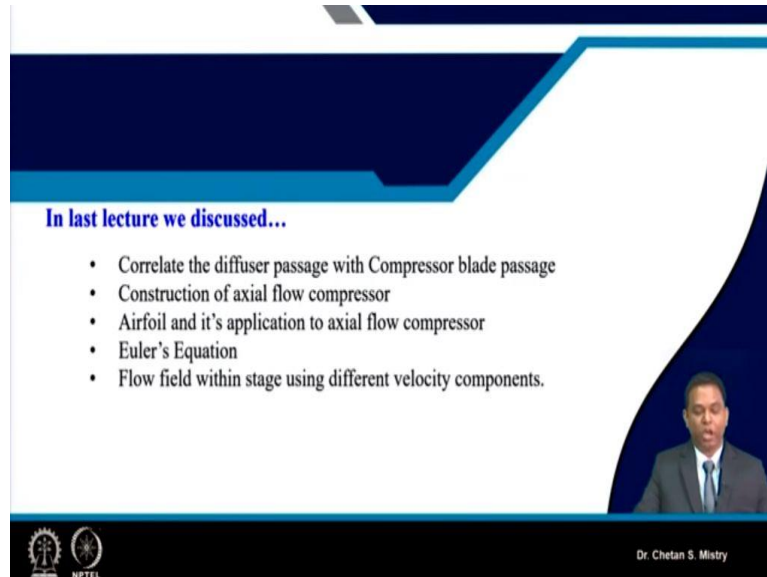


**Aerodynamic Design of Axial Flow Compressors and Fans**  
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**Lecture 5**  
**Introduction (Contd)**

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**In last lecture we discussed...**

- Correlate the diffuser passage with Compressor blade passage
- Construction of axial flow compressor
- Airfoil and it's application to axial flow compressor
- Euler's Equation
- Flow field within stage using different velocity components.

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Hello and welcome to lecture 5 for the introduction topic. In last lecture, we were discussing about the correlation of our diffusing passage with the compressor blade. And we have realized this compressor blade; it is basically making a passage, that's what is of diffuser size and diffuser shape. Then we were discussing about the construction of axial flow compressor.

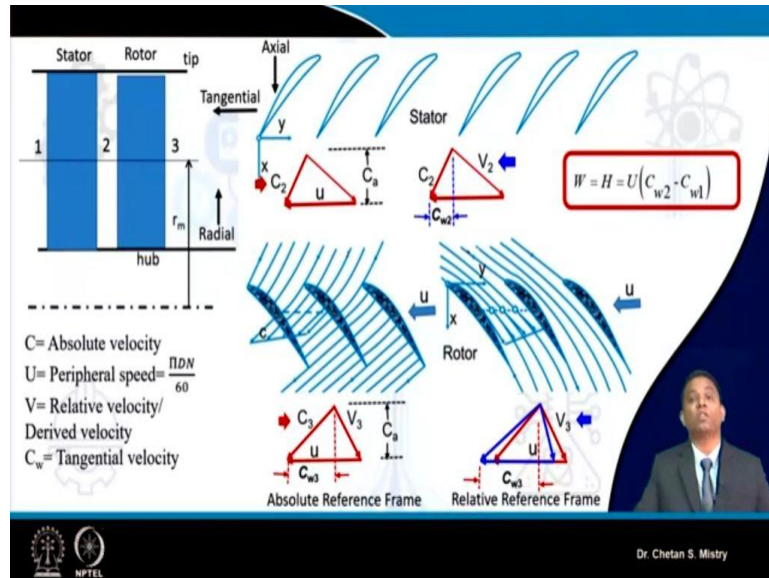
And we realized, this construction of axial flow compressor, we are having say... combination of passage, say... my blade passage, it is comprising of diffusing shape, and my flow passage or flow track we say; because our axial flow compressor it is comprising of number of stages, okay.

So, at the exit, we will be having our area to be lower compared to my entry area to the compressor stage. Then we were discussing about the concept of airfoil and that's what is applicable to our axial flow compressor. Many times people used to say what is the use of airfoil? And now you must realize, you know, like the lift force that's what is happening because of the streamlined body called airfoil; we are getting the benefit in sense of compressor.

Many times people used to bend the plate and that also they are considering as a blade. Yes, that also can be used as a blade for making of fan; but just realize one thing, what pressure rise we are expecting that's what is say large. For say curved blade, that pressure rise will not be

achieved as per our expectation, okay. Then, we were discussing about the Euler's equation and its application, okay.

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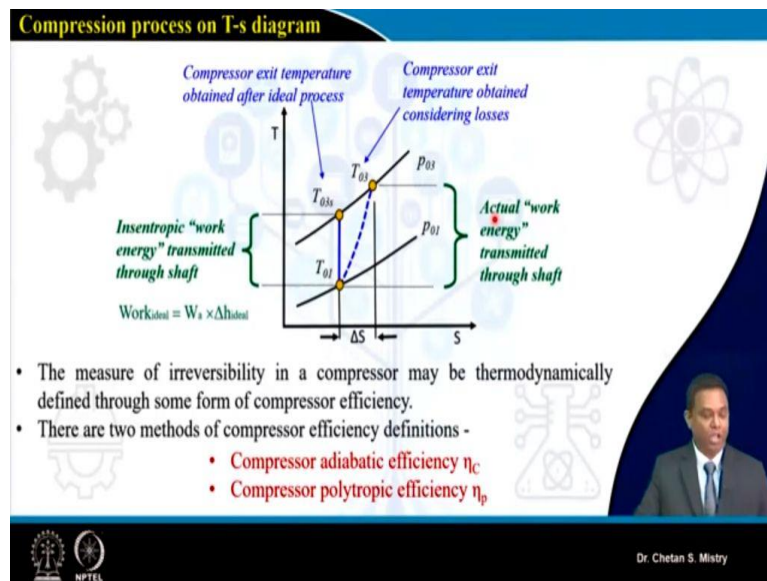


Then, finally, we have discussed; we have end up with our flow field. That is what we have discussed within our stage at particular station using our velocity triangle, okay. And now we are aware of different velocity components. Name a few saying like peripheral speed, that is what we are representing as say...  $U$ ; my absolute velocity that's what we are representing as say...  $C$ ; we are representing our relative velocity as say...  $V$ , okay. We are having axial flow velocity component; we can say it is a flow component; and that's what is represented by  $C_a$ . When we say we are having whirl component or tangential velocity component; that is what we are writing as say...  $C_w$ .

