Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 53
Transonic Compressors (Contd.)

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Hello, and welcome to lecture 53. We are discussing about transonic compressors. In last lecture, we started discussing about the effect of different flow parameters on say, flow behaviour within the blade passage. We were discussing about the formation of losses. We have discussed about the different losses, that's what is going to occur because of change of incidence angle.

Then we were discussing about the effect of change of area ratio. We were discussing about the effect of change of pitch to chord ratio. As we have discussed, now, we are more interested towards the development of these airfoils. So, today's lecture, that's what is mostly covering the part, that's what we say how to develop these transonic airfoils.

Now, in order to have detailed understanding for the development, we need to have some fundamentals of say, geometry, trigonometry as well as some mathematics what you have already studied during class 10 or say 12. Let us move ahead with.

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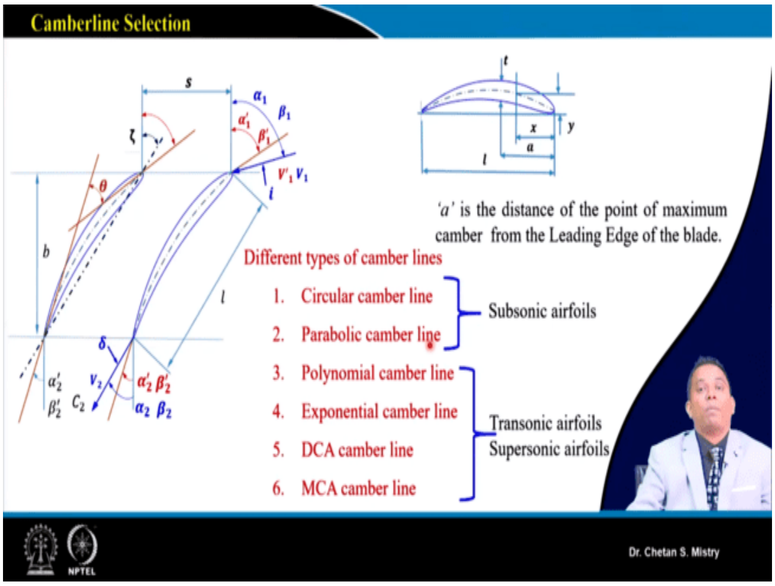

So here, this is what we are discussing in sense of development of say Camber line. So, if you recall, this is what we were discussing when we have discussed about say, subsonic airfoils or when we were discussing about subsonic, say, compressors design; where we have discussed about the different types of camber lines. We have said like circular arc camber line, parabolic camber line, polynomial camber line, exponential camber line, DCA camber line, MCA camber line.

Now, if you look at, this circular and parabolic chamber line, that's what has mainly been used for subsonic airfoils, even Double Circular Arc also, people, they started using that also we have discussed. Now for say, transonic and supersonic airfoils, we are looking for different kind of camber shapes or camber line shapes.

They are say, polynomial type or say exponential type, say DCA type or MCA type. So, let us try to learn how exactly we will be making these camber lines.

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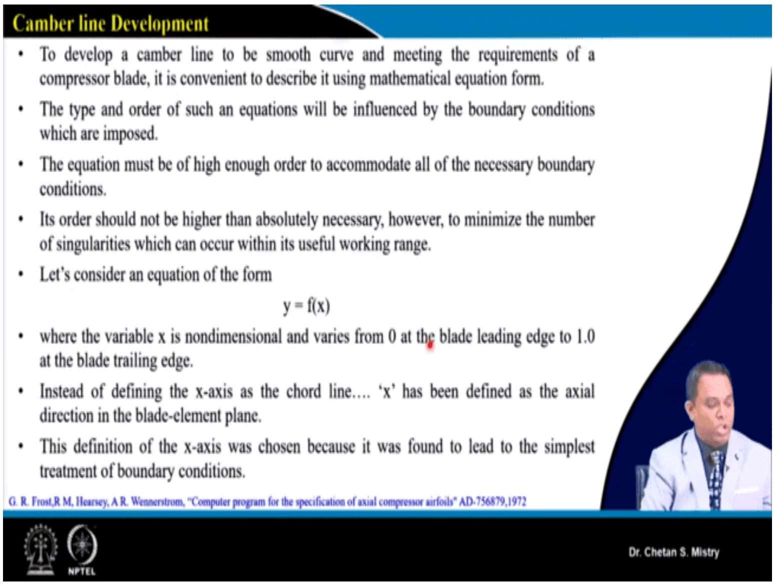

We are discussing about say development of the camber line for the transonic airfoil. So, we can say, what line we will be developing that line need to be a smooth line. When we say we are looking for say smooth line, that's what will be having say, number of points, that's what is set nearby. Secondly, we need to decide the formulation or the equation for that camber line and that camber line equation need to be such that it will give say…if you give say…certain boundary conditions, maybe at the leading edge or maybe somewhere in the mid chord or maybe in the trailing edge region then it should give the solution for that.

If we consider, we can represent say any camber line equation as say y is equal to function of x, where x we can consider, say, mainly people, they are considering say x as say chord length but the sake of brevity here, in this case, we are considering x as say location rather than, you know, defining that as a chord; because that's what will be helping us in solving the equations.

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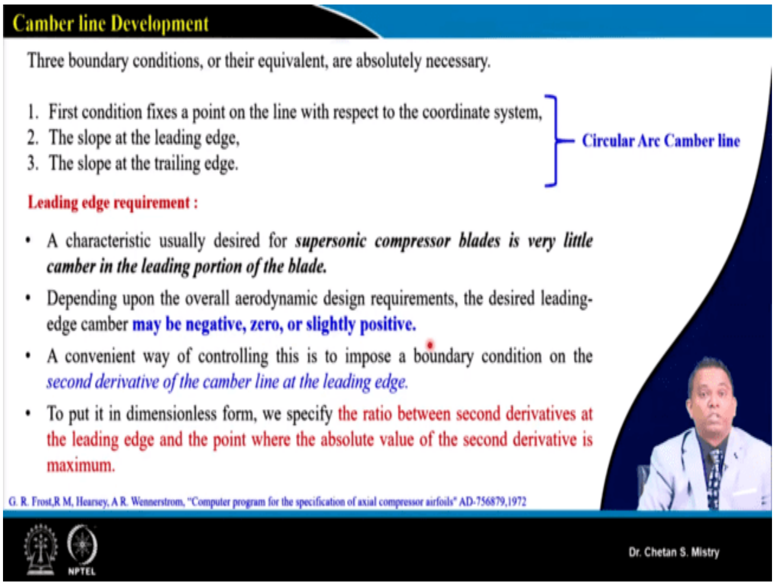

So, in case of solution of the equation, what all we are discussing those equations are say quadratic equations. Now, those equations when we say, we need to solve that equation. For solving that equation, we are looking for different kinds of boundary conditions. So, if you recall when we were discussing about subsonic airfoils that time we have discussed the first condition, that's what is say all points that need to be fit on the line for particular coordinate system.

If you recall, we were discussing about the slope at the leading edge and third boundary condition we were discussing, that's what is say slope at the trailing edge. Now these three-boundary conditions, that's what is sufficient when we are discussing about say circular arc camber line. We are discussing about the transonic airfoils development under that condition we are looking for some special conditions.

If you look at here, say many supersonic airfoil, that's what is required little cambered near the leading edge. This cambered, it may be zero or it may be negative or it may be positive. Now in order to have that cambered, in that case, we are looking for some boundary condition. That boundary condition we can say second derivative of the camber line at the leading edge. How to get that?

Say, in order to achieve that second derivative part, the ratio between the second derivative at the leading edge and point where the absolute value of second derivative is maximum, that's what we will be considering as a boundary condition. So, this is what will be my fourth boundary condition.

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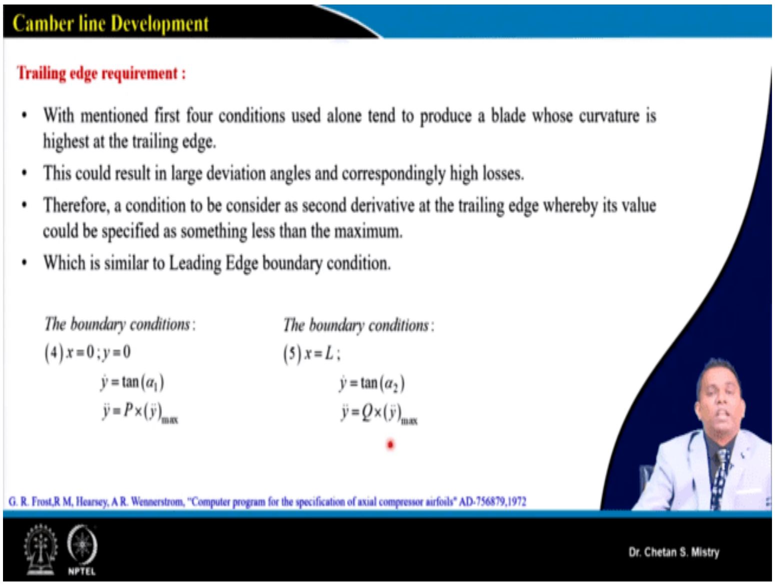

Now, similar to that we will be having trailing edge. And, for trailing edge also, we need to have the boundary condition because we can understand when the flow, that's what is flowing from my suction surface and the pressure surface near the trailing edge, we will be having the flow that's what will be subjected to some deviation angle. And, if we will not take care of the curvature at the trailing edge, it may be possible that we will be having rise of losses. We define that as say wake loss or the profile loss. So, for that also, we need to put the fifth boundary condition.

So, we can say, this boundary conditions as say at $x = 0$, $y = 0$, that is nothing but at the leading edge, my first derivative, I will be putting as say $\tan \alpha_1$. This α_1 is nothing but my inlet flow angle and $\ddot{y}$, that's what is say second derivative. We can say it is P, that's what is some constant into second derivative, that's what is having maximum value.

$$\textit{The boundary conditions:}$$

$$(4)\ \ x = 0; \qquad y = 0$$

$$\dot{y} = \tan(\alpha_1)$$

$$\ddot{y} = P \times (\ddot{y})_{max}$$

At $x = L$, we can say this is what is at the trailing edge; we can say, $\dot{y}$ that's what is say my first derivative, that's what we are writing as say $\tan \alpha_2$ and second derivative we are putting Q into this second derivative which is having maximum value.

$The\ boundary\ conditions$:

$$(5)\ x = L;$$

$$\dot{y} = \tan(\alpha_2)$$

$$\ddot{y} = Q \times (\ddot{y})_{max}$$

Now, this is what is more in sense of mathematics. Let us try to understand when we will be discussing the development of different kind of camber lines, that's what will be giving more clarity.

