Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 47
Design of Low Speed Contra Rotating Fan (Contd)

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Hello, and welcome to lecture 47. We are discussing design of low speed contra rotating fan.

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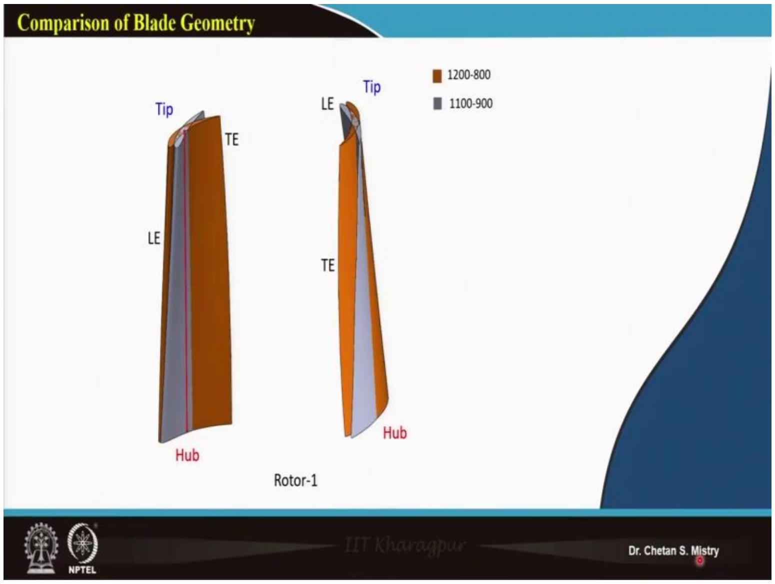

So, in last lecture, we started discussing about design of rotor-1. We have done our calculation at the mid station, we have done our calculation at the hub station, as well as we have done our calculation at a tip station. Then we were discussing how do we distribute our total pressure along the span at three different station, such that we will take care of what all constraints we have defined with.

Maybe in sense of say diffusion factor, De-Haller's factor, my camber angle, we will put an eye for this kind of configurations. Then, we have discussed about the comparison of say two different rotors; one, that's what is say having total pressure ratio expected to be 1100 Pa, and for presentation design, this is what is say 1200 Pa.

And we realize, when we are increasing our total pressure ratio expected from rotor-1, my blade twist that will be coming to be slightly on higher side. So here, this is what we have discussed for say 1100 and 1200 Pa pressure rise. You can see for 1200 Pa, near the tip region, we are having this angle or say my camber, that's what is coming to be slightly on higher side. Now, today, we will be discussing about the design for rotor-2, okay.

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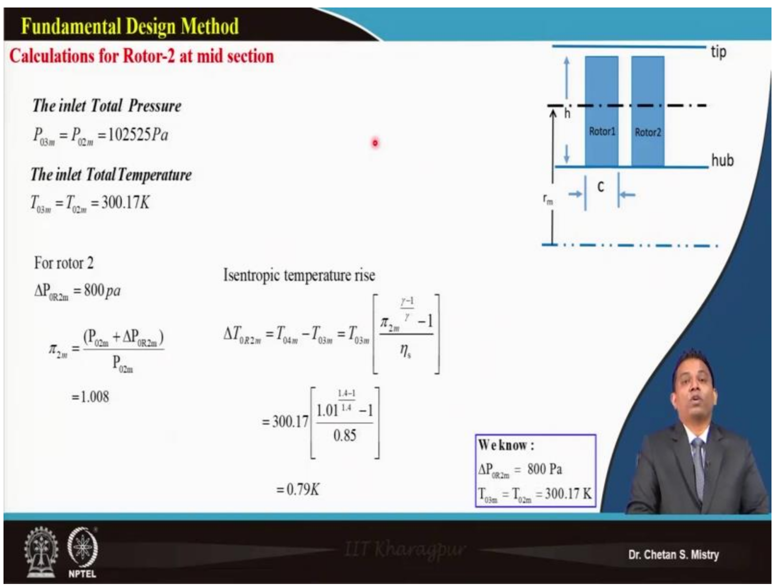

Now, say when we are talking about the design for rotor-2, we are looking for say total pressure at the entry of rotor-2 and total temperature at the entry of rotor-2. So, we can say my inlet pressure, that's what will be equal to say P_{02}, that's what is equal to P_{03}, okay. Same way, we can say, our temperature T_{03} at the mid station, that's what is equal to T_{02}.

So, you can say, my station 2 is nothing but that's what is representing exit of my rotor-1. And station 3, that's what is representing my entry of rotor-2.

So here in this case, if we are considering this as my pressure, we can say, we are expecting our pressure rise of 800 Pa at the mid station as per our design requirement. So, I will be calculating with say my pressure ratio of rotor-2 at the mid station, and that's what is coming 1.008. Same way, we can do our calculation for say ΔT_0, and this ΔT_0 for rotor-2 at mid station, that's what is coming say 0.79 K, okay.

$The\ inlet\ Total\ Pressure$

$$P_{03m} = P_{02m} = 102525\ Pa$$

$The\ inlet\ Total\ temperature$

$$T_{03m} = T_{02m} = 300.17\ K$$

$For\ rotr - 2,$

$$\Delta P_{0R2m} = 800\ Pa$$

$$\pi_{2m} = \frac{P_{02m} + \Delta P_{0R2m}}{P_{02m}}$$

$$= 1.008$$

$Isentropic\ temperature\ rise,$

$$\Delta T_{0R2m} = T_{04m} - T_{03m} = T_{03m}\left[\frac{\pi_{2m}^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s}\right]$$

$$= 300.17\left[\frac{\left(1.01^{\frac{1.4-1}{1.4}} - 1\right)}{0.85}\right]$$

$$= 0.79\ K$$

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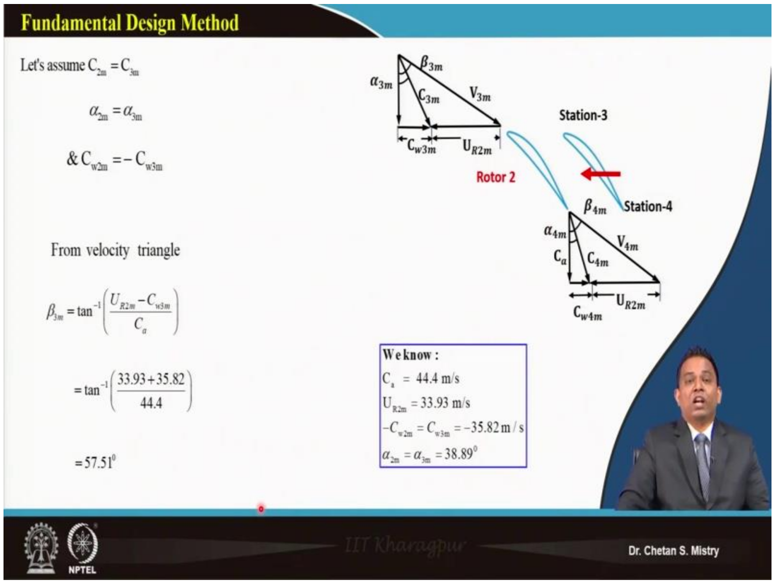

What we are doing here? That's what is say we are started doing design for rotor-2. And as we have discussed, we are making our assumption, that's what is say my absolute velocity coming out from rotor-1, that's what will be the entry velocity for the rotor-2. And that is the reason my α_2 at mid station, that's what will be equal to α_{3m}.

$$Let's\ \ assume\ C_{2m} = C_{3m}$$

$$\alpha_{2m} = \alpha_{3m}$$

$$\&\ C_{w2m} = -C_{w3m}$$

One more observation what we are having is in sense of whirl component coming out from rotor-2, that will be the entry whirl component for say rotor-2. And, you know, that's what is having opposite sign. So, this is what we need to take care of, okay. Now, if this is what is your case, we can say we can calculate what will be my β_3, that is nothing but my flow angle with which my air that's what is entering inside the rotor-2.

