are doing. Now, this ΔT_0, that's what will be helping us for calculating other parameters. What we have done in earlier design, in line to that only we are moving forward with.

$$Total\ temperature\ rise\ for\ the\ stage\ at\ mean\ span\ can\ be\ calculated\ from,$$

$$\Delta T_{0m} = \left[\left(\frac{P_{01} + \Delta P_{0,m}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \times \frac{T_{01}}{\eta_p}$$

$$= \left[\left(\frac{101325 + 63835}{101325}\right)^{\frac{1.4-1}{1.4}} - 1\right] \times \frac{298}{0.93}$$

$$\Delta T_{0m} = 48\ K$$

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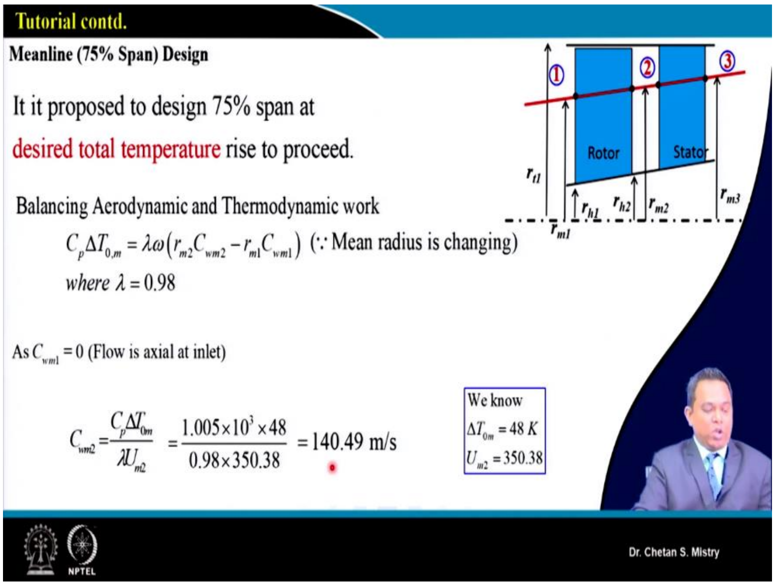

So, what it says? Now, at 75% span, we are assuming our pressure ratio to be 1.63. So, we will be balancing our aerodynamic work and thermodynamic work. So, you can say, it is $C_p\Delta T_{0,m}$. This is nothing but our λ, we are assuming this λ to be 0.98. This is what we can say in sense of changing the equation because my radius, that's what is changing at the entry as well as at the exit. So, we need to be very careful here.

Now, the C_{w1}, that's what we are taking that to be 0. So, if this is what is your case, you can do calculation what will be my whirl component at the exit or this is what is at the station 2 and this is what is coming as say 140.49 m/s.

$$Balancing\ Aerodynamic\ and\ Thermodynamic\ work,$$

$$C_p \Delta T_{0,m} = \lambda \omega (r_{m2} C_{wm2} - r_{m1} C_{wm1})$$

$$where\ \lambda = 0.98$$

$$As\ C_{wm1} = 0\ (flow\ is\ axial\ at\ inlet)$$

$$C_{wm2} = \frac{C_p \Delta T_{0m}}{\lambda U_{m2}} = \frac{1.005 \times 10^3 \times 48}{0.98 \times 350.38} = 140.49\ m/s$$

Now, you know, like we have our entry condition with us, we have our exit condition with us. So, we will be moving with say use of velocity triangle in order to do calculation for different velocity components, okay. So, the approach is for the design or the steps for the design, that's what is in line to what all we have done up till now. But again and again, we need to be very careful what location, what numbers we are selecting with, okay.

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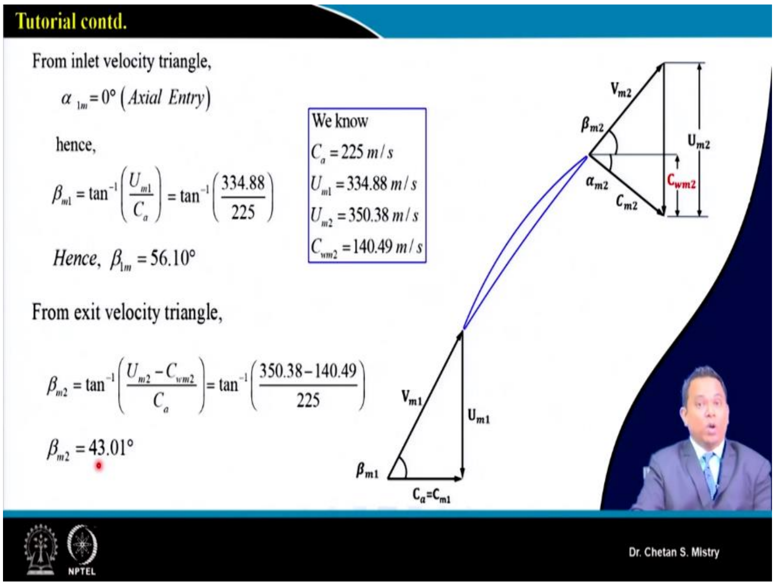

