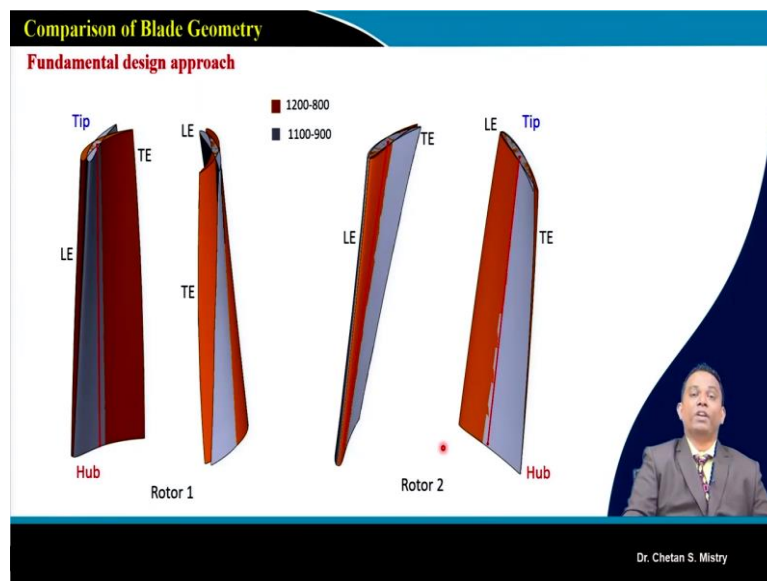


Aerodynamic Design of Axial Flow Compressor and Fans
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Lecture 48
Design of Low Speed Contra Rotating Fan (Contd.)

Hello, and welcome to lecture 48, we are discussing about Design of Low Speed Contra Rotating Fan.

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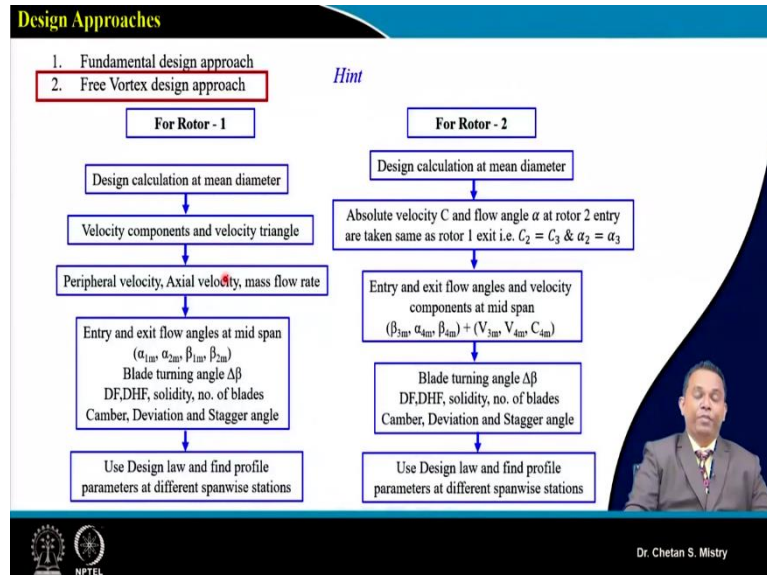
So, in last lecture we were discussing about Design of Low Speed Contra Rotating Fan for different aerodynamic loading. So, if we look at here, this is what is representing when we are considering our aerodynamic loading say 1200 for rotor 1 and 800 Pa that's what is for rotor 2. We also have discussed about when we are changing our aerodynamic loading, that's what is 1100 Pa for rotor 1 and 900 Pa for rotor 2.

Here in this case, if we look at by changing our aerodynamic loading for rotor 1 say 1200 and 1100 Pa, we can clearly see how my shape of the blade, that's what is changing all the way from hub towards say our tip region, okay, and since we are doing our design for tip loaded configuration in order to have say reasonable degree of reaction near the Hub region, we are having this curvature that's what we can see. Say for rotor 1, for 1200 Pa we will be having say larger turning of flow near the tip region.

If you are considering for say a rotor 2, not much variation that's what we can see but at the same time because of change of aerodynamic loading near the tip region for rotor 1 accordingly

the tip loading that's what will be changing for rotor 2. Now, this is what all we were discussing about the fundamental concept of fundamental method for the design.

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Let us discuss today for the next step. Say, we will be doing our design for this contra rotating fan using second concept that's what is say free vortex concept. We can understand, say we have discussed the design of say axial flow compressor with different design methods, similar kind of concept can be adopted for design of contra rotating fan. But for the sake of brevity we will be discussing here one more method for the design that's what is say free vortex design method.

So, for free vortex design method, the design procedure that's what will be similar to what all we have discussed for fundamental method till mid-section. So, we will not be repeating those calculation what we have done for the mid-section. Now, we know when we are talking about say low speed configuration, we are doing our calculation for mid-section at 50% of span, do not forget both the logics that's what is applicable for rotor 1 as well as for rotor 2.

Here in this case, once we have done our calculation at the mid-section then we will be adopting the low concept or we can say whirl distribution concept that's what method we will be opting for. Here in this case, we will be using say free vortex design concept for which we will be taking $C_w \cdot r = \text{constant}$ for rotor 1. Similarly, here for the design of rotor 2, we are making assumption that the flow which is coming out from rotor 1 with absolute velocity of C_2 , the same flow or same absolute velocity with which flow will be entering in my rotor 2.

That means, if we are considering our absolute flow angle at the exit of rotor 1, say α_2 that's what will be equal to α_3 . This is what we are making assumption in order to simplify our design process and later on we will be discussing and we know like we have adopted this method and with which we have done our design, that's what is working absolutely fine with.

Here in this case, if we consider now for rotor 2, we have discussed the flow which is coming out from rotor 1, that's what will be having some whirl component say C_{w2} . Now, this whirl component, that's what will be having the direction, that's what is opposite to the direction of the rotation. So, we can say, this whirl component, that's what will be getting added up, we will see all this calculation when we move ahead with. So, let us check with.

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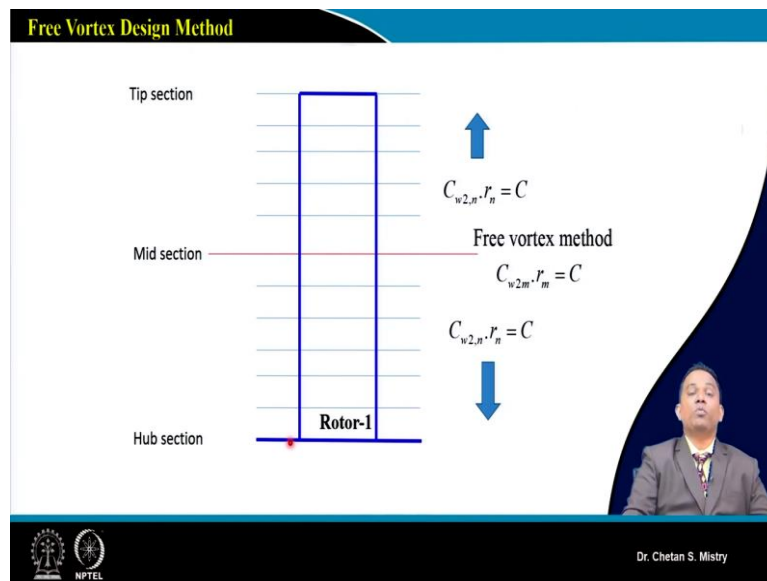
Free Vortex Design Method

| Rotor-1 | |
|---------------------------|---------|
| Solution | 6-Mid |
| r | 0.152 |
| r/h | 0.67 |
| Peripheral speed U | 33.93 |
| Axial velocity C_a | 44.42 |
| Total inlet temp T_{in} | 299 |
| ΔP_{in} | 1200 |
| $\eta_{in}(P_2/P_1)$ | 1.01 |
| ΔT_{in} | 1.1852 |
| β_1 (Deg) | 37.38 |
| w ($C_a \Delta T_{in}$) | 1191.13 |
| $\tan \beta_2$ | -0.04 |
| β_2 (Deg) | -2.44 |
| C_{w1} | 35.92 |
| Constant | 4.84 |
| C_{w2} | 35.82 |
| $\tan \beta_3$ | -0.04 |
| β_3 (Deg) | -2.44 |
| $\tan \alpha_2$ | 0.81 |
| α_2 (Deg) | 38.89 |
| $\Delta \beta$ | 39.83 |
| Power (kW) | 11.21 |
| V_1 | 55.89 |
| V_2 | 48.44 |
| C_u | 57.05 |
| D/H | 0.80 |
| m | 0.41 |
| Pitch s | 0.04 |
| Solidity σ | 1.01 |
| DP | 0.52 |
| Incidence | 0.00 |
| Camber angle | 67.87 |
| Deviation angle | 28.05 |
| Stagger angle | -3.45 |

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So, this is what we already have done calculation at the mid-section for the rotor 1. So, here in this case, we have taken our pressure rise as 1200 Pa. So, we will be using this free vortex design concept for 1200 Pa of rotor 1 and 800 Pa for rotor 2. So, if you are opting for that, we have done our calculation for the constant. We can say for free vortex, we are taking C_w at mid-section and my radius at the mid-section the product of that, that's what is coming to be constant and that's what we are calculating as 4.84.

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The diagram, titled "Free Vortex Design Method", illustrates a vertical rotor section labeled "Rotor-1". The section is divided into three horizontal regions: "Tip section" at the top, "Mid section" in the middle, and "Hub section" at the bottom. A horizontal red line marks the "Mid section". To the right of the rotor, three equations are listed, each associated with a region: $C_{w2n} \cdot r_n = C$ for the tip section (indicated by an upward-pointing blue arrow), "Free vortex method" for the mid section, and $C_{w2n} \cdot r_n = C$ for the hub section (indicated by a downward-pointing blue arrow). The NPTEL logo is visible in the bottom left corner, and the name "Dr. Chetan S. Mistry" is in the bottom right corner.

Now, this is what is a concept, that's what we have discussed for our stage design. Same concept we can use for our contra rotating fan design. Here in this case at the mid-section as we have discussed, we have done our calculation for $C_w \cdot r = \text{constant}$. Now, we will be using this free vortex concept, say above the mid-section towards the tip region and below the mid-section towards the Hub region, where we will be putting our $C_w \cdot r = \text{constant}$. We know, here in this case, my constant that's what will remain same, my radius that's what will be changing.

Now, since my radius is changing, that is the reason why my whirl component that's what will be changing. So, my whirl component, that's what will be changing all the way from Hub to tip, okay. And, based on that we are doing all our calculations. So, if we recall, basically by calculating our velocity components, say peripheral speed, my axial velocity, my tangential velocity, absolute velocity and relative velocity, we will be having the distribution of flow angle along the span, okay.

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Free Vortex Design Method

Calculations at other stations

At Hub We know that for Free vortex method
 $C_{w2m} \cdot r_m = \text{Constant} = 35.82 \times 0.135 = 4.84$

At radial distance of 'r'
 $C_{w2r} = \frac{\text{Constant}}{r_r}$ And $\beta_{2r} = \tan^{-1} \left(\frac{U_{R1r} - C_{w2r}}{C_a} \right)$

$C_{w2h} = \frac{\text{Constant}}{r_h} = \frac{4.84}{0.0675} = 71.65 \text{ m/s}$

$\beta_{2h} = \tan^{-1} \left(\frac{U_{R1h} - C_{w2h}}{C_a} \right) = \tan^{-1} \left(\frac{16.96 - 71.65}{44.4} \right) = -50.92^\circ$

$\tan \beta_{1h} = \frac{U_{R1h}}{C_a} = \frac{16.96}{44.4} = 0.382$
 $\beta_{1h} = 20.90^\circ$

$\tan \alpha_{2h} = \frac{C_{w2h}}{C_a} = \frac{71.65}{44.4} = 1.614$
 $\alpha_{2h} = 58.21^\circ$

We know :
 $C_a = 44.4 \text{ m/s}$
 $r_r = 0.0675 \text{ m}$
 $U_{R1h} = 16.96 \text{ m/s}$

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So, let us move ahead with the calculation at other stations. So, we will be doing our calculation at the Hub. So, what we know from our free vortex concept, we can say $C_w \cdot r = \text{constant}$, that's what is coming say 4.84.

We know that for Free vortex method,

$$C_{w2m} \cdot r_m = \text{constant} = 35.82 \times 0.135 = 4.84$$

Now, at any radial station, we can put C_{w2} at any radial station, that's what is constant by particular station radius. So, here in this case, C_{w2} at Hub we can calculate by using this radius as well as by concept. So, it is coming say 71.65 m/s.