So, it says my β_3 that is my relative flow angle, it is coming 57.51°, okay. Here, in this case, you can say this is what is say negative sign, that is the reason why it is getting added up.

$$From\ velocity\ triangle,$$

$$\beta_{3m} = \tan^{-1}\left(\frac{U_{R2m} - C_{w3m}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{33.93 + 35.82}{44.4}\right)$$

$$= 57.51°$$

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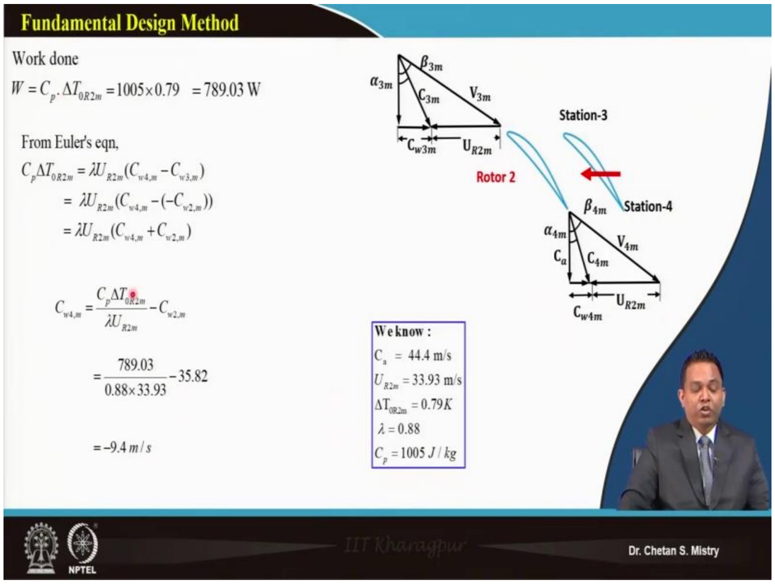

Now, based on our comparison for say Euler's equation, we can calculate what will be my C_{w4} at the exit of my rotor-2. So, if you are putting this, it says my velocity, that’s what is coming $-9.46\ m/s$.

$$Work\ done,$$

$$W = C_p \cdot \Delta T_{0R2m} = 1005 \times 0.79 = 789.03\ W$$

$$From\ Euler's\ equation,$$

$$C_p \Delta T_{oR2m} = \lambda U_{R2m}\left(C_{w4,m} - C_{w3,m}\right)$$

$$= \lambda U_{R2m}\left(C_{w4,m} - \left(-C_{w2,m}\right)\right)$$

$$= \lambda U_{R2m}\left(C_{w4,m} + C_{w2,m}\right)$$

$$C_{w4,m} = \frac{C_p \Delta T_{0R2m}}{\lambda U_{R2m}} - C_{w2,m}$$

$$= \frac{789.03}{0.88 \times 33.93} - 35.82$$

$$= -9.4\ m/s$$

We can calculate what will be our β_4, that is, you know like, the angle with which my flow that will be coming out. It is coming to be 44.42, okay. And my α_4 is nothing but my absolute exit flow angle. If you are putting, it is C_{w4} by C_a, that's what is coming say 11.94.

$$\tan \beta_{4m} = \frac{U_{R2m} + C_{w4m}}{C_a}$$

$$= \frac{33.93 - (-9.4)}{44.4}$$

$$= 0.98$$

$$\beta_{4m} = 44.42°$$

$$\alpha_{4m} = \tan^{-1}\left(\frac{C_{w4m}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{9.4}{44.4}\right)$$

$$= 11.94°$$

Now sometimes, when you are doing your design, it may be having constraint, it says your exit need to be axial one. So that's what will be putting my α_4, that's what is equal to 0. Maybe my C_{w4}, also, I will be taking that as say 0. And that's what will be putting whole lot of stress on design calculations, okay. So, we are not restricting our self with say axial outlet, and as a reason, you can see at mid station, we are having our α_4, that's what is coming say 11.94°.

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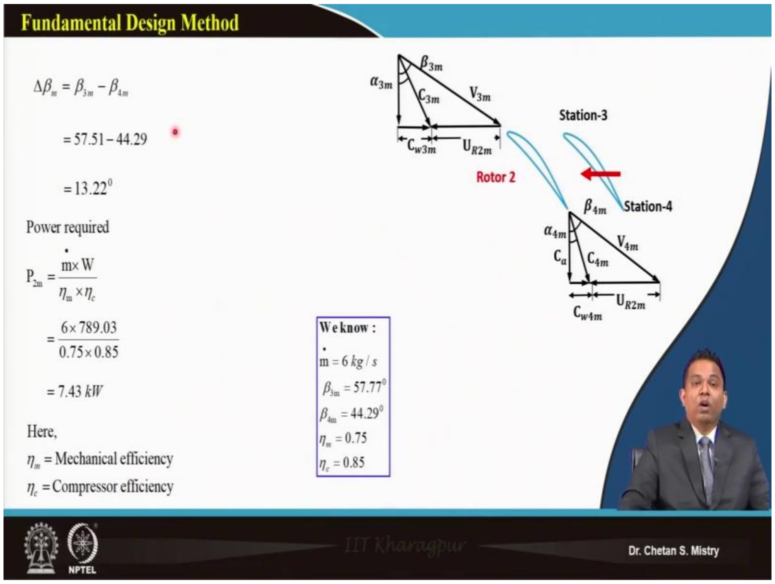

So, based on that we can do our calculation for say my $\Delta\beta$. Once, this $\Delta\beta$ is known to us, we can move forward with the calculation of other velocity components. Again, since, we are having our motors, they are having capacity of 15 kW, that's what is a constraint that what was given to us.

$$\Delta\beta_m = \beta_{3m} - \beta_{4m}$$

$$= 57.51 - 44.29$$

$$= 13.22°$$

So, we will be calculating what will be the power required for rotor-2. So here if you consider, again, we are assuming our mechanical efficiency for rotor-2 also to be 75%, and our efficiency for the compressor, we are assuming 85%, that's what is coming say 7.43 kW.

$$Power\ required,$$

$$P_{2m} = \frac{\dot{m} \times W}{\eta_m \times \eta_c}$$

$$= \frac{6 \times 789.03}{0.75 \times 0.85}$$

$$= 7.43\ kW$$

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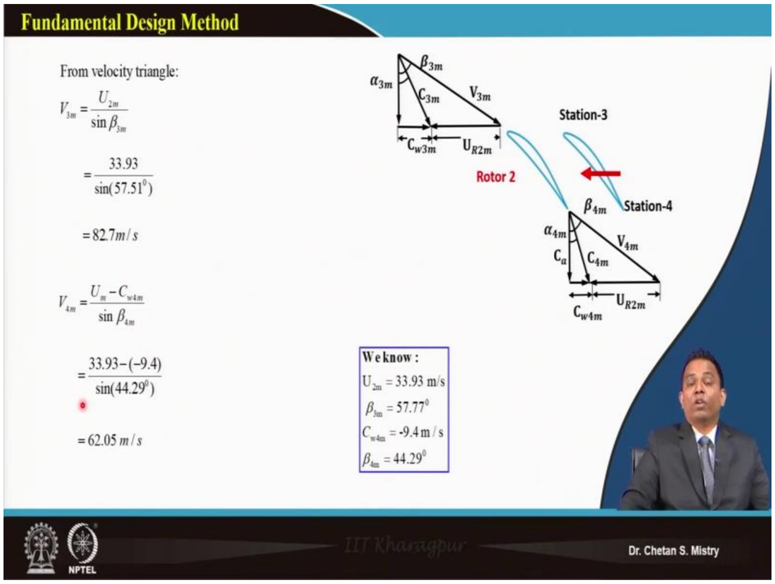

Now, once the flow angles, they are known to us, we can do our calculation for the relative velocity components. So, this relative velocity component, we can calculate based on say $U_2/\sin\beta_3$, okay, and that's what is coming say 87…82.7 m/s. So, we are having our relative velocity with which flow is entering. Same way, we can do our calculation for the V_4, that is nothing but that's what is my exit relative velocity from the rotor-2, and that's what is coming 62.05 m/s, okay.

$$From\ velocity\ triangle,$$

$$V_{3m} = \frac{U_{2m}}{\sin\beta_{3m}}$$

$$= \frac{33.93}{\sin(57.51°)}$$

$$= 82.7\ m/s$$

$$V_{4m} = \frac{U_m - C_{w4m}}{\sin\beta_{4m}}$$

$$= \frac{33.93 - (-9.4)}{\sin(44.29°)} = 62.05\ m/s$$

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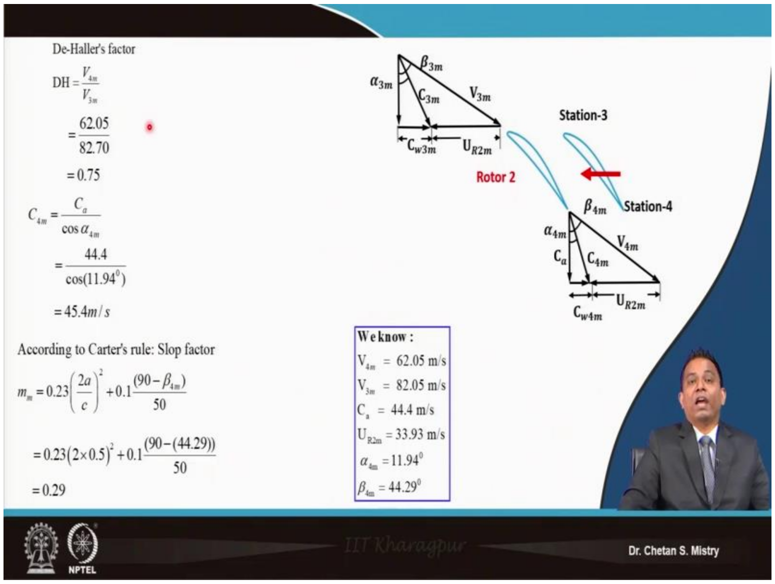

Once this is what is known to us, we can do our calculation for De-Haller's factor. That is nothing but it's a velocity ratio, relative velocity ratio, and that's what is coming 0.75, okay.

$$De - Haller's factor,$$

$$DH = \frac{V_{4m}}{V_{3m}}$$

$$= \frac{62.05}{82.7}$$

$$= 0.75$$

Now, we can calculate our absolute flow angle based on my known α_4 angle, and that is coming 45.4 m/s.

