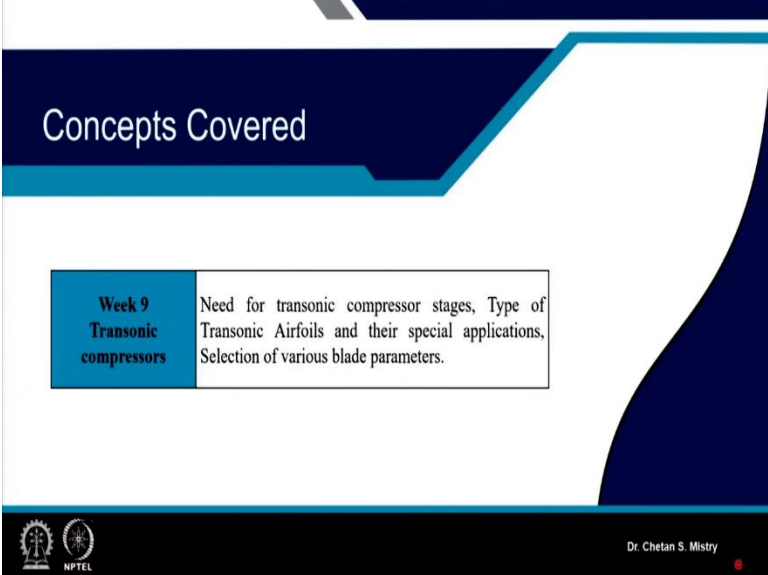


**Aerodynamic Design of Axial Flow Compressors & Fans**  
**Professor Chetankumar Sureshbhai Mistry**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology, Kharagpur**  
**Lecture 54**  
**Design of Transonic Compressor**

Hello, and welcome to lecture 54. This is week 10. We are starting with design of transonic compressor. In last week, we were discussing about many important aspects for transonic compressor, and which was making base for our future design for transonic compressor.

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The slide features a dark blue header with the text 'Concepts Covered' in white. Below the header is a table with two columns. The first column is blue with white text, and the second column is white with black text. At the bottom left, there are logos for IIT Kharagpur and NPTEL. At the bottom right, the name 'Dr. Chetan S. Mistry' is displayed.

Week 9 Transonic compressors	Need for transonic compressor stages, Type of Transonic Airfoils and their special applications, Selection of various blade parameters.
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So, what all we were discussing was say need of transonic compressor. So, we must realize for future requirement for the axial flow compressor, that's what is per stage pressure rise to be very high. And, that's what is fulfilling our requirement for the future engines with compactness, with lightweight and fuel efficiency; even that's what is taking care of wider operating range.

And, such kind of high-pressure ratio, that's what is possible by implementing this kind of say transonic cases. What we realize is maybe in order to have high pressure rise, we need to rotate our rotor at very high speed, or maybe we need to change the diameter and other parameters that's what all we have explored with.

Now, when we are having our flow at the entry to be supersonic, or maybe slightly on high subsonic side, then we can say, we will be having initial flow to be subsonic and that's what will be followed by say passage shock and later on that passage of we can say as a normal shock. And, that's what was giving say flow to be subsonic downstream. So, the change of flow parameters, flow properties from supersonic to say subsonic, because of presence of normal shock, that's what will be giving us very high pressure rise. And, that's what we are taking as a benefit; and that's what, people, they have explore after 60s.

So, this is what is of special kind with good aerodynamics understanding, good gas dynamics understanding; with which we realize, if we will be able to manage our flow at the entry, that means at the leading edge of our say transonic airfoil, that's what will be giving us benefit. Initially, we feel if we will be having sharp edge, that's what was giving benefit in sense of anchoring our shock.

But what is happening, say it will be working say worst when it is working under off design condition, we have seen in off design condition, it may be say change of speed or may be change of mass flow rate, and all those parameters; it may be possible that flow will go subsonic in that case. So, under that circumstances, this is what is not working fine and that is the reason why we are giving some rounding at the leading edge, as well as, as and when required, we will be giving say rounding at the trailing edge.

Then we were discussing about say different kinds of airfoils what we will be using for transonic say compressor. For transonic compressor we have realized, those airfoils are thin airfoils. Basically, we are managing our shock within the flow passage, and that's what was giving us benefit in sense of getting higher and higher pressure rise.

So, we were discussing about Double Circular Arc airfoil, Controlled Diffusion Airfoil; we also have discussed about Multiple Circular Arc airfoil, we have discussed about S type of airfoil. And, later on after understanding that flow structure, we try to get say our camber line, that's what is required for generation of these airfoils. We also have discussed about the distribution of the thickness over this camber line.

So, we have considered say circular arc thickness distribution, then we have discussed about say standard thickness distribution kind of thing. Then, after having all this idea, we have made our airfoils. Now, those airfoils which we will be using for our future design, this is what was giving brief idea about what all we were discussing in last session.

Now, I know, you are excited for the design of say transonic compressor because in open literature, you will not be finding such kind of discussion in detail. Maybe because of some proprietary nature of the things, that is one thing; secondly, that's what is little challenging and it required lot of understanding in sense of change of flow parameters. So, we will try to discuss in decent way.