And in last lecture only, we were discussing about our fundamental equation for Euler's. And from that equation, if you look at, it says, in order to increase your compression work or power input that's what is required for compression; that's what is a function of my peripheral speed and my tangential velocity component, okay. So, it says if I am looking for increase of my work, then exit whirl component, that is what we are looking to be large. And that is what we have discussed here; say... if we are looking for that kind of configuration, the change of velocity, that's what is say my relative velocity, that's what is very important, okay. With this, let us try to move ahead.

Now, what all we know, this is what is representing my aerodynamic configuration. Now let us try to understand this compression process using thermodynamic aspect, okay.

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So, here if you look at my process for compression, that is what I am representing in sense of say... we are having T-s diagram, okay. So, if I consider I will be having my pressure; that's what is say... entry pressure  $P_{01}$ ; and my exit pressure I am writing as say...  $P_{03}$ , okay. Ideally, I can assume my compression process as say... isentropic process; and that's what is say my entry temperature I am writing as say...  $T_{01}$ , my exit temperature we are writing as say...  $T_{03s}$ . This is what is representing my ideal condition.

So, you can say, this is what is the temperature at the exit of my compressor; and if I am assuming this process to be ideal process, okay. So, compression process, ideally we are assuming as isentropic process in which my entropy that's what is remains constant. Now, in actual condition, if you look at my scenario that will be different, okay. Let me show that process; so, here if you look at, this is what is representing my actual compression process. And if you look at carefully here, this temperature  $T_{03}$ ; that's what is say... higher compared to what is my exit temperature if I'm considering my isentropic process.

And this is what we say... it is the actual temperature, that's what we are getting at the exit of my compressor. Now, the question may arise, what is the reason why we are getting this much temperature rise? So, this is nothing, but that's what we are representing in sense of change of entropy. Now, you know, this rise of pressure that's what is because of the losses which are incorporated with the process. So, in compression process, we will be having, say... aerodynamic losses, we will be having thermodynamic losses. And this losses in terms of thermodynamics we are talking as say... irreversibility of the process.

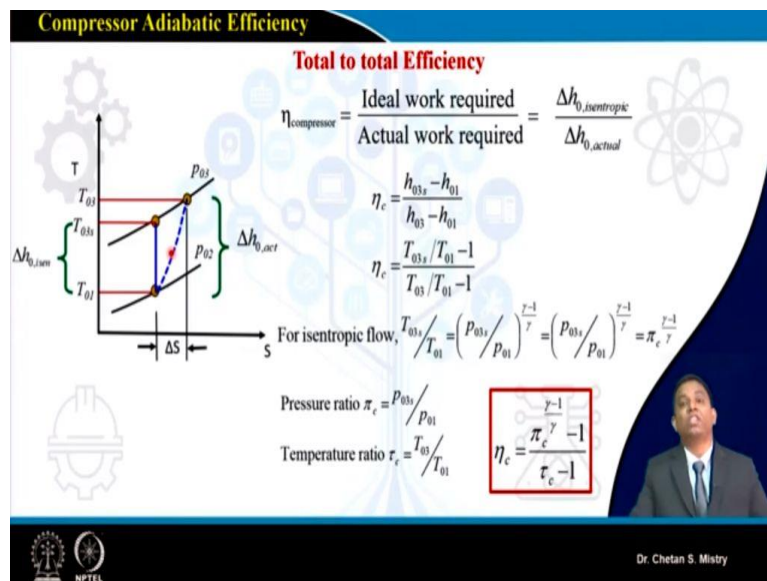
And this irreversibility that's what we are representing as entropy term. So, it says in actual process, I will be having irreversibility; that's what is increasing, so, my entropy, that's what is increasing. Now, here if you look at, if I want to represent the work done by my compression; that is what it is written in sense of say... ideal work done that is given by, say... mass flow rate into  $\Delta h$ .

$$Work_{ideal} = W_a \times \Delta h_{ideal}$$

Here, at this point, we can represent this is what is my actual work. Now, being an engineer, always we are looking for the terminology that's what is say efficiency. We engineers are very choosy, okay, and we are always differentiating; we are looking all devices in sense of efficiency.

So, for compressor, people, they have defined their compressor efficiency as say adiabatic efficiency. Suppose, if I am considering I will be having number of stages, for that, it is defined as say polytropic efficiency. So, let us try to look at what is the meaning of this efficiency, okay.

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So, if I am representing my compression process again here, you can this what is my representation for say... isentropic temperature rise; this is what is my actual temperature rise; or we can say... this is as say... my enthalpy rise, okay. Now, what we know, efficiency what we are writing as say... my ideal work divided by actual work required; do not get confused. Many times people used to say, if I will be taking this as a reciprocal, then my efficiency will be coming more than 100 percent; and that is the reason why we are putting this.