$$C_{4m} = \frac{C_a}{\cos \alpha_{4m}}$$

$$= \frac{44.4}{\cos(11.94°)}$$

$$= 45.4 \, m/s$$

So, once we are calculating our relative velocity ratio, our next target that will be to calculate the diffusion factor. And for calculation of diffusion factor, as we have discussed earlier, we need to calculate what is our m factor, we need to calculate what will be our solidity, okay. So, this is what we are putting in sense of my m-factor, and here you can say like m, that's what is coming 0.29.

$$\text{According to Carter's rule: Slop factor,}$$

$$m_m = 0.23\left(\frac{2a}{c}\right)^2 + 0.1\frac{90 - \beta_{4m}}{50}$$

$$= 0.23(2 \times 0.5)^2 + \frac{0.1\big(90 - (44.29)\big)}{50}$$

$$= 0.29$$

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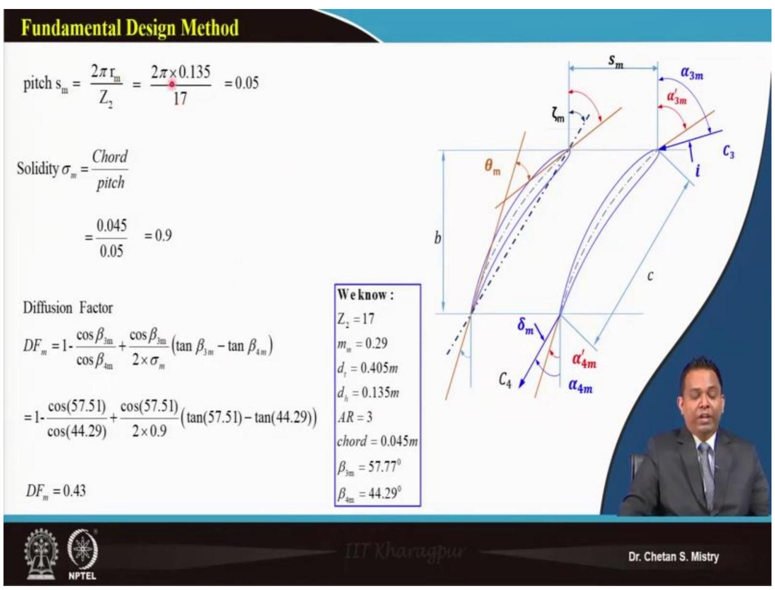

Now, for rotor-2 when we are doing our calculation for the pitch, so here in this case we are assuming our number of blade, that to be say 17, okay, because this is what is in line to what all design we are comparing with, already existing rotors, okay. And for that, we are having number of blades for rotor-2, that's what is coming to be 17.

No doubt, when you are making your mean line calculation, once you are ready with your Excel sheet, you can play with these numbers, not an issue, okay. So here, if you are putting that, it says my pitch, that's what is coming 0.05. We can do our calculation for the solidity based on chord. It says, it is coming 0.9.

$$pitch\ s_m = \frac{2\pi r_m}{Z_2}$$

$$= \frac{2\pi \times 0.135}{17}$$

$$= 0.05$$

$$Solidity, \sigma_m = \frac{Chord}{pitch}$$

$$= \frac{0.045}{0.05}$$

$$= 0.9$$

Once this solidity, that's what is known to us, we can do our calculation for diffusion factor at the mid station for rotor-2, and that's what is coming 0.43. Be careful in sense of putting the angles. β_1, β_2 here this will be β_3 and β_4, okay.

$Diffusion\ Factor,$

$$DF_m = 1 - \frac{\cos\beta_{3m}}{\cos\beta_{4m}} + \frac{\cos\beta_{3m}}{2 \times \sigma_m}(\tan\beta_{3m} - \tan\beta_{4m})$$

$$= 1 - \frac{\cos(57.51)}{\cos(44.29)} + \frac{\cos(57.51)}{2 \times 0.9}(\tan(57.51) - \tan(44.29))$$

$$\therefore DF_m = 0.43$$

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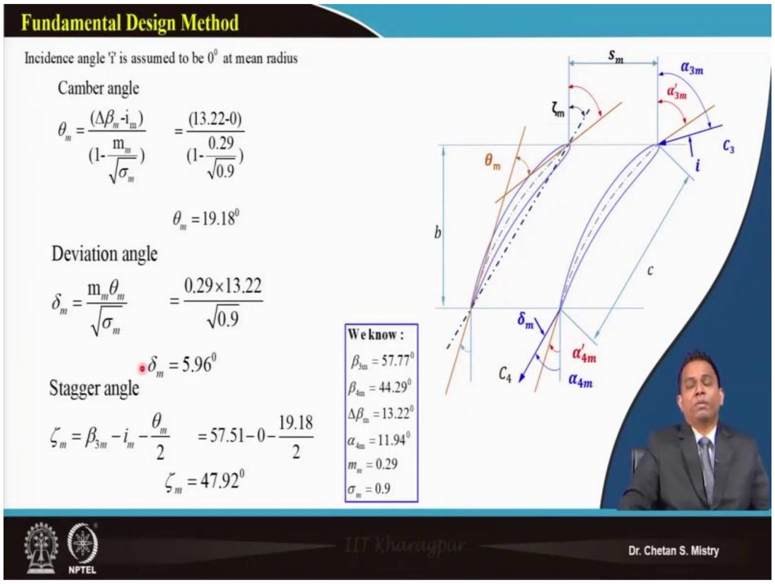

Now, in order to calculate other flow angles, say may be camber angle, deviation angle and the stagger angle, we need to have assumption of our incidence angle. So, in line to what we have discussed for rotor-1, here for rotor-2 also, our incidence angle at mid station, we are assuming to be 0, okay. If this is what is coming to be 0, we can say our camber angle, it is coming 19.18°. Same way, my deviation angle, that's what is coming 5.96°, and my stagger angle, that's what is coming 47.92°.

$$Incidence\ angle\ 'i'\ is\ assumed\ tobe\ 0\ at\ mean\ radius$$

$$Camber\ angle,$$

$$\theta_m = \frac{\Delta\beta_m - i_m}{1 - \frac{m}{\sqrt{\sigma_m}}} = \frac{13.22 - 0}{1 - \frac{0.29}{\sqrt{0.9}}}$$

$$\therefore \theta_m = 19.18°$$

$$Deviation\ angle,$$

$$\delta_m = \frac{m_m \theta_m}{\sqrt{\sigma_m}} = \frac{0.29 \times 13.22}{\sqrt{0.9}}$$

$$\therefore \delta_m = 5.96°$$

$Stagger\ angle,$

$$\zeta_m = \beta_{3m} - i_m - \frac{\theta_m}{2} = 57.51 - 0 - \frac{19.18}{2}$$

$$\therefore \zeta_m = 47.92°$$

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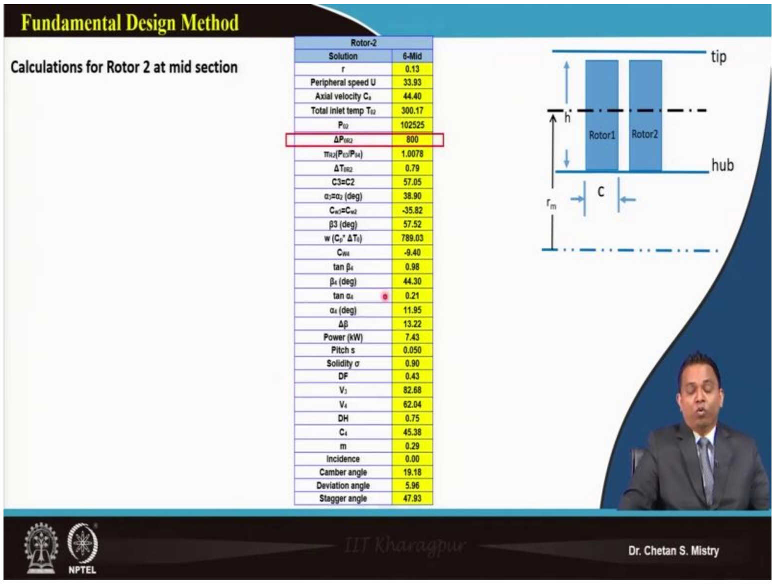

Rotor-2	
Solution	6-Mid
r	0.13
Peripheral speed U	33.93
Axial velocity C_a	44.40
Total inlet temp T_{02}	300.17
P_{02}	102525
ΔP_{0R2}	800
$\pi_{R2}(P_{03}/P_{04})$	1.0078
ΔT_{0R2}	0.79
C3=C2	57.05
$\alpha_3=\alpha_2$ (deg)	38.90
$C_{w3}=C_{w2}$	-35.82
β3 (deg)	57.52
w (C_p* ΔT_0)	789.03
C_{W4}	-9.40
tan β_4	0.98
β_4 (deg)	44.30
tan α_4	0.21
α_4 (deg)	11.95
Δβ	13.22
Power (kW)	7.43
Pitch s	0.050
Solidity σ	0.90
DF	0.43
V_3	82.68
V_4	62.04
DH	0.75
C_4	45.38
m	0.29
Incidence	0.00
Camber angle	19.18
Deviation angle	5.96
Stagger angle	47.93

So, by this calculation, we are able to have this Excel sheet at the mid station, ready with us. So, you can say, this is what is our expected pressure rise from rotor-2, that is nothing but 800 Pa, okay.