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**3-D flows through Axial Compressor**

tip  
blade  
hub

$\frac{r_h}{r_t} = 0.8$

tip  
blade  
hub

$\frac{r_h}{r_t} = 0.4$

(a)

2-D Flow  
↓  
No radial movement  
↓  
Holds true for small height blades  
Hub-tip ratio is greater than 0.8

Later stages of compressor have hub-tip ratio 0.8  
For Front stages hub-tip ratio is about 0.4  
↓  
Results into higher flow area at inlet  
low flow area at exit  
↓  
High mass flow can passed through machine

*The area variation also results into flow with small radial velocity component along with axial and tangential component*

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Let us see what all we know. This is what we were discussing in our week 3, let us introduce the parameter we were discussing about the hub to tip ratio, we have realized hub to tip ratio of 0.8 and hub to tip ratio of say 0.4. And, what we know, like maybe in the later stages of our axial flow compressor, in say multistage axial flow compressor, we will be having our say this ratio...hub to tip ratio in the range of say 0.4. Maybe on the front side, we will be having our hub to tip ratio in the range of 0.8. We say, when we are discussing about the fans, mainly those fans are transonic fans, for that hub to tip ratio will be in that range, maybe in the range of 0.4.

Then later on what we realize, like with increase of our pressure ratio, the properties in sense of density, that's what we will be changing under downstream of our rotor. If you are assuming our

axial velocity to be constant, and if that's what is your case, we will be having say shape of our passage, this will be like that, okay. Here in this case, you can see, my entry area, that's what is larger; my exit area, that's what will be say lower, okay. And, that's what is satisfying our continuity equation.

So, we were discussing about the flow three dimensionality, that's what is happening for such kind of configuration. If we consider these are the streamlines we can plot, that's what is making our flow, that's what is flowing through our axial flow compressor. So, when we say, we are discussing about the transonic compressor, this is what is now coming into the picture. And, as we discussed, say for subsonic compressor and transonic compressor, we will be having the strategies for the design, that's what will be somewhat different, okay.

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**Simple three dimensional flow analysis:**

Generalized vortex law may be written as

$$C_w \cdot r^n = \text{constant}$$

$n = 1$  is free vortex law.

Normally  $-1 > n > 2$ .

Where

- $0.75 > n \geq 1.0$  Near-free vortex or Relaxed-free-vortex designs with blade sections are **Overloaded** w.r.t. Free Vortex Theory;
- $n > 1$  the blade sections are **Under loaded** w.r.t. Free Vortex Theory;
- $n = -1$  is often known as **Forced vortex design**

*Permits designers to think differently from constrains of Free Vortex design process....!!!*

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So, let us move with. What we know? We have realized say in order to satisfy our radial equilibrium equation, we have come up with  $C_w \cdot r = \text{constant}$ , and later on what we realize is say people, they have modified the law of changing this say whirl velocity component or say load distribution. What they are doing is say, they are assuming  $C_w \cdot r = \text{constant}$ , if you are putting  $n = 1$ , that's what is saying like free vortex design concept.

Generally, this 'n', that's what is varying in the range of  $-1$  to  $2$ , okay. So, if we are considering say if we assume our 'n', that's what is in the range of  $0.7$  to say nearly equal to  $1$ , that's what is

giving say relaxed free vortex design. And, that's what is giving say overloaded blade design. Now, this is what is very important when we are discussing about the design aspects. What all designs we have discussed up till now, that's what is of say subsonic compressor design, then we have discussed about say contra rotating configuration. That also, we have discussed in sense of subsonic configuration.

Now, if you will be having 'n', that's what is greater than 1, we will be having our loading that's what we say it is under loaded kind of configuration; and, we are taking  $n = -1$ , we say that as say forced vortex design, okay. So, it is designer's choice what kind of vortex distribution he or she will be selecting at the entry of the rotor, at the exit of the rotor, okay.

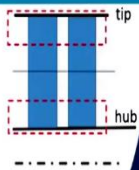
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**Simple three dimensional flow analysis:**

Generalized vortex law may be written as  $C_w \cdot r^n = \text{constant}$

Many most modern designs,

- The *tip sections and hub sections* are deliberately **Off-loaded by using  $n \approx 2$**   
*...to reduce the tip and other end-wall losses.*
- While *the mid sections* of the blades are Over loaded  
*...to compensate for loss of loading near the tip and hub by using  $n \approx 0.8$ .*
- This gives flexibilities to designer to meet special requirements as lower losses and improvement in efficiency.
- The special requirements under *off-design conditions and operational stabilities* need to be address as special case and will be based on past design experience and available design datasets with particular organization....



The diagram shows a vertical cross-section of a blade. The top part is labeled 'tip' and the bottom part is labeled 'hub'. A dashed red line outlines the tip region, and a dashed blue line outlines the hub region. The middle section is the main body of the blade.

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Now, what all we have learnt? It says, when we are talking about the most modern design, we are having lot of flexibility in sense of doing the design. And, as we have discussed, what we realize is say you are applying your law for the design and based on that whatever blades you will be making for rotor as well as stator and if it is working fine and it is giving what performance you are expecting, that's what is your design.

So, under that flexibility, what it says? Near tip and hub section, mostly we are having say off-loaded configuration where we are assuming our exponent to be 2. And, it says with the justification say, that's what is reducing your tip as well as other endwall losses. So, in week 3, we

were discussing about these aspects. So, you know better now, and if not maybe you can revise that part again in order to have detail understanding what we are talking about say end-wall losses.