At hub,

At radial distance of 'r'

$$C_{w2r} = \frac{\text{constant}}{r_r}$$

$$C_{w2h} = \frac{\text{Constant}}{r_h}$$

$$= \frac{4.84}{0.0675}$$

$$= 71.65 \text{ m/s}$$

Now, since we know what is our C_{w2} , we can calculate what will be our flow angles. So, we will be calculating our exit blade angle, that's what is say β_2 , that is given by say based on our velocity triangle, we can say C_1 it is nothing but U minus C_{w2} divided by C_a and that's what is coming -50.92 .

$$\begin{aligned}\beta_{2h} &= \tan^{-1} \left(\frac{U_{R1h} - C_{w2h}}{C_a} \right) \\ &= \tan^{-1} \left(\frac{16.96 - 71.65}{44.4} \right) \\ &= -50.92^\circ\end{aligned}$$

Same way, based on velocity triangle we can do our calculation for β_1 and that's what is coming 20.90.

$$\begin{aligned}\tan \beta_{1h} &= \frac{U_{R1h}}{C_a} \\ &= \frac{16.96}{44.4} \\ \beta_{1h} &= 20.9^\circ\end{aligned}$$

Be careful, when we are doing our calculation at different station, accordingly you need to modify your velocity triangle! Here, at the Hub station we are having velocity triangle, that's what is coming to be different, okay.

$$\begin{aligned}\tan \alpha_{2h} &= \frac{C_{w2h}}{C_a} \\ &= \frac{71.65}{44.4} \\ \alpha_{2h} &= 58.21^\circ\end{aligned}$$

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Free Vortex Design Method

From velocity triangle:

$$V_{1h} = \frac{U_{R1h}}{\sin \beta_{1h}}$$

$$= \frac{16.96}{\sin(20.91)}$$

$$= 47.52 \text{ m/s}$$

$$V_{2h} = \frac{U_{R1h} - C_{w2h}}{\sin \beta_{2h}}$$

$$= \frac{16.96 - 71.65}{\sin(-50.92)}$$

$$= 70.45 \text{ m/s}$$

$$C_{2h} = \frac{C_s}{\cos \alpha_{2h}}$$

$$= \frac{44.4}{\cos(58.21^\circ)}$$

$$= 84.28 \text{ m/s}$$

We know :

$C_s = 44.4 \text{ m/s}$

$U_{R1h} = 16.96 \text{ m/s}$

$\beta_{1h} = 20.90^\circ$

$\beta_{2h} = -50.92^\circ$

$C_{w2h} = 71.65 \text{ m/s}$

$\alpha_{2h} = 58.21^\circ$

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Now, say once we have done our calculation for these blade angles, we are interested in calculating our De-Haller's number and for that we are looking for calculation of our relative velocity. Now, this relative velocity at the entry based on signed rule, that's what is coming as say 47.52 m/s.

From velocity triangle:

$$V_{1h} = \frac{U_{R1h}}{\sin \beta_{1h}}$$

$$= \frac{16.96}{\sin(20.91)}$$

$$= 47.52 \text{ m/s}$$

Same way, for exit velocity triangle, we can say my relative velocity at the exit near the Hub region, that's what is coming 70.45 m/s, okay.

$$V_{2h} = \frac{U_{R1h} - C_{w2h}}{\sin \beta_{2h}}$$

$$= \frac{16.96 - 71.65}{\sin(-50.92)}$$

$$= 70.45 \text{ m/s}$$

Same way, we can do our calculation for absolute velocity component also. So, my C_2 , that's what is coming 84.28 m/s.

$$C_{2h} = \frac{C_a}{\cos \alpha_{2h}}$$

$$= \frac{44.4}{\cos(58.21^\circ)}$$

$$= 84.28 \text{ m/s}$$

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Free Vortex Design Method

De-Haller's factor

$$DH = \frac{V_{2h}}{V_{1h}}$$

$$= \frac{70.45}{47.52}$$

$$= 1.48$$

According to Carter's rule; Slop factor

$$m_h = 0.23 \left(\frac{2\alpha}{c} \right)^2 + 0.1 \frac{(90 - \beta_{2h})}{50}$$

$$= 0.23(2 \times 0.5)^2 + 0.1 \frac{(90 - (-50.92))}{50}$$

$$= 0.511$$

We know :
 $V_{1h} = 47.52 \text{ m/s}$
 $V_{2h} = 70.45 \text{ m/s}$
 $\beta_{2h} = -50.92^\circ$

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Now, we know our De-Haller's factor, that's what we are defining as a ratio of velocity, relative velocity ratio, that is nothing but V_2/V_1 . So, here this is what is coming to be 1.48, okay.

De – Haller's factor

$$DH = \frac{V_{2h}}{V_{1h}}$$

$$= \frac{70.45}{47.52}$$

$$= 1.48$$

This is what is coming slightly on the higher side, we will see as compared to our say fundamental method this number is coming on the higher side.

Now, after doing this calculation, we will be checking with what will be our diffusion factor for rotor 1. And in order to do that calculation, we are looking for say Carter's parameter or slope factor; that slope factor which is given by this equation it is

$$\begin{aligned}
 m_h &= 0.23 \left(\frac{2a}{c} \right)^2 + 0.1 \frac{(90 - \beta_{2h})}{50} \\
 &= 0.23(2 \times 0.5)^2 + 0.1 \frac{(90 - (-50.92))}{50} \\
 &= 0.511
 \end{aligned}$$

Now, if you will be putting the numbers it says my 'm', that's what is coming 0.511. Now, once we know this 'm' parameter, we can check with the calculation of our solidity.

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Free Vortex Design Method

pitch $s_h = \frac{2\pi r_h}{Z_1}$

We know number of blades for Rotor-1 $Z_1 = 19$

$$= \frac{2\pi \times 0.0675}{19}$$

$$= 0.0223$$

Solidity $\sigma_m = \frac{\text{Chord}}{\text{pitch}}$

$$= \frac{0.045}{0.0223}$$

$$= 2.017$$

We know :
 $r_m = 0.135m$
 $r_h = 0.0675m$
 $d_h = 0.135m$
 $AR = 3$
 $c = 0.045m$

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Now, for this calculation of the solidity, we can say that is nothing but it is given by $\frac{2\pi r_h}{Z}$. So, we can say the radius at the Hub it is known to us, as we have done calculation for the number of blades to be 19 for say rotor 1, the same number of blades we will be putting that's what will be helping us for say comparing our results, okay. So, if you are taking my number of blades for rotor 1 to be 19, we can say my pitch is coming 0.022, okay.

$$pitch\ s_h = \frac{2\pi r_h}{Z_1}$$

We know number of blades for rotor – 1 $Z_1 = 19$

$$= \frac{2\pi \times 0.0675}{19} = 0.0223$$

And, based on that since we already know our chord. So, we can calculate our solidity near the Hub region and that's what is coming 2.017.

$$\begin{aligned}
 \text{Solidity } \sigma_h &= \frac{\text{Chord}}{\text{pitch}} \\
 &= \frac{0.045}{0.0223} \\
 &= 2.017
 \end{aligned}$$

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Free Vortex Design Method

Diffusion Factor

$$DF_h = 1 - \frac{\cos \beta_{1h} + \cos \beta_{2h}}{\cos \beta_{2h}} + \frac{\cos \beta_{1h}}{2 \times \sigma_h} (\tan \beta_{1h} - \tan \beta_{2h})$$

$$= 1 - \frac{\cos(20.90)}{\cos(-50.92)} + \frac{\cos(20.90)}{2 \times 2.017} (\tan(20.90) - \tan(-50.92))$$

$$DF_h = -0.11$$

We know :

$\beta_{1h} = 20.90^\circ$

$\beta_{2h} = -50.92^\circ$

$\sigma_h = 2.017$

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Now, once this solidity is known to us; we know, we can calculate our Diffusion Factor. Now, diffusion factor correlation, that's what is in sense of β_1, β_2 and solidity. We know what is my β_1 and β_2 at the Hub station. Same way, be careful, this solidity...that solidity is not remaining constant throughout my span. Because solidity, that's what is a function of my radius and that is the reason why solidity, that's what will be varying all the way from Hub to tip region.

Diffusion Factor,

$$\begin{aligned}
 DF_h &= 1 - \frac{\cos \beta_{1h}}{\cos \beta_{2h}} + \frac{\cos \beta_{1h}}{2 \times \sigma_h} (\tan \beta_{1h} - \tan \beta_{2h}) \\
 &= 1 - \frac{\cos(20.90)}{\cos(-50.92)} + \frac{\cos(20.90)}{2 \times 2.018} (\tan(20.90) - \tan(-50.92))
 \end{aligned}$$

$$\therefore DF_h = -0.11$$

So, if you are putting that number, this diffusion factor, that's what is coming -0.11 . So, this is what is little disappointing in that sense; but, you know, like for free vortex concept we are not having our...we are not having control for these numbers. So, whatever numbers that's

what is coming we will be taking care of at the initial stage and maybe as and when required, we will be modifying these numbers, okay. So, do not get confused with diffusion factor, that's what is coming to be negative in this case, okay.

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Free Vortex Design Method

Let's assume Incidence angle i at hub as $+2^\circ$.

Camber angle

$$\theta_h = \frac{(\Delta\beta_h - i_h)}{\left(1 - \frac{m_h}{\sqrt{\sigma_h}}\right)} = \frac{(71.83 - 2)}{\left(1 - \frac{0.511}{\sqrt{2.017}}\right)} = 109.07^\circ$$

Deviation angle

$$\delta_h = \frac{m_h \theta_h}{\sqrt{\sigma_h}} = \frac{0.511 \times 109.07}{\sqrt{2.017}} = 39.24^\circ$$

Stagger angle

$$\zeta_h = \beta_{1h} - i_h - \frac{\theta_h}{2} = 20.90 - 2 - \frac{109.07}{2} = -35.63^\circ$$

We know :

- $\beta_{1h} = 20.90^\circ$
- $\Delta\beta_h = 71.83^\circ$
- $\sigma_h = 2.017$
- $m_h = 0.511$

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Now, you know, like once we have done our this calculation. So, we can go with say calculating our cascade parameters. Now, as we have discussed earlier, say we will be assuming our incidence angle at the Hub to be $+2^\circ$. So, we will be taking that as $+2$, we can calculate our camber angle at the Hub, that's what is nothing but

$$\theta_h = \frac{\Delta\beta_h - i_h}{1 - \frac{m_h}{\sqrt{\sigma_h}}}$$

And if you are putting that number, my camber angle is also coming to be large this is what is coming 109.07.

$$= \frac{(71.83 - 2)}{1 - \frac{0.511}{\sqrt{2.017}}} = 109.07^\circ$$

You can understand, like when we will be discussing all our results, we will be realizing what all are the limitations or what all are the challenges when we are opting with free vortex design concept. And that is the reason why intentionally initial calculation we are showing for the Hub station, okay.

Now, once we know what is our camber angle based on say by Carter's correlation we can calculate deviation angle; that is nothing but $\frac{m_h \theta_h}{\sqrt{\sigma_h}}$ and that's what is coming 39.24° .

$$\delta_h = \frac{m_h \theta_h}{\sqrt{\sigma_h}} = \frac{0.511 \times 109.07}{\sqrt{2.017}}$$

$$\therefore \delta_h = 39.24^\circ$$

Same way, we can calculate our blade setting angle or we can say stagger angle, that's what is in sense of $\beta_{1h} - i_h - \frac{\theta_h}{2}$ and that's what is coming -35.63° , okay.

$$\zeta_h = \beta_{1h} - i_h - \frac{\theta_h}{2} = 20.90 - 2 - \frac{109.07}{2}$$

$$\therefore \zeta_h = -35.63^\circ$$

So, these all are the calculations for my cascade parameters or airfoil parameters or blade parameters at particular station, that's what is at the hub.

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Free Vortex Design Method

Calculations at other stations

At Tip We know that for Free vortex method
 $C_{w2r} \cdot r_t = \text{Constant} = 35.82 \times 0.135 = 4.84$

At radial distance of 'r'
 $C_{w2r} = \frac{\text{Constant}}{r_t}$ And $\beta_{2r} = \tan^{-1} \left(\frac{U_{R1r} - C_{w2r}}{C_a} \right)$

At tip,
 $C_{w2t} = \frac{\text{Constant}}{r_t} = \frac{4.84}{0.2025} = 23.88 \text{ m/s}$

$\beta_{2t} = \tan^{-1} \left(\frac{U_{R1t} - C_{w2t}}{C_a} \right) = \tan^{-1} \left(\frac{50.89 - 23.88}{44.4} \right) = 31.31^\circ$

$\tan \beta_{1t} = \frac{U_{R1t}}{C_a} = \frac{50.89}{44.4} = 1.146$
 $\beta_{1t} = 48.82^\circ$

$\tan \alpha_{2t} = \frac{C_{w2t}}{C_a} = \frac{23.88}{44.4} = 0.538$
 $\alpha_{2t} = 28.27^\circ$

We know :
 $C_a = 44.4 \text{ m/s}$
 $r_t = 0.2025 \text{ m}$
 $U_{R1t} = 50.89 \text{ m/s}$

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Now, what we are looking for is we are also looking for the calculation what is happening near the tip region. So, in line to what all we have discussed at the Hub station, same logic that's what is applicable here. Now, we will be applying say our free vortex design concept near the tip region. So, we can say my C_{w2} at the tip that is nothing but it is $\frac{\text{constant}}{r_t}$, okay.

Now, if we are putting this as say C_{w2} it is coming 23.88 m/s .