So, if this is what is your case, you can say at mid station, this is what is say my airfoil, okay. And, we can say, this is what is my velocity triangle at the entry, my α_1, that's what is say 0 because we are considering that as axial entry, this U that's what we have calculated. This is what will be my say axial velocity triangle, okay. This is what is our exit velocity triangle, okay. Now, if this is what is your case, we can do calculation what will be my entry angle that's what is say β at mid station at the entry that is given by $\tan^{-1}\left(\frac{U}{C_a}\right)$.

Now, we have done our calculation for peripheral speed at the entry, we have done calculation for our peripheral speed at the exit. So, based on that we can do our calculation for β_1, and that's what is coming 56.10°, okay.

$$From\ inlet\ velocity\ triangle,$$

$$\alpha_{1m} = 0° \ (Axial\ Entry)$$

$Hence,$

$$\beta_{m1} = \tan^{-1}\left(\frac{U_{m1}}{C_a}\right) = \tan^{-1}\left(\frac{334.88}{225}\right)$$

$Hence,\ \ \beta_{1m} = 56.10°$

Now, at the exit, this is what is our velocity triangle. So, based on our trigonometry we can say this is what is given by $\tan^{-1}\left(\frac{U-C_w}{C_a}\right)$. So, this is what will be giving me say our β_2 at the exit and that's what is coming say 43.01°. So, now at mid station, what is my β_1 and what is my β_2 those angles are known to us.

$From\ exit\ velocity\ triangle,$

$$\beta_{m2} = \tan^{-1}\left(\frac{U_{m2} - C_{wm2}}{C_a}\right) = \tan^{-1}\left(\frac{350.38 - 140.49}{225}\right)$$

$$\beta_{m2} = 43.01°$$

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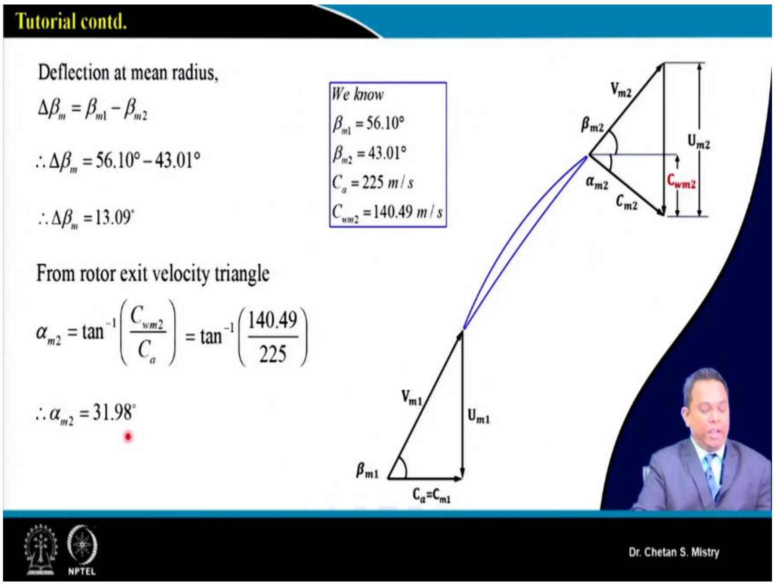

Now, this is what will be giving me what is my $\Delta\beta$ at mid station, it is coming 13.09.

$$Deflection\ at\ mean\ radius,$$

$$\Delta\beta_m = \beta_{m1} - \beta_{m2}$$

$$\therefore \Delta\beta_m = 56.10° - 43.01°$$

$$\therefore \Delta\beta_m = 13.09°$$

Same way, we can do our calculation for say α, we can do calculation for C_2 because those parameters are required when we will be doing our design for stator. And, that is the reason why these $\tan\alpha_2$ that's what is given by C_w/C_a C_w by C_a. We can say my α that's what is coming 31.98°, okay.