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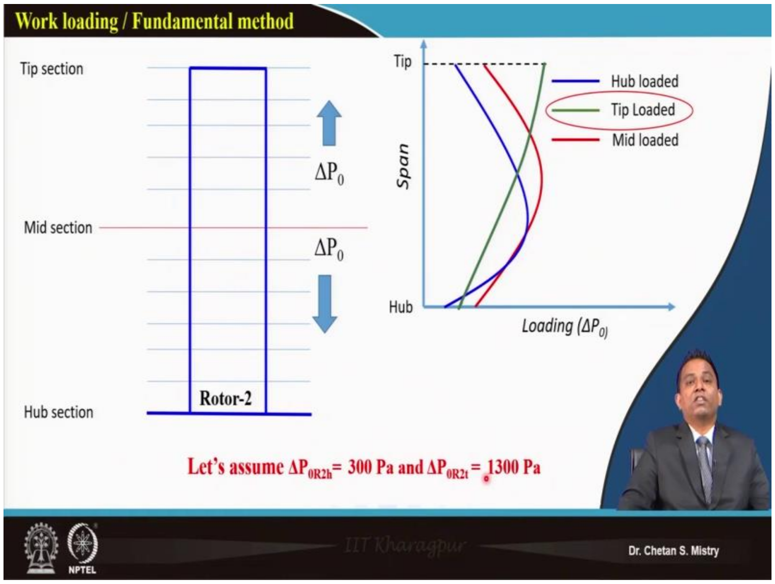

Now, what all we have learned, say for our fundamental method, we are assuming our ΔP_0 at different stations, and since we have designed our rotor-1 with the tip loaded configuration, same trend we will be using for rotor-2, that's what will be having say tip loaded configuration. And if this is what is your case, initially, we can assume say my expected pressure rise near the hub to be 300 Pa, and near the tip region, is say 1300 Pa, okay.

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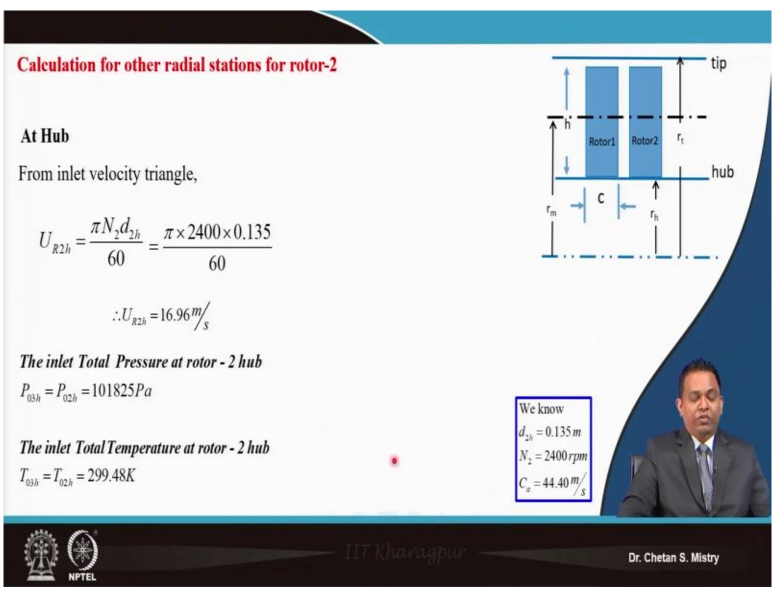

Now, if this is what is your case, what we can do is we will start doing calculation at the hub station, same way, we will be doing our calculation at the tip station. But we need to be very careful, we should not commit mistake. Like this design approach for rotor-2, for contra rating configuration, that's what is different, okay. And your small mistake in calculation, that's what will lead to give the numbers that may not be acceptable. So be careful about that part. So, if you are considering that as a case, we can say at the hub, we can calculate our peripheral speed, okay.

$$At\ hub,$$

$$From\ inlet\ velocity\ triangle,$$

$$U_{R2h} = \frac{\pi N_2 d_{2h}}{60} = \frac{\pi \times 2400 \times 0.135}{60}$$

$$\therefore U_{R2h} = 16.96\ m/s$$

Now near the hub, I need to have my pressure as well as temperature because we are looking for calculation of pressure ratio as well as what will be my ΔT_0. So be careful here, near the hub region, whatever is my total pressure at the outlet, near the hub, that's what will be the entry pressure for my rotor-2. Same way, what is my total temperature at the outlet of the hub for rotor-1, that will be my entry temperature for rotor-2, okay. So, these are the numbers what we are putting in sense.

$$The\ inlet\ Total\ Pressure\ at\ rotor-2\ hub,$$

$$P_{03h} = P_{02h} = 101825\ Pa$$

$$The\ inlet\ Total\ Temperature\ at\ rotor-2\ hub,$$

$$T_{03h} = T_{02h} = 299.48\ K$$

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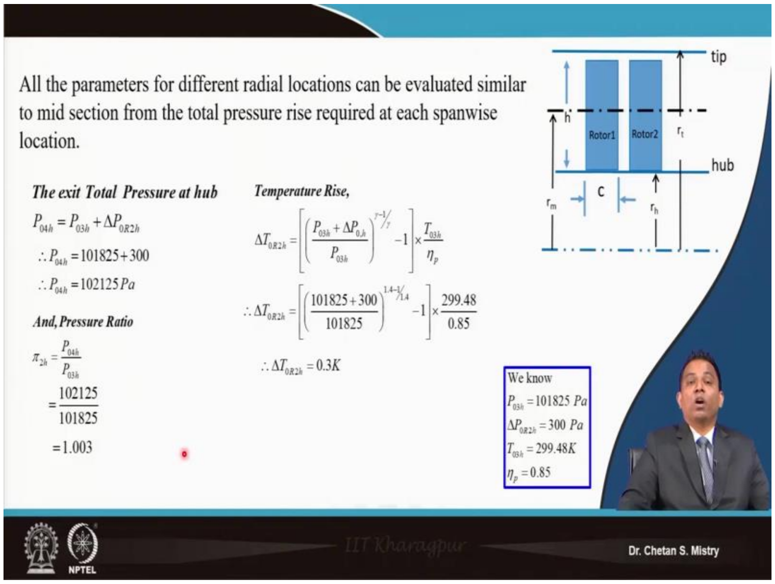

Now, once this is known to us, we can do our calculation for the pressure ratio. And the pressure ratio near the hub, that's what is coming 1.003, okay. Same way, my ΔT_0 at the hub, you can say that's what is coming 0.3 K, okay. Remember, what pressure rise we are expecting, that's what is lower. We are designing for tip loaded rotor, and that is the reason why my ΔT_0, that's what is coming lower in the numbers.

The exit total Pressure at hub

$$P_{04h} = P_{03h} + \Delta P_{0R2h}$$

$$\therefore P_{04h} = 101825 + 300$$

$$\therefore P_{04h} = 102125\ Pa$$

Pressure Ratio

$$\pi_{2h} = \frac{P_{04h}}{P_{03h}}$$

$$= \frac{102125}{101825}$$

$$= 1.003$$

Temperature Rise,

$$\Delta T_{0R2h} = \left[\left(\frac{P_{03h} + \Delta P_{0R2h}}{P_{03h}} \right)^{\frac{(\gamma-1)}{\gamma}} - 1 \right] \times \frac{T_{03h}}{\eta_p}$$

$$\therefore \Delta T_{0R2h} = \left[\left(\frac{101825 + 300}{101825} \right)^{\frac{1.4-1}{1.4}} - 1 \right] \times \frac{299.48}{0.85}$$

$$\therefore \Delta T_{0R2h} = 0.3\ K$$

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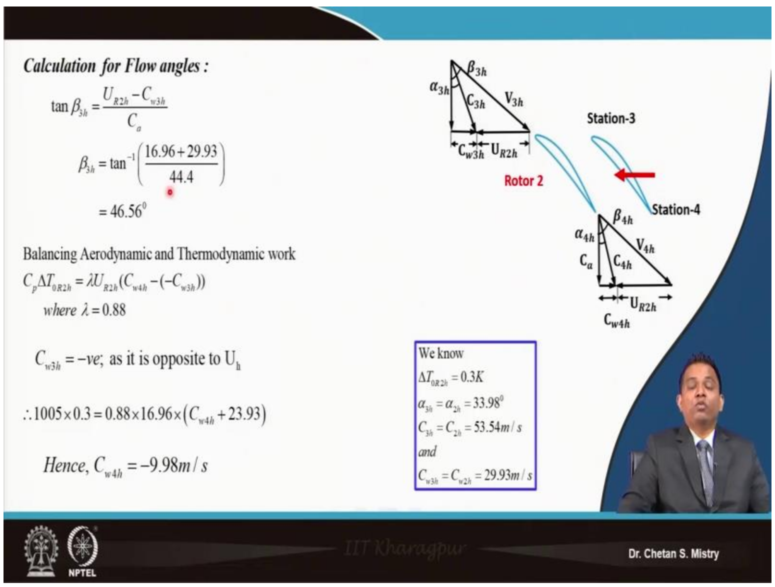

So, now based on this, we can start doing calculation for our β_3, and as we have seen, my C_{w2} at the hub, that's what is equal to my C_{w3} at the hub. And that's what we are putting here, that's what says my β_3 at mid station is coming 46.56°, okay. And now, once this is known to us, we can do our calculation for the whirl component.