We know that for Free vortex method,

$$C_{w2m} \cdot r_m = \text{constant} = 35.82 \times 0.135 = 4.84$$

At tip,

At radial distance of 'r'

$$C_{w2r} = \frac{\text{constant}}{r_r}$$

$$C_{w2t} = \frac{\text{Constant}}{r_t}$$

$$= \frac{4.84}{0.2025}$$

$$= 23.88 \text{ m/s}$$

We can do our calculation for the blade angles β_1 and β_2 at the tip station, okay. Here in this case...here in this case, we will be calculating β_2 , that's what is coming 31.31° , my β_1 at the tip, that's what is coming 48.82° and my α_2 , that's what is coming 28.27° .

$$\beta_{2t} = \tan^{-1} \left(\frac{U_{R1t} - C_{w2t}}{C_a} \right)$$

$$= \tan^{-1} \left(\frac{50.89 - 23.88}{44.4} \right)$$

$$= 31.31^\circ$$

$$\tan \beta_{1t} = \frac{U_{R1t}}{C_a}$$

$$= \frac{50.89}{44.4}$$

$$\beta_{1t} = 48.82^\circ$$

$$\tan \alpha_{2t} = \frac{C_{w2t}}{C_a}$$

$$= \frac{23.88}{44.4}$$

$$\alpha_{2t} = 28.27^\circ$$

So, these all, what we say, that's what is based on our velocity triangle and based on trigonometry this is what all we have discussed earlier also. So, without taking much time, let us move ahead with the calculation for the relative velocity ratio near the tip station.

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Free Vortex Design Method

From velocity triangle:

$$V_{1t} = \frac{U_{R1t}}{\sin \beta_{1t}}$$

$$= \frac{50.89}{\sin(48.82^\circ)}$$

$$= 67.61 \text{ m/s}$$

$$V_{2t} = \frac{U_{R1t} - C_{w2t}}{\sin \beta_{2t}}$$

$$= \frac{50.89 - 23.88}{\sin(31.31^\circ)}$$

$$= 51.97 \text{ m/s}$$

$$C_{a2t} = \frac{C_a}{\cos \alpha_{2t}}$$

$$= \frac{44.4}{\cos(28.27^\circ)}$$

$$= 50.41 \text{ m/s}$$

We know :

- $C_a = 44.4 \text{ m/s}$
- $U_p = 50.89 \text{ m/s}$
- $\beta_{1t} = 48.82^\circ$
- $\beta_{2t} = 31.31^\circ$
- $C_{w2t} = 23.88 \text{ m/s}$
- $\alpha_{2t} = 28.27^\circ$

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So, here, this is what we can say we are having my inlet relative velocity as 67.61 m/s, my outlet relative velocity as say 51.97, and absolute velocity is coming 50.41 m/s, okay.

From velocity triangle:

$$V_{1t} = \frac{U_{R1t}}{\sin \beta_{1t}}$$

$$= \frac{50.89}{\sin(48.82^\circ)}$$

$$= 67.61 \text{ m/s}$$

$$V_{2t} = \frac{U_{R1t} - C_{w2t}}{\sin \beta_{2t}}$$

$$= \frac{50.89 - 23.88}{\sin(31.31^\circ)}$$

$$= 51.97 \text{ m/s}$$

$$C_{2t} = \frac{C_a}{\cos \alpha_{2t}}$$

$$= \frac{44.4}{\cos(28.27^\circ)}$$

$$= 50.41 \text{ m/s}$$

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Free Vortex Design Method

De-Haller's factor

$$DHF = \frac{V_{2t}}{V_{1t}}$$

$$= \frac{51.97}{67.61}$$

$$= 0.76$$

According to Carter's rule; Slop factor

$$m_t = 0.23 \left(\frac{2a}{c} \right)^2 + 0.1 \frac{(90 - \beta_{2t})}{50}$$

$$= 0.23(2 \times 0.5)^2 + 0.1 \frac{(90 - (31.31))}{50}$$

$$= 0.347$$

We know :
 $V_{1t} = 67.61 \text{ m/s}$
 $V_{2t} = 51.97 \text{ m/s}$
 $\beta_{2t} = 31.31^\circ$

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Now, when we are taking this ratio, that's what is coming as say 0.76, okay. And, as we know, that need to be greater than 0.72 and that's what is happening here, okay. Now, in order to calculate our diffusion factor, the 'm' parameter that's what has been calculated near the tip station and that's what is coming 0.347.

De – Haller's factor

$$DHF = \frac{V_{2t}}{V_{1t}}$$

$$= \frac{51.97}{67.61} = 0.76$$

According to Carter's rule; slop factor

$$m_t = 0.23 \left(\frac{2a}{c} \right)^2 + 0.1 \frac{(90 - \beta_{2t})}{50}$$

$$= 0.23(2 \times 0.5)^2 + 0.1 \frac{90 - (31.31)}{50}$$

$$= 0.347$$

(Refer Slide Time: 16:47)

Free Vortex Design Method

pitch $s_t = \frac{2\pi r_t}{Z_1}$

We know number of blades for Rotor-1 $Z_1=19$

$$= \frac{2\pi \times 0.2025}{19}$$

$$= 0.0669$$

Solidity $\sigma_t = \frac{\text{Chord}}{\text{pitch}}$

$$= \frac{0.045}{0.0669}$$

$$= 0.672$$

We know :
 $r_m = 0.135m$
 $r_t = 0.2025m$
 $AR = 3$
 $c = 0.045m$

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Now, once this 'm' parameter is known to us, we can do our calculation for the solidity and the Diffusion Factor. Now, for solidity as we have discussed, we have our number of blades to be 19 for rotor 1. Now, we will be putting say $2\pi r_t$ into my tip radius as say 0.2025 and 19 number of blades that's what is giving pitch to be 0.066, okay. And, the solidity is coming 0.672.

$$\text{Pitch, } s_t = \frac{2\pi r_t}{Z_1}$$

We have number of blades for Rotor – 1 $Z_1 = 19$

$$\therefore s_t = \frac{2\pi \times 0.2025}{19}$$

$$\therefore s_t = 0.0669 \text{ m}$$

Solidity at tip station,

$$\sigma_t = \frac{\text{Chord}}{\text{Pitch}}$$

$$= \frac{0.045}{0.0669}$$

$$\therefore \sigma_t = 0.672$$

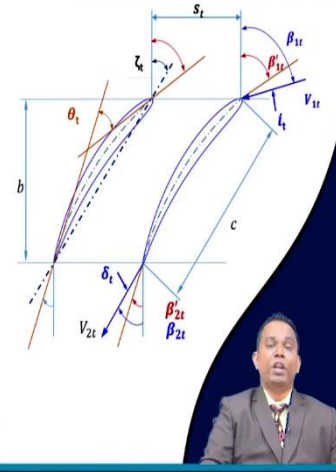
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Free Vortex Design Method

Diffusion Factor

$$DF_t = 1 - \frac{\cos \beta_{1t}}{\cos \beta_{2t}} + \frac{\cos \beta_{1t}}{2 \times \sigma_t} (\tan \beta_{1t} - \tan \beta_{2t})$$

$$= 1 - \frac{\cos(48.82)}{\cos(31.31)} + \frac{\cos(48.82)}{2 \times 0.672} (\tan(48.82) - \tan(31.31))$$

$$DF_t = 0.489$$


We know :

- $\beta_{1t} = 48.82^\circ$
- $\beta_{2t} = 31.31^\circ$
- $\sigma_t = 0.672$

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Now, based on this solidity and my blade angles β_1 and β_2 near the tip station, we are able to calculate the diffusion factor at the tip and that's what is coming 0.489.

Diffusion factor,

$$DF_t = 1 - \frac{\cos \beta_{1t}}{\cos \beta_{2t}} + \frac{\cos \beta_{1t}}{2 \times \sigma_t} (\tan \beta_{1t} - \tan \beta_{2t})$$

$$= 1 - \frac{\cos(48.82^\circ)}{\cos(31.31^\circ)} + \frac{\cos(48.82^\circ)}{2 \times 0.672} (\tan(48.82^\circ) - \tan(31.31^\circ))$$

$$\therefore DF_t = 0.489$$

So, you know, like this is what will give idea how my say De-Haller's Factor, how my diffusion factor, that's what is happening or that's what is coming near the tip station.

(Refer Slide Time: 17:51)

Free Vortex Design Method

Let's assume Incidence angle i at hub as -2° .

Camber angle

$$\theta_t = \frac{(\Delta\beta_t - i_t)}{\left(1 - \frac{m}{\sqrt{\sigma_t}}\right)} = \frac{(17.58 + 2)}{\left(1 - \frac{0.347}{\sqrt{0.672}}\right)} = 33.98^\circ$$

Deviation angle

$$\delta_t = \frac{m_t \theta_t}{\sqrt{\sigma_t}} = \frac{0.347 \times 33.98}{\sqrt{0.672}} = 14.4^\circ$$

Stagger angle

$$\zeta_t = \beta_t - i_t - \frac{\theta_t}{2} = 48.82 + 2 - \frac{33.98}{2} = 33.90^\circ$$

We know :

- $\beta_{1t} = 48.82^\circ$
- $\Delta\beta_t = 17.58^\circ$
- $\sigma_t = 0.672$
- $m = 0.347$

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In order to have the parameters for the generation of blade, we are looking for the blade angles, we are looking for say airfoil angles that's what we are defining as a camber angle, stagger angle and say deviation angle, that's what we are calculating. As we have discussed here, we are assuming our incidence angle near the tip region to be -2° . When we say this is what is say -2° , we can calculate our camber angle and that camber angle that's what is coming 33.98.

Camber angle,

$$\begin{aligned} \theta_t &= \frac{\Delta\beta_t - i_t}{1 - \frac{m_t}{\sqrt{\sigma_t}}} \\ &= \frac{(17.58 + 2)}{\left(1 - \frac{0.347}{\sqrt{0.672}}\right)} \\ &= 33.98^\circ \end{aligned}$$

We are having our deviation angle, that's what is coming to be 14.4 and the stagger angle is coming 33.90.

Deviation angle,

$$\begin{aligned} \delta_t &= \frac{m_t \theta_t}{\sqrt{\sigma_t}} = \frac{0.347 \times 33.98}{\sqrt{0.672}} \\ \therefore \delta_t &= 14.4^\circ \end{aligned}$$

Stagger angle,

$$\zeta_t = \beta_{1t} - i_t - \frac{\theta_t}{2}$$

$$= 48.82 + 2 - \frac{33.98}{2}$$

$$= 33.9^\circ$$

(Refer Slide Time: 18:43)

Free Vortex Design Method

| | Rotor-1 | | |
|---|---------|---------|---------|
| Solution | 1-Hub | 6-Mid | 11-Tip |
| r | 0.067 | 0.135 | 0.203 |
| r/R | 1.33 | 0.67 | 1.00 |
| Peripheral speed U | 16.96 | 33.63 | 50.89 |
| Axial velocity C _z | 44.40 | 44.40 | 44.40 |
| Total inlet temp T ₀₁ | 299 | 299 | 299 |
| ΔP _{Pass} | 1200 | 1200 | 1200 |
| m ₀₁ (P ₀₁ /P ₀₂) | 1.01 | 1.01 | 1.01 |
| ΔP _{Pass} | 1.1852 | 1.1852 | 1.1852 |
| β ₁ (deg) | 20.91 | 37.38 | 48.90 |
| w (C _z ΔT ₀) | 1191.13 | 1191.13 | 1191.13 |
| tan β ₂ | -0.34 | | |
| β ₂ (deg) | -2.44 | | |
| C _{w2} | 35.82 | | |
| Constant | 4.84 | 4.84 | 4.84 |
| C _{w0} | 71.85 | 35.82 | 23.88 |
| tan β ₀ | -1.23 | -0.04 | 0.61 |
| β ₀ (deg) | -40.92 | -2.44 | 31.31 |
| tan α ₂ | 1.61 | 0.81 | 0.54 |
| α ₂ (deg) | 58.21 | 38.89 | 28.27 |
| Δβ | 71.83 | 39.83 | 17.58 |
| Power (kW) | 11.21 | 11.21 | 11.21 |
| V ₁ | 47.53 | 55.80 | 67.54 |
| V ₂ | 70.44 | 44.44 | 51.87 |
| C ₂ | 84.28 | 57.05 | 50.42 |
| DH | 1.48 | 0.80 | 0.77 |
| m | 0.51 | 0.41 | 0.35 |
| Pitch s | 0.02 | 0.04 | 0.07 |
| Solidity σ | 2.02 | 1.01 | 0.67 |
| DP | -0.11 | 0.52 | 0.49 |
| incidence | 2.00 | 0.00 | -2.00 |
| Camber angle | 109.19 | 67.87 | 33.98 |
| Deviation angle | 39.36 | 28.05 | 14.40 |
| Stagger angle | -35.69 | 3.45 | 33.90 |

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Now, as we have discussed earlier for the calculation for stage, in line to that we will be having all this parameter, that's what is coming, okay, combinedly we can say Hub, mid and tip station, if we make the observation my C_{w2} , that's what is coming 35.82. And here my $\Delta\beta$ near the Hub station, that's what is coming to be large, that's what is coming say 70.83. Now, diffusion factor variation we can say it is -0.11 to 0.49 and this is what is a variation of my $\Delta\beta$. Now, this is what all we are discussing about say free vortex design concept at three different station Hub, mid and tip station.