Remember one thing, this compressor device that is nothing but, you know, it is a power consuming or work consuming device. And for that the reason why we are representing our efficiency in sense of say... ideal work done divided by actual work done. This is what we can represent in sense of my enthalpy. I say,

$$\eta_c = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}}$$

This is what if I am assuming my  $C_p$  to be constant, that what we can represent in sense of my temperature ratio, okay.

$$\eta_c = \frac{T_{03s}/T_{01} - 1}{T_{03}/T_{01} - 1}$$

This temperature ratio, here, this process of compression as we have discussed, it is an isentropic process; so, we can use our isentropic relation in order to calculate what will be my temperature ratio.

So, that is what we can correlate using my pressure ratio. So here, this is what  $P_{03s}/P_{01}$ .

$$\text{For isentropic flow, } T_{03s}/T_{01} = (p_{03s}/p_{01})^{\frac{\gamma-1}{\gamma}} = (p_{03}/p_{01})^{\frac{\gamma-1}{\gamma}} = \pi_c^{\frac{\gamma-1}{\gamma}}$$

This is nothing but that is what I am writing as say my pressure ratio, okay. So, this is what is say my pressure ratio. If I am putting this together, it says my adiabatic efficiency.

$$\text{Pressure ratio, } \pi_c = \frac{p_{03s}}{p_{01}}$$

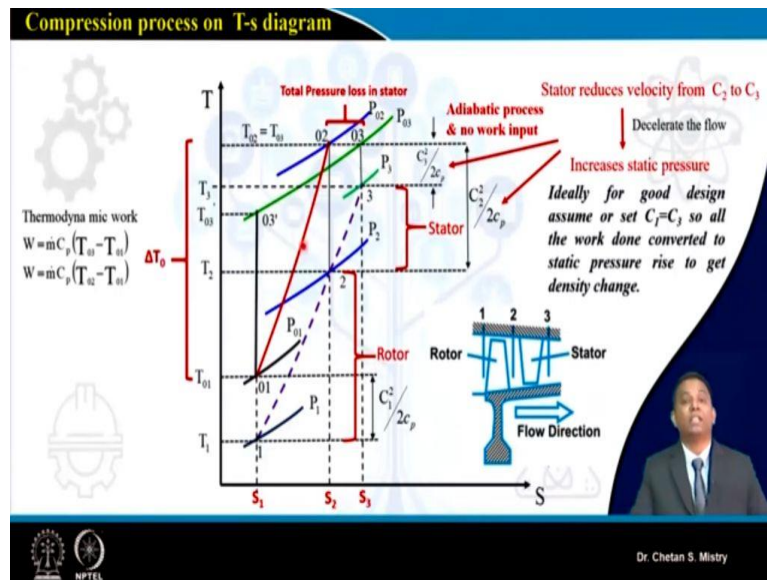
$$\text{Temperature ratio, } \tau_c = \frac{T_{03}}{T_{01}}$$

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1}$$

People sometimes used to say that as say... total to total efficiency for the compressor. It is a function of my pressure ratio and temperature ratio. So, this is what is a correlation that is what we are using for defining our adiabatic efficiency, okay. So, this is what is representing the adiabatic efficiency of our compressor. As we go along this term that will be coming repeatedly and that is the reason why we need to be fundamentally clear from the beginning.

That is the reason why we are having this definition, okay. Now, here what we are discussing, that is what you will say, this is what is say... my pressure ratio. This pressure ratio I am talking about the overall pressure ratio. Let us try to understand what is happening when we say we are having stage.

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So here, say... this is what we have discussed; I say, I am having my rotor and stator, okay. My entry condition I say, it is a station 1; exit condition from the rotor is station 2; and exit of my stator that's what is say... my station 3. If I am considering, I will be having my air that will be sucked from the atmosphere that will be having, say... pressure  $P_1$ ; it will be having my temperature, say...  $T_1$ , okay. Now, what is happening? My flow, that's what will be get sucked inside my compressor rotor specially, okay. When it is getting inside, what will happen? It will be achieving some kind of kinetic energy.

That is what I say... it is  $\frac{C_1^2}{2C_p}$ ; so you can say, this is what is say...  $T_1 + \frac{C_1^2}{2C_p}$ . That's what is representing my total enthalpy or my total temperature; and somewhere here, I am putting this pressure as say...  $P_{01}$ , okay. So, this is nothing but that's what is called total pressure. So, my flow in compressor, it will be entering with total pressure  $P_{01}$ . So, as we go along, we need to remember we are more interested in total property compared to our static property, okay; because my fluid that's what is flowing through this machine; it is a continuous fluid, okay.

So, our interest, that's what is now on, say... total property. So, I will say, my pressure at the entry of my compressor, that's what is say...  $P_{01}$ . Now, what is happening? During my compression, I will be having the rise of pressure. So, you can say... my pressure rising from

$P_{01}$  to  $P_{02}$ , okay. So, at that station I will be writing I will be having my temperature say...  $T_{01}$ ; or I can say, my enthalpy as say...  $h_{02}$ , okay. So, this is what is representing my total enthalpy or total temperature  $T_{02}$ . Now, what is happening? Now, I will be having, say... this process that's what is happening in my stator; say rotor... that's what is we can say... it is doing my compression work.

There is no work that's what has been done by my stator, okay. So, we can assume the process to the stator as say adiabatic process, okay. So, we can say, my  $T_{02}$  and  $T_{03}$ , they both will be same; so my total enthalpy that will remain same. But, here if you look at, I will be having some losses that's what is happening in my stator. These losses are because of frictional losses, it is because of my aerodynamic losses; because you can understand my stator blade it is made up of, say... solid body. So, you can say, there will be drop off pressure from  $P_{02}$  to  $P_{03}$ . So, this is what is representing my total pressure loss, okay.