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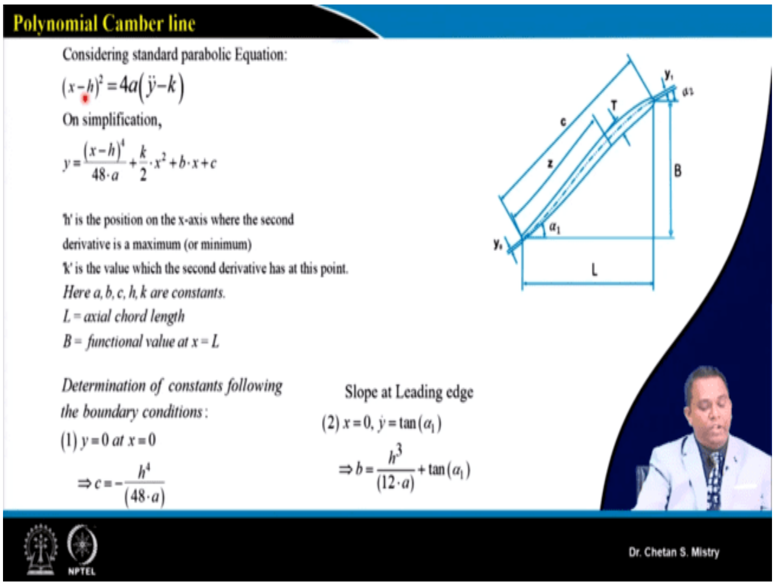

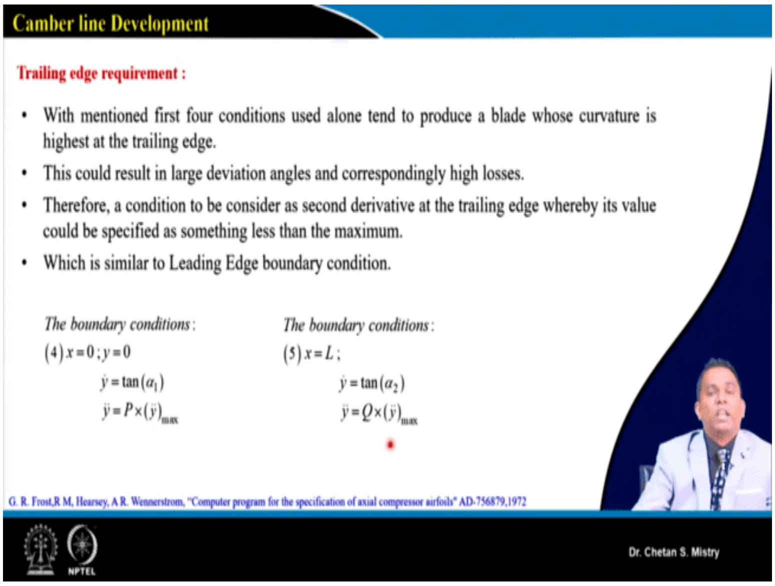

Now, very first, let us try to develop say, polynomial camber line. So, here in this case, say this is what is a kind of camber line what we are looking for. Say, this is what is say my assumed camber line which is having say inflow angle, we can say is α_1, my outlet flow angle, that's what is say α_2. We can say this is what is say my chord length c, this is my axial chord and this is what is representing B, that's what we can say some kind of location. Say, we can say it is a function of $x = L$.

Now, in order to develop this polynomial camber line, we need to select some equation. Now, there is a good literature, that's what is available, that's what was very old literature but still, that's what is giving very good understanding and explanation for the development of such kind of camber lines. So, here what all we are discussing, that's what will be giving you some idea. Later on, as per your expectation or as per your requirement, you can modify the formulation of this line.

Say, here we are assuming our polynomial camber line equation as say $(x-h)^2 = 4a(\ddot{y} - k)$. That's what we can say, it is a parabolic equation. Now, this is what is my second derivative. If I will be simplifying that, this is what will be giving me the equation in sense of $y = f(x)$.

$$Considering\ standard\ parabolic\ equation:$$

$$(x-h)^2 = 4a(\ddot{y} - k)$$

$$On\ simplification,$$

$$y = \frac{(x-h)^4}{48 \cdot a} + \frac{k}{2} \cdot x^2 + b \cdot x + c$$

Here in this case, 'h' is nothing but the position on the x axis where the second derivative, that's what is maximum, that's what is of our interest. 'k' is the value which the second derivative has at that point and a, b, c, h and k, they all are the constants.

$$L = axial\ chord\ length$$

$$B = functional\ value\ at\ x = L$$

Now, in order to get the solution for this equation, we need to find this constant, say, what constants we are looking at, that's what is say a, b, c, h and k, those all constants we can achieve by putting different boundary conditions. So, we are putting very first boundary condition. We can say at $y = 0$ and $x = 0$, we will be getting the value of some constant c.

$$Determination\ of\ constants\ following\ the\ boundary\ conditions:$$

$$(1)\quad y = 0\ at\ x = 0$$

$$\Rightarrow c = -\frac{h^4}{(48 \cdot a)}$$

Same way, if we are considering our boundary condition at say leading edge, what we have defined at say $x = 0$, my first derivative, that's what is equal to tan α_1. So, that's what is giving me the value of constant B.

$$Slope\ at\ Leading\ edge$$

$$(2)\quad x = 0, \dot{y} = \tan(\alpha_1)$$

$$\Rightarrow b = \frac{h^3}{(12 \cdot a)} + \tan(\alpha_1)$$

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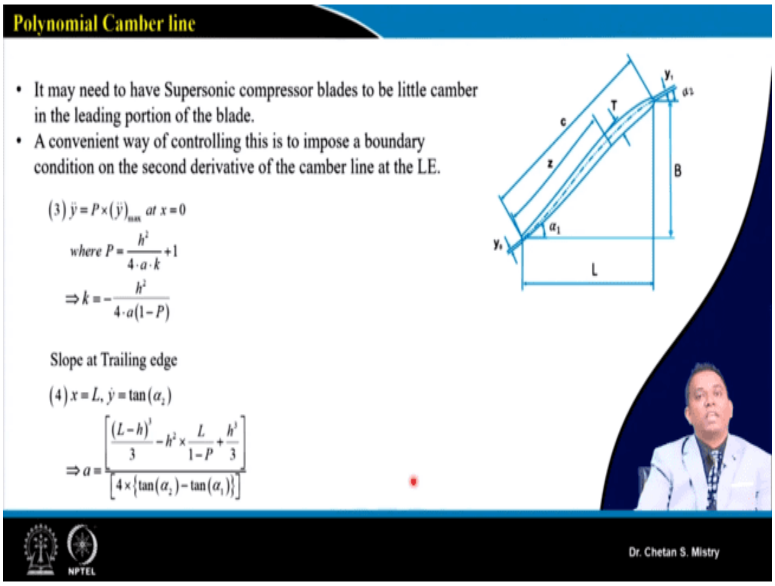

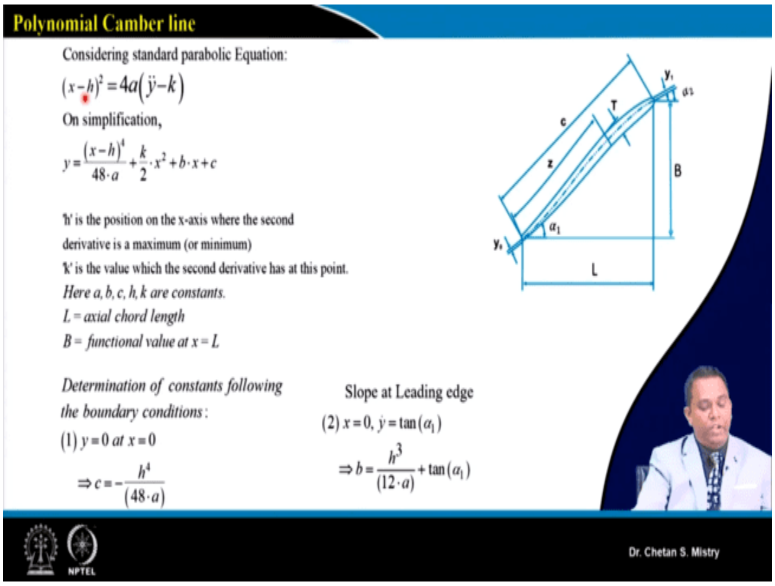

Now, what we know, as we have discussed for many, say airfoils, we need to have slight camber, that’s what need to be provided at the leading edge. In order to have that as a configuration, we are putting our boundary condition, as we have discussed, this is what is at $x = 0$, my second derivative, that’s what I am writing as $P \times (\ddot{y})_{max}$, that’s what will lead to give me the value of ‘k’ as well as the value of ‘P’.

$$(3)\ \ \ddot{y} = P \times (\ddot{y})_{max}\ at\ x = 0$$

$$where\ P = \frac{h^2}{4 \cdot a \cdot k} + 1$$

$$\Rightarrow k = -\frac{h^2}{4 \cdot a(1 - P)}$$

Now, we know what is the slope at our trailing edge. Based on that, we can calculate what will be the value of ‘a’. So, here in this case, we will see, we are having say constants we are calculating a, b, c and k.

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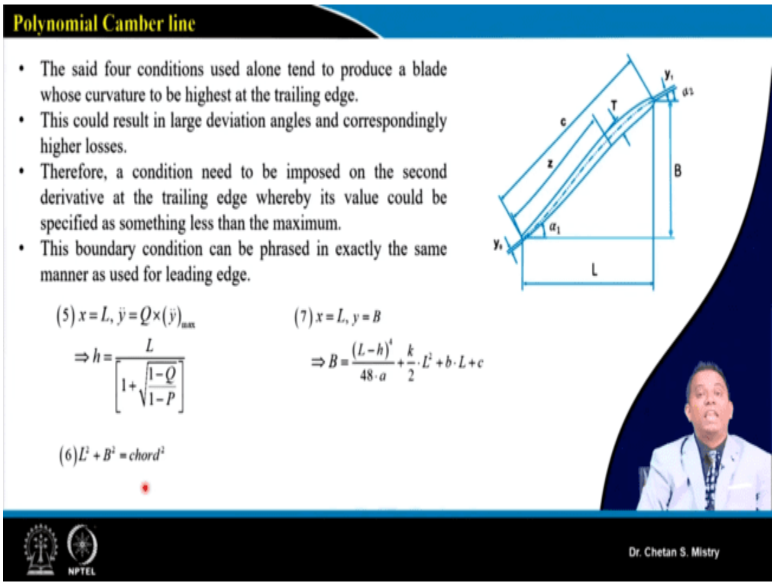

Now, as we have discussed, we are also looking for say trailing edge configuration. We need to take care of what is happening with our say deviation angle. In order to have that inline to what we have done at the leading edge, we will be doing, say putting boundary condition as say leading edge what we have done. So, here in this case, at $x = L$, my second derivative I am writing as some value of Q into second derivative which is having maximum value, that's what will lead to give me the constant 'h'.

$$(5)\ \ x = L, \ddot{y} = Q \times (\ddot{y})_{max}$$

$$\Rightarrow h = \frac{L}{\left[1+\sqrt{\frac{1-Q}{1-P}}\right]}$$

Now, you know, we are having five constants and we are having, say different equations. We can say $L^2 + B^2 = chord^2$, that's what is based on my trigonometry.

$$(6)\ \ L^2 + B^2 = chord^2$$

And, here in this case, at $x = L\ and\ y = B$, if I will be putting those numbers, we will be getting the value of this B.

$$(7)\ \ x = L, \qquad y = B$$

$$\Rightarrow B = \frac{(L-h)^4}{48 \cdot a} + \frac{k}{2} \cdot L^2 + b \cdot L + c$$

Now, what all we know? We know all these constants. Now for a particular value of x, we can calculate what will be the value of y. And that is how we will be generating the coordinates, say coordinates, say x coordinates and y coordinates.

So, you know, in order to have the development of our pressure surface as well as say for suction surface. In line to this for camber line; here in this case, this camber line, that's what can be developed by using this particular equation.