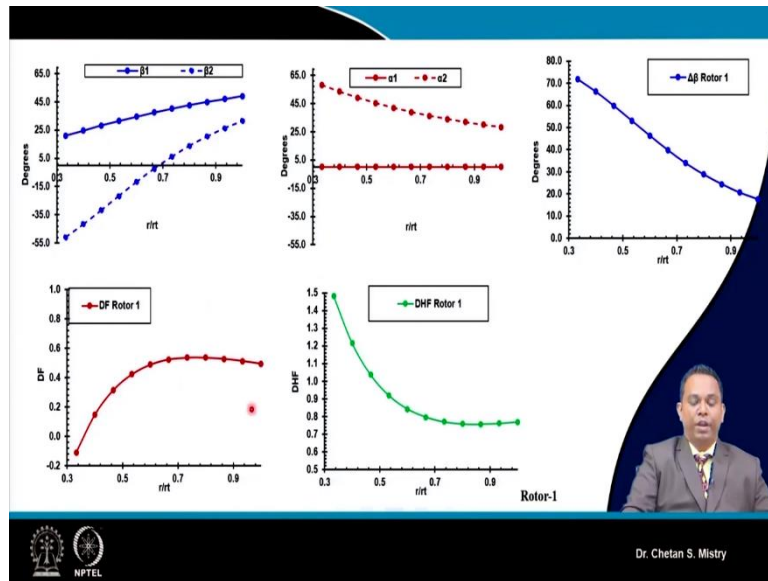
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| | Rotor-1 | | | | | | | | | | |
|---------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Solution | 1-Hub | 2.00 | 3.00 | 4.00 | 5.00 | 6-Mid | 7.00 | 8.00 | 9.00 | 10.00 | 11-Tip |
| r | 0.087 | 0.081 | 0.094 | 0.108 | 0.121 | 0.135 | 0.148 | 0.162 | 0.175 | 0.189 | 0.203 |
| r/h | 0.33 | 0.40 | 0.47 | 0.53 | 0.60 | 0.67 | 0.73 | 0.80 | 0.87 | 0.93 | 1.00 |
| Peripheral speed U | 16.96 | 20.36 | 23.75 | 27.14 | 30.54 | 33.93 | 37.32 | 40.72 | 44.11 | 47.50 | 50.89 |
| Axial velocity C_a | 44.40 | 44.40 | 44.40 | 44.40 | 44.40 | 44.40 | 44.40 | 44.40 | 44.40 | 44.40 | 44.40 |
| Total inlet temp T_{in} | 299 | 299 | 299 | 299 | 299 | 299 | 299 | 299 | 299 | 299 | 299 |
| ΔP_{in} | 1200 | 1200 | 1200 | 1200 | 1200 | 1200 | 1200 | 1200 | 1200 | 1200 | 1200 |
| $m(P_{in}/P_{hub})$ | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| ΔT_{in} | 1.1852 | 1.1852 | 1.1852 | 1.1852 | 1.1852 | 1.1852 | 1.1852 | 1.1852 | 1.1852 | 1.1852 | 1.1852 |
| β (deg) | 20.91 | 24.83 | 28.14 | 31.44 | 34.52 | 37.38 | 40.05 | 42.52 | 44.81 | 46.93 | 48.90 |
| $w(C_a/\Delta T)$ | 1191.13 | 1191.13 | 1191.13 | 1191.13 | 1191.13 | 1191.13 | 1191.13 | 1191.13 | 1191.13 | 1191.13 | 1191.13 |
| tan β | | | | | | | | | | | |
| β (deg) | | | | | | | | | | | |
| C_w | | | | | | | | | | | |
| Constant | 4.84 | 4.84 | 4.84 | 4.84 | 4.84 | 4.84 | 4.84 | 4.84 | 4.84 | 4.84 | 4.84 |
| C_w | 71.85 | 58.70 | 51.18 | 44.78 | 39.30 | 35.82 | 32.57 | 29.85 | 27.58 | 25.59 | 23.88 |
| tan β | -1.23 | -0.89 | -0.62 | -0.40 | -0.21 | -0.04 | 0.11 | 0.24 | 0.37 | 0.49 | 0.61 |
| β (deg) | -56.82 | -41.54 | -31.70 | -21.69 | -11.79 | -2.44 | 6.11 | 13.75 | 20.44 | 26.27 | 31.31 |
| tan α | 1.51 | 1.34 | 1.15 | 1.01 | 0.90 | 0.81 | 0.73 | 0.67 | 0.62 | 0.58 | 0.54 |
| α (deg) | 56.21 | 53.38 | 49.05 | 45.24 | 41.87 | 38.89 | 36.26 | 33.91 | 31.82 | 29.95 | 28.27 |
| $\Delta\beta$ | 71.83 | 66.17 | 59.84 | 53.10 | 46.30 | 39.83 | 33.93 | 28.77 | 24.38 | 20.66 | 17.58 |
| Power (W) | 11.21 | 11.21 | 11.21 | 11.21 | 11.21 | 11.21 | 11.21 | 11.21 | 11.21 | 11.21 | 11.21 |
| V_1 | 47.53 | 48.85 | 50.35 | 52.04 | 53.89 | 55.88 | 58.01 | 60.34 | 62.95 | 65.82 | 67.54 |
| V_2 | 70.44 | 59.33 | 52.19 | 47.78 | 45.36 | 44.44 | 44.66 | 45.71 | 47.39 | 49.52 | 51.97 |
| C_u | 84.29 | 74.41 | 67.75 | 63.06 | 59.83 | 57.05 | 55.07 | 53.51 | 52.28 | 51.25 | 50.42 |
| DH | 1.48 | 1.21 | 1.04 | 0.92 | 0.84 | 0.80 | 0.77 | 0.76 | 0.76 | 0.76 | 0.77 |
| m | 0.51 | 0.49 | 0.47 | 0.45 | 0.43 | 0.41 | 0.40 | 0.38 | 0.37 | 0.36 | 0.35 |
| Pitch τ | 0.02 | 0.03 | 0.03 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.06 | 0.06 | 0.07 |
| Solidity σ | 2.03 | 1.88 | 1.64 | 1.38 | 1.12 | 1.01 | 0.92 | 0.84 | 0.78 | 0.72 | 0.67 |
| Incidence | -0.1% | 0.15 | 0.32 | 0.42 | 0.49 | 0.52 | 0.54 | 0.54 | 0.53 | 0.51 | 0.49 |
| Incidence | 2.00 | 1.60 | 1.20 | 0.80 | 0.40 | 0.00 | -0.40 | -0.80 | -1.20 | -1.60 | -2.00 |
| Camber angle | 109.19 | 104.22 | 96.85 | 87.73 | 77.76 | 67.87 | 58.74 | 50.75 | 44.02 | 38.47 | 33.98 |
| Deviation angle | 39.38 | 39.85 | 38.21 | 35.43 | 31.86 | 28.05 | 24.41 | 21.18 | 18.45 | 16.21 | 14.40 |
| Stagger angle | -35.69 | -29.98 | -21.48 | -13.23 | -4.77 | 3.45 | 11.06 | 17.94 | 24.00 | 29.29 | 33.90 |

Now, in order to understand, we will be dividing our whole span into say number of stations; say for the sake of simplicity we have divided our whole span into 11 stations. So, you can see this is what is representing my different radial locations. And as we have discussed, we are having our C_w into r, that's what is coming to be constant, okay. And based on that, we are calculating what will be my C_{w2} .

Now, once the C_{w2} is known to us we can do calculation for different flow angles, okay. And, that's what is giving me my $\Delta\beta$ to be varying from say 71.83° to 17° . So, this is what is large $\Delta\beta$, that's what we are getting for the free vortex concept for rotor 1. And my diffusion factor near the Hub that is coming to be say -0.11 , maybe if required we need to do certain modification in sense of loading such that we will be getting this number to be positive, okay. And this camber angle also is coming to be very large in the case of free vortex concept for rotor 1.

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Now, if you look at here, this is what is representing how my β_1 and β_2 that's what is varying along my say span. So, this is what is representing this span, if you look at here for the free vortex concept and rotor 1 design, we are having larger $\Delta\beta$ that's what is coming here near the Hub region and near the tip region this angle that's what is getting reduced, okay. So, this is what is representing.

So, near the Hub region, we will be having this $\Delta\beta$, that's what is coming around 70° and near the tip region, that's what is coming around 21° . Since, we are having our α_1 , that's what is axial entry and this is what is representing my α_2 and this is what is representing my diffusion factor for the rotor 1 and De-Haller's factor for rotor 1, okay.

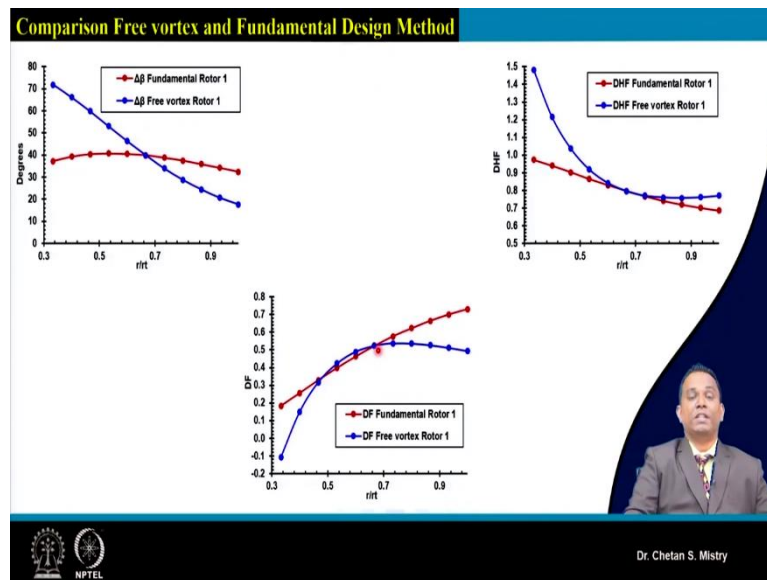
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Now, let us try to compare with our fundamental designs, okay. Here in this case, if we are observing carefully say my β_1 , that's what is same for both the design cases for fundamental approach as well as for free vortex approach but here in this case my β_2 if you are looking at then my β_2 , that's what is coming to be larger for a free vortex concept. So, here my β_2 , that's what is coming to be larger, okay. So, $\Delta\beta$ is coming to be higher.

Now, if we compare our fundamental design method, it says my $\Delta\beta$ is you know it is nearly half in the region; and, you know, if you are comparing our $\Delta\beta$ for fundamental design approach it is not giving very high twist to rotor 1 blade; but we are considering if we are considering a free vortex concept for that we will be having our $\Delta\beta$ to be coming as large from Hub to tip.

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Now, this is what we were discussing about. Same way, if we are comparing our De- Haller's factor that is also coming in a great control, when we are talking about the fundamental design approach, when we are talking about the Diffusion Factor, it shows a smooth diffusion, that's what is will be going to happen.

Now, we are not criticizing any design method, okay; whatever design method you will be adopting for design and development of rotors, that's what is absolutely fine, if it is working as per the expectations. So, it is designer's choice to opt for which kind of method or what rotor and how the design method or whirl distribution he will be opting for. If it is giving what results are expected or what performance are expected and if it is working absolutely fine, that's what

is the design approach, okay. So, do not misunderstand in sense of free design say free vortex design concept and fundamental design method concept, okay.

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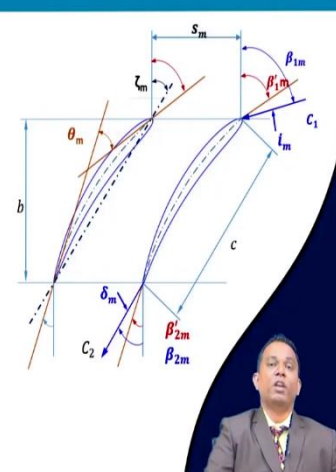
Corrected Flow Angles


At mid section
Let's assume $\delta_{assume} = 4^\circ$


Corrected deviation
 $\delta_{m,corr} = 28.05 + 4 = 32.05^\circ$

Corrected camber
 $\theta_{corr} = \Delta\beta + \delta_{corr} - i$
 $= 39.83 + 32.05 - 0$
 $\theta_{m,corr} = 71.87^\circ$

Corrected stagger
 $\zeta_{m,corr} = 37.38 - 0 - \frac{71.8}{2}$
 $\zeta_{m,corr} = 1.45^\circ$




 Dr. Chetan S. Mistry



Now, we have discussed about the corrected flow angles. So, here also in order to have similar discussion in sense of my deviation angle. We have discussed what all we will be getting using the computational method or computational analysis accordingly we need to modify our deviation angle. So, let us assume the variation of say deviation angle we are taking that to be 4° , okay.