$$From\ rotor\ exit\ velocity\ triangle,$$

$$\alpha_{m2} = \tan^{-1}\left(\frac{C_{wm2}}{C_a}\right) = \tan^{-1}\left(\frac{140.49}{225}\right)$$

$$\therefore \alpha_{m2} = 31.98°$$

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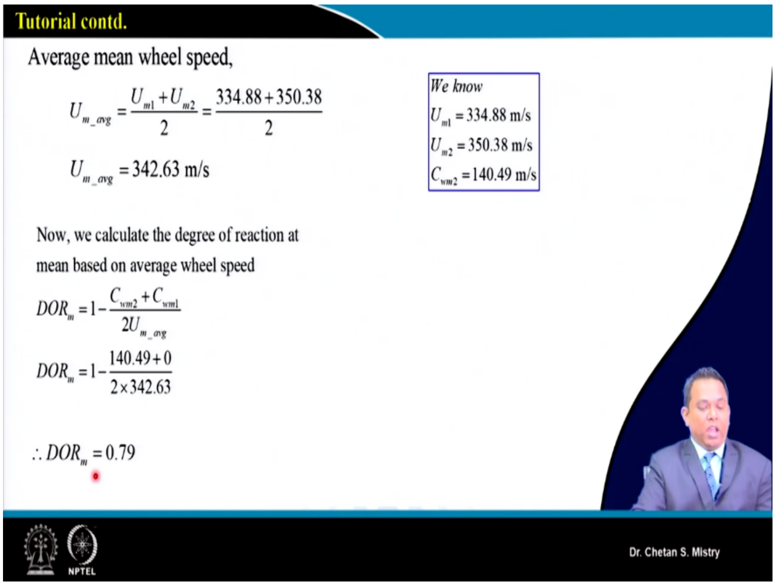

Now, in order to do the calculation for degree of reaction; so, degree of reaction equation that's what we are writing in sense of my peripheral speed, okay. So, in a cruel way, we can rather having say selection of U at entry, U at the exit, rather taking which number or to avoid that kind of confusion, we will be taking the average value. Let us say we will be calculating average value of peripheral speed. If we are assuming or we are calculating our average value, we can do our calculation for degree of reaction at the mid station and if you put that number this is coming as say 0.79, okay.

$$Average\ mean\ wheel\ speed,$$

$$U_{m\_avg} = \frac{U_{m1} + U_{m2}}{2} = \frac{334.88 + 350.38}{2}$$

$$U_{m\_avg} = 342.63 m/s$$

$$Now, we\ calculate\ the\ degree\ of\ reaction\ at\ mean\ based\ on\ average\ wheel\ speed,$$

$$DOR_m = 1 - \frac{C_{wm2} + C_{wm1}}{2U_{m_{avg}}}$$

$$DOR_m = 1 - \frac{140.49 + 0}{2 \times 342.63}$$

$$\therefore DOR_m = 0.79$$

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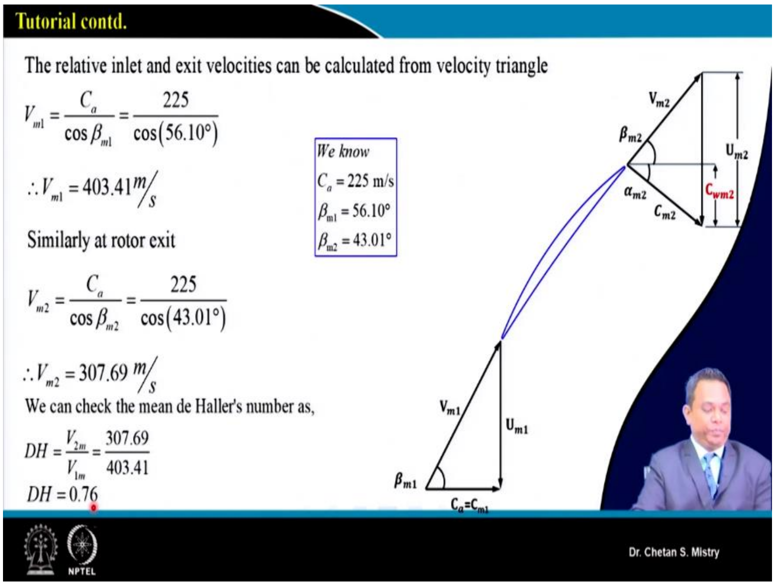