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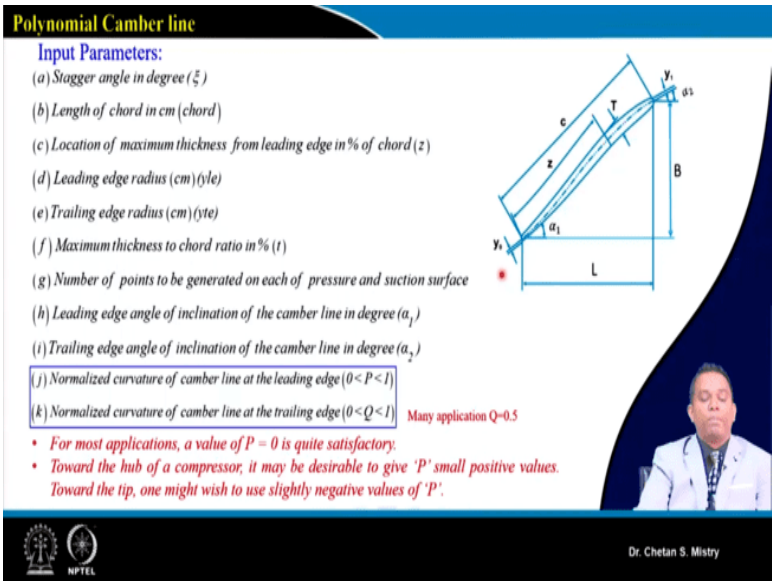

Now, let us see what all input parameters we are looking for in order to develop this camber line. Suppose say we are looking for polynomial camber line. We are looking for data as say stagger angle. We are looking for say length. We are looking for say, maximum thickness in sense of percentage chord, leading edge radius, trailing edge radius, maximum thickness to chord ratio. We will be looking at number of points to be developed for each, okay. We are looking for say leading edge angle, that's what is α_1, trailing edge angle (α_2). We are also looking for say normalized curvature of camber line at the leading edge, that's what we say as P, that's what is in the range of 0 to 1.

Same way, normalized curvature of camber line at the trailing edge, that's what we say as Q. Conventionally, it says like Q, that's what is having say value in the range of 0.5. For most of the application, we are not considering say we will be having say camber or say slight camber at the leading edge and that is the reason why P equal to 0, that's what is quite satisfactory.

Towards the hub region when we are developing the camber line for say hub region where we need to have P value to be slightly smaller or say that's what is a positive value we need to consider; near the tip region, we need to consider this P value to be negative. So, this is what will give us idea how do we develop say polynomial camber line. I am sure, this is what will give you idea how exactly we will be developing this line.

Now, in order to develop this kind of camber line, you need to write the program, say you know, suppose if I consider I am having blade, for that blade, I will be having number of stations and for all those stations we are looking for the camber line and based on my α_1 and α_2, if I will be putting these numbers, taking care of all this what all we have discussed, we will be able to develop say polynomial kind of say camber line.

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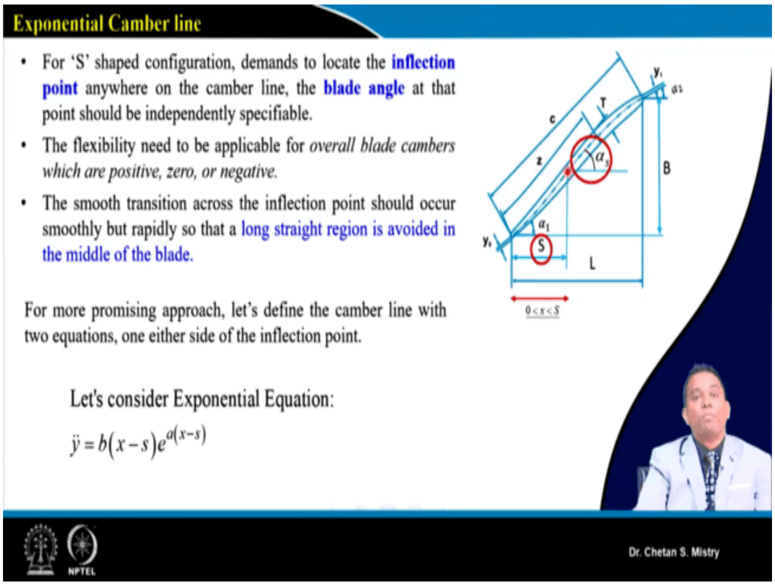

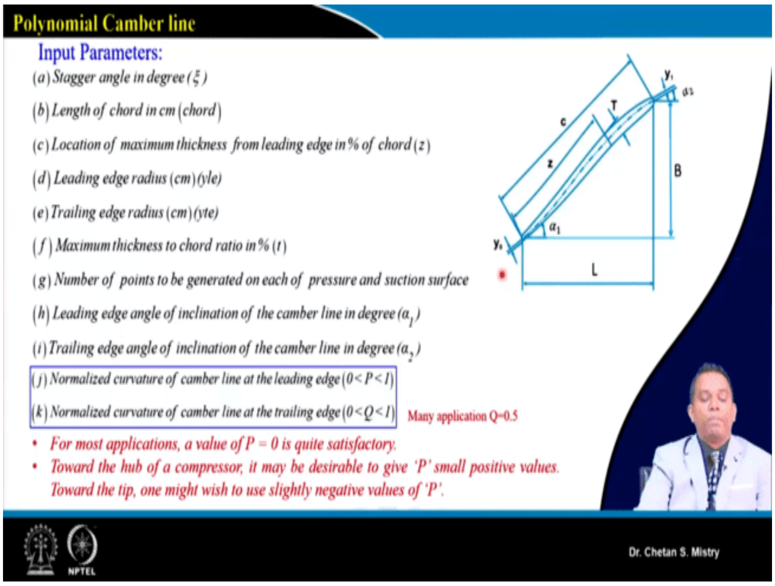

Now, second camber line, that's what we say is an exponential kind of camber line. Now, what is happening here is for say polynomial kind of camber line, we are not having much control in sense of formation of say curve for specially camber line. Suppose, say I want to develop Multiple Circular Arc kind of configuration or Multiple Circular Arc kind of airfoil. Under that condition, we need to put some extra location here. So, this is what we can say, this is what is a location at say some point s. We say that as say inflection point and this is what is some angle α_s.

So, what we will be doing? We will be putting some point, that point, we will be having control on that. So, we will be having say one line up to some location. We will be having second line up to some location and that is how we can go with the development of this kind of say camber line. So, here in this case, we will be using two different equations. As I discussed, we will be having say one equation of camber line up to this inflection point. We will be having second camber line equation that's what is from say inflection point to the trailing edge.

So, let us consider, say exponential kind of equation. So, this is what is the equation, what we are considering, say

$$\ddot{y} = b(x - s)e^{a(x-s)}$$

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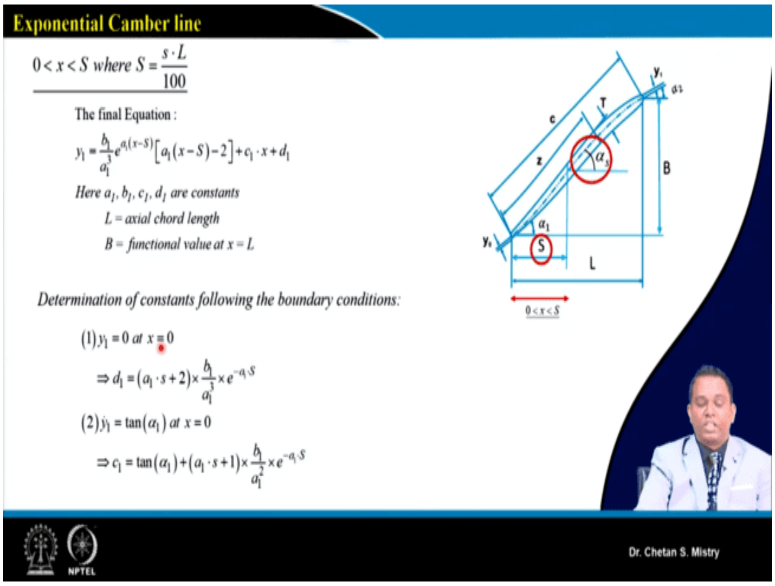

Now, suppose if we consider, as we are discussing, we are considering the location from my leading edge to some inflection point, this equation we can write down, say as y_1. We will be having some constants a_1, b_1, c_1 and d_1 and L as we have discussed, it is an axial chord and this is what is a function of say $x = L$.

$$0 < x < S \; where \; S = \frac{s \cdot L}{100}$$

$The\ final\ equation$:

$$y_1 = \frac{b_1}{a_1^3} e^{a_1(x-S)}[a_1(x-S)-2] + c_1 \cdot x + d_1$$

$$Here\ a_1, b_1, c_1, d_1\ are\ constants$$

$$L = axial\ chord\ length$$

$$B = functional\ value\ at\ x = L$$

Now, in order to solve this, we need to put certain boundary conditions as we have discussed. This boundary conditions, that's what will remain similar to what all we have discussed in earlier case. We are looking for five boundary conditions.

First, that's what we have discussed is $y_1 = 0\ at\ x = 0$, that's what will be giving me constant d_1. When I am taking say $x = 0$, this is nothing but this is what is my slope as we have discussed. That's what will be giving me my second constant as say c_1. You can do pen paper calculation in order to check with what all numbers we are putting here.

$Determination\ of\ constants\ following\ the\ boundary\ conditions$:

(1) $y_1 = 0\ at\ x = 0$

$$\Rightarrow d_1 = (a_1 \cdot s + 2) \times \frac{b_1}{a_1^3} \times e^{-a_1 \cdot S}$$

(2) $\dot{y}_1 = \tan(\alpha_1)\ at\ x = 0$

$$\Rightarrow c_1 = \tan(\alpha_1) + (a_1 \cdot s + 1) \times \frac{b_1}{a_1^2} \times e^{-a_1 \cdot S}$$

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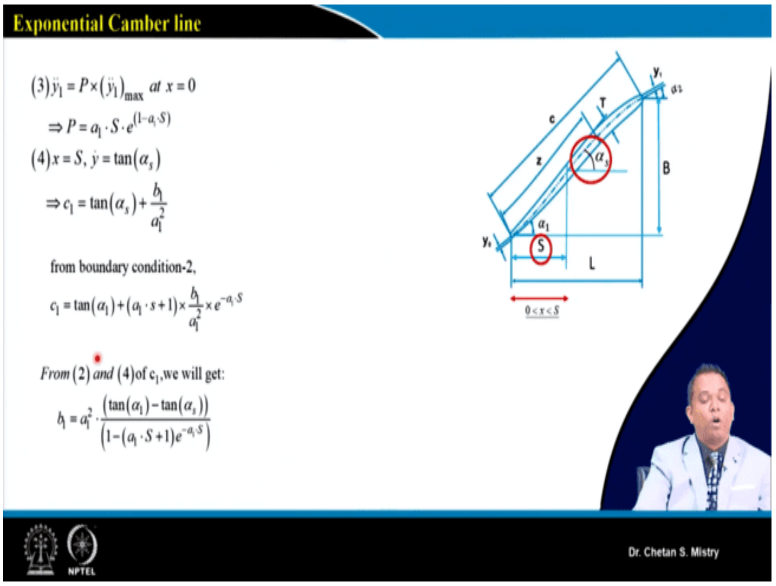

Now, as we have discussed near this leading edge, we may need to look for say slight camber and that is the reason why we are incorporating this boundary condition where we can calculate what will be the value of say constant P.

$$(3)\ \ddot{y}_1 = P \times (\ddot{y}_1)_{max}\ at\ x = 0$$

$$\Rightarrow P = a_1 \cdot S \cdot e^{(1-a_1 \cdot S)}$$

Same way, now, we are reaching at location $x = S$. So, in previous case for polynomial camber line, what we have done, that’s what is say at leading edge and trailing edge we have given the boundary condition.