So, now you can say at the exit, I will be getting my pressure as say...  $P_{03}$ . Now, in line to what we have discussed, say... if I will be considering my absolute velocity, that's what is coming out for my stage; that is what I am putting it here as, say...  $\frac{C_3^2}{2C_p}$ . Based on our fundamental, I can locate my static pressure at the exit of my stage, okay. Similarly, if I will be talking about say... exit of my rotor, I will be having my absolute velocity; that is what it say...  $C_2^2$ . So, I can locate my point say... my point 2; this is what is representing my pressure  $P_2$ . So, just understand I am having say...  $P_1$ ,  $P_2$  and  $P_3$ .

So, if I will be representing this as a process, then I can say, from 1 to 2, I will be having a rise of pressure; that's what is my static pressure rise, okay. And this is what is happening inside my rotor. I will be having the rise of pressure from station 2 to 3; that's what is happening in my stator, okay. Now, you know, if we are talking as I told, we are more interested in total property. So, basically if you look at carefully; here, this is what is say... station 01 to 03', that's what is representing my isentropic process. And you can understand if I will be joining say... 01 to 03, that's what is representing my actual process. That's what is happening in my stage, okay.

So, if I say here, if you look at carefully, this absolute velocity components  $\frac{C_2^2}{2C_p}$ ; or you can say...  $C_3$  and  $C_2$ , if you compare, what is happening when it is flowing through my stator? This absolute velocity it is going to be reduced, okay. And that's what is representing the rise of my pressure. Now, in many designs people used to prefer this inlet absolute velocity and exit

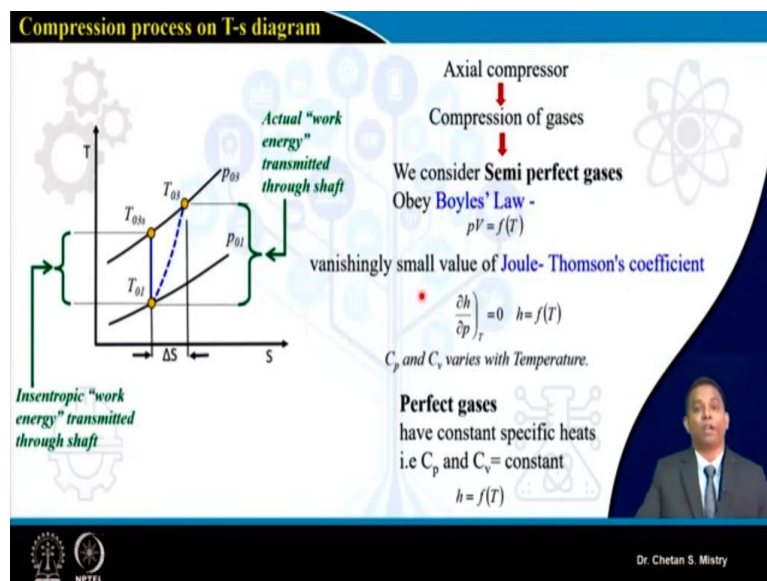


absolute velocity to be same. We will be discussing about this aspect very soon. But, what they say... this is what is we are expecting our work done to convert the static pressure rise to get the density change.

And that is what will be giving me my expected absolute velocity  $C_1$  and  $C_3$  to be same, okay! Now, here in this case, if I say, I want to define my compression work, then thermodynamically, that's what is being represented in sense of rise of total temperature. So, this total temperature I can write down say...  $\Delta T_0$ , okay. So, let me put here, this is what it says my thermodynamic work. So, here if you look at, my thermodynamic work, that's what is represented as say...  $\dot{m}C_p\Delta T_0$ , okay. Now, you know, many times people they are getting confused in sense of  $T_{03}$  and  $T_{02}$ . But, remember what we have assumed; our process, that's what is happening in my stator and that's what is say, the adiabatic process, okay.

And that is the reason why I am writing that as a  $\dot{m}C_p(T_{02} - T_{01})$ . So, this is what is all about, what is happening in my single stage of axial flow compressor.

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Now, you know, like this is what is a representation as we have discussed that's what is talking about, say... my compression process, that's what is happening in my axial flow compressor. Now, this axial flow compressor, that's what is mainly been used for compressing of this gas, okay. Now, when we say air we are considering as a working fluid, we can take that as say... Semi perfect gas, okay. So, when I say it is a Semi perfect gas, it will be followed your Boyles' Law; that means your  $pV$  that's what is a function of my temperature, or product of pressure and volume, that's what is a function of my temperature.



At the same time, you will be having vanishingly small Joule-Thompson's coefficient; it says  $\left(\frac{\partial h}{\partial p}\right)_T = 0$ . It says my enthalpy also is a function of my temperature, okay. But, for this configuration, we can say, our specific heat what we say...  $C_p$  and  $C_v$ , that's what is varying with the temperature, okay. Now, when we are considering our ideal condition, for that we are assuming our working fluid as a perfect gas. And when that's what is your case, you can say, you will be having your  $C_p$  and  $C_v$  to be constant. You can my enthalpy it is a function of my temperature, okay. So, you know, like this is what is very fundamental and that's what is required.

So, you know, when I have started with the class, that time we have discussed, we are having this axial flow compressors, they are having various kinds of applications. One, that's what is say... for your land-based power plant; second, that's what is for your aero application. And we have discussed some of the say... industrial application, where my working fluid, it is other than air; it may be gas, okay. So, we have discussed about the supercritical CO<sub>2</sub> cycle or we have discussed about supercritical CO<sub>2</sub> power plant.

So, you can understand under that condition, you need to have your fundamental understanding of this all terminologies.