Now, what next parameter we are interested in is say our de-Haller's factor. So, in order to do the calculation for de-Haller's factor, we are interested in calculating our relative velocity component at the entry, my relative velocity component at the exit. So, based on trigonometry we can say, my relative velocity at the entry, that's what is $C_a/\cos\beta_1$ and at the exit we can say, this is say $C_a/\cos\beta_2$. And, that's what is giving my entry relative velocity as 403.41 m/s and my exit relative velocity as 307.69 m/s, okay. Now, if we are putting this number, it says my degree of reaction that's what is coming 0.76, okay.

The relative inlet and exit velocities can be calculated from velocity triangle,

$$V_{m1} = \frac{C_a}{\cos\beta_{m1}} = \frac{225}{\cos(56.10°)}$$

$$\therefore V_{m1} = 403.41\ m/s$$

Similarly at rotor exit,

$$V_{m2} = \frac{C_a}{\cos\beta_{m2}} = \frac{225}{\cos(43.01°)}$$

$$\therefore V_{m2} = 307.69\ m/s$$

$$We\ can\ check\ the\ mean\ de - Haller's\ number\ as,$$

$$DH = \frac{V_{2m}}{V_{1m}} = \frac{307.69}{403.41}$$

$$DH = 0.76$$

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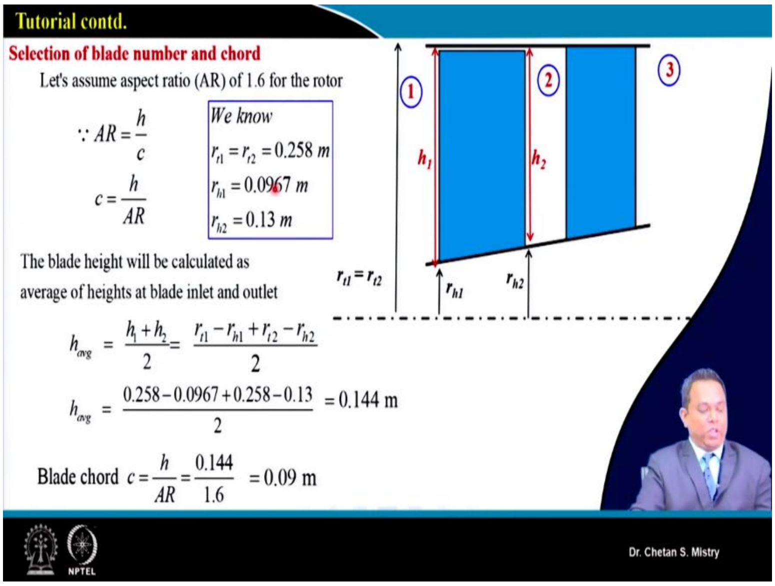

Now, once we have done this calculation, we are interested in calculating what will be our number of blades, what will be our chord. We are also interested what is our diffusion factor that means, we are interested in calculation of the solidity, okay. So, you know, like here in this case, we need to have certain assumptions to be made, okay. So, for this design, let me assume the aspect ratio of this blade as say 1.6. There is nothing wrong you can assume that to be 1, you can assume 1.2, even you can go with say 0.8 or so on, okay.

So, for the design…for this particular design, let me assume my aspect ratio to be 1.6. What aspect ratio is that is nothing but what is my span to chord ratio. Now, when I say span, that is nothing but my height. Now, that height, the confusion here is which height needs to be selected. And that is the reason what we will be doing, we will be doing our calculation for height at the entry, we will be doing our calculation for height at the exit. And we will be taking the average of these. So here, my span or by height of the blade, that's what is $\frac{h_1+h_2}{2}$. And, this h_1 and h_2 that's what we can write down in sense of difference of my say radius, okay.