$$\tan \beta_{3h} = \frac{U_{R2h} - C_{w3h}}{C_a}$$

$$\beta_{3h} = \tan^{-1}\left(\frac{16.96 + 29.93}{44.4}\right)$$

$$= 46.56°$$

Here in this case, we need to be very careful. Say, here my work done factor, that's what I have taken as 0.88. For my rotor-1, we have considered that as say 0.98. And here, this is what is say 0.88. This is what is based on what all we have discussed in our earlier class, okay. And that's what is giving me my C_{w4} as $-9.98\ m/s$. So minus in sense, you need to be very careful about what will be my angle and what will be the direction. Accordingly, you need to do modification in your triangles, okay.

$$Balancing\ Aerodynamic\ and\ Thermodynamic\ work,$$

$$C_p \Delta T_{0R2h} = \lambda U_{R2h}\left(C_{w4h} - (-C_{w3h})\right)$$

$$where\ \lambda = 0.88$$

$$C_{w3h} = -ve;\ as\ it\ is\ opposite\ to\ U_h$$

$$\therefore 1005 \times 0.3 = 0.88 \times 16.96 \times (C_{w4h} + 23.93)$$

$$Hence, C_{w4h} = -9.98\ m/s$$

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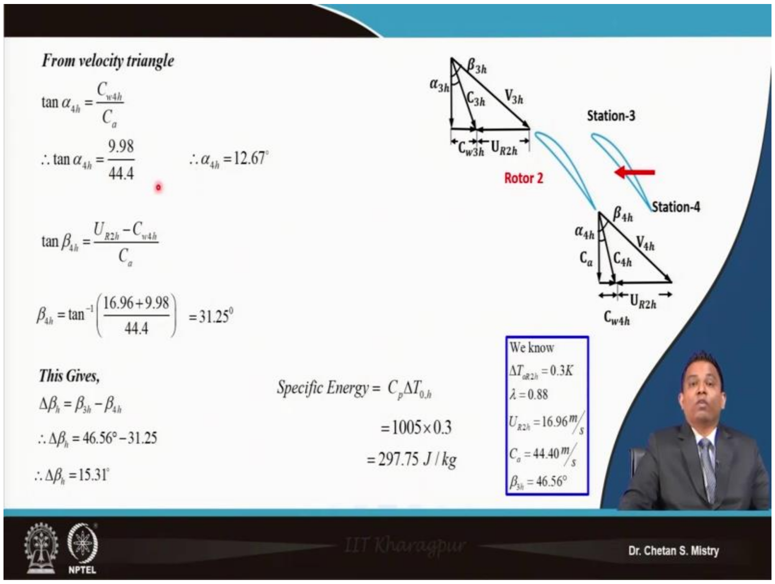

Now, if this is known to us, we can calculate what will be my α_4, okay. My α_4, that's what is coming 12.67 near the hub region. If you recall, at the mid station also, that was coming around 11°. And we can do our calculation for β_4.

$From\ velocity\ triangle,$

$$\tan \alpha_{4h} = \frac{C_{w4h}}{C_a}$$

$$\therefore \tan \alpha_{4h} = \frac{9.98}{44.4}$$

$$\therefore \alpha_{4h} = 12.67°$$

$$\tan \beta_{4h} = \frac{U_{R2h} - C_{w4h}}{C_a}$$

$$\beta_{4h} = \tan^{-1}\left(\frac{16.96 + 9.98}{44.4}\right) = 31.25°$$

Now, for rotor, when we are saying, we are always interested in $\Delta\beta$. So, at $\Delta\beta$ at the hub for rotor-2, it is coming 15.31, okay.

$This\ gives,$

$$\Delta\beta_h = \beta_{3h} - \beta_{4h}$$

$$\therefore \Delta\beta_h = 46.56° - 31.25$$

$$\therefore \Delta\beta_h = 15.31°$$

And my specific energy based on our understanding, that's what is say $C_p \Delta T_0$, if you are putting this is what is a number, that's what is coming, okay. It is 0.29 kJ/kg.

$$Specific\ energy = C_p \Delta T_{0R2h}$$

$$= 1005 \times 0.3$$

$$= 297.75\ J/kg$$

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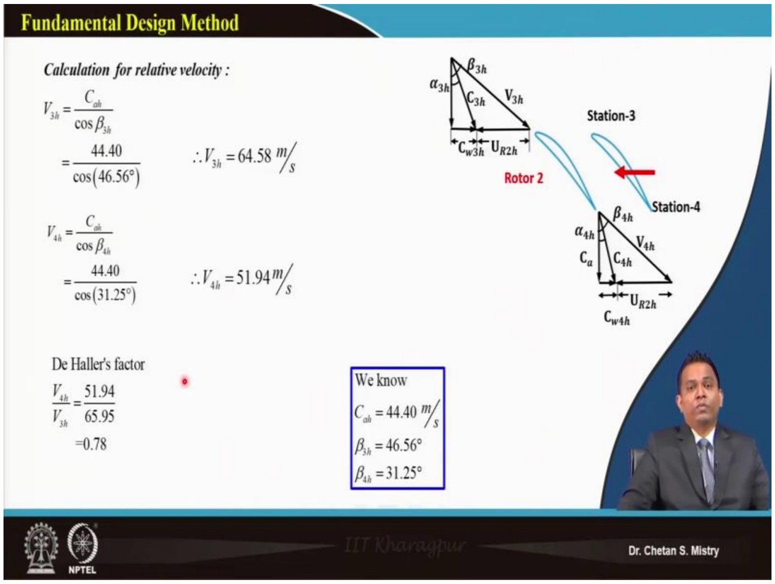

Now, once we have calculated our flow angles, we can calculate what will be our relative velocity because we are interested in calculating our De-Haller's factor near the hub region. So, if you are calculating that, my V_3 at the hub, it is coming 64.58, and my V_4 at the hub, that's what is coming 51.94 m/s. That's what is giving me my De-Haller's factor to be 0.78. So near hub, we are having this De-Haller's factor, that's what is coming 0.78, okay.

$$Calculation\ for\ relative\ velocity:$$

$$V_{3h} = \frac{C_{ah}}{\cos\beta_{3h}}$$

$$= \frac{44.4}{\cos(46.56°)}$$

$$\therefore V_{3h} = 64.58\ m/s$$

$$V_{4h} = \frac{C_{ah}}{\cos\beta_{4h}}$$

$$= \frac{44.40}{\cos(31.25°)}$$

$$\therefore V_{4h} = 51.94\ m/s$$

$$De - Haller's factor$$

$$\frac{V_{4h}}{V_{3h}} = \frac{51.94}{65.95}$$

$$= 0.78$$

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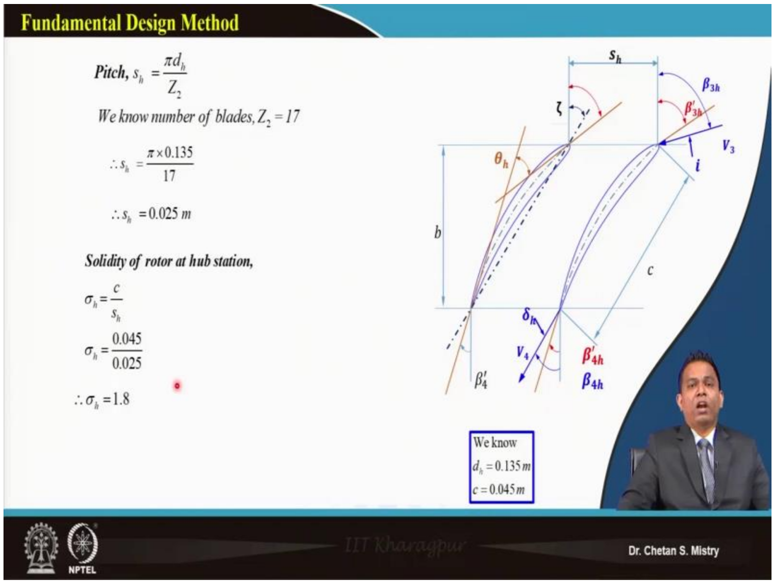

Now, based on that later part, as we are doing calculation, the sequence of calculation that's what will remain same when we are designing say rotor or rotor-2, in line to what we have done calculation for rotor-1. So, we will be calculating our pitch. And near the hub, as we have discussed, my pitch, that's what will be coming to be lower. Here, for rotor-2, we have assumed our number of blades to be 17. So, that's what is giving me my solidity at the hub, it is say 1.8, okay.