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**Altered Gas Properties**

When gases other than air are considered...  
 The change of molecular weight...  
 And therefore variation of Gas constant as well as specific heat to be taken into consideration

Gas	Molecular weight	Gas constant J/kgK	Cp J/kgK	$\gamma$	Speed of sound At 288K
Air	28.97	287	1005	1.4	340
Argon	39.94	208	518	1.67	316
CO <sub>2</sub>	44.01	189	652	1.29	275
Helium	4.00	2079	5277	1.66	994
Hydrogen	2.02	4116	14155	1.41	1293
Methane	16.04	518	2368	1.28	436
Freon-11	137.4	60.5	666	1.1	138

Universal Gas Constant  $R=8314 \text{ J/kg mole K}$

When operating near saturation, as in steam and refrigeration plant, gases tend to deviate significantly from the Perfect gas law... The parameters other than molecular weight varies considerably...  
 It demands for "Mollier diagram" to bring the change of state.



Dr. Chetan S. Mistry

Let us move, this is what we know. When we say, when gases are other than say... air, what we are considering, then there will be change in, say... your molecular weight. And that is the reason why you will be having variation in your gas constant as well as you will be having variation in your specific heat, okay. So, here if you look at, this is what is a table, that's what

is representing different kinds of gases; that is what people they are using these days. So, specially say... these compressors, they are being used for these different kinds of gases; we will see what all are the importance.

So here, this is what is representing my gases, my molecular weight, gas constant, my specific heat, my specific heat ratio, and speed of sound. That is what we can it is  $\sqrt{\gamma RT}$ ; but that temperature, it is say... 288 K. Now, here in this case, if you compare the value of my specific heat for Freon, that's what is say... it is around 1.1. And when we are talking about Argon, that is what is having specific heat of 1.67. Same way, if I am comparing my say, speed of the sound, that's what is say... around 138 for my Freon; and if I look it at, say... my Hydrogen, that is what will be having my speed of sound to be very large.

So, you know, like we are having the variation of this property, that's what is a drastic variation in the property, okay. Now, you know like, if I compare these two, say... air as well as for Freon; and if we look at carefully, my speed of the sound, that's what is almost say three times kind of situation. Now, let me tell you, what all is the use of this? People they are doing different kinds of testing for different applications, okay. Now, suppose say, I want to test my compressor at high Mach number condition, okay. So, if I will be using air as my working fluid, maybe I need to rotate my wheel at very high speed, okay. And it may be having some constraints in sense of, you know, like, mechanical constraints.

So, in order to avoid that kind of situation, many times people, they are using, say, Freon-11 as a gas for the initial testing, okay. So same kind of situation, what we have observed for our wind tunnel testing, it is a kind of same, okay. Same way if you look at, say... people they are using helium gas; so, these days people they are moving towards say... application of your cryogenic fluid. For that application for cooling application, we are looking for helium gas, where you are looking for, say... different kind of say... compressors. So, helium gas compressor, that's what is very common in application.

Now, in order to test those kinds of say... helium compressors, what they are doing? They are rotating or they are using, say... axial flow compressor using air as a working fluid, okay. That is what will be giving the benefit and that will be rotating at comparatively low speed as compared to when you are rotating that wheel using your helium gas, okay. So, just understand, like based on the understanding of these properties, people they are making their testing. So, testing facility development, that is also a one kind of skill, and that's what is demand for.

So, in Europe and US, if you look at many universities, they are having their own testing facility, which are being sponsored by different companies, okay. And that initial testing of their compressor or their blades, that's what is happening in their university level testing, okay. Now, as I was saying, like when we are having our property to be different; under that condition, we need to go with different property test, okay. And for that you need to use your Mollier diagram for different working fluids.

So be careful, you know, some assumptions what we are making that may lead to put you in trouble, okay. So, when I say, design of axial flow compressor, you will say... Sir, you are teaching axial flow compressor design for air. You have not discussed anything about the working fluid, but you must realize, this is what is a fundamental what you need to learn; and later on, you need to apply it for your working fluid or your kind of working fluid, what you are looking for, okay!

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**Polytropic or Small Stage Efficiency**

$$\eta_c = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}}$$

$$\eta_c = \frac{T_{03s}/T_{01} - 1}{T_{03}/T_{01} - 1}$$

- In Multistage compressors, the designer tries to obtain same efficiency for each stage.
- The **small stage or polytropic efficiency** is defined as the isentropic efficiency of an elemental stage (infinitesimal) of the compressor, which remains constant throughout the whole process of compression.

Dr. Chetan S. Ministry

Now, say... as I was discussing, this is what is representing, my say... your overall pressure ratio for the compressors stage. If I say, my compressor it is comprising of number of stages. If I say for axial flow compressor it is made up of say... maybe 10 stages, okay. So, this is what is representing overall pressure rise or you can say this is what is representing overall compression process. What we learn is suppose say... I want to do my compression from say... pressure  $P_{02}$  to  $P_{03}$ ; I will go with number of stages. As we have discussed earlier, say... my axial flow compressor it is very sensitive device; and it is having aerodynamic limitation.