Now, at the mid station we can understand at the hub and tip we are mildly loading, that's what is say off-loaded kind of configuration; that means at the mid station, we need to load slightly on a higher side. And, it says under that configuration, your exponent, you need to select as say 0.8. There is nothing wrong if you will be going with say free vortex design concept. But, we know the limitations of free vortex design concept and in order to get rid of those limitations, we are moving with some other kind of configuration, okay.

Now, this flexibility, that's what is lowering your losses that means it is improving your efficiency. At the same time, we are also looking for say better performance in off -design condition; as well as, we are looking for say wider operating range. So, all this configuration, that's what will be coming after the experience or maybe if we are talking about engine design manufacturing companies, they are having their own database, they are having their own design philosophies.

So, for all the stages, as we discussed earlier also, all the stages, they are not same, they are different. You need to design each and individual stage indigenously and that is the reason why we are having a whole lot of research and development activities, that's what is going on for the design of axial flow compressor.


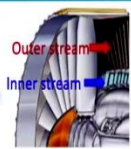
So, when we say, we are launching...say company is planning to launch new engine that means that's what we will be giving whole lot of different design aspects approaches; and, you know, having detailed understanding. And, we will be having say inclusion of universities, as well as research center, as well as company, they all together they are developing this kind of compressors and fans, okay.

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Simple three dimensional flow analysis:

While it is normally desirable to retain the constant specific work-input condition,

- The fan for a turbofan engine of high bypass ratio might provide an exception to this rule.
- The inner stream, feeding the high pressure core, may be designed for a lower pressure ratio than the outer stream...
- While for fan facing the bypass duct and where the blade speed is high for comparatively high Pressure ratio.
- The work then varies with radius,  $dh_0/dr$  is not zero....

$$\frac{dh_0}{dr} = C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w^2}{r}$$


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Now, when we are discussing about the high bypass ratio fan, it says this design, that's what is with some different design approach. When we say different design approach, what is the reason? It says our inner stream, what we are talking of, suppose if I consider this is what is our fan, for that fan, my inner stream, that's what is supplying my compressed air to say downstream core region, or say maybe downstream say compressor. And, my upper stream, that's what is bypassing their air to duct.

So, under that condition, this lower part, that's what has been designed for low loading, okay. At the same time, my upper part, that's what we can load on higher side, because we will be taking our benefit in sense of having higher diameter. That means, we are able to increase our peripheral speed and that's what is giving the benefit in sense of doing the design.


So, now here in this case, under special conditions, it says we are violating our radial equilibrium equation. It says work done, that's what is varying with radius and that's what is non-zero. So, that's what is a special kind of design. We will be approaching or we will be discussing about such kind of design configuration in this week, okay.

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**Tutorial**

The startup company is planning to design an engine for UAV application. The designer has selected an axial-flow compressor stage designed based on the fundamental design law. Air enters the rotor at temperature of 298 K and Pressure of 101325 Pa. The tip radius  $R_t = 260$  mm and assume hub to tip ratio at inlet to be 0.375, the rotor rotational speed must be kept under 16100 rpm based on turbine rotational speed and preferred to have the Relative Mach no. need not to exceed the value of 1.4 at the tip of blade. The stage need to generate the pressure ratio of 1.63, with the mass flow rate of 38.69 kg/s. Take  $C_p = 1005$  kg/KJ K.

Given	Design Constrains
$\pi_{r,stage} = 1.63$	$r_{tip} = 0.260$ m
$T_{01} = 298$ K	$r_h/r_{tip} = 0.375$
$P_{01} = 101325$ Pa	$RPM \leq 16100$
$\dot{m} = 38.69$ kg/s	$M_{tip} = 1.4$
$RPM \leq 16100$	
$C_p = 1.005$ kJ/kg.K	



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Now, in order to start with, let us take one of the design for our calculation. So, what it says? We are having our startup company, that's what is planning to design the engine for UAV application. Now, this is what is recent trend. People, they are moving towards say Unmanned Aerial Vehicles. So, let us take one of the design configuration for such application. So, designer has selected axial flow compressor stage, that's what is designed based on fundamental design approach. The air enters the rotor at a temperature of 298 K and pressure of 101325 Pa pascal. So, you can say, that's what is your atmospheric condition.

Now, and that too its on ground, okay. Now, my tip radius, that's what is say 260 mm and assume up to tip radius ratio at the entry to be 0.375. So, as we have discussed, when we are doing design for our front stages, that's what is in this range only, okay. The rotational speed must be kept under 16,100 rpm based on turbine rotational speed and preferred to have the relative Mach number should not be exceed by 1.4 at the tip, okay. The stage need to generate the pressure ratio of 1.63. That's what is slightly on the higher side with mass flow rate of 38.69 kg/s. And take the value of  $C_p$  as  $1.005 \frac{kJ}{kg \cdot K}$ , okay.

Now, here in this case, what all that's what is given to us, that's what is say we are looking for compressor to be designed, that's what is having pressure ratio of 1.63. And, our temperature and pressure, that's what is given; our rotational speed that's what is given as a constraint, it says like



we should not increase our rotational speed beyond 16,100, okay. Now, under this condition, we can say, we are having constraint with our diameter of the casing, we have, you know, like inlet dimension or inlet radius ratio that is also given to us and it says my tip Mach number should not be exceeded by 1.4. So, we can say these are our design constraints, okay.