Here in this case, we will be putting this boundary condition at say inflection point. So, this is what we are putting at the inflection point.

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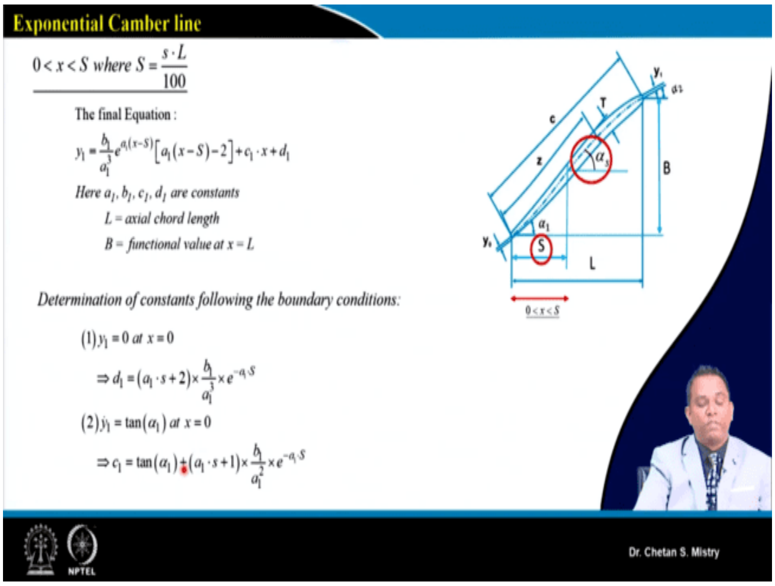

Now, if you look at, say we are having two values of constant c_1, that's what we have received by considering $x = 0$ and slope factor.

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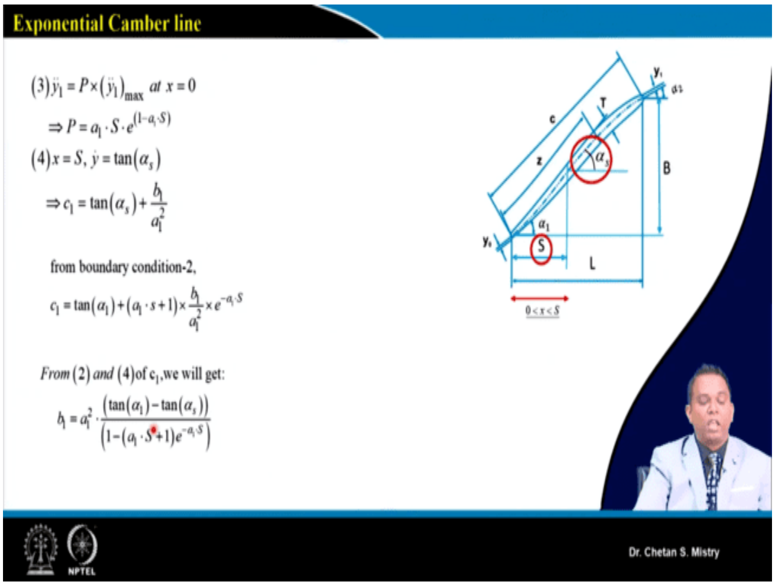

And, this is what is say my second boundary condition. So, based on that if we are solving, we will be getting the constant b_1.

$$(4)\ \ x = S,\ \ \dot{y} = \tan(\alpha_s)$$

$$\Rightarrow c_1 = \tan(\alpha_s) + \frac{b_1}{a_1^2}$$

$$from\ boundary\ condition - 2,$$

$$c_1 = \tan(\alpha_1) + (a_1 \cdot s + 1) \times \frac{b_1}{a_1^2} \times e^{-a_1 \cdot S}$$

$$From\ (2)\ and\ (4)\ of\ c_1, we\ will\ get:$$

$$b_1 = a_1^2 \cdot \frac{(\tan(\alpha_1) - \tan(\alpha_s))}{1 - (a_1 \cdot S + 1)e^{-a_1 \cdot S}})$$

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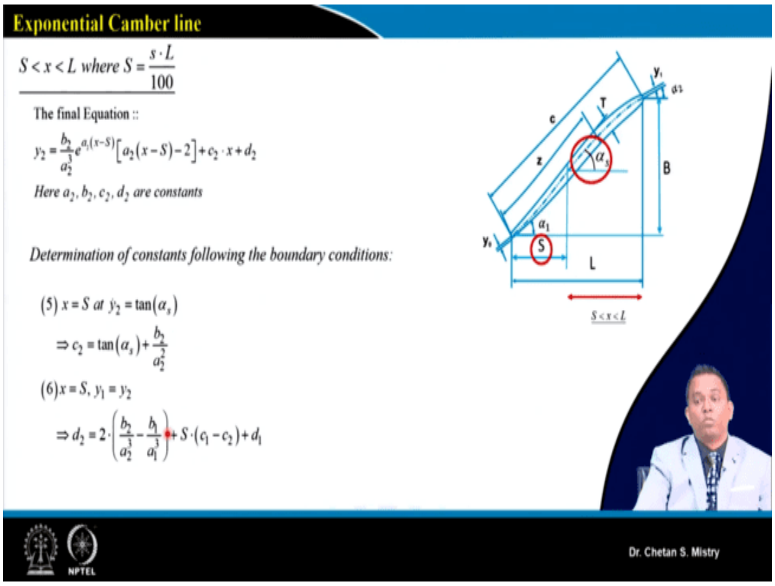

Now, let us move towards the next location. So, this is what is my next location. So, my equation, that's what will be written as say y_2. We can say, this is what all are the constants a_2, b_2, c_2 and d_2. Again, we will be putting our boundary condition at say inflection point. So, at $x = s$, we will be having $\dot{y}_2 = \tan(\alpha_s)$, that's what will be giving me my constant c_2.

$$0 < x < S\ where\ S = \frac{s \cdot L}{100}$$

$$The\ final\ equation:$$

$$y_2 = \frac{b_2}{a_2^3} e^{a_2(x-S)} [a_2(x - S) - 2] + c_2 \cdot x + d_2$$

$$Here\ a_2, b_2, c_2, d_2\ are\ constants$$

Now, when we are putting this location, this particular location, we need to be very careful what point we are discussing about 's' that needs to be a smooth point. So, you know, like the line what we are drawing, that line need to be smooth at this connection point; and that's what we are putting here. At $x = s$, we are putting $y_1 = y_2$, and that's what will be giving me second constant as say d_2.

$$(5)\ \ x = S\ at\ \dot{y}_2 = \tan(\alpha_s)$$

$$\Rightarrow c_2 = \tan(\alpha_s) + \frac{b_2}{a_2^2}$$

$$(6)\ \ x = S,\ \ y_1 = y_2$$

$$\Rightarrow d_2 = 2 \cdot \left(\frac{b_2}{a_2^3} - \frac{b_1}{a_1^3}\right) + S \cdot (c_1 - c_2) + d_1$$

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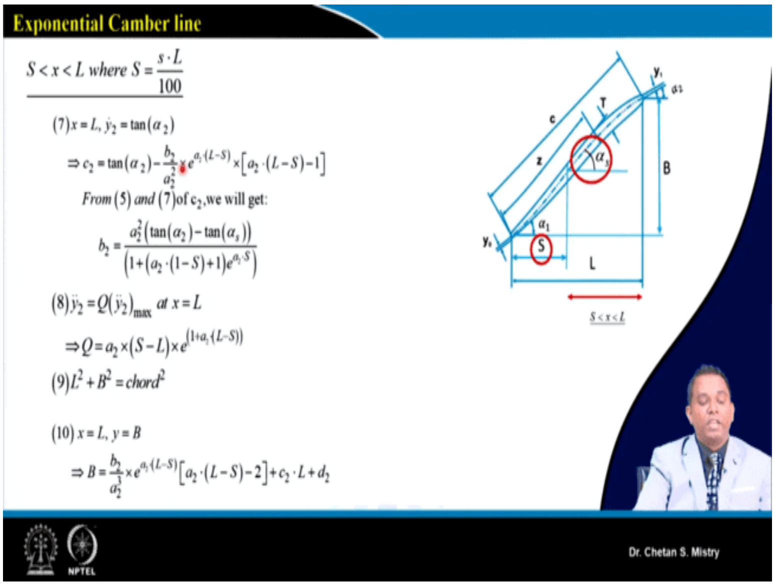

Now, you know, we will be putting say next boundary condition at $x = L$ near the trailing edge, that's what will lead to give me say second value of c_2. Again, if you will be solving these two values of constant c_2, we will be able to achieve the value of constant b_2. And, same boundary condition in order to consider your deviation angle configuration, we will be putting the boundary condition of second derivative and its maximum value, that's what will lead to give me the value of Q.

$$S < x < L\ where\ S = \frac{s \cdot L}{100}$$

$$(7)\ x = L,\ \ \dot{y}_2 = \tan(\alpha_2)$$

$$\Rightarrow c_2 = \tan(\alpha_2) - \frac{b_2}{a_2^2} \times e^{a_2 \cdot (L-S)} \times [a_2 \cdot (L - S) - 1]$$

$$From\ (5)\ and\ (7)\ of\ c_2, we\ will\ get:$$

$$b_2 = \frac{a_2^2(\tan(\alpha_2) - \tan(\alpha_s))}{(1 + (a_2 \cdot (1 - S) + 1)e^{a_2 \cdot S})}$$

$$(8)\ \ \ddot{y}_2 = Q(\ddot{y}_2)_{max}\ \ at\ x = L$$

$$\Rightarrow Q = a_2 \times (S - L) \times e^{(1 + a_2 \cdot (L - S))}$$

We know what is my $L^2 + B^2$, that's what is $chord^2$ and at $x = L$, and $y = B$, that's what will be giving me the value of B.

$$(9)\ \ L^2 + B^2 = chord^2$$

$$(10)\ x = L,\ \ y = B$$

$$\Rightarrow B = \frac{b_2}{a_2^3} \times e^{a_2 \cdot (L - S)}[a_2 \cdot (L - S) - 2] + c_2 \cdot L + d_2$$

Now, all these parameters, that's what will be helpful to us in order to develop the exponential camber line.