At mid section,

$$\text{Let's assume } \delta_{assume} = 4^\circ$$

Corrected deviation,

$$\delta_{m,corr} = 28.05 + 4 = 32.05^\circ$$

Corrected camber,

$$\begin{aligned} \theta_{corr} &= \Delta\beta + \delta_{corr} - i \\ &= 39.83 + 32.05 - 0 \end{aligned}$$

$$\theta_{m,corr} = 71.87^\circ$$

Corrected stagger,

$$\zeta_{m,corr} = 37.38 - 0 - \frac{71.8}{2}$$

$$\zeta_{m,corr} = 1.45^\circ$$

(Refer Slide Time: 24:34)

Corrected Flow Angles

| Solution | Rotor-1 | | | | | | | | | | |
|-------------------------|---------|--------|--------|--------|-------|-------|-------|-------|-------|-------|--------|
| | 1-Hub | 2.00 | 3.00 | 4.00 | 5.00 | 6-Mid | 7.00 | 8.00 | 9.00 | 10.00 | 11-Tip |
| r | 0.067 | 0.081 | 0.094 | 0.108 | 0.121 | 0.135 | 0.148 | 0.162 | 0.175 | 0.189 | 0.203 |
| r/h | 0.33 | 0.40 | 0.47 | 0.53 | 0.60 | 0.67 | 0.73 | 0.80 | 0.87 | 0.93 | 1.00 |
| Incidence | 2.00 | 1.50 | 1.20 | 0.80 | 0.40 | 0.00 | -0.40 | -0.80 | -1.20 | -1.60 | -2.00 |
| Camber angle | 106.19 | 104.22 | 96.85 | 87.73 | 77.76 | 67.87 | 58.74 | 50.75 | 44.02 | 38.47 | 33.98 |
| Deviation angle | 39.36 | 39.85 | 38.21 | 35.43 | 31.86 | 28.05 | 24.41 | 21.18 | 18.45 | 16.21 | 14.40 |
| Stagger angle | -35.69 | -29.08 | -21.48 | -13.23 | -4.77 | 3.45 | 11.68 | 17.94 | 24.00 | 29.29 | 33.90 |
| Deviation (assumed) | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 |
| Corrected deviation | 43.38 | 43.65 | 42.21 | 39.43 | 35.86 | 32.05 | 28.41 | 25.18 | 22.45 | 20.21 | 18.40 |
| Corrected camber | 115.19 | 108.22 | 100.85 | 91.73 | 81.76 | 71.87 | 62.74 | 54.75 | 48.02 | 42.47 | 37.98 |
| Corrected Stagger angle | -37.69 | -31.08 | -23.48 | -15.23 | -6.77 | 1.45 | 9.68 | 15.94 | 22.00 | 27.29 | 31.90 |

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If you are assuming that to be 4° , accordingly we will be having corrected camber and corrected stagger. And as we discussed, this corrected camber and corrected stagger angle, that's what will be used for making of the blades, okay.

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Generation of blades – Free vortex method

Rotor 1 Blade

Fundamental design 1200-800

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So, if you look at here, this is what is representing, say you know, by having, this is what is representing my free vortex design concept. So, we can see, we are having highly twisted blade that's what is coming for rotor 1 and if you are comparing this is what is our fundamental design method for say 1200 Pa and 800 Pa distribution, my blade twist is coming to be lower.

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Free Vortex Design Method

| Rotor-2 | |
|--------------------------------|--------|
| Solidity | 6.8861 |
| r | 0.13 |
| Peripheral speed U | 33.93 |
| Axial velocity C_x | 44.42 |
| Total inlet temp T_{01} | 300 |
| P_{01} | 102525 |
| ΔP_{02} | 500 |
| ΔT_{02} | 1.91 |
| ΔT_{02} | 0.79 |
| $C_3=C_2$ | 57.05 |
| $\alpha_3(\text{deg})$ | 38.89 |
| $C_{w3}=C_{w2}$ | 35.82 |
| β_1 (deg) | 57.51 |
| w ($C_x \cdot \Delta T_1$) | 789.03 |
| C_{w1} | -8.40 |
| Constant | -1.27 |
| C_{w2} | -8.40 |
| $\tan \beta_1$ | 0.88 |
| β_1 (deg) | 44.29 |
| $\tan \alpha_1$ | 0.21 |
| α_1 (deg) | 11.94 |
| ΔP | 12.22 |
| Power (kW) | 7.43 |
| V_1 | 82.70 |
| V_2 | 62.05 |
| C_1 | 45.40 |
| DH | 0.75 |
| m | 0.29 |
| Pitch s | 0.05 |
| Solidity σ | 0.90 |
| OP | 0.43 |
| Incidence | 0.95 |
| Camber angle | 19.18 |
| Deviation angle | 5.96 |
| Stagger angle | 47.92 |

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Now, same design concept or same methodology we can opt for rotor 2. So, here in this case, we need to be very careful when we are doing our design for contra rotating fan as we made the statement earlier and as we have discussed at the initial stage also, we are assuming our absolute velocity say my C_2 and C_3 to be same and my α_2 and α_3 to be same. If you are considering that as a case, we will be having our C_{w2} and C_{w3} , that's what will be coming to be same.

Now, my whirl component at the exit, that's what is important when we are doing our design for free Vortex concept. So, we will be doing our calculation for $C_{w4} \cdot r = \text{constant}$ and that method, that's what we will be opting for designing or calculating different flow parameters, different flow angles, velocity components at different stations.

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Free Vortex Design Method

Calculations at hub

Let's assume $C_{2h} = C_{3h}$

$$\alpha_{2h} = \alpha_{3h}$$

$$\& C_{w2h} = -C_{w3h}$$

From velocity triangle

$$\beta_{3h} = \tan^{-1} \left(\frac{U_{R2h} - C_{w3h}}{C_a} \right)$$

$$= \tan^{-1} \left(\frac{16.96 + 71.65}{44.4} \right)$$

$$= 63.38^\circ$$

We know :
 $C_a = 44.4 \text{ m/s}$
 $U_{R2h} = 16.96 \text{ m/s}$

The diagram illustrates the velocity triangles at Station-3 and Station-4. At Station-3, the velocity triangle shows the axial velocity C_{a3h} , the tangential velocity U_{R2h} , and the resultant velocity V_{3h} at an angle α_{3h} to the axial direction. The flow angle at the rotor exit is β_{3h} . At Station-4, the velocity triangle shows the axial velocity C_{a4h} , the tangential velocity U_{R2h} , and the resultant velocity V_{4h} at an angle α_{4h} to the axial direction. The flow angle at the rotor inlet is β_{4h} . The schematic shows two rotors, Rotor-1 and Rotor-2, between a hub and a tip. The hub radius is r_h , the tip radius is r_t , and the chord length is c . The flow direction is indicated by arrows.

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Now, same in line to what all we have discussed for say design, here we will be calculating the flow parameters at hub station. So, as we have discussed here in this case, we are taking $C_2 = C_3$, my $\alpha_2 = \alpha_3$ and that is the reason my $C_{w2} = -C_{w3}$. Be careful about that part when we are considering that as a case, we will be able to calculate what is our β_3 .

Let's assume

$$C_{2h} = C_{3h}$$

$$\alpha_{2h} = \alpha_{3h}$$

$$\& C_{w2h} = -C_{w3h}$$

Here, in this case, this is what is $U - C_{w3}$ but this C_{w3} , that's what is nothing but it is $-C_{w2}$ and that is the reason why my β_3 is coming to be 63.38° .

From velocity triangle,

$$\beta_{3h} = \tan^{-1} \left(\frac{U_{R2h} - C_{w3h}}{C_a} \right)$$

$$= \tan^{-1} \left(\frac{16.96 + 71.65}{44.4} \right)$$

$$\beta_{3h} = 63.38^\circ$$

The same logic we have discussed for our fundamental design approach also. So, do not get confused why this sign is coming as a negative, okay.

(Refer Slide Time: 27:26)

Free Vortex Design Method

We know that for Free vortex method

$$C_{w4m} \cdot r_m = \text{Constant} = -9.4 \times 0.135 = -1.27$$

At radial distance of 'r' And

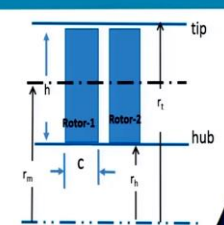
$$C_{w4r} = \frac{\text{Constant}}{r} \quad \beta_{4r} = \tan^{-1} \left(\frac{U_{R2r} - C_{w4r}}{C_a} \right)$$

$$C_{w4h} = \frac{\text{Constant}}{r_h} \quad \beta_{4h} = \tan^{-1} \left(\frac{U_{R2h} - C_{w4h}}{C_a} \right)$$

$$= \frac{-1.27}{0.0675} = -18.79 \text{ m/s} = \tan^{-1} \left(\frac{16.96 - (-18.79)}{44.4} \right) = 38.83^\circ$$

$$\Delta\beta_s = \beta_{4h} - \beta_{4r} = 63.38 - 38.83 = 24.54^\circ$$

We know :
 $C_{w4m} = -9.4 \text{ m/s}$
 $\beta_{4h} = 63.38^\circ$



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Now, we will be opting for our free vortex design concept at the exit of rotor 2. So, we will be putting $C_w \cdot r = \text{constant}$, since my C_{w4} that's what we have calculated at the mid station and based on that my constant is coming -1.27 .

We know that for Free vortex method,

$$C_{w4m} \cdot r_m = \text{Constant} = -9.4 \times 0.135 = -1.27$$

At radial distance of 'r'

$$C_{w4r} = \frac{\text{Constant}}{r_r}$$

$$C_{w4h} = \frac{\text{Constant}}{r_h}$$

$$= -\frac{1.27}{0.0675}$$

$$= -18.79 \text{ m/s}$$

And,

$$\beta_{4r} = \tan^{-1} \left(\frac{U_{R2r} - C_{w4r}}{C_a} \right)$$

$$\beta_{4h} = \tan^{-1} \left(\frac{U_{R2h} - C_{w4h}}{C_a} \right)$$

$$= \tan^{-1} \left(\frac{16.96 - (-18.79)}{44.4} \right)$$

$$= 38.83^\circ$$

$$\Delta\beta_h = \beta_{3h} - \beta_{4h}$$

$$= 63.38 - 38.83$$

$$= 24.54^\circ$$

So, if you will be using that we are able to calculate what will be our blade angles β_3 and β_4 for rotor 2, okay. So, here in this case my β_4 , that's what is coming 38.83 and my $\Delta\beta$, that's what is coming say 24.54, okay.

(Refer Slide Time: 28:10)

Free Vortex Design Method

$$\tan \alpha_{4h} = \frac{C_{w4h}}{C_a}$$

$$= \frac{18.79}{44.4}$$

$$\alpha_{4h} = 22.93^\circ$$

Power required

$$P = \frac{\dot{m} \times C_p \times \Delta T_{OR2}}{\eta_m \times \eta_c}$$

$$= \frac{6 \times 789.03}{0.75 \times 0.85}$$

$$= 7.43 \text{ kW}$$

Here,
 η_m = Mechanical efficiency
 η_c = Compressor efficiency

We know:
 $C_a = 44.4 \text{ m/s}$
 $C_{w4h} = -18.79 \text{ m/s}$

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Now, in line to that we will be able to calculate what will be our flow angles say α_4 .

$$\tan \alpha_{4h} = \frac{C_{w4h}}{C_a}$$

$$= \frac{18.79}{44.4}$$

$$\alpha_{4h} = 22.93^\circ$$

Power required,

$$P = \frac{\dot{m} \times C_p \times \Delta T_{OR2}}{\eta_m \times \eta_c}$$

$$= \frac{6 \times 789.03}{0.75 \times 0.85}$$

$$= 7.43 \text{ kW}$$

(Refer Slide Time: 28:19)

Free Vortex Design Method

From velocity triangle:

$$V_{3h} = \frac{U_{R2h}}{\sin \beta_{3h}}$$

$$= \frac{16.96}{\sin(63.83^\circ)}$$

$$= 99.12 \text{ m/s}$$

$$V_{4h} = \frac{U_{R2h} - C_{w4h}}{\sin \beta_{4h}}$$

$$= \frac{16.96 - (-18.79)}{\sin(38.83^\circ)}$$

$$= 57.02 \text{ m/s}$$

We know :

$U_{R2h} = 16.96 \text{ m/s}$

$\beta_{3h} = 63.83^\circ$

$C_{w4h} = -18.79 \text{ m/s}$

$\beta_{4h} = 38.83^\circ$

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Free Vortex Design Method

De-Haller's factor

$$DH_h = \frac{V_{4h}}{V_{3h}}$$

$$= \frac{57.02}{99.12}$$

$$= 0.58$$

$$C_{i4} = \frac{C_a}{\cos \alpha_{4h}}$$

$$= \frac{44.4}{\cos(22.93^\circ)}$$

$$= 48.23 \text{ m/s}$$

According to Carter's rule: Slop factor

$$m_h = 0.23 \left(\frac{2a}{c} \right)^2 + 0.1 \frac{(90 - \beta_{4h})}{50}$$

$$= 0.23(2 * 0.5)^2 + 0.1 \frac{(90 - (38.83))}{50}$$

$$= 0.28$$

We know :

$V_{3h} = 99.12 \text{ m/s}$

$V_{4h} = 57.02 \text{ m/s}$

$\alpha_{4h} = 22.93^\circ$

$\beta_{4h} = 38.83^\circ$

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Free Vortex Design Method

pitch $s_h = \frac{2\pi r_h}{Z_2}$
 $= \frac{2\pi \times 0.067}{17}$
 $= 0.02$

Solidity $\sigma_h = \frac{\text{Chord}}{\text{pitch}}$
 $= \frac{0.045}{0.02}$
 $= 1.8$

Diffusion Factor
 $DF_h = 1 - \frac{\cos\beta_{3h}}{\cos\beta_{4h}} + \frac{\cos\beta_{3h}}{2 \times \sigma_h} (\tan\beta_{3h} - \tan\beta_{4h})$
 $= 1 - \frac{\cos(63.38)}{\cos(38.83)} + \frac{\cos(63.38)}{2 \times 1.8} (\tan(63.38) - \tan(38.83))$
 $DF_h = 0.57$

We have number of blades for Rotor 2
 $Z_2 = 17$

We know :
 $r_h = 0.067m$
 $c = 0.045m$
 $\beta_{3h} = 63.38^\circ$
 $\beta_{4h} = 38.83^\circ$

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We can calculate what will be our say relative velocity components. So, my V_3 and V_4 based on our velocity triangle we can calculate.