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**Design Approaches**

*Hint*

```

graph TD
    A[Estimation of size and speed] --> B[Design calculation at 75% span of blade]
    B --> C[Velocity components and velocity triangle]
    C --> D[Peripheral velocity, Axial velocity, mass flow rate]
    D --> E["Entry and exit flow angles at mid span  
(α1m, α2m, β1m, β2m)  
Blade turning angle Δβ  
DOR, DF, DH, solidity, no. of blades  
Camber, Deviation and Stagger angle"]
    E --> F[Use Design law and find profile parameters at different spanwise stations]
  
```

Given Data		
Inlet total temperature	$T_{01}$	298 K
Inlet total pressure	$P_{01}$	101325 Pa
Avg. Pressure Ratio	$\pi$	1.63
Efficiency (assumed)	$\eta$	93%
Mass flow rate	$\dot{m}$	38.69 kg/s
Tip diameter	$d_t$	520 mm
Rotational speed	$N$	< 16100
Tip Mach number	$M_{tip}$	1.4

Assumed data		
Ratio of specific heat	$\gamma$	1.4
Work factor	$\lambda$	0.98
Inlet flow angle	$\alpha_1$	0
Specific heat (const. pr.)	$C_p$	1005 J/kg K

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Now, in order to start approaching this design, let us see what all we can do here. So, based on available data, as we have discussed, we are having constraints with 3, that's what is my casing diameter, we have constrained with our rotational speed, we have constrained with our tip Mach number.

So, how do we start doing design for such transonic compressor? You can say, my Mach number at the tip, it is given 1.4. That means, this is what is an indication that this compressor is of transonic nature, okay. So, what we can do? We need to estimate the size and the speed because we are not aware of what will be the dimensions for my blades, both for rotor as well as for the stator, okay.

Now, what all calculation we will be doing for transonic compressor, we know, we will be doing our calculation at 75% of our span. Now, at 75% of span, we can say, that's what is a mid-section, okay. At that mid-section, we are looking for different velocity components to be known. Now, all those velocity components, that's what will be helping us for calculating different flow angles.

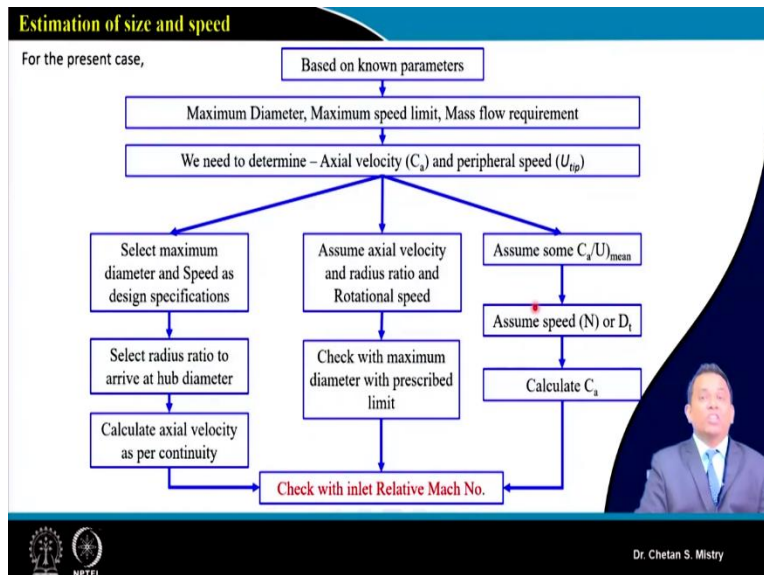
Then, we will be calculating our parameters like diffusion factor, degree of reaction, de-Haller's factor, all those parameters, that's what we need to decide with.

Now, once we are deciding with these numbers, we are also interested in what will be the number of blades for rotor as well as for the stator, okay. And, we are looking for different angles which we will be using for making of our say compressor; in sense, camber angle, stagger angle, deviation angle, all those parameters.

So, at mid station we will be doing all our calculation. Now, after doing this calculation at the mid station, we will be applying our design law at different locations at the entry as well as at the exit. Now, here in this case, it says like we need to go with the fundamental design approach. So, we will be moving with that part only.

Now, for practice, you can even try with free vortex design, even you can try with the forced vortex design, even you can go with constant reaction design; it is you need to decide what approach you will be selecting with. So, specifically for this design, we will be moving with the fundamental design approach, okay.

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Now, as we have discussed, we are more interested in calculating say geometrical dimension that means what will be the size, okay. Now, based on input parameters what we have, we need to decide with the maximum diameter, maximum speed and mass flow rate; these three parameters,

that's what is defined to us. Now, in order to do the calculation for different velocity components, that's what is of our interest, maybe we are interested in calculation of axial velocity, we are looking for peripheral velocity, we are looking for say velocity component of relative velocity, we are also interested in say absolute velocity component.

So, in order to do start with this design, you know, we have different approaches which are possible. Now, here in this case, let us discuss one by one, that's what and out of which we will be selecting one of the approach, okay. So, what all are the possibilities with us, it says maybe first, you can select the maximum diameter and speed as per the design dimension. So, you can say, our given diameter, that's what is known to us and rotational speed is also given to us. So, we can say, this is what we can use for our initial design calculation.