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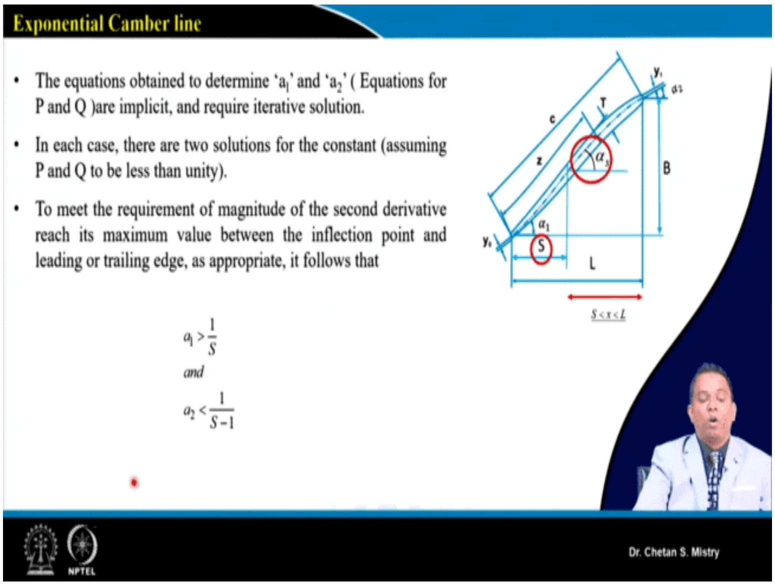

So, let us look at this exponential camber line. Now, as we were discussing, we are having say, different values of P and Q. In order to have say, special requirement at the inflection point, we need to be little careful of and it says we need to have our a_1 value, that's what is my constant value a_1 and constant value of a_2 that need to be selected in this particular range.

$$a_1 > \frac{1}{S}$$

$$and$$

$$a_2 < \frac{1}{S-1}$$

Because we can say, I will be making my curve here, I can make my second curve here. But when we are having this connection, it should not have any jump, it should be smooth one and that's what is very important. And, that's what we can consider by providing particular boundary condition and selecting these numbers, okay.

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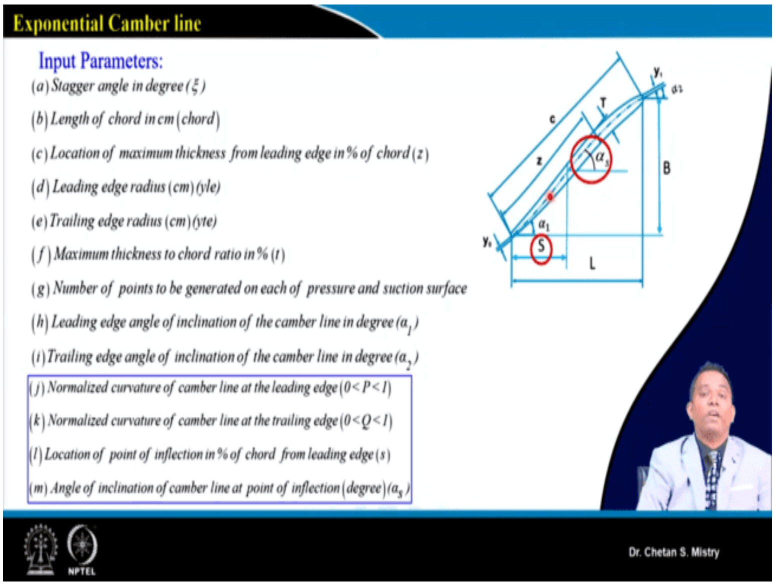

Now, suppose say, if you are looking for development of, say, exponential camber line; again, what all we are looking for is say our stagger angle. We are looking for the length of the chord, location of the maximum thickness, leading edge radius, trailing edge radius, maximum thickness to chord ratio. We are looking for say leading edge angle, that's what is say, α_1. We are looking for α_2. Same way, here, we are looking for say, constant value of P and Q. Two additional points we need to care for here, that's what is the location point of inflection in percentage chord. So, we are looking for where exactly we want to put our inflection point.

Are we looking for this inflection point towards the leading edge? Are we looking for, say, this inflection towards the trailing point? At what location? That's what is very important and that's what we are defining in sense of percentage chord. At the same time, you know, we are also interested in putting the inclination, okay. At what inclination we are putting this line that will make the shape of my airfoil. So, if you recall when we were discussing about the Multiple Circular Arc airfoil, we are having say different kind of configuration where we required for managing our flow towards the trailing edge. Okay.

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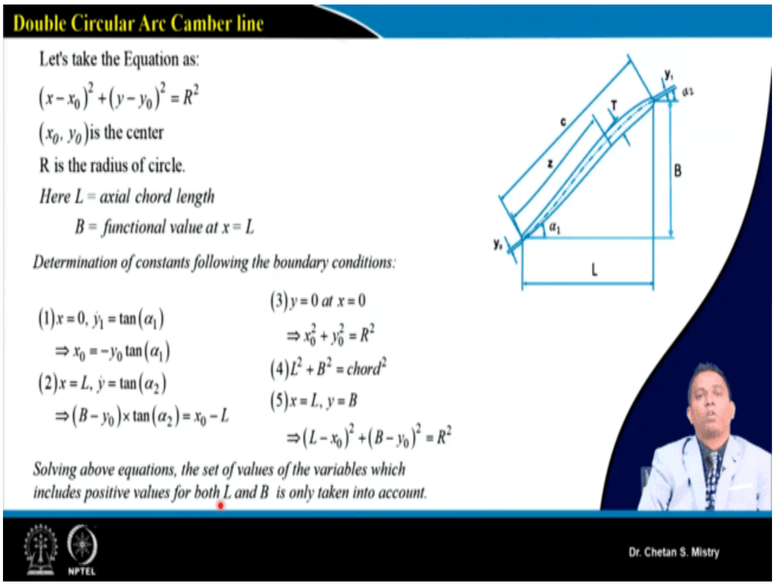

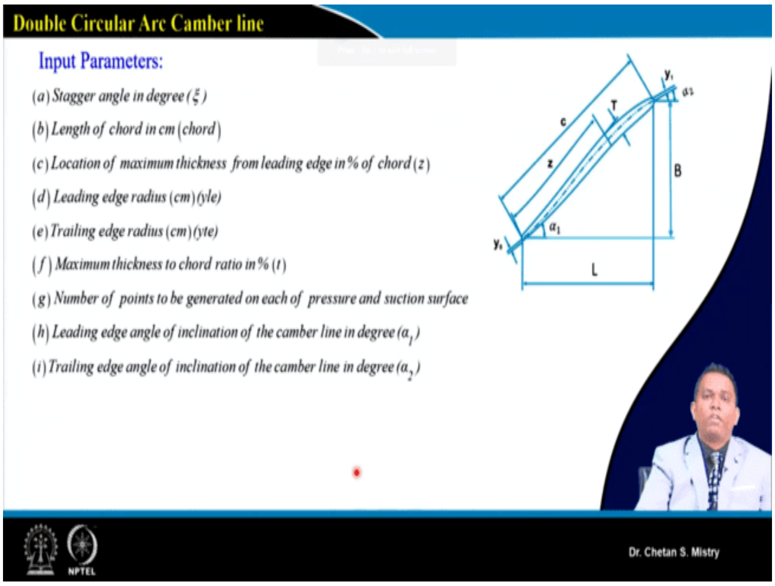

Now, Double Circular Arc airfoil or say Double Circular Arc camber line, what we are looking for, that's what can be represented in sense of this equation:

$$(x-x_0)^2+(y-y_0)^2=R^2$$

$$(x_0, y_0)\ is\ the\ center$$

$$R\ is\ the\ radius\ of\ circle$$

$$Here, L = axial\ chord\ length$$

$$B = functional\ value\ at\ x = L$$

And based on that, if we will be putting our boundary conditions, the same boundary condition, what all we have discussed in sense of condition 1, condition 2 and say this condition 3 based on that and we will be having our Pythagoras law, we will be able to achieve what all we are looking for in sense of coordinates for my camber line. So, this is what is for Double Circular Arc camber line.

$$Determination\ of\ constants\ following\ the\ boundary\ conditions:$$

$$(1)\ x = 0,\ \ \dot{y}_1 = \tan(\alpha_1)$$

$$\Rightarrow x_0 = -y_0 \tan(\alpha_1)$$

$$(2)\ x = L,\ \ \dot{y} = \tan(\alpha_2)$$

$$\Rightarrow (B - y_0) \times \tan(\alpha_2) = x_0 - L$$

$$(3)\ \ y = 0\ at\ x = 0$$

$$\Rightarrow x_0^2 + y_0^2 = R^2$$

$$(4)\ \ L^2 + B^2 = chord^2$$

$$(5)\ \ x = L,\ \ y = B$$

$$\Rightarrow (L - x_0)^2 + (B - y_0)^2 = R^2$$

$$Solving\ above\ equations, the\ set\ of\ values\ of\ the\ variables\ which\ includes$$

$$positive\ values\ for\ both\ L\ and\ B\ is\ only\ taken\ into\ account.$$

Now, there are different methods, people, they are opting with different kinds of Double Circular Arc camber line. This is what is one of the methods. One more method, that's what has been discussed we will see. Now in order to develop this Double Circular Arc camber line, here in this case, this is what is straightforward. We are having say three boundary conditions.

That's what is sufficient for us and based on that, we can say, we can put our stagger angle. We can put where we will be having maximum say thickness, we know, that's what is at the 50% of my chord. What will be my leading edge radius, trailing edge radius and, you know, what will be my angles $\alpha_1 and\ \alpha_2$. That's what will be sufficient condition for development of Double Circular Arc camber line.

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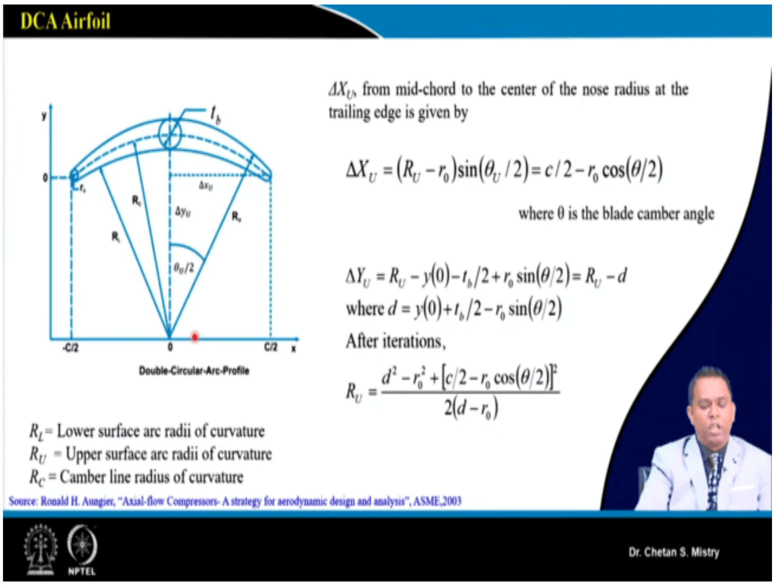

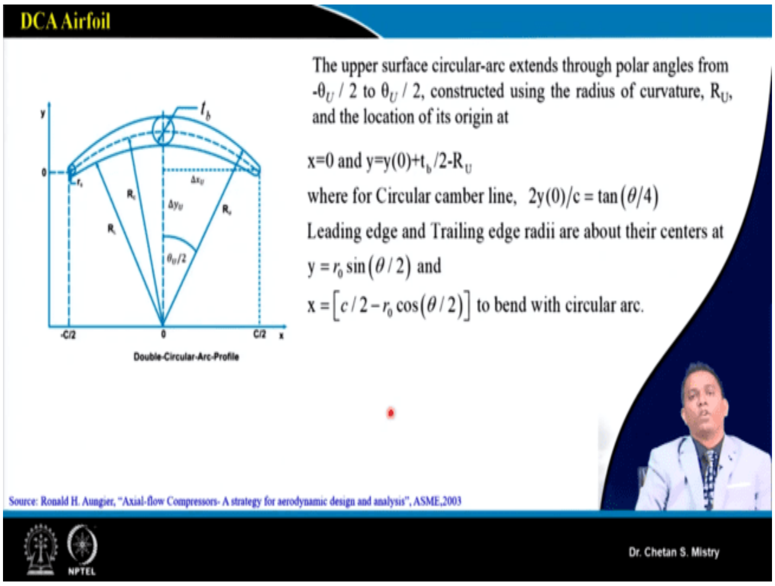

Now, as I was discussing, this is what has been discussed by Aungier in his book. This is also for the development of Double Circular Arc airfoil, rather defining in sense of Double Circular Arc camber line. So, what they have done? Here in this case, we will be having maximum thickness, that's what has been placed at the 50% of chord and upper surface and lower surface coordinates that's what is he has been kept with. This is what is representing the upper radius. This is what is representing the lower radius.