From velocity triangle,

$$V_{3h} = \frac{U_{R2h}}{\sin\beta_{3h}}$$

$$= \frac{16.96}{\sin(63.83^\circ)}$$

$$= 99.12 \text{ m/s}$$

$$V_{4h} = \frac{U_{R2h} - C_{w4h}}{\sin\beta_{4h}}$$

$$= \frac{16.96 - (-18.79)}{\sin(38.38^\circ)} = 57.02 \text{ m/s}$$

Once we are calculating this, this is what will be helpful to us in order to calculate the De-Haller's Factor, that's what is coming 0.58, okay.

De – Haller's factor,

$$DH_h = \frac{V_{4h}}{V_{3h}}$$

$$= \frac{57.02}{99.12}$$

$$= 0.58$$

$$\begin{aligned}
C_{4h} &= \frac{C_a}{\cos \alpha_{4h}} \\
&= \frac{44.4}{\cos(22.93^\circ)} \\
&= 48.23 \text{ m/s}
\end{aligned}$$

Now, in order to calculate our Diffusion Factor, we will be doing our calculation for slope Factor, that's what is coming 0.28 for the Hub and diffusion factor, that's what is coming to be 0.57.

According to Carter's rule: Slop factor,

$$\begin{aligned}
m_h &= 0.23 \left(\frac{2a}{c} \right)^2 + 0.1 \frac{(90 - \beta_{4h})}{50} \\
&= 0.23(2 \times 0.5)^2 + \frac{0.1(90 - (38.83))}{50} \\
&= 0.28
\end{aligned}$$

$$\begin{aligned}
\text{pitch } s_h &= \frac{2\pi r_h}{Z_2} \\
&= \frac{2\pi \times 0.067}{17} \\
&= 0.02
\end{aligned}$$

$$\begin{aligned}
\text{Solidity, } \sigma_h &= \frac{\text{Chord}}{\text{pitch}} \\
&= \frac{0.045}{0.02} \\
&= 1.8
\end{aligned}$$

Diffusion Factor,

$$\begin{aligned}
DF_h &= 1 - \frac{\cos \beta_{3h}}{\cos \beta_{4h}} + \frac{\cos \beta_{3h}}{2 \times \sigma_h} (\tan \beta_{3h} - \tan \beta_{4h}) \\
&= 1 - \frac{\cos(63.38)}{\cos(38.83)} + \frac{\cos(63.38)}{2 \times 1.8} (\tan(63.38) - \tan(38.83))
\end{aligned}$$

$$\therefore DF_h = 0.57$$

Now, this is nothing but it is a repetition of calculation we need to be very careful when we are doing our calculation, we need to select the radius as per the station. So, it is preferred that as and when you are doing your calculation, you just carry forward the data what all that's what is available here. So, here you can see I am putting my r_h , I am putting my chord, I am putting my $\beta_2, \beta_3, \beta_4$. So, that's what will be helping us in order to avoid the mistakes that will be happening when we are doing pen paper calculation. Anyway, this all calculation we will be cross verifying using the Excel sheet. So, there is no problem at all for this kind of design calculations, okay.

(Refer Slide Time: 29:44)

Free Vortex Design Method

Let's assume Incidence angle i at hub as $+2^\circ$.

Camber angle

$$\theta_h = \frac{(\Delta\beta_h - i_h)}{(1 - \frac{m_h}{\sqrt{\sigma_h}})} = \frac{(24.54 - 2)}{(1 - \frac{0.28}{\sqrt{1.8}})} = 28.57^\circ$$

Deviation angle

$$\delta_h = \frac{m_h \theta_h}{\sqrt{\sigma_h}} = \frac{0.28 \times 28.57}{\sqrt{1.8}} = 6.02^\circ$$

Stagger angle

$$\zeta_h = \beta_{3h} - i_h - \frac{\theta_h}{2} = 63.39 - 2 - \frac{28.57}{2} = 47.90^\circ$$

We know :
 $\beta_{3h} = 63.38^\circ$
 $\Delta\beta_h = 24.54^\circ$
 $\sigma_h = 1.8$
 $m_h = 0.28$

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Free Vortex Design Method

Calculations at tip

Let's assume $C_2 = C_3$

$\alpha_{2t} = \alpha_{3t}$
 $\& C_{w2t} = -C_{w3t}$

From velocity triangle

$$\beta_{3t} = \tan^{-1} \left(\frac{U_{R2t} - C_{w3t}}{C_a} \right)$$

$$= \tan^{-1} \left(\frac{50.89 + 23.88}{44.4} \right)$$

$$= 59.29^\circ$$

We know :
 $C_a = 44.4 \text{ m/s}$
 $U_{R2t} = 50.89 \text{ m/s}$
 $C_{w3t} = -23.88 \text{ m/s}$

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Now, based on that we can do our calculation for say angle θ , deviation angle and the stagger angle, at the Hub station.

Let's assume *Incidence angle 'i' at hub as + 2°*

Camber angle,

$$\theta_h = \frac{(\Delta\beta_h - i_h)}{\left(1 - \frac{m_h}{\sqrt{\sigma_h}}\right)} = \frac{(24.54 - 2)}{1 - \frac{0.28}{\sqrt{1.8}}}$$

$$\therefore \theta_h = 28.57^\circ$$

Deviation angle,

$$\delta_h = \frac{m_h \theta_h}{\sqrt{\sigma_h}} = \frac{0.28 \times 28.57}{\sqrt{1.8}}$$

$$\therefore \delta_h = 6.02^\circ$$

Stagger angle,

$$\zeta_h = \beta_{3h} - i_h - \frac{\theta_h}{2} = 63.39 - 2 - \frac{28.57}{2}$$

$$\therefore \zeta_h = 47.90^\circ$$

Similarly, we can do our calculation for say our tip region. So, you know, in line to what all we have discussed for the Hub station, we will be opting same for the tip station.

So, the thing is the logic will be my absolute velocity at the exit of my rotor 1 at the tip, that's what is equal to absolute velocity at the entry of my rotor 2 at the tip and that is the reason my α_2 and α_3 , that's what will be coming to be same, okay. And remaining design methodology, that's what will be remaining same. So, be careful, here in this case, the calculation for β_3 , that's what is mainly been based on what we say what whirl component we are calculating, okay.

Calculations at tip

Let's assume

$$C_{2t} = C_{3t}$$

$$\alpha_{2t} = \alpha_{3t}$$

$$\& C_{w2t} = -C_{w3t}$$

From velocity triangle,

$$\beta_{3t} = \tan^{-1} \left(\frac{U_{R2t} - C_{w3t}}{C_a} \right)$$

$$= \tan^{-1} \left(\frac{50.89 + 23.88}{44.4} \right)$$

$$\beta_{3t} = 59.29^\circ$$

(Refer Slide Time: 30:44)

Free Vortex Design Method

Calculations at other stations

We know that for Free vortex method

$$C_{w4m} \cdot r_m = \text{Constant} = -9.4 \times 0.135 = -1.27$$

At radial distance of 'r' And

$$C_{w4r} = \frac{\text{Constant}}{r_r} \quad \beta_{4r} = \tan^{-1} \left(\frac{U_{R2r} - C_{w4r}}{C_a} \right)$$

$$C_{w4t} = \frac{\text{Constant}}{r_t} \quad \beta_{4t} = \tan^{-1} \left(\frac{U_{R2t} - C_{w4t}}{C_a} \right)$$

$$= \frac{-1.27}{0.2025} \quad = \tan^{-1} \left(\frac{50.89 - (-6.26)}{44.4} \right)$$

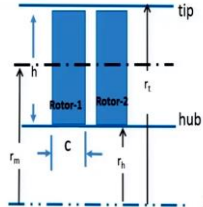
$$= -6.26 \text{ m/s} \quad = 52.15^\circ$$

$$\Delta\beta = \beta_{3t} - \beta_{4t}$$

$$= 59.29 - 52.15$$

$$= 7.14^\circ$$

We know :

$$\beta_{3t} = 59.29^\circ$$


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We know that for Free vortex method,

$$C_{w4m} \cdot r_m = \text{Constant} = -9.4 \times 0.135 = -1.27$$

At radial distance of 'r'

$$C_{w4r} = \frac{\text{Constant}}{r_r}$$

$$C_{w4t} = \frac{\text{Constant}}{r_t}$$

$$= -\frac{1.27}{0.2025}$$

$$= -6.26 \text{ m/s}$$

And,

$$\beta_{4r} = \tan^{-1} \left(\frac{U_{R2r} - C_{w4r}}{C_a} \right)$$

$$\begin{aligned}\beta_{4t} &= \tan^{-1} \left(\frac{U_{R2t} - C_{w4t}}{C_a} \right) \\ &= \tan^{-1} \left(\frac{50.89 - (-6.26)}{44.4} \right) \\ &= 52.15^\circ \\ \Delta\beta_t &= \beta_{3t} - \beta_{4t} \\ &= 59.29 - 52.15 \\ &= 7.14^\circ\end{aligned}$$

Free Vortex Design Method

$\tan \alpha_{4t} = \frac{C_{w4t}}{C_a}$
 $= \frac{6.26}{44.4}$
 $\alpha_{4t} = 8.03^\circ$

Power required
 $P = \frac{\dot{m} \times C_p \times \Delta T_{OR2}}{\eta_m \times \eta_c}$
 $= \frac{6 \times 789.03}{0.75 \times 0.85}$
 $= 7.43 \text{ kW}$

Here,
 η_m = Mechanical efficiency
 η_c = Compressor efficiency

We know :
 $C_a = 44.4 \text{ m/s}$
 $C_{w4t} = -6.26 \text{ m/s}$

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$$\begin{aligned}\tan \alpha_{4t} &= \frac{C_{w4t}}{C_a} \\ &= \frac{6.26}{44.4} \\ \alpha_{4t} &= 8.03^\circ\end{aligned}$$

Power required,

$$\begin{aligned}P &= \frac{\dot{m} \times C_p \times \Delta T_{OR2}}{\eta_m \times \eta_c} \\ &= \frac{6 \times 789.03}{0.75 \times 0.85} \\ &= 7.43 \text{ kW}\end{aligned}$$

Free Vortex Design Method

From velocity triangle:

$$V_{3t} = \frac{U_{R2t}}{\sin \beta_{3t}}$$

$$= \frac{50.89}{\sin(59.29^\circ)}$$

$$= 86.97 \text{ m/s}$$

$$V_{4t} = \frac{U_{R2t} - C_{w4t}}{\sin \beta_{4t}}$$

$$= \frac{50.89 - (-6.26)}{\sin(52.15^\circ)}$$

$$= 72.39 \text{ m/s}$$

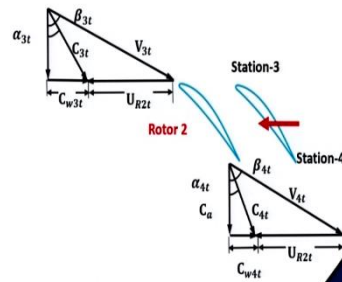
We know :

$$U_{R2t} = 50.89 \text{ m/s}$$

$$\beta_{3t} = 59.29^\circ$$

$$C_{w4t} = -6.26 \text{ m/s}$$

$$\beta_{4t} = 52.15^\circ$$



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From velocity triangle,

$$V_{3t} = \frac{U_{R2t}}{\sin \beta_{3t}}$$

$$= \frac{50.89}{\sin(59.29^\circ)}$$

$$= 86.97 \text{ m/s}$$

$$V_{4t} = \frac{U_{R2t} - C_{w4t}}{\sin \beta_{4t}}$$

$$= \frac{50.89 - (-6.26)}{\sin(52.15^\circ)} = 72.39 \text{ m/s}$$

De – Haller's factor,

$$DH_t = \frac{V_{4t}}{V_{3t}}$$

$$= \frac{72.39}{86.97}$$

$$= 0.83$$

$$C_{4t} = \frac{C_a}{\cos \alpha_{4t}}$$

$$= \frac{44.4}{\cos(8.03^\circ)}$$

$$= 44.86 \text{ m/s}$$

According to Carter's rule: Slop factor,

$$\begin{aligned}
 m_t &= 0.23 \left(\frac{2a}{c} \right)^2 + 0.1 \frac{(90 - \beta_{4t})}{50} \\
 &= 0.23(2 \times 0.5)^2 + \frac{0.1(90 - (52.15))}{50} \\
 &= 0.29
 \end{aligned}$$

Free Vortex Design Method

pitch $s_t = \frac{2\pi r_t}{Z_2}$ We have number of blades for Rotor 2 $Z_2=17$

$$= \frac{2\pi \times 0.2025}{17} = 0.07$$

Solidity $\sigma_t = \frac{\text{Chord}}{\text{pitch}} = \frac{0.045}{0.07} = 0.6$