Now, you know, this radius ratio at the entry, that's what is given to us. So, we can calculate what will be our hub diameter, based on that we will be calculating our axial velocity, we can use our continuity equation for this calculation. Once this axial velocity we can calculate, our next target that's what we need to check with inlet relative Mach number.

So, whether it is meeting with our requirement or not. So, this is what is one of the approach, you can go in an iterative way also, okay. So, this is what is one of the approach, say there is nothing wrong in doing this kind of calculation. Second approach, it says you can assume your axial velocity; this radius ratio and rotational speed also you can assume. Suppose if I say, radius ratio at the entry, that's what is given to you, that's what you can use. Suppose say it is not given, you can assume your radius ratio.

Rotational speed also you can assume. Suppose, if you are doing design for high bypass ratio fan, you can assume this rotational speed to be in the range of 3000 rpm or 3500 rpm, for we are discussing about commercial aircraft, okay. Suppose say it is a military aircraft, we know this will be in the range of maybe 7000 to 12,000 rpm. So, just select those numbers, okay.

Now, with this, we need to check with the maximum diameter whether it is meeting with our requirement or not, okay. Then after, again we will be checking with our relative inlet Mach number, okay. Now, there is third approach, what it says maybe you can assume  $C_a/U$  value or

flow coefficient at the mid station. This is also one of the good approach. Many design or many designers, they prefer to go with this kind of configuration, they are assuming  $C_a/U$  value, okay.

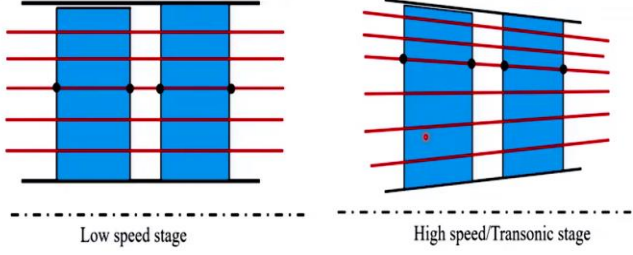
Based on that you will be selecting or maybe assuming say rotational speed. Based on that they will be doing the calculation for tip diameter, they will check with whether it is meeting with the requirement or not. If not then maybe accordingly this  $C_a/U$  value or maybe the speed can be changed. You can calculate your axial velocity. Again, you need to check with your inlet relative Mach number.

So, this is what all we can say. Here, there is one more possibility, maybe someone will say, we will be assuming the degree of reaction at the mid station. So, it is similar to what all we are discussing here. So, you will be assuming your degree of reaction, that's what will be giving you  $C_a/U$  value at the mid station and again you will be doing the same kind of configuration.

So, the design method what all we are discussing for transonic compressor, where we are not having information, that's what is available with us, we need to assume and we need to follow certain rules, okay. So, these are some of the possibilities, since for this design, some of the parameters, that's what has been constraint with. So, that's what will be making our life easy and that will be making say design iterations to be less, okay.

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Tutorial contd.




Low speed stage

High speed/Transonic stage

- For a constant streamwise axial velocity,
- The streamwise passage area remains almost same as the changes in density across a single stage is very less for incompressible stages.
- The streamwise passage area **must decrease** to accommodate the same mass flow as the changes in density across a single stage is significant for transonic stages.

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So, let us move with the approach. Now, before going into the detail, we must understand, we are discussing the design of say transonic axial flow compressor. Now, we have discussed about the subsonic compressor design, in which if you consider, my pressure ratio, that's what is say lower. When I say my pressure rise expected, that's what is lower that means my inlet area and outlet area, we can say that's what is safely to be same. And, that is the reason why we are having this kind of configuration where we are doing all our calculation at the mid station and then after we are doing our calculation at different stations.

Now, this mid station for subsonic compressor we have discussed in our all the designs, that's what is done at say 50 percent span, okay. Then after, we will be dividing this whole span into equal number of divisions. Maybe you can take 10 station, 20 station, maybe you can take more than 20 also, no one will stop.

It is you, as a designer you need to decide with. More will be the station it will be, you know, more easy to make those blades, okay; and, that's what will be giving you the smooth curvature for the blade. If you will be having less number of station then it will be making some difficulty when you are start making solid model for that blade. And, it may be possible that you will not be having great control for loading or expected performance from particular airfoil at particular span, okay.

So, it is preferred you just go with say more number of say stations. Now, when we are talking about say transonic, as we have discussed, what is happening, this is what is say my inlet area. Now, because we are expecting our pressure ratio to be high, we can say my exit area, that's what will be lower. If we are assuming our axial velocity to be constant, in order to satisfy our continuity equation, it will be giving me my exit area that's what to be different. Now, if this is what is your case, what we need to do? We will be having these conditions at the entry as well as at the exit. Now, under that condition, if you look at carefully, say here this is what is say constant mean diameter kind of configuration, okay.

So, we will be doing our calculation at 75% of my span. So here, we will be doing our calculation. So, what you need to do? Say, this is what is say my height of the blade, we need to calculate this location or we need to locate this point. Same way, at the exit of the rotor, we need to locate this point; same way for stator, we need to locate this particular point at say station 2. At the same way, at the exit station also, we need to locate this point.