$$Pitch, s_h = \frac{\pi d_h}{Z_2}$$

$$We\ have\ number\ of\ blades, Z_2 = 17$$

$$s_h = \frac{\pi d_h}{Z_2}$$

$$\therefore s_h = \frac{\pi \times 0.135}{17}$$

$$\therefore s_h = 0.025\ m$$

$$Solidity\ of\ rotor\ at\ hub\ station,$$

$$\sigma_h = \frac{c}{s_h}$$

$$= \frac{0.045}{0.025}$$

$$\therefore \sigma_h = 1.8$$

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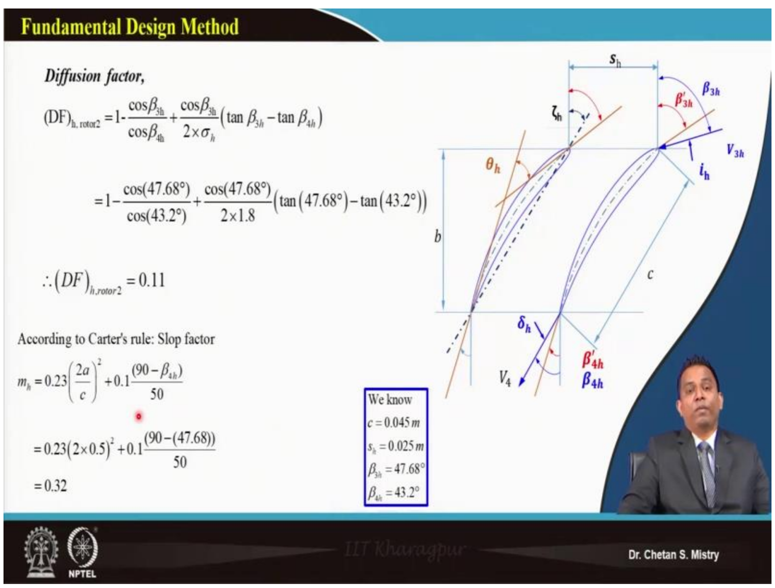

Now, once we are calculated with the solidity, we know what will be our β_3, what will be our β_4. We can do our calculation for the diffusion factor near the hub region, and if you look at, this number, that's what is coming 0.11, okay. Now for say calculation of our angles, we are looking for say our m factor or say…what we say Carter's factor, that's what is coming to be 0.32, okay.

Diffusion factor,

$$(DF)_{h,rotor2} = 1 - \frac{\cos\beta_{3h}}{\cos\beta_{4h}} + \frac{\cos\beta_{3h}}{2\times\sigma_h}(\tan\beta_{3h} - \tan\beta_{4h})$$

$$= 1 - \frac{\cos(47.68°)}{\cos(43.2°)} + \frac{\cos(47.68°)}{2\times 1.8}(\tan(47.68°) - \tan(43.2°))$$

$$\therefore (DF)_{h,rotor2} = 0.11$$

According to Carter's rule; slop factor

$$m_h = 0.23\left(\frac{2a}{c}\right)^2 + 0.1\frac{(90-\beta_{4h})}{50}$$

$$= 0.23(2\times 0.5)^2 + 0.1\frac{90-(47.68)}{50}$$

$$= 0.32$$

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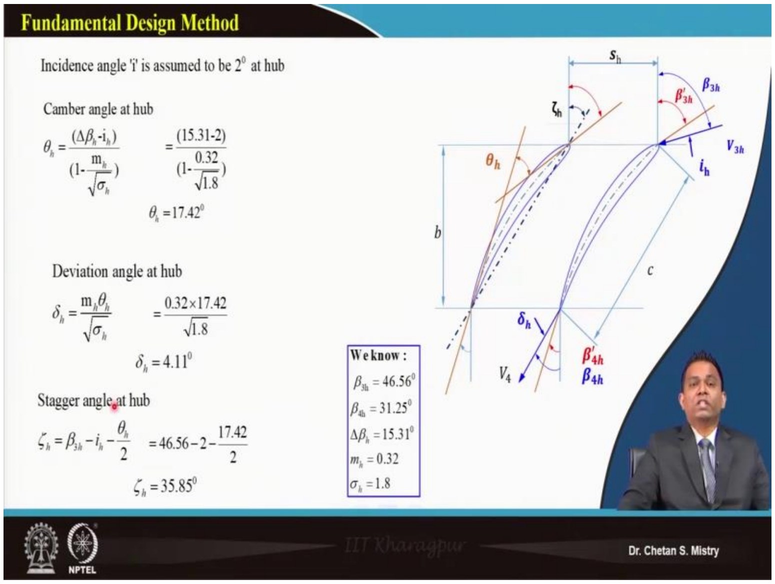

Now, once this is what is known to us, again, do not forget when we are doing our calculation for say camber angle, say deviation angle and stagger angle, we need to have certain amount of incidence to be assumed. And as we have discussed, near the hub region, we are assuming our incidence to be positive incidence, and that too we are taking, $+2°$.

So, if we are putting this, it says my camber angle at the hub, it is coming 17.42, my deviation angle is 4.11, and my stagger angle that's what is coming 35.85°, okay. You can cross verify this once you are doing your pen and paper calculation. That's what will give you the idea for the calculation, okay.

$$Let's\ assume\ Incidence\ angle\ 'i'\ tobe + 2° \ at\ hub$$

$Camber\ angle\ at\ hub,$

$$\theta_h = \frac{\Delta\beta_h - i_h}{1 - \frac{m_h}{\sqrt{\sigma_h}}} = \frac{15.31 - 2}{1 - \frac{0.32}{\sqrt{1.8}}}$$

$$\therefore \theta_m = 17.42°$$

$Deviation\ angle\ at\ hub,$

$$\delta_h = \frac{m_h \theta_h}{\sqrt{\sigma_h}} = \frac{0.32 \times 17.42}{\sqrt{1.8}}$$

$$\therefore \delta_h = 4.11°$$

$Stagger\ angle\ at\ hub,$

$$\zeta_h = \beta_{3h} - i_h - \frac{\theta_h}{2} = 46.56 - 2 - \frac{17.42}{2}$$

$$\therefore \zeta_h = 35.85°$$

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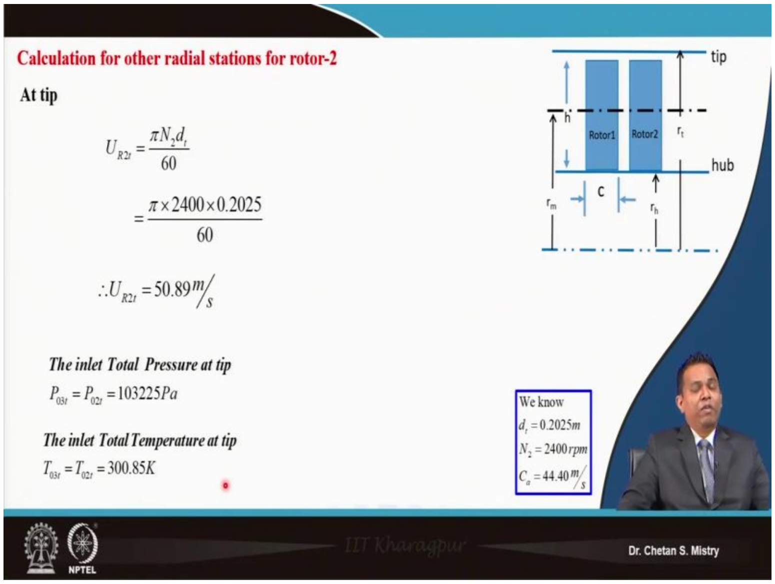

Now, similarly, we will be doing our calculation near the tip region. We will be having the same step. That's what we need to follow. We will be calculating our peripheral speed. Same way, near the tip region we need to have our total pressure and temperature, that's what is coming from rotor-1, that will be the entry condition for my rotor-2.

$$At\ tip,$$

$$From\ inlet\ velocity\ triangle,$$

$$U_{R2t} = \frac{\pi N_2 d_{2t}}{60} = \frac{\pi \times 2400 \times 0.2025}{60}$$

$$\therefore U_{R2t} = 50.89\ m/s$$

$$The\ inlet\ Total\ Pressure\ at\ tip,$$

$$P_{03t} = P_{02t} = 103225\ Pa$$

$$The\ inlet\ Total\ Temperature\ at\ tip,$$

$$T_{03t} = T_{02t} = 300.85\ K$$

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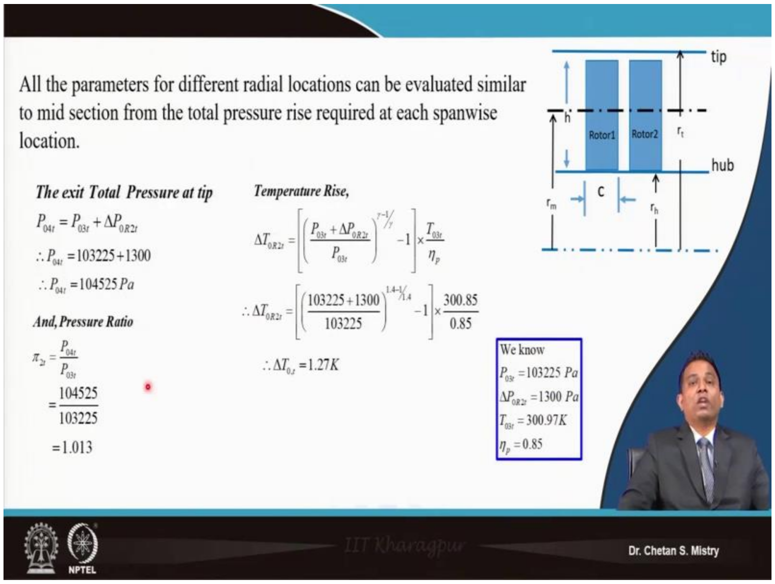

Now, once this pressure and temperature, that's what is known to us, we can do our calculation for say what will be my pressure ratio at the tip, and that's what is coming 1.013. My temperature rise ΔT_0, at the tip, that's what is coming 1.27 K, okay.