So, this is what is $r_{t1} - r_{h1}$, that's what will be giving me h_1, this is same, $r_{t2} - r_{h2}$, that's what will be giving me my height h_2. If that's what is your case, you can calculate your average height, that's what is coming 0.144 m, okay. Now, once this is what is known, we need to come up with say very important parameter, that's what is what will be the chord and that's what chord we can calculate based on h/AR, this is what is coming as say 0.09 m, okay.

$$Let's\ assume\ aspect\ ratio\ (AR)\ of\ 1.6\ for\ the\ rotor$$

$$\because AR = \frac{h}{c}$$

$$c = \frac{h}{AR}$$

$$The\ blade\ height\ will\ be\ calculated\ as\ average\ of\ heights\ at\ blade\ inlet\ and\ outlet$$

$$h_{avg} = \frac{h_1 + h_2}{2} = \frac{r_{t1} - r_{h1} + r_{t2} - r_{h2}}{2}$$

$$h_{avg} = \frac{0.258 - 0.0967 + 0.258 - 0.13}{2} = 0.144\ m$$

$$Blade\ chord, c = \frac{h}{AR} = \frac{0.144}{1.6} = 0.09\ m$$

Now, many times when you are going with this assumption. This number that may be coming slightly on the lower side; and, you realize one thing, when we were discussing about the contra rotating fan design, we have realized that part. What will happen, my chord will be coming very small. When this chord is coming small that means, you need to do whole lot of diffusion with small chord. Under that configuration, my camber angle will also be coming to be large.

So, this is what is giving you hint what number need to be selected with. So, be careful, this chord it should not happen that's what is coming very small, maybe 20 mm 25 mm, that's what is not acceptable in that range, okay. For this particular pressure ratio, be careful, listen to me carefully, okay! So, this is what we can say is a safe in that range, okay. So, 0.09 m that's what we can say. So, here the assumption of this aspect ratio that's what you need to be very careful in sense of calculating the chord. If this is what is not coming in the range, accordingly you modify your aspect ratio, okay.

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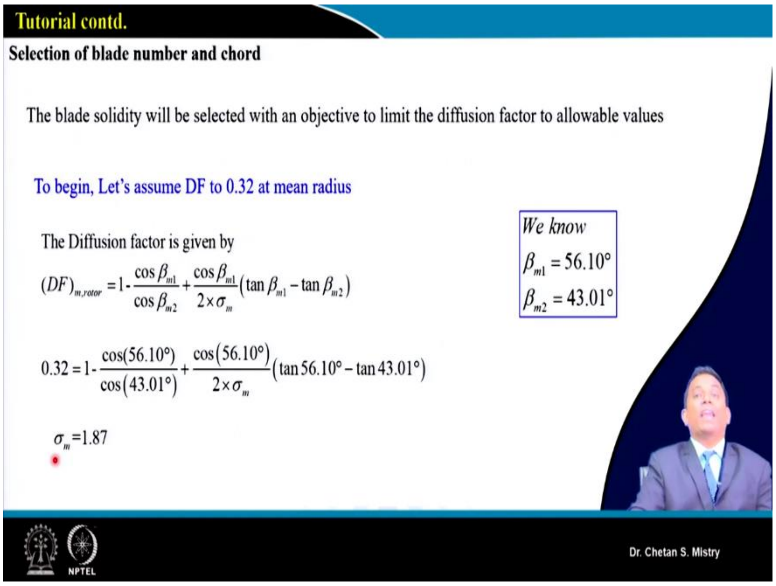

Now, once this is what is known to us in sense of chord, we are not aware of what will be our solidity, that solidity, that's what we will be using for the calculation of our diffusion factor; even that's what we are using for the calculation of number of blade. So, again, here in this case, you can assume number of blade and based on that you can do your calculation for solidity, you can do your calculation for diffusion factor, that's what we have done in our earlier design.

Now, when we were discussing that design for subsonic compressor, that time we have assumed number of blades and we have done calculation for diffusion factor. Now here, let us take different approach that's what will be giving you some different feeling. So, what we will be doing? We are assuming our deffusion factor to be 0.32 at mid station, okay. So, when we are assuming this diffusion factor that to be say 0.32 at the mid station, that's what will be helping us in order to calculate what will be our solidity, okay. Now, this is it says my solidity at mid station it is coming 1.87.