We can say, pressure surface as well as the suction surface and this is what is for say my camber line equation. So, my coordinates Δx, that's what we can calculate based on trigonometric rule. Δy for upper surface also can be calculated based on the trigonometry. Now, this θ, that is nothing but which is representing my camber angle.

$\Delta X_U, from\ mid - chord\ to\ the\ center\ of\ the\ nose\ radius\ at\ the\ trailing\ edge$

$is\ given\ by$

$$\Delta X_U = (R_U - r_0)\sin\left(\frac{\theta_U}{2}\right) = {}^{c}/_{2} - r_0\cos\left(\frac{\theta}{2}\right)$$

$$where\ \theta\ is\ the\ blade\ camber\ angle$$

$$\Delta Y_U = R_U - y(0) - {}^{t_b}/_{2} + r_0\sin\left({}^{\theta}/_{2}\right) = R_U - d$$

$$where\ d = y(0) + {}^{t_b}/_{2} - r_0\sin\left({}^{\theta}/_{2}\right)$$

$After\ iterations,$

$$R_U = \frac{d^2 - r_0^2 + \left[{}^{c}/_{2} - r_0\cos\left({}^{\theta}/_{2}\right)\right]^2}{2(d - r_0)}$$

$$R_L = Lower\ surface\ arc\ radii\ of\ curvature$$

$$R_U = Upper\ surface\ arc\ radii\ of\ curvature$$

$$R_C = Camber\ line\ radius\ of\ curvature$$

Now, in order to achieve this curvature or say radius of my upper surface, we need to go with the number of iterations solving this upper surface and lower surface. And, that's what will lead to give the radius of my upper surface, that's what is as a function of this d, what is my r_0, that is nothing but my radius at the leading edge, what is my chord, what will be my camber line or camber angle. So, this is how we can achieve our upper surface or upper radius. In line to that we can achieve what will be our lower radius. So, this is also one of the method.

Other people, they are developing the Double Circular Arc airfoil with some different approach, different methodology. So, here in this case, this is what is representing Double Circular Arc kind of camber line.

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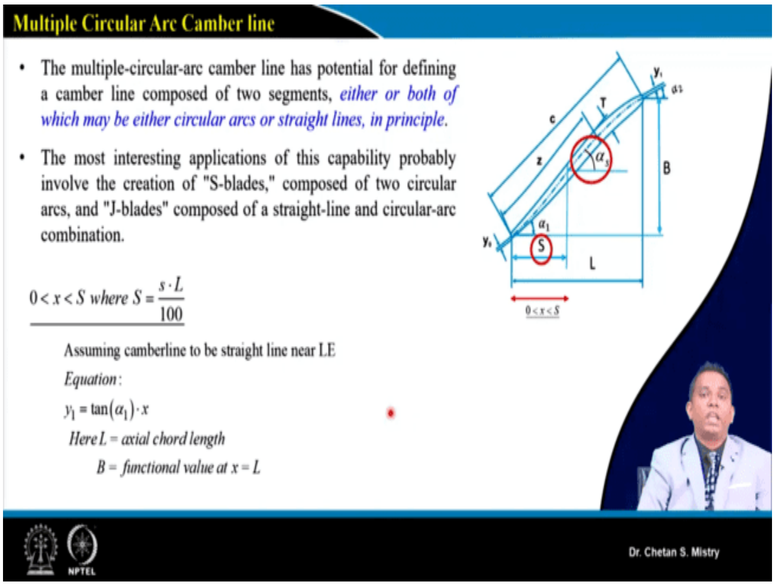

Now, let us move it what all we are more interested in or say for future development of our transonic airfoil, that's what is say Multiple Circular Arc. We can say, that's what is of S kind of shape. So, in line to what all we have discussed for our exponential kind of say camber line, we will be using the similar kind of logic but here, this is what has been developed in a different way. So, here in this case, also we are looking for say, inflection point. So, this is what is my location for inflection point and this is what is my inflection angle.

Now, up to from leading edge to my inflection location, we will be considering one curve, one line or we can say one camber line and from say inflection point to trailing edge, we will be considering other line, okay. Now, there are different methodologies, different authors, different researchers, different companies, different universities, they are developing their own camber lines for Multiple Circular Arc. Here in this case, for the sake of simplicity, what we have done, say up to our inflection point, we have considered this as a straight line.

So here, this equation, we can say, that's what is given by $y_1 = \tan(\alpha_1) \cdot x$, we can say this is what is the equation of straight line.

$$0 < x < S \; where \; S = \frac{s \cdot L}{100}$$

$$Assuming \; camberline \; to \; be \; straight \; line \; near \; LE$$

$$Equation: y_1 = \tan(\alpha_1) \cdot x$$

$$Here, L = axial \; chord \; length; \; and \; B = functional \; value \; at \; x = L$$

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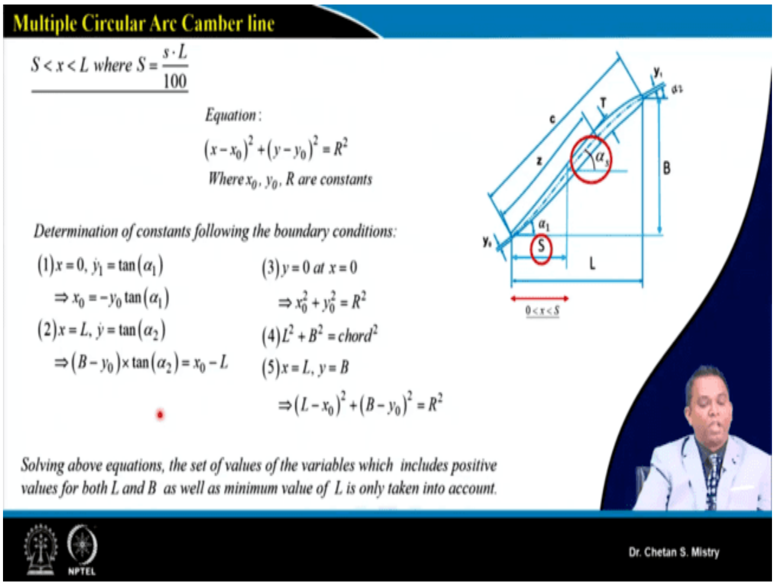

Now, in order to have the other line, say this is what is from inflection point towards the trailing edge, that's what we can consider as a circular arc. So, this is what is

$$0 < x < S \; where \; S = \frac{s \cdot L}{100}$$

$$Equation:$$

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$where \; x_0, y_0, R \; are \; constants$$

Now, in line to what all boundary conditions what we have discussed, we will be putting those boundary conditions here. So here in this case, if we are putting our boundary condition, that's what will lead to give us all the variables, all the constants. Now, once we have derived with the constant, we will be calculating our x and y coordinate for the camber line, okay.

$$Determination \; of \; constants \; followng \; the \; boundary \; conditions:$$

$$(1) \; x = 0, \;\; \dot{y}_1 = \tan(\alpha_1)$$

$$\Rightarrow x_0 = -y_0 \tan(\alpha_1)$$

$$(2) \; x = L, \;\; \dot{y} = \tan(\alpha_2)$$

$$\Rightarrow (B - y_0) \times \tan(\alpha_2) = x_0 - L$$

$$(3) \;\; y = 0 \; at \; x = 0$$

$$\Rightarrow x_0^2 + y_0^2 = R^2$$

(4) $L^2 + B^2 = chord^2$

(5) $x = L, \;\; y = B$

$$\Rightarrow (L - x_0)^2 + (B - y_0)^2 = R^2$$

Solving above equations, the set of values of the variables which includes positive values for both L and B as well as minimum value of L is only taken into account.

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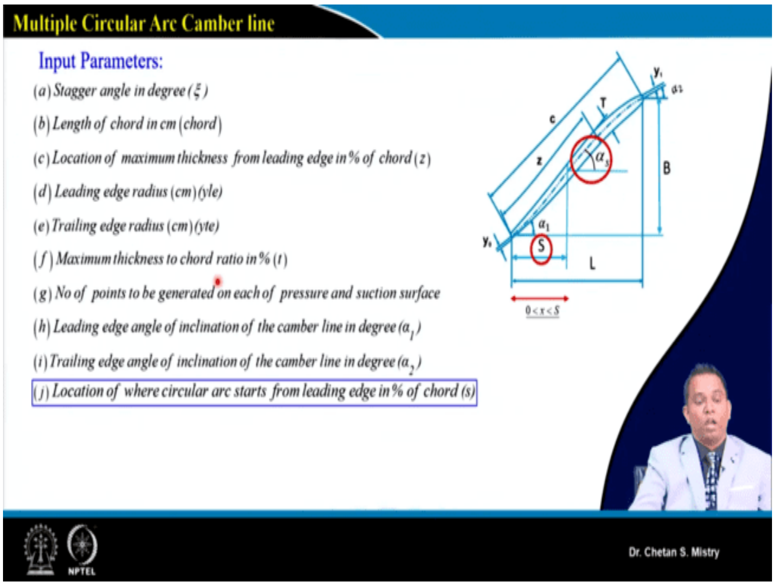

Now, in order to develop this Multiple Circular Arc camber line, additional parameter what we are looking for is a location from where the arc that will be started with. So, as I told, like for development of Multiple Circular Arc airfoil, many people they are considering two circular arcs, say initial circular arc, second circular arc and based on that they are developing say Multiple Circular Arc camber line. This is one of the methods what we have discussed. We will be having straight line, that's what will be followed by the curvature or we can say that will be followed by circular camber line.

So, as per how we want to manage our flow on the suction surface of our blade, we need to modify or we need to make the airfoil or we need to make our camber line. So, once we are ready with our camber line, very next question that's what will be coming is how do we put our thickness?

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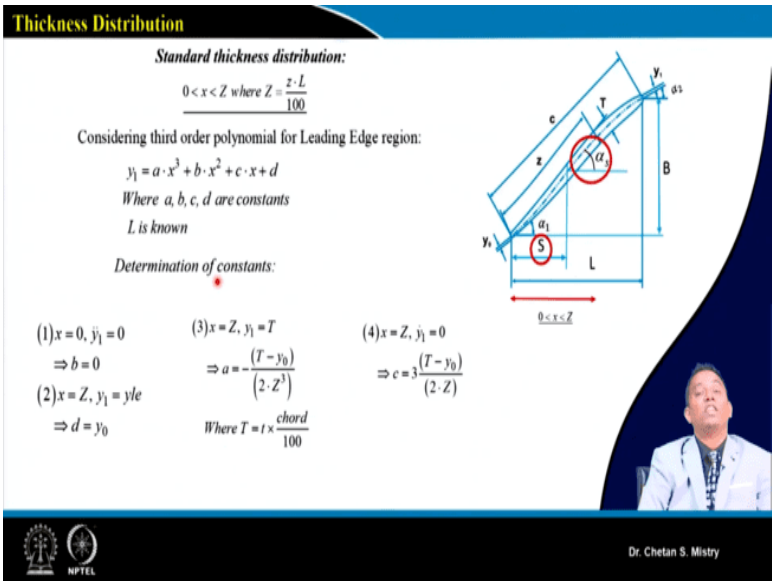

So, let us move towards say, having the distribution of the thickness. So, for transonic airfoils, say, specially when we are considering the thickness distribution, that's what has been defined in sense of say, polynomial, say third order polynomial. So, here we are considering, suppose if you are considering the location somewhere, up to somewhere some location, somewhere location we can say, that's what is say my maximum thickness location. We can say this is what is my z location.