Now, this is what will be giving you one stream line. We know these stream lines, they are the imaginary lines, okay, these lines are imaginary lines. So, that is the reason why if you are considering this, so, accordingly we need to divide say this whole span into equal number of stations. Be careful, what all I am talking here. Suppose say this is what is my entry, I will divide this whole into equal number of station. Same way, from here to here I will be dividing into equal number of station. Same way for stator, as well as, for entry, as well as for the exit. Then after we will be joining that with the streamlines, okay.

So, we will realize here, there is a difference in sense of design. Here in this case, what all we are discussing where we say my  $U_1 = U_2$ . Here in this case, that's what is not the case, my entry peripheral speed and my exit peripheral speed both are different, okay. Here in this case, I can say, my airfoil to be two dimensional airfoil in sense, okay.

So, this is what is say flat kind of airfoil. Here in this case, if you look at, my airfoil will not be like the way in which we are discussing, that's what we will be having third dimension, okay. And, that is the reason why the making of this blade, making of these airfoils for such kind of transonic compressor, that's what is a little challenging.

Now, based on available tools, computational tools, what we are discussing, that's what will be helping us in order to have say perfect shape of the airfoil. So, you need to start with some airfoil and later on based on your flow physics, detail flow understanding, you will be modifying the shape of your airfoil. That's what we already have discussed with, okay.

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**Tutorial contd.**

**Determination of tip diameter and axial velocity**

The mass flow rate through the compressor is given by,

$$\dot{m} = \rho_1 \pi r_t^2 \left(1 - \frac{r_h^2}{r_t^2}\right) C_a$$

This can be rearranged to give tip radius at the entry as


$$r_{t,1}^2 = \frac{\dot{m}}{\pi \times \rho_1 \times C_a \times \left(1 - \frac{r_h^2}{r_t^2}\right)}$$

The inlet density depends on the static pressure and temperature. Also changes with axial velocity.

Let's check the exit dimensions by assuming different values of axial velocities

```

graph TD
    A[Given mass flow rate] --> B[Calculate annular area using mass continuity]
    B --> C[Express tip radius explicitly]
    
```



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Now, here in this case, what we will be doing? Say, let us see, we will be having different kind of configuration, that's what is possible. What we say? We know what is our mass flow rate, so that mass flow rate, I say, that's what is *density × area × axial velocity*. Now, this I am representing in terms of say my tip diameter, or tip radius, this is what is my entry tip radius. So, if you look at, it says this is what is a function of mass flow rate, it is a function of my density, it is a function of my axial velocity and this is what is my radius ratio.

*The mass flow rate through the compressor is given by,*

$$\dot{m} = \rho_1 \pi r_t^2 \left(1 - \frac{r_h^2}{r_t^2}\right) C_a$$

*This can be rearranged to give tip radius at the entry as*

$$r_{r,1}^2 = \frac{\dot{m}}{\pi \times \rho_1 \times C_a \times \left(1 - \frac{r_h^2}{r_t^2}\right)}$$

So, if you recall, this mass flow rate, that's what is known to us; even this radius ratio that is also known to us. Now, unknown parameter for us, that's what is say my axial velocity, and even I do not know what is my density, okay. So, entry density also, that's what is not known to us. So, what

we can do? We need to calculate our inlet density, we need to calculate what will be my axial velocity.

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**Tutorial contd.**

**Static properties at inlet:**

Assuming  $C_a = 160 \text{ m/s}$

$$T_1 = T_{01} - \frac{C_a^2}{2C_p} = 298 - \frac{160^2}{2 \times 1.005 \times 10^3}$$

$$\Rightarrow T_1 = 285.26 \text{ K}$$

We can calculate static pressure using isentropic relation,

$$P_1 = P_{01} \left( \frac{T_1}{T_{01}} \right)^{\frac{\gamma}{\gamma-1}} = 101325 \left( \frac{285.26}{298} \right)^{\frac{\gamma}{\gamma-1}} = 86956.64 \text{ Pa}$$

The density is thus given by using Equation of state,  $\rho_1 = \frac{P_1}{RT_1} = \frac{86956.64}{287 \times 285.26}$

$$\rho_1 = 1.0621 \text{ kg/m}^3$$

We know

$P_{01} = 101325 \text{ Pa}$

$c_p = 1.005 \text{ kJ/kgK}$

$C_2 = C_1$

$T_{01} = 298 \text{ K}$

```

    graph TD
      1((1)) --> 2((2))
      2 --> 3((3))
      subgraph Rotor
        1
      end
      subgraph Stator
        2
      end
  
```

Assume an axial velocity

↓

Calculate static properties at inlet

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So, what we will be doing? Say, we will be assuming our axial velocity in order to do calculation for our entry density. So, now here in this case, what we are assuming? Say, let us assume our axial velocity to be say  $160 \text{ m/s}$ , okay. Now, if nothing that's what is given here, in sense of how my flow, that's what is entering, we can safely assume our entry, that's what is axial. That means, we can say my absolute velocity and axial velocity at the entry, that's what is same. And, that's what we have assumed as say  $160$ . So, if we will be using our correlation for total temperature, that's what will be giving me what will be my temperature at the entry.

So, we say, this is what is my entry temperature...entry static temperature. Be careful, do not get confused with the total temperature and do not get confused with the static temperature. So, we are interested in calculation of our density that density we are talking about the static density that means we are looking for static property. Density we are writing, that's what is say  $P_1/RT_1$ , that's what is a static property.