$$\textit{The exit total Pressure at tip}$$

$$P_{04t} = P_{03t} + \Delta P_{0R2t}$$

$$\therefore P_{04t} = 103225 + 1300$$

$$\therefore P_{04t} = 104525\ Pa$$

$$\textit{Pressure Ratio}$$

$$\pi_{2t} = \frac{P_{04t}}{P_{03t}}$$

$$= \frac{104525}{103225}$$

$$= 1.013$$

$$Temperature\ Rise,$$

$$\Delta T_{0R2t} = \left[\left(\frac{P_{03t} + \Delta P_{0R2t}}{P_{03t}} \right)^{\frac{(\gamma-1)}{\gamma}} - 1 \right] \times \frac{T_{03t}}{\eta_p}$$

$$\therefore \Delta T_{0R2t} = \left[\left(\frac{103225 + 1300}{103225} \right)^{\frac{1.4-1}{1.4}} - 1 \right] \times \frac{300.85}{0.85}$$

$$\therefore \Delta T_{0R2t} = 1.27\ K$$

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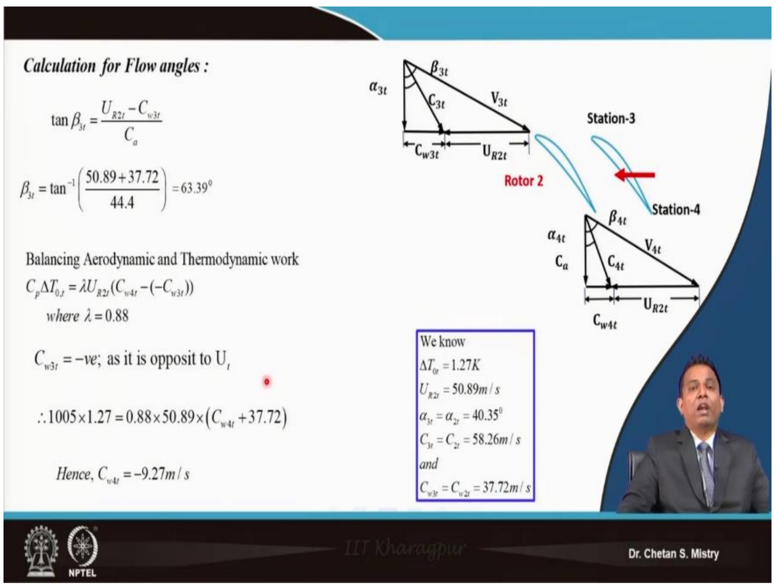

Now, based on all these calculations, now later part, we will be calculating what will be our blade angles or relative flow angles. So, my β_3 at the tip, that's what is coming 63.39, okay. And when we are comparing, our aerodynamic and thermodynamic work, that's what will be helping us for calculating what will be my C_{w4} at the tip region, and that's what is coming $-9.26\ m/s$, okay.

$$\tan\beta_{3t} = \frac{U_{R2t} - C_{w3t}}{C_a}$$

$$\beta_{3t} = \tan^{-1}\left(\frac{50.89 + 37.72}{44.4}\right)$$

$$= 63.39°$$

$$Balancing\ Aerodynamic\ and\ Thermodynamic\ work,$$

$$C_p \Delta T_{0R2t} = \lambda U_{R2t}\big(C_{w4t} - (-C_{w3t})\big)$$

$$where\ \lambda = 0.88$$

$$C_{w3t} = -ve;\ as\ it\ is\ opposite\ to\ U_t$$

$$\therefore 1005 \times 1.27 = 0.88 \times 50.89 \times (C_{w4t} + 37.72)$$

$$Hence, C_{w4t} = -9.26\ m/s$$

Now, once this relative velocity, that's what is known to us, relative flow angles known to us, my absolute components are known to us, later on we will be calculating the parameters, called De-Haller's factor, and my diffusion factor. So, let us move in that direction.

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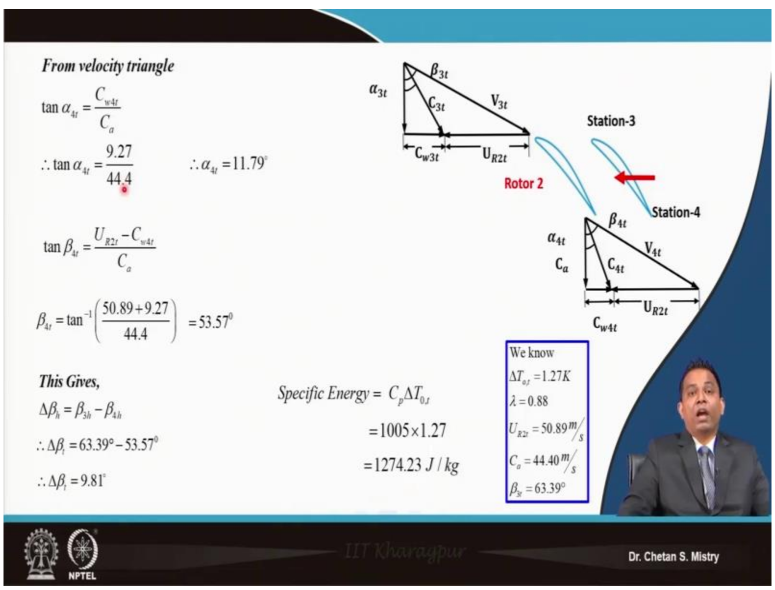

So, it says, I will be calculating what will be my α_4 at the tip region. We will be calculating what will be our $\Delta\beta$. And if you look at, near the tip region, my $\Delta\beta$ is coming 9.81, okay. So, this numbers, that's what is having certain meanings. We will see when we will be discussing about say finalizing of this design, what all is the meaning of these numbers.

$From\ velocity\ triangle,$

$$\tan\alpha_{4t} = \frac{C_{w4t}}{C_a}$$

$$\therefore \tan\alpha_{4t} = \frac{9.27}{44.4}$$

$$\therefore \alpha_{4t} = 11.79°$$

$$\tan\beta_{4t} = \frac{U_{R2t} - C_{w4t}}{C_a}$$

$$\beta_{4t} = \tan^{-1}\left(\frac{50.89 + 9.27}{44.4}\right) = 53.57°$$

$This\ gives,$

$$\Delta\beta_t = \beta_{3t} - \beta_{4t}$$

$$\therefore \Delta\beta_t = 63.39° - 53.57°$$

$$\therefore \Delta\beta_t = 9.81°$$

$$Specific\ Energy = C_p\Delta T_{0,T}$$

$$= 1005 \times 1.27$$

$$= 1274.23\ J/kg$$

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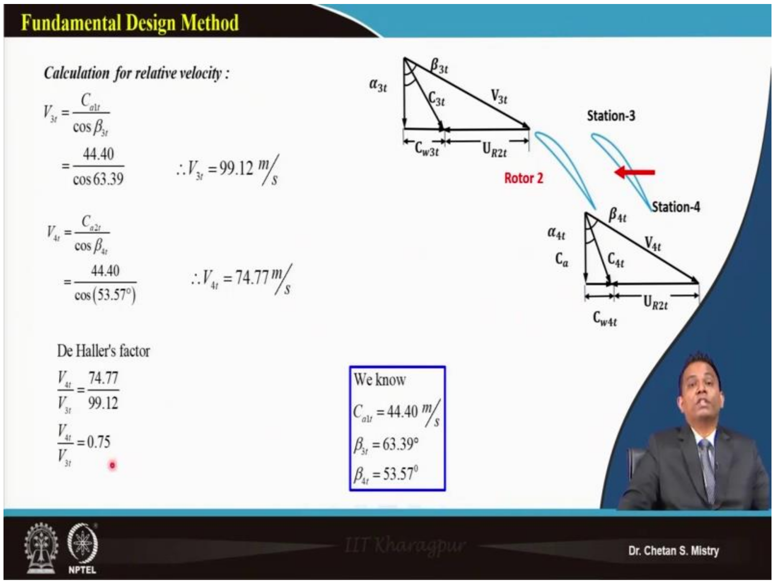

Now, as we have discussed, we are looking for our relative velocity ratio or say De-Haller's factor. So, we will be calculating what will be my V_3 at the tip, what will be my V_4 at the tip, and that's what is coming 99.12 m/s. And V_4 is coming say 74.77, okay. If you keep an eye with these numbers, you will realize my V_3, that's what is coming to be higher, my V_4, that's what is coming to be lower. And that's what is giving me my relative velocity ratio as 0.75, okay.