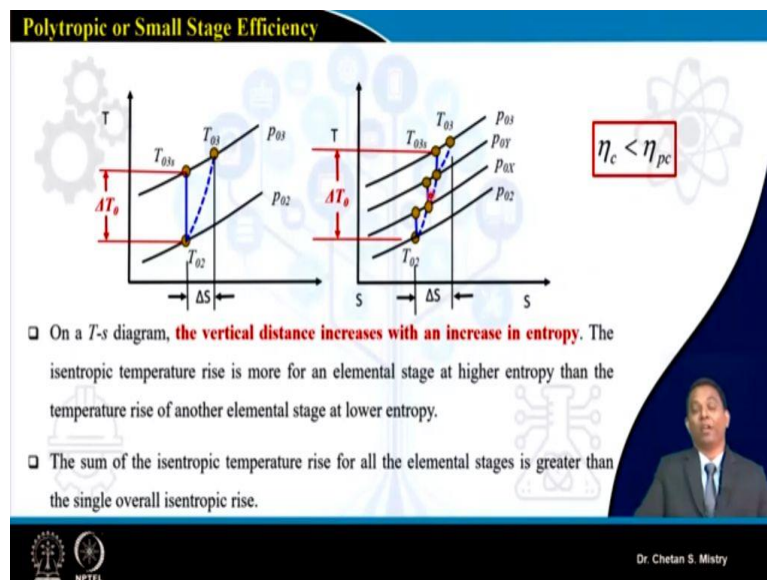
And that is the reason why we are going with number of stages; so that is what we are doing. Suppose if I consider, I am making my compression process by using three number of stages,

different number of stages. Let me put this process what we have learned from our T-s diagram. So, for my first stage you can say, this is what is representing my isentropic compression, this is what is representing my actual compression. Now, this temperature that's what is coming out from my first stage that is what will be entering into my second stage.

That is what will be going for say... isentropic compressor. In actual I can say, I will be having my temperature to be large. Similar to that if will be putting, say... for my third stage, I will be having this temperature rise that's what is say...  $T_{03s}$  and this is what is representing my  $T_{03}$ . Now, remember one thing, you know, like these compressors they are being designed for particular efficiency; and that efficiency that's what we are defining as a polytropic efficiency; that's what is per stage efficiency. Here this is what is representing overall efficiency; here, we are looking for say... polytropic efficiency.

So, designers they are preferring when they are doing their design for say... LP stage or say... for HP stage, they people, they are trying to put that polytropic efficiency to be constant, okay. And that is how they are doing their compression work. So, here if you look at, you can clearly compare this  $\Delta S$ . So, here for this case, I am having my  $\Delta S$  that's what is coming to be large. Let me come to the point.

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What it says? Here if you consider, say... vertical distance, if I will be putting; I will be adding together for my isentropic process. It says from my  $T_{02}$  to  $T_{03s}$ , my  $\Delta T_0$  that's what is coming to the large, okay. Now, here for my, say... actual compression process for overall stage, if I will be considering my  $\Delta T_0$ , that's what is coming to be lower, okay. Now, what it says? Here in this case, it says my elemental rise of temperature, that's what is giving me my  $\Delta T_0$  to be

large. What is happening because of that? Suppose say, I am doing my calculation for overall efficiency based on these temperatures. And if I will be measuring or by calculating my efficiency based on these temperatures, it clearly says I will be getting my overall efficiency, okay, that's what is lower than that of my polytropic efficiency, okay.

So, you can say, my polytropic efficiency, that's what is coming to be large and that's what is large efficiency; and my adiabatic efficiency, that's what is say, lower efficiency. So, you know like, when people, they are bidding for supplying of their engine, or say... when they are supplying their compressor, they used to define some number. Say... as a customer, you will be asking what is my, say... what is your compression ratio and what is your compressor efficiency? Remember one thing, what efficiency they are defining. If they are saying say... efficiency is 90 percent, just realize and ask straightway which efficiency are you talking of?

Is it polytropic efficiency or adiabatic efficiency, okay? So, you know, like this, will give you misguiding sometimes.

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**Compressor Polytropic Efficiency**

Adiabatic efficiency take care of finite steps ( $\Delta h_0$ ).

$$\eta_c = \frac{\Delta h_{0s}}{\Delta h_0}$$

It takes infinitesimal steps ( $dh_0$ ).

$$\eta_{pc} = \frac{dh_{0s}}{dh_0}$$

From Second law  $T_0 ds = dh_0 - \frac{dp_0}{\rho_0}$

$ds = 0$  and  $dh_0 = dh_{0s}$

$$dh_{0s} = \frac{dp_0}{\rho_0}$$

On Integration,

$$\frac{dp_0}{p_0} = \frac{\gamma}{\gamma-1} \eta_{pc} \frac{dT_0}{T_0}$$

$$\frac{p_{03}}{p_{02}} = \pi_c = \left( \frac{T_{03}}{T_{02}} \right)^{\frac{\gamma}{\gamma-1} \eta_{pc}} = (\tau_c)^{\frac{\gamma \eta_{pc}}{\gamma-1}}$$

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1}$$

The slide also features a T-s diagram with isentropes and a small video inset of Dr. Chetan S. Mistry.

Now, let me put this in sense of say... calculation of my polytropic efficiency. So, here if you look at, my efficiency, I am writing in sense of

$$\eta_c = \frac{\Delta h_{0s}}{\Delta h_0}$$

If I am talking about small elemental part, I am putting that as say

$$\eta_{pc} = \frac{dh_{0s}}{dh_0}$$

We know from our fundamentals of second law of thermodynamics, we can write down

$$T_0 ds = dh_0 - \frac{dp_0}{\rho_0}$$

We know this is what is my isentropic process; for that I am putting

$$\text{For isentropic process, } ds = 0 \text{ and } dh_0 = dh_{0s}$$

It says I am calculating my isentropic enthalpy rise. That's what is given by

$$dh_{0s} = \frac{dp_0}{\rho_0}$$

Now, let me put in sense of definition of my polytropic efficiency. Here if you look at, so this is what we are writing, okay. I am putting that as say...  $dh_{0s}/dh_0$ ; this I am putting as say...  $dp_0/\rho_0$ . This  $\rho$  I can represent in sense of  $p$  upon  $RT$ . Let me put it here, this  $h_0$  I am writing in sense of  $c_p dT_0$ , okay.