So, it says in between 0 to z or say from leading edge to some location, we can say, we will be selecting third order polynomial thickness distribution. That's what is say, $a \cdot x^3 + b \cdot x^2 + c \cdot x + d$; where a, b, c, d they all are the constants. And as we know, in order to solve these equations and in order to get this constant, we need to put different boundary conditions.

$$Standard\ thickness\ distribution:$$

$$0 < x < Z\ where\ Z = \frac{z \cdot L}{100}$$

$$Considering\ third\ order\ polynomial\ for\ Leading\ Edge\ region:$$

$$y_1 = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$$

$$where\ a, b, c, d\ are\ constants$$

$$L\ is\ known$$

So, this boundary conditions are say, at $x = 0$, we will be putting say second derivative of this equation to be 0 and that's what will be giving constant B. At $x = Z$ and y_1, we will be putting our leading edge. We will be having $d = y_0$. Same way, at $x = Z$ and T, this capital T, that's what we are defining in sense of thickness to chord ratio, okay. And, this is what is my third boundary condition. In line to that, we will be putting our fourth boundary condition and by putting this boundary conditions, we will be able to achieve what are the constant a, b, c, d for this equation.

$$Determination\ of\ constants:$$

$$(1)\ \ x = 0,\ \ \ddot{y}_1 = 0$$

$$\Rightarrow b = 0$$

$$(2)\ \ x = Z,\ \ y_1 = yle$$

$$\Rightarrow d = y_0$$

$$(3)\ \ x = Z,\ \ y_1 = T$$

$$\Rightarrow a = -\frac{(T - y_0)}{(2 \cdot Z^3)}$$

$$where\ T = t \times \frac{chord}{100}$$

$$(4)\ \ x = Z,\ \ \dot{y}_1 = 0$$

$$\Rightarrow c = 3\frac{(T - y_0)}{(2 \cdot Z)}$$

Now, as we know, this is what we are saying as an axial location. It is not the chord, this is what is my axial location. So, we will be selecting different x values, and for those x values by having this constant, we will be able to achieve the y values, okay. So, that is how we will be calculating our thickness distribution on suction surface. We will be able to calculate the thickness distribution on our pressure surface, specially up to some location Z.

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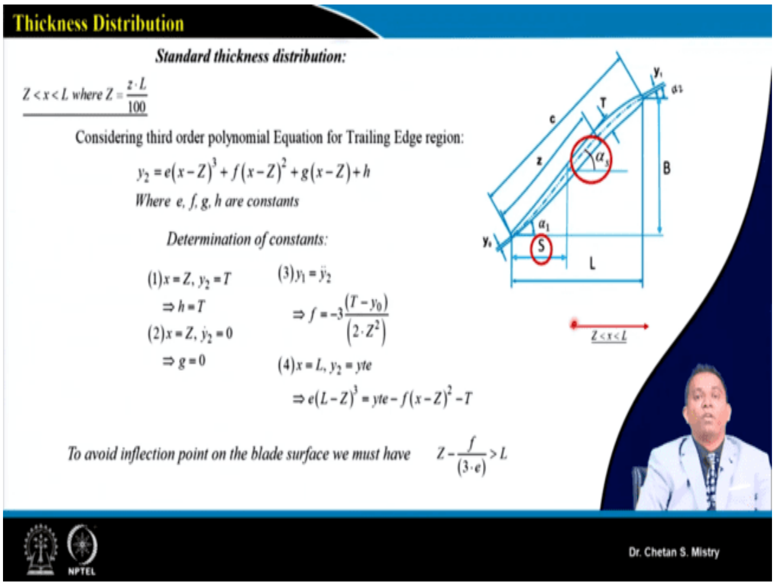

Now, you know, later part, say towards the trailing edge, there also we are taking say third order polynomial but we know in that particular region, we are looking for the management of shock in a different way. So, third order polynomial equation, that's what is been defined in a different way. So, this is what is the equation, that's what people, they have defined with. As I told, there is nothing, that's what is fixed. You can use your own equation based on your requirement.

Maybe based on your computational study, you can modify this distribution, okay. So, this is what is representing how my value of y, that's what is varying with the x, we will be putting different boundary conditions at Z and this is what is at, say my trailing edge location. We will be able to achieve the distribution of the thickness on say, suction surface as well as on the pressure surface towards the trailing edge side, okay.

$$Standard\ thickness\ distribution:$$

$$Z < x < L\ where\ Z = \frac{z \cdot L}{100}$$

$$Considering\ third\ order\ polynomial\ Equation\ for\ Trailing\ Edge\ region:$$

$$y_2 = e(x-Z)^3 + f(x-Z)^2 + g(x-Z) + h$$

$$Where\ e, f, g, h\ are\ constants$$

$$Determination\ of\ constants:$$

$$(1)\ x = Z,\ \ y_2 = T$$

$$\Rightarrow h = T$$

$$(2)\ \ x = Z,\ \ \dot{y}_2 = 0$$

$$\Rightarrow g = 0$$

$$(3)\ \ y_1 = \ddot{y}_2$$

$$\Rightarrow f = -3\frac{(T - y_0)}{(2 \cdot Z^2)}$$

$$(4)\ \ x = L,\ \ y_2 = yte$$

$$\Rightarrow e(L - Z)^3 = yte - f(x - Z)^2 - T$$

$$To\ avoid\ inflection\ point\ on\ the\ blade\ surface\ we\ must\ have$$

$$Z - \frac{f}{3 \cdot e} > L$$

Now, in order to avoid the inflection point on the blade surface, it says we need to take care of this number. So, $Z - \frac{f}{3 \cdot e} > L$. We can understand when we are discussing, we are selecting say Multiple Circular Arc; for that Multiple Circular Arc, we are having the inflection point, we have inflection angle. And, when we are doing our distribution of the thickness for suction surface as well as for the pressure surface, it should give the smooth curve and that's what is need to be managed.

Any jump or any sharp edge, that's what has not been allowed or permitted when we are discussing say transonic kind of airfoils or when our flow that's what is say supersonic flow, okay.

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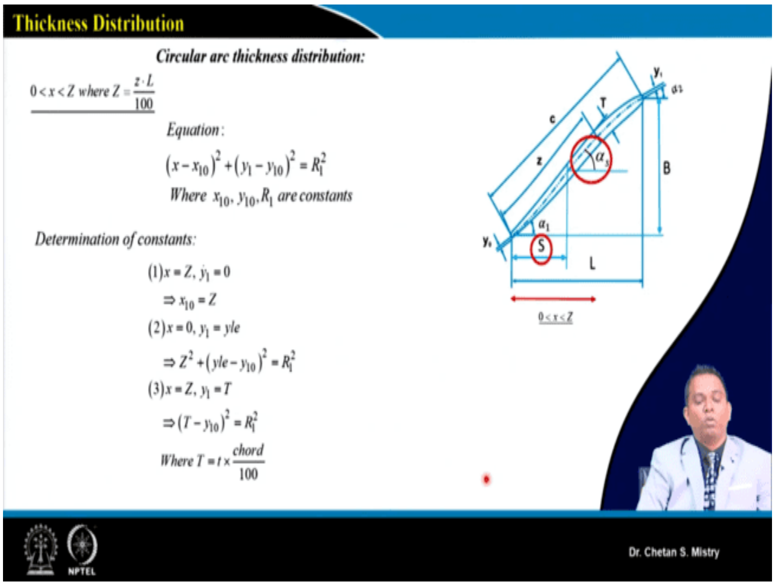

Now, some of the literature, it says you can go with say circular thickness distribution also. So, we can say, we will be going with say circular arc thickness, that's what is given by x minus x and this is what is y. So, in order to avoid the confusion, there is nothing at x_{10}, it is just for segregating from other constants, okay. And, we can use this circular arc thickness distribution up to some location as we have discussed by providing different boundary conditions, we will be able to achieve different constants. When we achieve, we derived the constants, we will be getting say values of x as well as y.

$$Circular\ arc\ thickness\ distribution:$$

$$0 < x < Z\ where\ Z = \frac{z \cdot L}{100}$$

$$Equation:$$

$$(x - x_{10})^2 + (y_1 - y_{10})^2 = R_1^2$$

$$Where\ x_{10}, y_{10}, R_1\ are\ constants$$

$$Determination\ of\ constants:$$

$$(1)\ \ x = Z,\ \ \dot{y}_1 = 0$$

$$\Rightarrow x_{10} = Z$$

$$(2)\ \ x = 0,\ \ y_1 = yle$$

$$\Rightarrow Z^2 + (yle - y_{10})^2 = R_1^2$$

$$(3)\ x = Z, y_1 = T$$

$$\Rightarrow (T - y_{10})^2 = R_1^2$$

$$where\ T = t \times \frac{chord}{100}$$

Now, this is what all we are discussing that's what is very hectic process when we are doing pen paper calculations. So, it is advised that you just learn some coding…some coding tools, maybe by using say your C, C++ or MATLAB. Using those say program language, you can develop your own code, you can develop your coordinates x and y for say suction surface, you can develop your coordinates for x and y for say your pressure surface. Even you can develop your own camber line.

You can play with the equations and that's what will be giving you what all we are looking for. As I told, there is no superficial rule for having say thickness distribution to be followed by this equation only, okay. As per the expectation what all you are getting, if it is solving your purpose, your expectation, your required result that's what is your airfoil. So, maybe it may be possible that you will be going through the development of airfoil you can name that airfoil as your own name. Yes, this is what is possible. So, whole lot of scope, that's what has been laid in sense of development of these airfoils.

So, people initially they are going with say computational study; using CFD they are developing this kind of airfoils. Later on, in order to build the confidence, they will be doing cascade testing, you know better now. Based on those cascade testing, they will be having performance map and that performance map, that's what will be used for the future development of blades.