So, we need to have our static pressure, we need to have our static temperature, okay. So, once we are calculating our static temperature based on our isentropic relation, we can calculate what will



be our static pressure. So, if we are putting  $T_1$  and  $T_{01}$ , if they are known to us, we can say this is what is coming say 86.95 kPa.

*Static properties at inlet:*

*Assuming  $C_a = 160$  m/s*

$$T_1 = T_{01} - \frac{C_a^2}{2C_p} = 298 - \frac{160^2}{2 \times 1.005 \times 10^3}$$

$$\Rightarrow T_1 = 285.26 \text{ K}$$

*We can calculate static pressure using isentropic relation,*

$$P_1 = P_{01} \left( \frac{T_1}{T_{01}} \right)^{\frac{\gamma}{\gamma-1}} = 101325 \left( \frac{285.26}{298} \right)^{\frac{1.4}{1.4-1}} = 86956.64 \text{ Pa}$$

Now, once this is what is known to us, we can safely take our density as say  $P_1/RT_1$ . Be careful, these units suppose you are taking kilo pascal and pascal, so careful in taking these units, otherwise, this density will not come, okay. So, this density, that's what is coming as say 1.0621. So, you can say, this entry density that is static density, that's what is known to us, okay.

*The density is thus given by using Equation of state,*

$$\rho_1 = \frac{p_1}{RT} = \frac{86956.64}{287 \times 285.26}$$

$$\rho_1 = 1.0621 \text{ kg/m}^3$$

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**Tutorial contd.**

The tip radius can be calculated by using the prescribed value of hub-to-tip ratio,

$$r_{t,1}^2 = \frac{\dot{m}}{\pi \times \rho_1 \times C_a \times \left(1 - \frac{r_h^2}{r_t^2}\right)} = \frac{38.69}{\pi \times 1.0621 \times 160 \times (1 - 0.375^2)}$$

$$r_{t,1} = 0.299 \approx 0.3 \text{ m} \gg 0.26 \text{ m (upper constrain on tip radius)}$$

We also need to check for the rotational speed based on selected tip speed against rpm and tip relative Mach no.

Let's assume  $U_{ti} = 450 \text{ m/s}$

The rotational speed can be calculated for the current tip radius as,

$$N = \frac{60U_{ti}}{2\pi r_{t,1}} = \frac{60 \times 450}{2\pi \times 0.3} = 14324 \text{ rpm (Acceptable)}$$


We know

$\dot{m} = 38.69 \text{ kg/s}$

$C_a = 160 \text{ m/s}$

$\left(\frac{r_h}{r_t}\right)_{inlet} = 0.375$

$T_{01} = 298 \text{ K}$



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Now, what all we know? We know our say radius at the tip, at the entry, that's what is a function of mass flow rate, density and axial velocity. So, we can say if we are putting these numbers, what we have calculated in sense of density, that's what is 1.0621; my axial velocity, we have assumed that's what is 160 m/s. If you are putting that it says my radius at the tip, that's what is coming nearly 0.3 m and what we are expecting is 0.26 m. So, you can say, this is what is not meeting with our requirement.

*The tip radius can be calculated by using the prescribed value of hub to tip ratio,*

$$r_{t,1}^2 = \frac{\dot{m}}{\pi \times \rho_1 \times C_a \times \left(1 - \frac{r_h^2}{r_t^2}\right)} = \frac{38.69}{\pi \times 1.0621 \times 160 \times (1 - 0.375^2)}$$

$$r_{t,1} = 0.299 \approx 0.3 \text{ m} \gg 0.26 \text{ m (upper constrain on tip radius)}$$

So, again, what you need to do is maybe you can go in a reverse way, you assume your axial velocity, and do your iteration. Now, in order to reduce those iterations, what we have done? We have assumed say tip peripheral speed as say 450 m/s. So, let us say, let us assume this say  $U_{tip}$ , that's what is say 450 m/s, okay. When we are assuming that, we can do our calculation for the peripheral speed, and based on that rotational speed, we can calculate, and that's what is coming 14,324. Now, when I say if this is what is coming, this is what is say in acceptable range.

Let's assume  $U_{t1} = 450 \text{ m/s}$

The rotational speed can be calculated for the current tip radius as,

$$N = \frac{60U_{t1}}{2\pi r_{t1}} = \frac{60 \times 450}{2\pi \times 0.3} = 14324 \text{ rpm}$$

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**Tutorial contd.**

In order to control the shock induced losses we also need to check for the relative tip Mach no

Based on inlet velocity triangle,


Relative tip speed,  $V_{tr} = \sqrt{C_a^2 + U_{t1}^2} = \sqrt{160^2 + 450^2} = 477.59 \text{ m/s}$

Sonic speed at inlet,  $a_1 = \sqrt{\gamma RT_1} = \sqrt{1.4 \times 287 \times 285.26} = 338.55 \text{ m/s}$


Hence, Tip Mach No.,  $M_{t1} = \frac{V_{tr}}{a_1} = \frac{477.59}{338.55} = 1.41 > 1.4$  (Slightly Exceeds the limit)

Similar calculations need to be performed iteratively using different axial velocities to arrive at allowable values

We know  
 $C_a = 160 \text{ m/s}$   
 $U_t = 450 \text{ m/s}$   
 $T_1 = 285.26 \text{ K}$



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Now, once we have done the calculation for this ratio, once in sense of my axial velocity, my peripheral speed, we will be checking with what is happening with our tip Mach number. Because that's what is our main target. It says, my tip Mach number should not exceed more than 1.4. So, we will be calculating that part. In order to do that calculation, we will be calculating our relative velocity, this relative velocity, we can say based on our velocity triangle, that's what is say axial entry. We can straight way write down,  $C_a^2 + U_{tip}^2$ .