$$\eta_{pc} = \frac{dh_{0s}}{dh_0} = \frac{\frac{dp_0}{\rho_0}}{dh_0} = \frac{\frac{dp_0}{p_0}}{\frac{dh_0}{RT_0}} = \frac{\frac{dp_0}{p_0}}{\frac{c_p dT_0}{RT_0}}$$

$$\frac{dP_0}{P_0} = \frac{\gamma}{\gamma - 1} \eta_{pc} \frac{dT_0}{T_0}$$

If I will be putting together and if I will be making my integration; it says my pressure ratio per stage or for a particular elemental stage, that's what is represented in sense of my temperature ratio. And this is what in sense of my polytropic efficiency.

$$\frac{p_{03}}{p_{02}} = \pi_c = \left( \frac{T_{03}}{T_{02}} \right)^{\frac{\gamma \times \eta_{pc}}{\gamma - 1}} = (\tau_c)^{\frac{\gamma \times \eta_{pc}}{\gamma - 1}}$$

So, my overall efficiency, that's what is a function of my, you know, pressure ratio, what pressure ratio we are talking of and this is what is a function of what is my temperature ratio, okay.

$$\eta_c = \frac{\pi_c^{\frac{\gamma - 1}{\gamma}} - 1}{\tau_c - 1}$$

So, let us try to understand let what is the meaning and what is the physical interpretation of this equation?



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### Compressor Polytropic Efficiency

- In a cycle analysis, we usually assume the Polytropic efficiency as the figure of merit for a compressor (and turbine) and then we can maintain *Polytropic efficiency* as constant in our engine off-design analysis.
- Compressor adiabatic efficiency for a finite size compressor is lower than the efficiency of a small individual stage compressor.
- Adding additional stage of compressor is considered as **burden** as it's power consuming device and additional stage increases losses.
- Typical values for the polytropic efficiency in modern compressors are in the range 88-92%.**
- If the machine Mach number at the exit from each stage is same throughout the compressor, then the "Static-to-Static" efficiency of each stage is same as "Total-to-Total efficiency" of the stage.
- For aircraft engines it is conventional to use overall efficiency in terms of stagnation states (Total-to-Total).

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\pi_c^{\frac{\gamma-1}{\gamma \eta_{pc}}} - 1}$$

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### Compressor Polytropic Efficiency

Adiabatic efficiency take care of finite steps ( $\Delta h_0$ ).

$$\eta_c = \frac{\Delta h_{0s}}{\Delta h_0}$$

It takes infinitesimal steps ( $dh_0$ ).

$$\eta_{pc} = \frac{dh_{0s}}{dh_0}$$

From Second law  $T_0 ds = dh_0 - \frac{dp_0}{\rho_0}$

$ds = 0$  and  $dh_0 = dh_{0s}$

$$dh_{0s} = \frac{dp_0}{\rho_0}$$

$$\eta_{pc} = \frac{dh_{0s}}{dh_0} = \frac{dp_0}{\rho_0 dh_0} = \frac{dp_0}{dh_0} \frac{1}{\rho_0}$$

$$\frac{dp_0}{P_0} = \frac{\gamma}{\gamma-1} \eta_{pc} \frac{dT_0}{T_0}$$

On Integration,

$$\frac{P_{03}}{P_{02}} = \pi_c = \left( \frac{T_{03}}{T_{02}} \right)^{\frac{\gamma \eta_{pc}}{\gamma-1}} = (\tau_c)^{\frac{\gamma \eta_{pc}}{\gamma-1}}$$

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1}$$

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Let me put it here. So, here if you look at, this is what is representing my compression ratio and this is what is representing what is my adiabatic efficiency. So, here it says my polytropic efficiency it is, say 0.9, and here I am assuming my polytropic efficiency to be 0.92, okay. So, let us try to understand, suppose if I consider, I am looking for my pressure ratio of 4. So, for that pressure ratio, if my polytropic efficiency or per stage efficiency is 90%, that's what is giving me my adiabatic efficiency as 88%, okay. So, remember, 90% and 85% that's what is a big difference, okay. Same way here if you look at, if I am taking my polytropic efficiency as say 92%, it says I am able to get my adiabatic efficiency to be say 91%.

So, just understand, what it says, if you are able to design your compressor stage in such a way that you will be having your polytropic efficiency to be large, okay; that means



aerodynamically you are designing your compressor in such a way that the losses are less, then you can say your polytropic efficiency is coming to be high. If that is what is your case, you can say your adiabatic efficiency you are getting to be large. So, what it indicates? It says when we are considering, then if we are able to increase, let me go back; here, it says if I will be dividing my compression process with more number of compressor stages, then we can say we are able to achieve this temperature rise to be small.

And that is what will be giving me very high temperature ratio; and that is what will be giving me higher efficiency. So, it says if you are adding the number of stages to your compressor, then you are able to increase your efficiency. Now, when I am adding my number of stages to, say... my axial flow compressor for engine; that is what will be increasing my length of the engine. Suppose if I am targeting, say... we are having our aero engine, that's what is in target, where if I will be increasing my number of stages; why I am increasing my number of stages? Because I am looking for my polytropic efficiency to be higher; because I am looking for my compressor efficiency to be higher.

Then what will happen? My number of stages that will be increasing. It will be increasing the length of my engine. Again, that is what will be increasing my drag. And when we are talking about say... commercial aircraft, where my efficiency is of major concern; and our fuel economy that's what is a major concern. So, it says you are restricted with the increase of number of stages. Even when we are talking about say... military application; for that military application what happens? My length of the engine will be increasing; we cannot accommodate that engine when within the fuselage, that is what has been provided for military applications.