$$The\ Diffusion\ factor\ is\ given\ by,$$

$$(DF)_{m,rotor} = 1 - \frac{\cos\beta_{m1}}{\cos\beta_{m2}} + \frac{\cos\beta_{m1}}{2\times\sigma_m}(\tan\beta_{m1} - \tan\beta_{m2})$$

$$0.32 = 1 - \frac{\cos(56.10°)}{\cos(43.01°)} + \frac{\cos(56.10°)}{2\times\sigma_m}(\tan 56.10° - \tan 43.01°)$$

$$\sigma_m = 1.87$$

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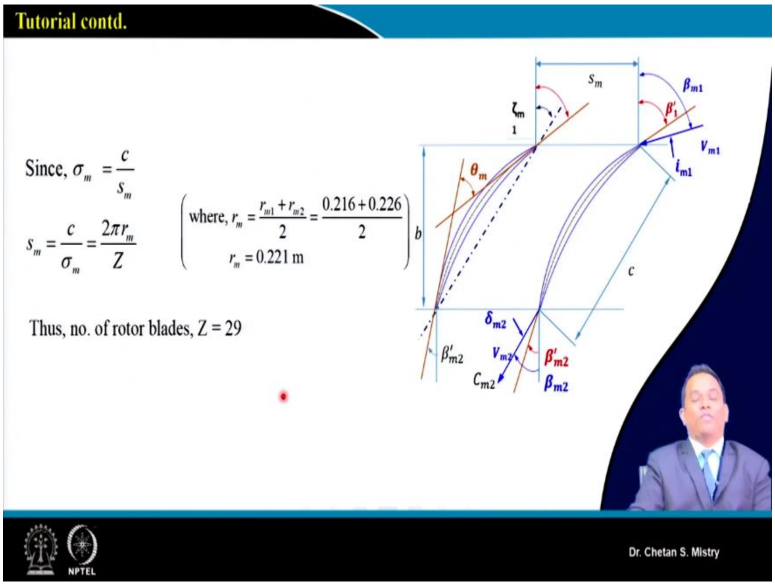

Now, once this solidity, that's what is known to us that's what will be helping us in order to calculate what will be my number of blades, okay. So, my number of blades that's what we are putting let say $\frac{2\pi r_m}{Z}$ based on my pitch calculation. It says my number of rotor blades that's what is coming 29, okay. So, what diffusion factor you are selecting, based on that you will be calculating your solidity and based on that you will be coming with say your number of blades.

Now, here it may be possible that your designer will be putting some constraint in sense of number of blades to be placed. Now, mechanical engineer, they will be having their own constraints in sense of accommodating number of blades on hub, because suppose say we are having our hub diameter to be small and in that if you are planning to fix this more number of blades or say maybe say number of blades that's what is coming say larger. It may be very difficult for them to design the slots, okay.

Suppose if you are considering say 'Blisk' kind of configuration there in order to do machining, it may be possible that tool may not be go inside, okay. So, here also, we are having the constraint, okay. For this design, this preliminary design, we are safely considering my diffusion factor to be 0.32 and what number of blades that's what is coming we will be selecting as it is, okay. But we need to be very careful here in sense of selection of number of blades. Again, it may be possible by number of blades are coming to be high, you can assume this number of blades to be small, then you recalculate what will be your solidity and based on that you check with your diffusion factor, okay.

$$Since, \sigma_m = \frac{c}{s_m}$$

$$s_m = \frac{c}{\sigma_m} = \frac{2\pi r_m}{Z}$$

$$where, r_m = \frac{r_{m1} + r_{m2}}{2} = \frac{0.216 + 0.226}{2}$$

$$r_m = 0.221\ m$$

$$Thus, number\ of\ rotor\ blades, Z = 29$$

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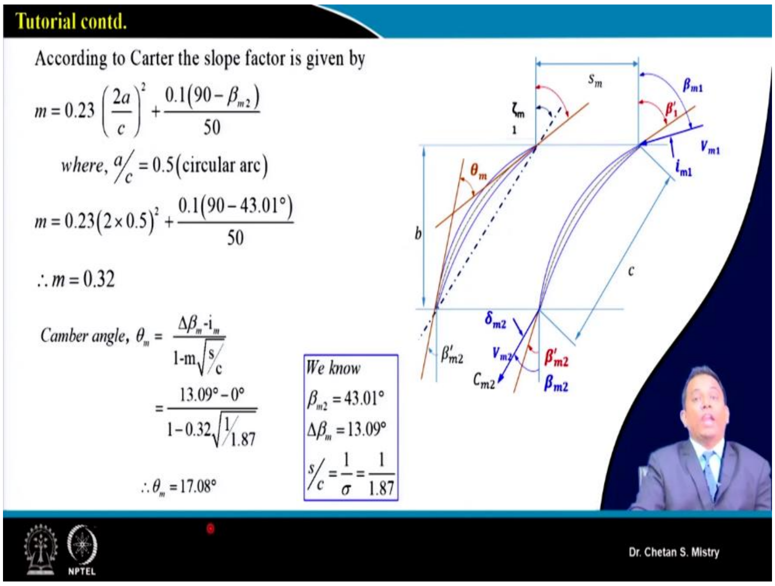