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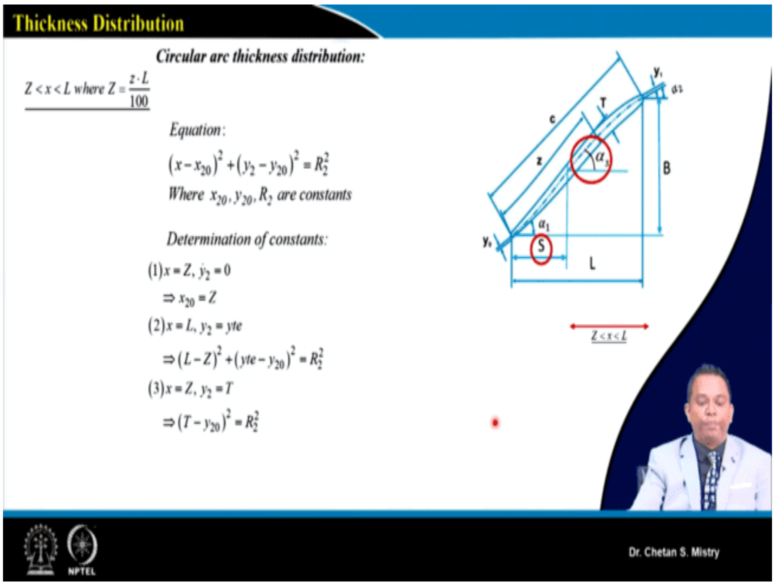

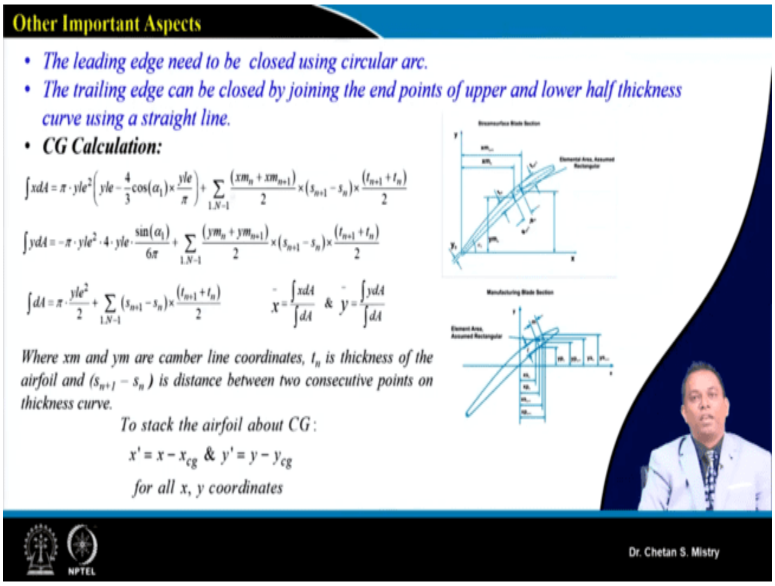

Now, towards the trailing edge also, you can use the same circular arc thickness distribution. Similarly, you will be getting your x and y coordinates. And, this is how you can have the distribution of thickness for transonic airfoils.

$$Z < x < L \; where \; Z = \frac{z \cdot L}{100}$$

$Equation:$

$$(x - x_{20})^2 + (y - y_{20})^2 = R_2^2$$

$$where \; x_{20}, y_{20}, R_2 \; are \; constants$$

$$Determination\ of\ constants:$$

$$(1)\ \ x = Z,\ \ \dot{y}_2 = 0$$

$$\Rightarrow x_{20} = Z$$

$$(2)\ \ x = L,\ \ y_2 = yte$$

$$\Rightarrow (L - Z)^2 + (yte - y_{20})^2 = R_2^2$$

$$(3)\ \ x = Z,\ \ y_2 = T$$

$$\Rightarrow (T - y_{20})^2 = R_2^2$$

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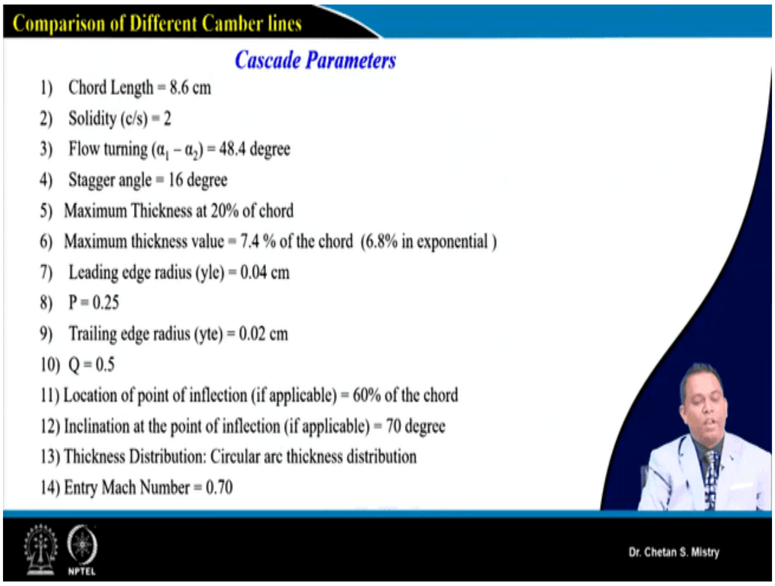

With this all understanding let us take the case for the development of say cascade. Now, let us take some of the cascade parameters. Say, let us we are assuming say chord length as say 8.6 cm, solidity as say 2, turning angle 48.4°, stagger angle 16°, maximum thickness at 20% of chord, maximum thickness value is 7.4% of the chord, leading edge radius we can say is 0.04 cm, value of P is 0.25. Trailing edge radius is 0.02 cm, Q value is say 0.5.

The location of the inflection point, that's what is at the 60% of the chord. The inclination at the point it is say inflection angle we can say is 70° as and where it is applicable. The thickness distribution, we can say, the circular arc thickness distribution, we will be considering our inlet Mach number to be 0.7. So, with this all input data, let us try to generate the airfoils what all we have learned.

$$Cascade\ Parameters$$

1) $Chord\ Length = 8.6\ cm$
2) $Solidity\ \left(\frac{c}{s}\right) = 2$
3) $Flow\ turning\ (\alpha_1 - \alpha_2) = 48.4\ degree$
4) $Stagger\ angle = 16\ degree$
5) $Maximum\ Thickness\ at\ 20\%\ of\ chord$
6) $Maximum\ thickness\ value = 7.4\%\ of\ the\ chord\ (6.8\%\ in\ exponential)$
7) $Leading\ edge\ radius\ (yle) = 0.04\ cm$
8) $P = 0.25$
9) $Trailing\ edge\ radius\ (yte) = 0.02\ cm$
10) $Q = 0.5$
11) $Location\ of\ point\ of\ inflection\ (if\ applicable) = 60\%\ of\ the\ chord$
12) $Inclination\ at\ the\ point\ of\ inflection\ (if\ applicable) = 70\ degree$
13) $Thickness\ Distribution: Circular\ arc\ thickness\ distribution$
14) $Entry\ Mach\ number = 0.7$

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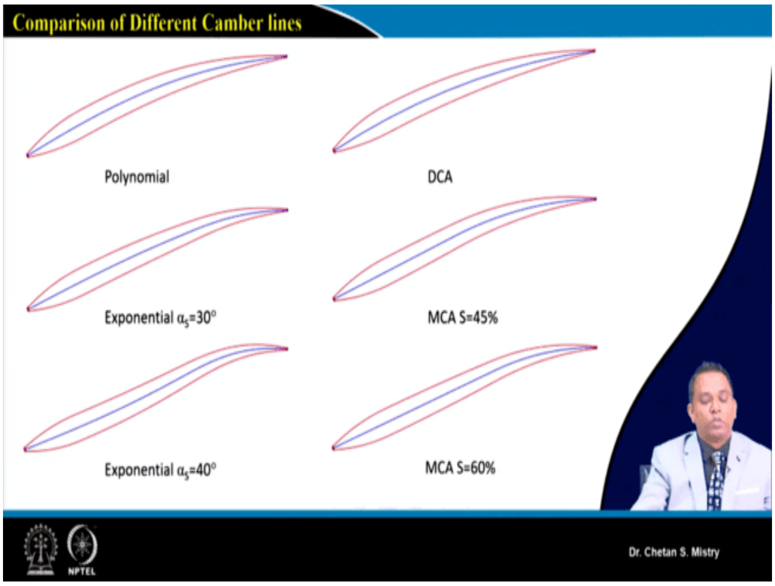

So, here in this case, if we will be putting all those input parameters, this is what is representing different kind of say camber lines with say thickness distribution. So here, this is what is representing my polynomial kind of camber line. This is what is representing say exponential camber line. This is what is a 70%. Location, we say, it is an inflection point, it is placed 70% and my α_s, that's what is angle we are putting 30°.

Suppose if you are putting that as a 40°, you can see the variation of the shape of this airfoil. You can clearly see the difference between these two. Same way, if you are considering say Double Circular Arc kind of configuration, this is what is Double Circular Arc configuration. Now when we are considering say Multiple Circular Arc, as we have discussed, this is what is my straight line. We can say my point that's what is placed at say 45% of my chord. So, this is what is representing my second circular camber line, okay.

And this is what is the same circular distribution of my thickness. This is what is when we are having, say, my inflection point, that's what has been placed at the 60% of my chord. So, we can see the clear difference, clear variation that's what is happening by changing the position of my inflection point for say Multiple Circular Arc camber line.

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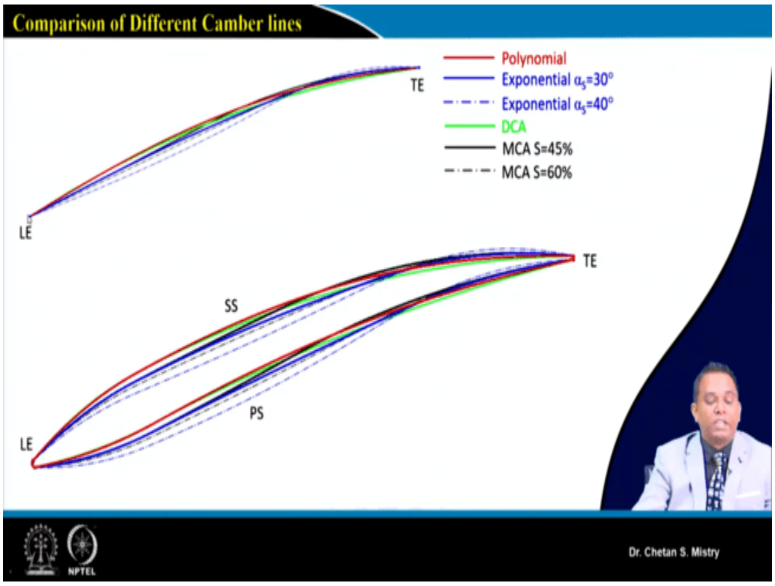

Now, here in this case, this is what is a representation of different camber lines. So, you can say, this red one, that's what is representing my polynomial camber line with all inputs, what we have discussed. We will be having say exponential. Here in this case for the exponential, when we are having our α_s, that's what is inflection angle, that's what is 30° and 40°. We will be having clear variation, that's what is happening upstream condition, just look at, even towards the downstream condition also.

So, the location of this inflection point, that's what is very important. We need to be very careful about that point. This is what is representing my, say, Double Circular Arc. Same way for, say, this black colour, that's what is representing Multiple Circular Arc and this is what is representing how my camber line for the same thickness distribution, how the blades or the airfoils they are being generated with.

Now, with this, we can say, we are able to generate the airfoils and after generating this airfoil, we will be managing our flow passage. And, by managing our flow passage, basically we are managing our shock structure. We are managing the placement of our normal shock and that is how we are managing our diffusion within the flow passage. And, that's what will be helping us for development of future more efficient airfoils.

So, this is what will give some feeling of special kind of application with different kind of camber lines and circular arc thickness distribution. Now, in next week, we will be discussing about how do we design those transonic compressors. Thank you very much for your kind attention! And I am sure, this is what all will be giving you some different kind of feeling with say, development of transonic airfoils and transonic compressors. Thank you. Thank you very much!