Diffusion Factor

$$DF_t = 1 - \frac{\cos \beta_{3t}}{\cos \beta_{4t}} + \frac{\cos \beta_{3t}}{2 \times \sigma_t} (\tan \beta_{3t} - \tan \beta_{4t})$$

$$= 1 - \frac{\cos(59.29)}{\cos(52.15)} + \frac{\cos(59.29)}{2 \times 0.6} (\tan(59.29) - \tan(52.15))$$

$$DF_t = 0.34$$

We know :

- $r_t = 0.2025m$
- $c = 0.045m$
- $\beta_{3t} = 59.29^\circ$
- $\beta_{4t} = 52.15^\circ$

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$$\begin{aligned}
 \text{pitch } s_t &= \frac{2\pi r_t}{Z_2} \\
 &= \frac{2\pi \times 0.2025}{17} \\
 &= 0.07
 \end{aligned}$$

$$\begin{aligned}
 \text{Solidity, } \sigma_t &= \frac{\text{Chord}}{\text{pitch}} \\
 &= \frac{0.045}{0.07} \\
 &= 0.6
 \end{aligned}$$

Diffusion Factor,

$$DF_t = 1 - \frac{\cos \beta_{3t}}{\cos \beta_{4t}} + \frac{\cos \beta_{3t}}{2 \times \sigma_t} (\tan \beta_{3t} - \tan \beta_{4t})$$

$$= 1 - \frac{\cos(59.29)}{\cos(52.15)} + \frac{\cos(59.29)}{2 \times 0.6} (\tan(59.29) - \tan(52.15))$$

$$\therefore DF_t = 0.34$$

Now, when we are opting for our free vortex design concept, we can do our calculation for $\Delta\beta$, we can do our calculation what we say in sense of relative velocity components, we can calculate what will be our De-Haller's number, okay. And, based on our 'm' parameter and the diffusion factor parameter, that's what can be calculated using the calculation of solidity and, you know, pitch. Here this diffusion factor near the tip region, that's what is coming to be 0.34 for rotor 2.

(Refer Slide Time: 31:24)

Free Vortex Design Method

Let's assume Incidence angle i at tip as -2° .

Camber angle

$$\theta_t = \frac{(\Delta\beta_t - i_t)}{\left(1 - \frac{m_t}{\sqrt{\sigma_t}}\right)} = \frac{(7.14 + 2)}{\left(1 - \frac{0.29}{\sqrt{0.6}}\right)} = 14.64^\circ$$

Deviation angle

$$\delta_t = \frac{m_t \theta_t}{\sqrt{\sigma_t}} = \frac{0.29 \times 14.64}{\sqrt{0.6}} = 5.5^\circ$$

Stagger angle

$$\zeta_t = \beta_{3t} - i_t - \frac{\theta_t}{2} = 59.29 + 2 - \frac{14.64}{2} = 53.97^\circ$$

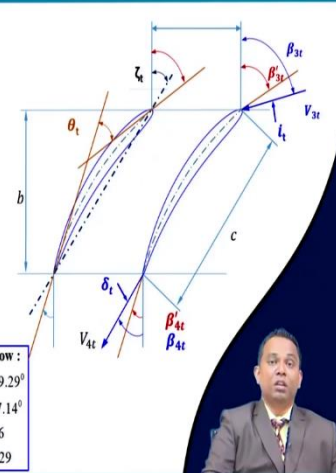
We know :

$\beta_{3t} = 59.29^\circ$

$\Delta\beta = 7.14^\circ$

$\sigma_t = 0.6$

$m_t = 0.29$



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Now, these are the calculation for my camber angle, deviation angle, as well as for the stagger angle.

Let's assume Incidence angle 'i' at tip as -2°

Camber angle,

$$\theta_t = \frac{(\Delta\beta_t - i_t)}{\left(1 - \frac{m_t}{\sqrt{\sigma_t}}\right)} = \frac{(7.14 + 2)}{1 - \frac{0.29}{\sqrt{0.6}}}$$

$$\therefore \theta_t = 14.64^\circ$$

Deviation angle,

$$\delta_t = \frac{m_t \theta_t}{\sqrt{\sigma_t}} = \frac{0.29 \times 14.64}{\sqrt{0.6}}$$

$$\therefore \delta_t = 5.5^\circ$$

Stagger angle,

$$\zeta_t = \beta_{3t} - i_t - \frac{\theta_t}{2} = 59.29 + 2 - \frac{14.64}{2}$$

$$\therefore \zeta_t = 53.97^\circ$$

(Refer Slide Time: 31:34)

Free Vortex Design Method

| | Rotor-2 | | |
|------------------------------------|---------|--------|--------|
| Solution | s-Hub | 8-Mid | 11-Tip |
| r | 0.07 | 0.13 | 0.20 |
| Peripheral speed U | 15.86 | 23.93 | 50.89 |
| Axial velocity C _a | 44.42 | 44.42 | 44.42 |
| Total inlet temp T ₀₁ | 300 | 300 | 300 |
| P ₀₁ | 102525 | 102525 | 102525 |
| #P _{stages} | 800 | 800 | 800 |
| m ₀₁ (kg/s) | 1.01 | 1.01 | 1.01 |
| ΔT _{stg} | 0.79 | 0.79 | 0.79 |
| C ₃ =C ₂ | 84.29 | 57.05 | 50.42 |
| α ₂ (deg) | 58.21 | 38.89 | 28.27 |
| C _u =C _{u2} | -71.85 | -35.82 | -23.88 |
| β ₁ (deg) | 83.38 | 57.51 | 58.29 |
| w(C ₁ ΔT ₁) | 789.03 | 789.03 | 789.03 |
| C _{av} | -0.40 | | |
| Constant | -1.27 | -1.27 | -1.27 |
| C _{av} | -18.79 | -4.40 | -8.26 |
| tan β ₁ | 0.80 | 0.86 | 1.28 |
| β ₁ (deg) | 38.83 | 44.28 | 52.15 |
| tan α ₂ | 0.42 | 0.21 | 0.14 |
| α ₂ (deg) | 22.83 | 11.94 | 8.03 |
| Δβ | 24.54 | 13.22 | 7.14 |
| Power (kW) | 7.43 | 7.43 | 7.43 |
| V ₁ | 99.12 | 52.70 | 86.97 |
| V ₂ | 57.02 | 62.05 | 72.39 |
| C ₁ | 48.23 | 45.40 | 44.88 |
| DH | 0.58 | 0.75 | 0.83 |
| m | 0.28 | 0.28 | 0.28 |
| Pitch s | 0.02 | 0.05 | 0.07 |
| Solidity σ | 1.80 | 0.90 | 0.80 |
| DF | 0.57 | 0.43 | 0.34 |
| Incidence | 2.00 | 0.00 | -2.00 |
| Camber angle | 26.57 | 18.18 | 14.64 |
| Deviation angle | 6.02 | 5.96 | 5.50 |
| Stagger angle | 47.09 | 47.92 | 53.97 |

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Now, compiling all this data, okay; say with the interest of time I am not repeating the same words what all we have done since these slides will be available with you just cross verify with the numbers, okay. And, this is what we will be making in sense of at different stations say Hub, mid, and tip station, okay.

Now, these constants what we have calculated it is -1.27 . We have done our calculation for diffusion factor, you can see near the Hub region, I will be having diffusion factor to be large, near a tip region diffusion factor that's what is coming to be lower, okay. $\Delta\beta$ is also coming in the range of 24° at the Hub and 7° near the tip region.

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Free Vortex Design Method

| Solution | Rotor-2 | | | | | | | | | | |
|---------------------------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 1-Hub | 2.00 | 3.00 | 4.00 | 5.00 | 6-Hub | 7.00 | 8.00 | 9.00 | 10.00 | 11-Tip |
| r | 0.07 | 0.08 | 0.09 | 0.11 | 0.12 | 0.13 | 0.15 | 0.16 | 0.18 | 0.19 | 0.20 |
| Peripheral speed U | 16.90 | 20.36 | 23.75 | 27.14 | 30.54 | 33.93 | 37.32 | 40.72 | 44.11 | 47.50 | 50.89 |
| Axial velocity C_a | 44.42 | 44.42 | 44.42 | 44.42 | 44.42 | 44.42 | 44.42 | 44.42 | 44.42 | 44.42 | 44.42 |
| Total inlet temp T_{02} | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 |
| P_{02} | 102525 | 102525 | 102525 | 102525 | 102525 | 102525 | 102525 | 102525 | 102525 | 102525 | 102525 |
| ΔP_{02} | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 |
| $w_{02}(P_{02}/\rho)$ | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| ΔT_{02} | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 |
| C_{3aD2} | 84.29 | 74.41 | 67.75 | 63.06 | 59.83 | 57.05 | 55.07 | 53.51 | 52.28 | 51.25 | 50.42 |
| α_{3aD2} (deg) | 59.21 | 53.38 | 49.05 | 45.24 | 41.97 | 39.29 | 36.28 | 33.91 | 31.92 | 29.95 | 28.27 |
| C_{u2}/C_{u1} | 71.65 | 69.79 | 67.18 | 64.78 | 62.50 | 60.32 | 58.27 | 56.35 | 54.56 | 52.89 | 51.43 |
| β_1 (deg) | 63.38 | 60.98 | 59.34 | 58.30 | 57.73 | 57.51 | 57.56 | 57.81 | 58.21 | 58.71 | 59.29 |
| $w(C_a \Delta T_1)$ | 789.03 | 789.03 | 789.03 | 789.03 | 789.03 | 789.03 | 789.03 | 789.03 | 789.03 | 789.03 | 789.03 |
| C_{02} | | | | | -8.40 | | | | | | |
| Constant | -1.27 | -1.27 | -1.27 | -1.27 | -1.27 | -1.27 | -1.27 | -1.27 | -1.27 | -1.27 | -1.27 |
| $\tan \beta_1$ | 18.78 | 11.58 | 7.82 | 5.75 | 4.44 | 3.40 | 2.54 | 1.85 | 1.32 | 0.91 | 0.65 |
| β_1 (deg) | 38.83 | 30.04 | 26.92 | 24.20 | 21.89 | 19.82 | 17.94 | 16.21 | 14.61 | 13.11 | 11.71 |
| $\tan \alpha_1$ | 0.42 | 0.35 | 0.30 | 0.26 | 0.24 | 0.21 | 0.19 | 0.18 | 0.16 | 0.15 | 0.14 |
| α_1 (deg) | 22.93 | 19.42 | 16.81 | 14.81 | 13.23 | 11.94 | 10.89 | 10.00 | 9.24 | 8.59 | 8.02 |
| L/D | 24.54 | 21.94 | 19.41 | 17.10 | 15.04 | 13.22 | 11.64 | 10.27 | 9.08 | 8.04 | 7.14 |
| Power (kW) | 7.43 | 7.43 | 7.43 | 7.43 | 7.43 | 7.43 | 7.43 | 7.43 | 7.43 | 7.43 | 7.43 |
| V_1 | 99.12 | 91.58 | 87.10 | 84.53 | 83.19 | 82.70 | 82.81 | 83.38 | 84.31 | 85.53 | 86.97 |
| V_2 | 57.92 | 57.19 | 57.92 | 59.04 | 60.43 | 62.05 | 63.85 | 65.80 | 67.89 | 70.09 | 72.39 |
| C_u | 48.23 | 47.10 | 48.40 | 49.95 | 51.83 | 54.00 | 56.33 | 58.80 | 61.40 | 64.02 | 66.66 |
| DH | 0.58 | 0.62 | 0.65 | 0.70 | 0.73 | 0.75 | 0.77 | 0.79 | 0.81 | 0.82 | 0.83 |
| m | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 |
| Pitch s | 0.02 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 | 0.05 | 0.06 | 0.06 | 0.07 | 0.07 |
| Solidity σ | 1.80 | 1.50 | 1.29 | 1.13 | 1.00 | 0.90 | 0.82 | 0.75 | 0.69 | 0.64 | 0.60 |
| DF | 0.57 | 0.54 | 0.50 | 0.47 | 0.45 | 0.43 | 0.41 | 0.39 | 0.37 | 0.35 | 0.34 |
| Inclination | 2.00 | 1.90 | 1.80 | 1.69 | 1.60 | 1.50 | 1.40 | 1.30 | 1.20 | 1.10 | 1.00 |
| Camber angle | 28.57 | 26.59 | 24.50 | 22.52 | 20.74 | 19.18 | 17.86 | 16.76 | 15.87 | 15.17 | 14.64 |
| Deviation angle | 6.02 | 6.25 | 6.29 | 6.22 | 6.10 | 5.96 | 5.82 | 5.69 | 5.59 | 5.53 | 5.50 |
| Stagger angle | 47.09 | 46.08 | 45.89 | 46.24 | 46.96 | 47.82 | 48.03 | 50.23 | 51.47 | 52.73 | 53.87 |

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Now, this is what is representing the whole Excel sheet, it is preferred you just go through this lecture thereafter you just cross verify with the numbers, try to make Excel sheet program and do modification, and check with whether you are getting what all we are discussing with, and that's what will be building your confidence, okay.

Now, this is what we were discussing in sense of variation of different parameters, say diffusion factor, camber angle, De-Haller's number. The question may arise in your mind, sir, for stage design, you were discussing about degree of reaction, here the degree of reaction term, that's what is not coming.