Now, once these parameters are known to us, we are interested in calculating what will be my say airfoil parameters. Now, for calculation of airfoil parameter, we need to do calculation for say Carter's parameter, that's what is say slope factor 'm'. We need to calculate what will be my say camber angle, what will be the deviation angle as well as what will be the stagger angle. So, we can say this is what is my circular arc. Be careful here, my Mach number that's what is coming in the range of 1.4. So, we can say, we will be going with say Double Circular Arc kind of configuration, and our camber line will be circular arc camber line.

So, here a/c we are taking as say 0.5. If that's what is your case, we can have calculation for 'm' parameter to be 0.32. If this is what is known, now here, in calculation of camber angle, we need to assume the incidence angle. Again, at mid station, we are assuming our incidence angle to be 0. Same logic what all we have discussed for subsonic compressor, we are opting here. We will be assuming our say incidence angle at hub to be $+2°$, we will be assuming our incidence angle at the tip tend to be $-2°$, at mid station we are considering that to be say 0. And that's what is giving me my camber angle to be 17.08°.

$$According\ to\ Carter, the\ slope\ factor\ is\ given\ by$$

$$m = 0.23\left(\frac{2a}{c}\right)^2 + \frac{0.1(90-\beta_{m2})}{50}$$

$$where, \frac{a}{c} = 0.5(circular\ arc)$$

$$m = 0.23(2\times 0.5)^2 + \frac{0.1(90-43.01°)}{50}$$

$$\therefore m = 0.32$$

$$Camber\ angle, \theta_m = \frac{\Delta\beta_m - i_m}{1 - m\sqrt{{}^{s}/_{c}}}$$

$$= \frac{13.09° - 0°}{1 - 0.32\sqrt{{}^{1}/_{1.87}}}$$

$$\therefore \theta_m = 17.08°$$

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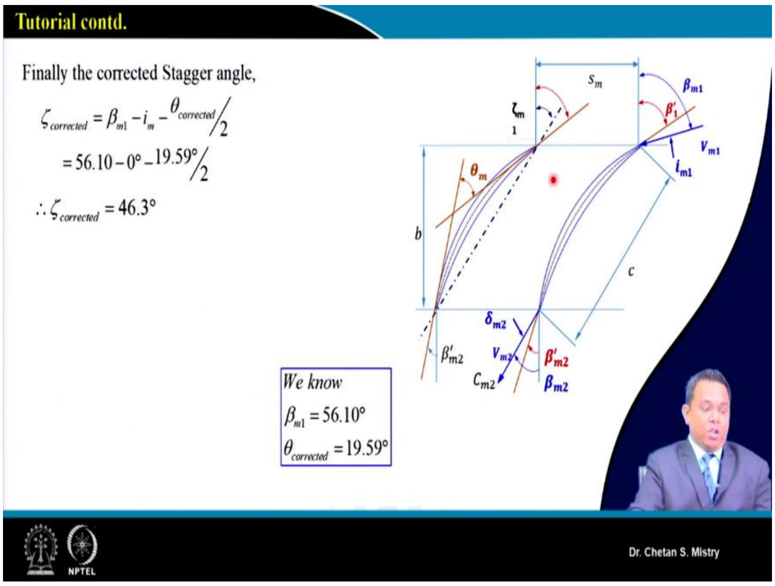

We can do our calculation for the deviation angle based on Carter's correlation and that's what is coming say it is say 4°, okay. Now, as we have discussed, you can go with say modifying your correction for say deviation angle. So, let say suppose we are correcting that with say 2.5°, we will be having corrected camber, we will be having corrected deviation angle.

$$Deviation\ angle,$$

$$\delta_m = m\theta_m\sqrt{s_m/c}$$

$$= 0.32 \times 17.08°\sqrt{1/1.87}$$

$$\therefore \delta_m = 4°$$

$$Considering\ an\ additional\ deviation\ of\ 2.5°, the\ corrected\ deviation\ is\ thus\ given\ by,$$

$$\delta_{corrected} = 4° + 2.5° = 6.5°$$

$$Corrected\ Camber,$$

$$\theta_{corrected} = \Delta\beta_m - i + \delta_{corrected}$$

$$= 13.09° - 0° + 6.5°$$

$$\theta_{corrected} = 19.59°$$

Same way, we can do our calculation for say corrected say stagger angle. So, these are the angles what we are calculating. So, here in this case, this is what is representing my transonic cascade and these all are the parameters what we have calculated with.