Based on inlet velocity triangle,

$$\text{Relative tip speed, } V_{1t} = \sqrt{C_a^2 + U_{t1}^2} = \sqrt{160^2 + 450^2} = 477.59 \text{ m/s}$$

We can calculate our sonic velocity at the entry, that's what is  $\sqrt{\gamma RT_1}$ . And, if you are calculating that's what is coming it is 1.41. And, this is what is nearly what we are looking for, but it says it

is slightly exceeding the range, okay. So, let us do some more iteration in sense, in order to achieve what we are looking for, okay.

$$\text{Sonic speed at inlet, } a_1 = \sqrt{\gamma RT_1} = \sqrt{1.4 \times 287 \times 285.26} = 338.55 \text{ m/s}$$

$$\text{Hence, Tip Mach number, } M_{1t} = \frac{V_{1t}}{a_1} = \frac{477.59}{338.55} = 1.41 > 1.4 \text{ (slightly exceeds the limit)}$$

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**Tutorial contd.**

For different values of  $C_a$  and  $U_{tip}$  combinations, we can tabulate the values of tip diameter,  $N$  and Mach no.

$C_a$	$U_{tip}$	$D_t$	$N$	$M_t$
160	450	0.5807	14798	1.411
170	440	0.5675	14807	1.397
180	440	0.5557	15120	1.405
225	400	0.516	14805	1.35

So we will proceed with the following parameters  
 $C_a = 225 \text{ m/s}$   
 $U_t = 400 \text{ m/s}$   
 $N = 14805 \text{ rpm}$

Tip diameter	$d_t$	520 mm
Rotational speed	$N$	< 16100
Tip Mach number	$M_{tip}$	1.4



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Now here, are some of the iterations what we have done with different combination of axial velocity and say your peripheral speed. Here again, let me tell you, one can go with say flow coefficient assumption by that way also you can do this calculation. So, in order to showcase what all parameter we need to select with this is the reason why we have selected these numbers.

So, if you look at, these are the combination. It says, we are looking for our speed, that speed, it is coming in the range what we are looking for. It should not be more than 16,100 for all assumption of axial velocity and tip speed, my speed...rotational speed, that's what is coming in the range.

But at the same time, you know, like we are constraint with our diameter. So, we are looking for diameter in the range of say 520 mm. So, that is the reason why we are safely selecting our axial velocity to be 225 m/s. Let us assume peripheral speed to be 400, that's what is giving my tip diameter of 0.516 and at the same time, this is what is giving us our tip Mach number to be in the

range of say less than 1.4. So, it is coming 1.35. So, now, for our calculation, we will be selecting these numbers, okay. So, these are the numbers, that's what we are selecting with.

$C_a$	$U_{tip}$	$D_t$	$N$	$M_t$
160	450	0.5807	14798	1.411
170	440	0.5675	14807	1.397
180	440	0.5557	15120	1.405
225	400	0.516	14805	1.35

<b>Tip diameter</b>	$d_t$	520 mm
<b>Rotational speed</b>	$N$	< 16100
<b>Tip Mach number</b>	$M_{tip}$	1.4

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**Tutorial contd.**

$C_a$	$U_{tip}$	$D_t$	$N$	$M_t$
160	450	0.5807	14798	1.411
170	440	0.5675	14807	1.397
180	440	0.5557	15120	1.405
225	400	0.516	14805	1.35

thus,  
 $r_{ti} = 0.258 \text{ m}$

also  
 $r_{hi} = \frac{r_{ti}}{r_t} \times 0.258 = 0.275 \times 0.258 = 0.0967 \text{ m}$

We know,  
 $\left(\frac{r_{hi}}{r_t}\right)_{inlet} = 0.375$

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Now, once we are selected with these numbers, what we know? We know, what is our radius ratio, so the radius ratio at the entry, this station radius ratio that's what is given to us. So, that's what we will be putting say 0.375 into this radius what we have calculated it is 0.258. That's what is giving me my hub radius at the entry and my tip radius at the entry, okay.

*thus,*

$$r_{t1} = 0.258 \text{ m}$$

*Also,*

$$r_{h1} = \frac{r_h}{r_t} \times 0.258 = 0.275 \times 0.258 = 0.0967 \text{ m}$$

So, now we can say based on our understanding, we are finalizing our entry dimensions at the tip, we are finalizing our entry dimension at the hub and this is what we are doing calculation at the say entry.

Now here, we are stopping with, in next lecture, we will be discussing how do we do our calculation for station 2 as well as for station 3. I am sure, this is what is giving you idea what all can be done in sense of starting with the initial design part at the entry. Maybe, as we have discussed, you can go with some other approach and you can come up with these numbers, okay.

Maybe you can assume your  $C_a/U$  value, you can assume your degree of reaction, you can assume your tip diameter, and that is how we will be doing our calculation. So, here we are stopping with. Thank you. Thank you very much for your kind attention! We will be continuing our design in the next lecture. Thank you.