Be careful, degree of reaction term, that's what is applicable when we are having rotor and stator combination, when we are talking about the stage, okay. Here we are having both the rotors and that is the reason why degree of reaction terminology that's what is not applicable for contra rotating fan. There are some open literatures where people they are trying to opt for say degree of reaction derivation but it seems fundamentally wrong, because degree of reaction terminology we are defining in sense of what is happening with rotor and what is happening with the stage, okay.

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Corrected Flow Angles

At mid section
 Let's assume $\delta_{assume} = 4^\circ$

Corrected deviation
 $\delta_{m,corr} = 5.96 + 4 = 9.96^\circ$

Corrected camber
 $\theta_{corr} = \Delta\beta + \delta_{corr} - i$
 $= 13.22 + 9.96 - 0$
 $\theta_{m,corr} = 23.18^\circ$

Corrected stagger
 $\zeta_{m,corr} = 57.51 - 0 - \frac{23.18}{2}$
 $\zeta_{m,corr} = 45.92^\circ$

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Corrected Flow Angles

| Solution | Rotor-2 | | | | | | | | | | |
|-------------------------|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| | 1-Hub | 2.00 | 3.00 | 4.00 | 5.00 | 6-Mid | 7.00 | 8.00 | 9.00 | 10.00 | 11-Tip |
| r | 0.07 | 0.08 | 0.09 | 0.11 | 0.12 | 0.13 | 0.15 | 0.16 | 0.18 | 0.19 | 0.20 |
| Incidence | 2.00 | 1.80 | 1.20 | 0.80 | 0.40 | 0.00 | -0.40 | -0.80 | -1.20 | -1.80 | -2.00 |
| Camber angle | 28.57 | 28.59 | 24.50 | 22.52 | 20.74 | 19.10 | 17.86 | 16.78 | 15.87 | 15.17 | 14.64 |
| Deviation angle | 6.02 | 6.25 | 6.29 | 6.22 | 6.10 | 5.96 | 5.82 | 5.69 | 5.59 | 5.53 | 5.50 |
| Stagger angle | 47.09 | 48.08 | 45.89 | 46.24 | 46.96 | 47.82 | 48.03 | 50.23 | 51.47 | 52.73 | 53.97 |
| Deviation (assumed) | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 |
| Corrected deviation | 10.02 | 10.25 | 10.29 | 10.22 | 10.10 | 9.96 | 9.82 | 9.69 | 9.59 | 9.53 | 9.50 |
| Corrected camber | 32.57 | 30.59 | 28.50 | 26.52 | 24.74 | 23.10 | 21.86 | 20.78 | 19.87 | 19.17 | 18.64 |
| Corrected Stagger angle | 45.09 | 44.08 | 43.89 | 44.24 | 44.96 | 45.92 | 47.03 | 48.23 | 49.47 | 50.73 | 51.97 |

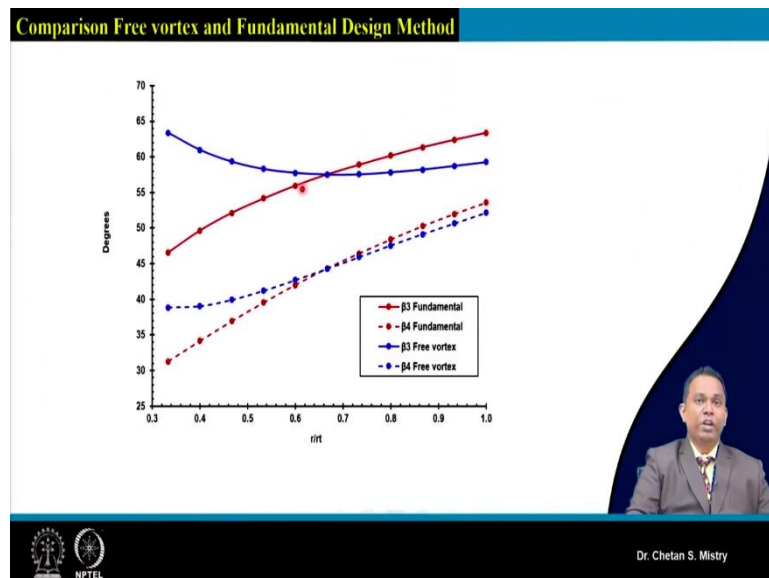
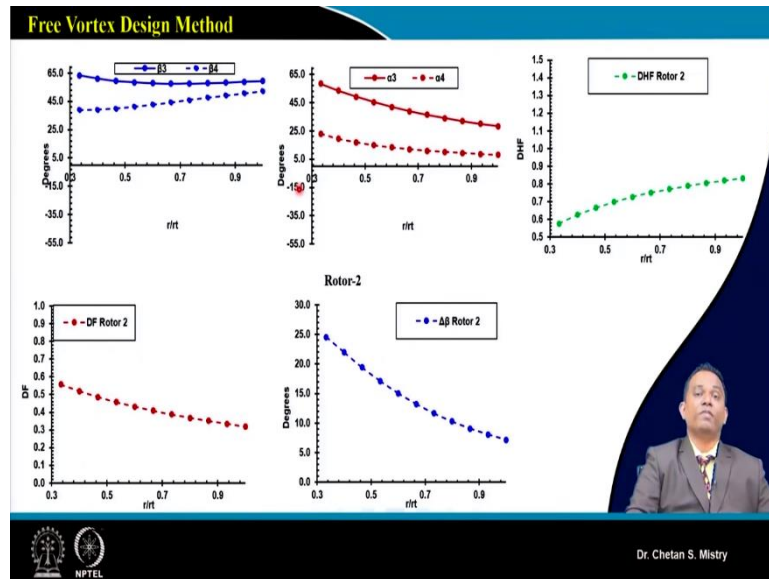
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Generation of blades – Free vortex method

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Now, we have done our calculation with assuming the deviation angle to be 4° even for rotor 2 and based on that we have done the calculation for the corrected camber as well as say corrected stagger. And this is what is coming out as rotor 2 blade, okay. Now, here in this case, if we look at my twist for rotor 2 blade seems to be lower. When we are comparing here, for our fundamental design approach, the twist, that's what is coming to be slightly larger when we are comparing these two design approaches, okay.

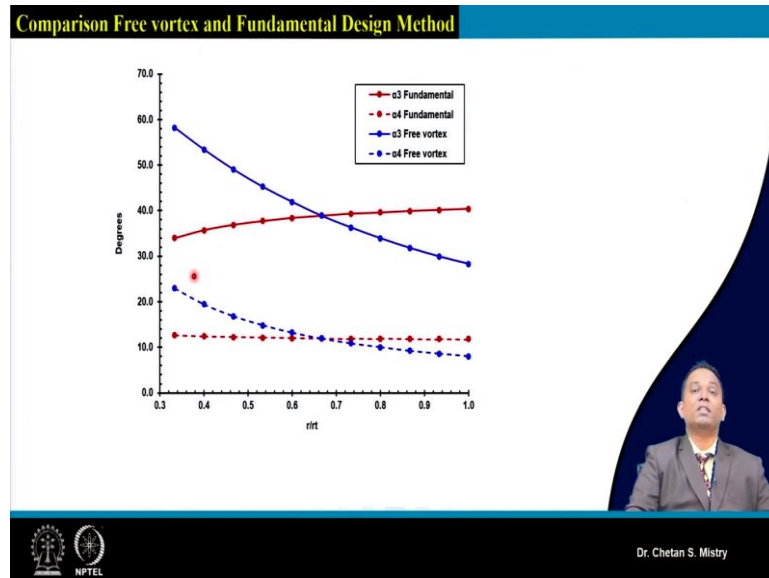
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Now, this is what is representing how my flow parameters, how my blade angles, diffusion factor, De-Haller's Factor, that's what is varying, we are more interested in sense of comparison. So, here in this case, if we look at, this red color one, that's what is representing the fundamental design approach for $\Delta\beta$, okay, for rotor 2.

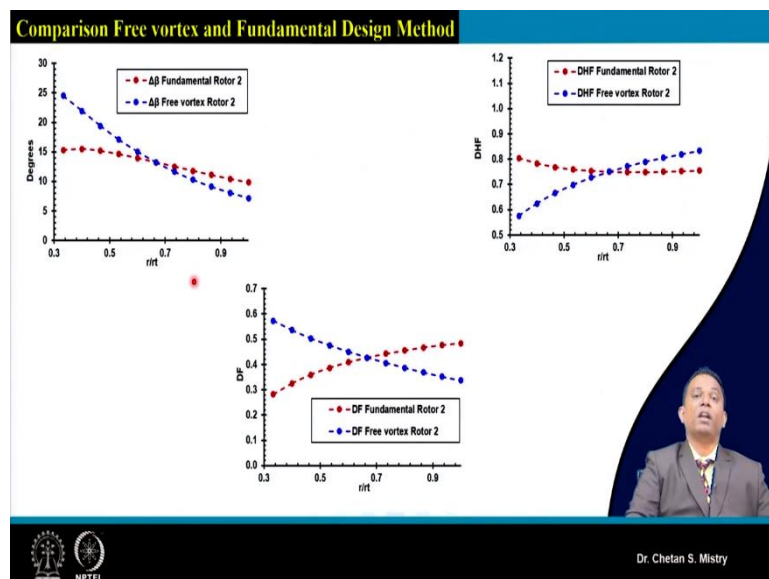
So, here this is what is representing, how my $\Delta\beta$, that's what is varying near the Hub region and near the tip region. If we look at carefully, my $\Delta\beta$ for the free vortex near the Hub that's what is coming to be larger compared to what all we are discussing for the fundamental design approach, okay.

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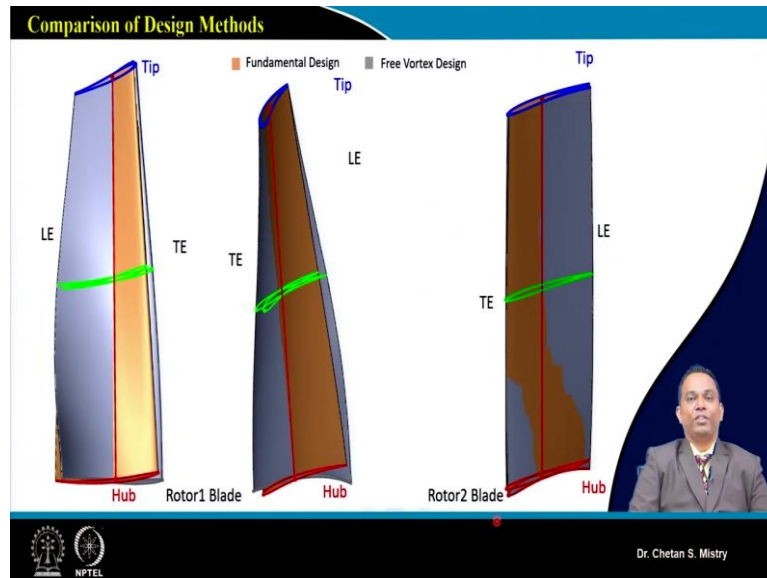
This is what is representing, how my say absolute flow angle, that's what is varying; and it is interestingly we can see my α_4 that is nothing but the angle with which my flow that's what is coming out from rotor 2 or my contra rotating stage that's what is having say angle in the range roughly see 11° or 12° . When we are opting for fundamental approach and when we are opting for the free vortex approach, this angle is coming to be large near the Hub region, okay.

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Now, this is what is representing how my $\Delta\beta$, my De-Haller's factor and diffusion factor that's what is varying along the span.

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So, finally if we look at the two-design approach or methodology what we have discussed, say fundamental design approach and free vortex design approach, if you are comparing the blade shapes you can see here this is what is representing my rotor 1 blade, okay; this is what is a view from one side this is what is from the other side.

And we can see carefully near the Hub region for the free vortex concept we are having $\Delta\beta$, that's what is coming to be large. That means, I will be having higher turning of the flow that's what is happening near the Hub and because of my free vortex concept, I will be having my blade to be highly twisted blade, all the way from Hub to say tip region. In line to that when we are opting for say free design concept and say for fundamental design approach for say rotor 2, not much variation that's what can be seen here, okay.

Now, this is what is all we need to discuss with, mainly the variation that's what is happening near the Hub region, that's what is creating the design to be more challenging, both for rotor 1 as well as for rotor 2. Specially when we are discussing at low speed compressor design, the design for the Hub or Hub region that's what is always a challenging task. As we have discussed earlier, this design methodology, that's what is say for this case we have assumed rotational speed for both the rotors to be same, you can go with variation of rotational speed maybe you can go with say higher rotational speed of rotor 1 or maybe higher rotational speed of rotor 2 compared to each other, okay; that is possible, you can go with this, okay.

So, many design possibilities what all we have, for contra rotating fan. Now, in sense of having this to be applicable for many purposes say for future EVTOL engines, say for futures tunnel fans, okay. So, looking to all the applications, it seems like more focus or more design calculation or more development need to be done for the contra rotating fan concept. So, here we are stopping with, I am sure, now based on what all we have discussed about the design methodology for fundamental design or fundamental approach and free view vortex design concept, you are able to design say low speed contra rotating fan. Thank you very much for your kind attention, see you in the next lecture.