$$Finally\ the\ corrected\ Stagger\ angle,$$

$$\zeta_{corrected} = \beta_{m1} - i_m - \theta_{corrected}/2$$

$$= 56.1° - 0° - 19.59°/2$$

$$\therefore \zeta_{corrected} = 46.3°$$

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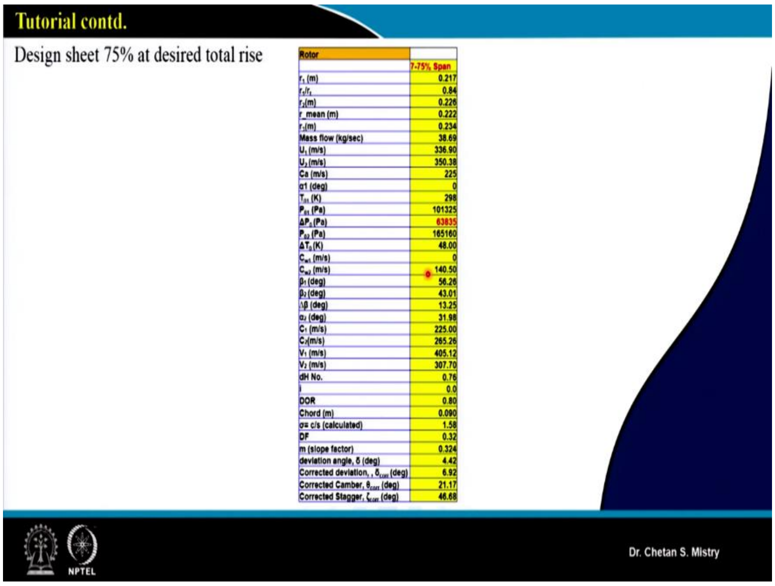

Rotor	
	7-75% Span
r_1 (m)	0.217
r_h/r_t	0.84
r_2(m)	0.226
r_mean (m)	0.222
r_3(m)	0.234
Mass flow (kg/sec)	38.69
U_1 (m/s)	336.90
U_2 (m/s)	350.38
Ca (m/s)	225
α1 (deg)	0
T_{01} (K)	298
P_{01} (Pa)	101325
ΔP_0 (Pa)	63835
P_{02} (Pa)	165160
ΔT_0 (K)	48.00
C_{w1} (m/s)	0
C_{w2} (m/s)	140.50
β_1 (deg)	56.26
β_2 (deg)	43.01
$\Delta\beta$ (deg)	13.25
α_2 (deg)	31.98
C_1 (m/s)	225.00
C_2(m/s)	265.26
V_1 (m/s)	405.12
V_2 (m/s)	307.70
dH No.	0.76
i	0.0
DOR	0.80
Chord (m)	0.090
σ= c/s (calculated)	1.58
DF	0.32
m (slope factor)	0.324
deviation angle, δ (deg)	4.42
Corrected deviation, , δ_{corr} (deg)	6.92
Corrected Camber, θ_{corr} (deg)	21.17
Corrected Stagger, ζ_{corr} (deg)	46.68

Now, after doing all this calculation, what we have done? We have done all our calculation at 75% of span. So, here you can say, what all we have done, based on parameter we have assumed our pressure ratio at the mid station or 75% span, as what we are looking for. Based on that we have calculated $\Delta P_0, \Delta T_0$. We have calculated what will be our velocity components at the entry; velocity components at the exit. What will be my flow angles at the entry, flow angles at the exit? We have calculated our checking parameter like diffusion factor, degree of reaction, de-Haller's factor, then we have calculated our cascade parameters, okay.

So, this is what all we say is in sense of doing the design at the mid station, okay. So, here we are stopping with. With this fundamental understanding for mid station calculation. We will be proceed further with the calculation of parameters at hub, parameters at tip section, and based on that we will try to finalize the design for this constant tip diameter configuration. So, thank you, thank you very much, maybe you can start doing design you can make your excel sheet design program for say mid station calculation and later on in next lecture, we will be discussing with what to do with other stations. Thank you. Thank you very much.