Now, you say, sir, this is what is benefit for my land-based power plant. Yes! So, for land-based power plant, we are not having any restriction with the length of my engine, gas turbine engine. You can say... you can increase more number of stages that is what people, they are doing, okay. They are putting more number of stages for land based power plant; so that, they are able to achieve higher efficiency. They are able to achieve what they are looking for in sense of performance. But, you know, like present day, even for land-based power plant people they are moving towards the concept what they already have used for the Aero engine.

They are also moving towards the compactness, okay. So, understand one thing, so this is what is a first hint what you need to do in sense of selection of number of stages, okay. It says typical value of this polytropic efficiency as on today for the modern compressors, that's what is ranging from 88% to 92%. You can understand this number when we say 92%, it is a big

number. We are reaching towards our extreme now, okay. It also says in order to avoid the confusion in sense of understanding; it says my Mach number at the exit of each stage that's what is remained constant throughout.

And as the reason in place of using static-to-static efficiency, we are putting that as say total-to-total efficiency, okay. And for, you know, aircraft engine, conventionally we are writing overall efficiency as say total-to-total efficiency.

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**Polytropic Index 'n'**

Polytropic index 'n' is defined such that

$$\frac{\gamma-1}{\gamma} \frac{1}{\eta_p} = \frac{n-1}{n} \quad \text{or} \quad \eta_p = \frac{\gamma-1}{\gamma} \times \frac{n}{n-1}$$

From consideration of small stage efficiency

$$\frac{T_{02}}{T_{01}} = \left( \frac{P_{02}}{P_{01}} \right)^{(\gamma-1)/\eta_p \gamma}$$

For ideal compression process

$$\frac{T'_{02}}{T_{01}} = \left( \frac{P_{02}}{P_{01}} \right)^{(\gamma-1)/\gamma}$$

Stage polytropic efficiency can now be written as

$$\eta_p = \frac{\gamma-1}{\gamma} \times \frac{n}{n-1} = \frac{\gamma-1}{\gamma} \frac{\ln(P_{02}/P_{01})}{\ln(T'_{02}/T_{01})}$$

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Now, you know, we have discussed working of different fluids, okay. Now, when I say, I am using different kinds of fluids, I will be having my polytropic index, that is what will be different, okay. Now, I want to represent different compression process on P-v diagram. So, let me put those processes between pressure P<sub>1</sub> and P<sub>2</sub>. Now, I can do my compression using different thermodynamic process. So, you have fundamentally studied in your thermodynamics, that if you are looking for the compression, you have different processes. One of the process that's what we have discussed, it says we can use isentropic process.

We have discussed, we can use polytropic process or polytropic compression process. We can go with say... isothermal process, okay. And for all these, 'n' you can represent that's what is say... for isentropic process, this polytropic index I am putting as say gamma γ; and by polytropic process, it will be within this range. And for isothermal process, we are considering my product of pressure and volume that's what is constant. Now, what it says from our fundamentals, my work required for compression using this isothermal process that's what is say low; and that's the reason why for many industrial application, people they are going with isothermal process.

Now, the question is... how do we do this compression, this process or compression process, say, isothermally? So, reciprocating compressor what we are using, that's what is working nearly on this quasi-static way. And that is what we can assume as say... isothermal process, okay. But, remember here, we are talking about the rotating device which are rotating at very high speed compared to your reciprocating compressor; and that is the reason to achieve this kind of isothermal compression process using this rotating device, that's what is not possible.

We are moving towards say... polytropic index, okay. So, if my working fluid that's what is somewhat different, I can represent my polytropic efficiency in this form.

$$\frac{\gamma - 1}{\gamma} \frac{1}{\eta_p} = \frac{n - 1}{n}$$

$$\eta_p = \frac{\gamma - 1}{\gamma} \times \frac{n}{n - 1}$$

It says  $\frac{\gamma-1}{\gamma} \times \frac{n}{n-1}$ ; so, we can represent that in sense of my pressure ratio.

*From consideration of small stage efficiency,*

$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\eta_p \gamma}}$$

*For ideal compression process*

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

And finally, my stage polytropic efficiency, we are representing as say...

$$\eta_p = \frac{\gamma - 1}{\gamma} \times \frac{n}{n - 1} = \frac{\gamma - 1}{\gamma} \frac{\ln(p_{02}/p_{01})}{\ln(T_{02}/T_{01})}$$

We can say if I am having my process, that's what is say... isentropic process for which my n I am putting as say  $\gamma$ , that is what will be coming in sense, okay.

So, this is what is all we are looking for our understanding, okay. So, you know what all we have discussed today that's what is we are talking our compression process as say thermodynamic process. And we have represented our all stages at the entry and exit of our stage, or entry and exit of our number of stages in overall sense, we are representing on T-s

diagram. Then, we have understood what is happening in my rotor and stator, okay, in sense of my T-s diagram. Then, we have discussed about what we understand for adiabatic efficiency and what we mean by our polytropic efficiency.

Later on, we have realized your polytropic efficiency, it has gained its own importance, and that's what is giving you idea by increasing your number of stages for the compressor, that's what will be giving you higher overall efficiency, higher adiabatic efficiency. But, we are considering adding the stages in your axial flow compressor, that's what is a burden, okay. And people they are avoiding that kind of situation, okay. So, in next lecture, we will be doing with one of the tutorial, that's what will be giving you idea how exactly you need to use this formula for the application.

Since this is what is a design course, you need to have some understanding from the fundamental; straightway we cannot go with the design aspects, okay. So, we will be taking some calculations, some tutorials; I will be giving you assignment also. That is what will be giving you the confidence in sense of use of what all derivations we are discussing here. Thank you. Thank you very much!