$$Calculation\ for\ relative\ velocity:$$

$$V_{3t} = \frac{C_{a1t}}{\cos\beta_{3t}}$$

$$= \frac{44.4}{\cos(63.39^\circ)}$$

$$\therefore V_{3t} = 99.12\ m/s$$

$$V_{4t} = \frac{C_{a2t}}{\cos\beta_{4t}}$$

$$= \frac{44.40}{\cos(53.57^\circ)}$$

$$\therefore V_{4t} = 74.77\ m/s$$

$$De - Haller's factor$$

$$\frac{V_{4t}}{V_{3t}} = \frac{74.77}{99.12}$$

$$\frac{V_{4t}}{V_{3t}} = 0.75$$

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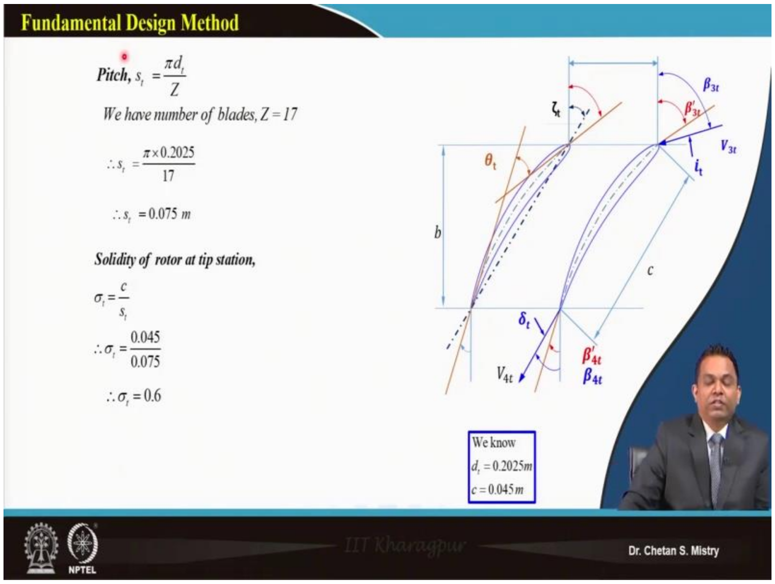

Now, in order to do the calculation for the pitch, we will be having say it is, my pitch is given by $\frac{\pi d_t}{Z}$. We can say our number of blades, what we have selected, is say 17. So, that's what is giving me my pitch, that's what is coming 0.075, okay. And my solidity, if we are putting our chord to be 0.045, solidity, that's what is coming 0.6.

$$Pitch, s_t = \frac{\pi d_t}{Z_2}$$

$$We\ have\ number\ of\ blades, Z_2 = 17$$

$$s_t = \frac{\pi d_t}{Z_2}$$

$$\therefore s_t = \frac{\pi \times 0.2025}{17}$$

$$\therefore s_t = 0.075\ m$$

$$Solidity\ of\ rotor\ at\ tip\ station,$$

$$\sigma_t = \frac{c}{s_t}$$

$$= \frac{0.045}{0.075}$$

$$\therefore \sigma_t = 0.6$$

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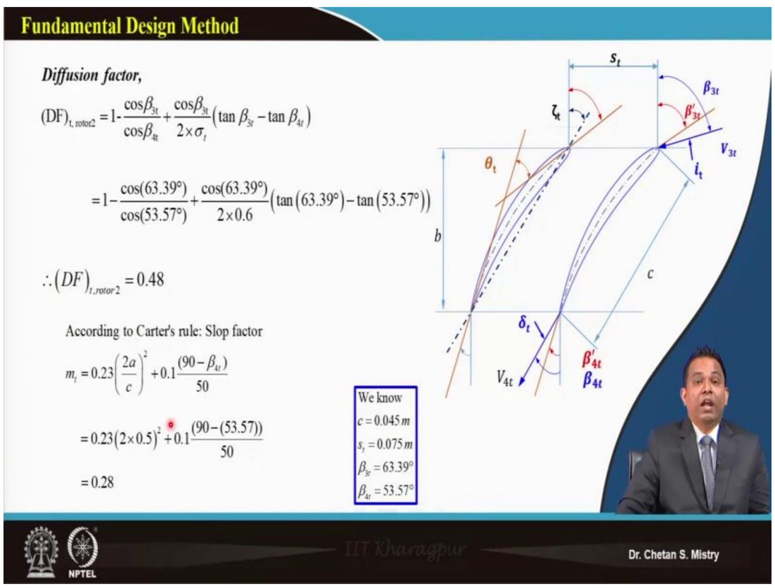

We can do our calculation for a diffusion factor at say our tip region. And that's what if we are putting our numbers say β_3, β_4 and my solidity, it is coming 0.48 near the tip region, okay. We will be calculating our m factor because we are looking for other parameters or flow angles to be calculated.

$Diffusion\ factor,$

$$(DF)_{t,rotor2} = 1 - \frac{\cos\beta_{3t}}{\cos\beta_{4t}} + \frac{\cos\beta_{3t}}{2\times\sigma_t}(\tan\beta_{3t} - \tan\beta_{4t})$$

$$= 1 - \frac{\cos(63.39°)}{\cos(53.57°)} + \frac{\cos(63.39°)}{2\times0.6}(\tan(63.39°) - \tan(53.57°))$$

$$\therefore (DF)_{t,rotor2} = 0.48$$

$According\ to\ Carter's\ rule;slop\ factor$

$$m_t = 0.23\left(\frac{2a}{c}\right)^2 + 0.1\frac{(90-\beta_{2h})}{50}$$

$$= 0.23(2\times 0.5)^2 + 0.1\frac{90-(53.57)}{50}$$

$$= 0.28$$

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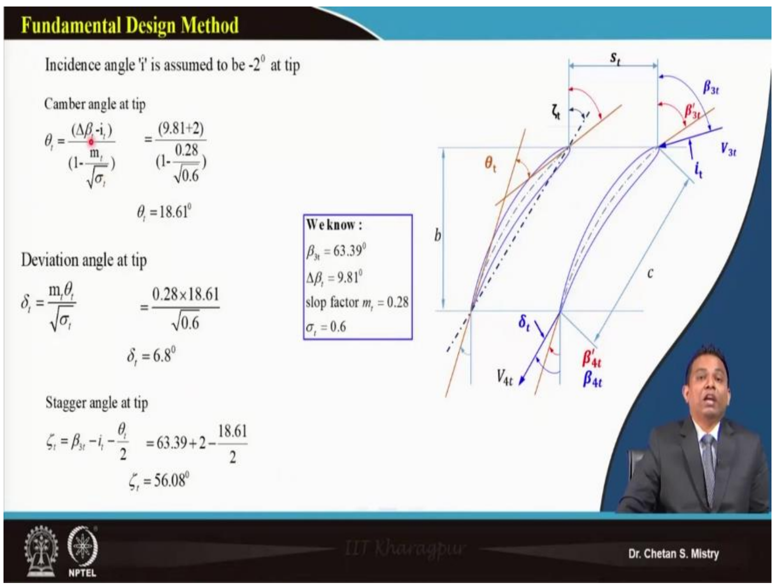

So, if we are putting this, here, when we are calculating our camber angle, say my incidence angle at the tip, that's what need to be assumed. And as we have discussed earlier, for rotor-1, in line to that, for rotor-2 also, we are assuming our incidence angle to be $-2°$, okay.

And if you are putting this as a number, it says my camber angle, it is coming 18.61, my deviation angle, it is coming 6.8, and my stagger angle, that's what is coming 56.08°, okay.

$$Let's\ assume\ Incidence\ angle\ 'i'\ tobe - 2° \ at\ tip$$

$$Camber\ angle\ at\ tip,$$

$$\theta_t = \frac{\Delta\beta_t - i_t}{1 - \frac{m_t}{\sqrt{\sigma_t}}} = \frac{9.81 + 2}{1 - \frac{0.28}{\sqrt{0.6}}}$$

$$\therefore \theta_t = 18.61°$$

$$Deviation\ angle\ at\ tip,$$

$$\delta_t = \frac{m_t\theta_t}{\sqrt{\sigma_t}} = \frac{0.28 \times 18.61}{\sqrt{0.6}}$$

$$\therefore \delta_t = 6.8°$$

$$Stagger\ angle\ at\ hub,$$

$$\zeta_t = \beta_{1t} - i_t - \frac{\theta_t}{2} = 63.39 + 2 - \frac{18.61}{2}$$

$$\therefore \zeta_t = 56.08°$$

So now, at mid station we have done our calculation, at the hub station we have done our calculation, at tip station we have done our calculation. So, like, let us see what all we have assumed, and like how, this is what is affecting.

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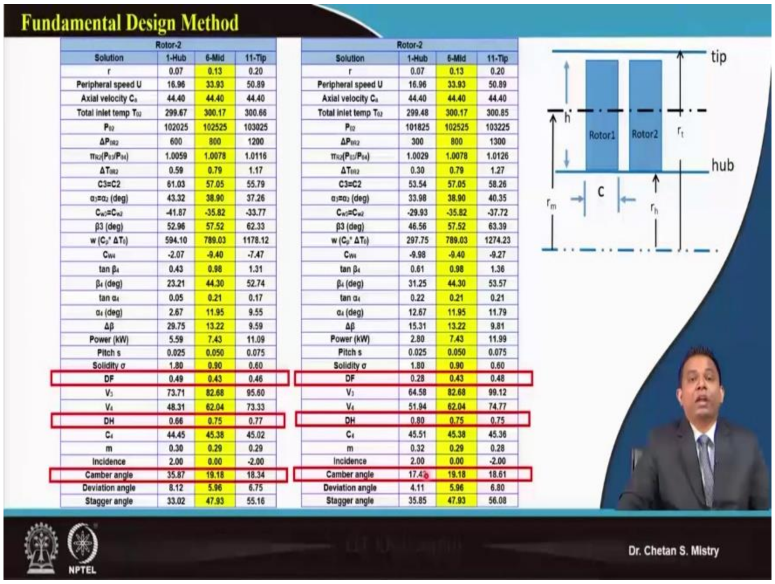

So here, if you look at, this is what is say the initial case, what we have planned for. Say, it is having say…pressure rise of…say near the hub region, it is 600, at mid station, we have kept 800, and at the tip, that's what has been say 1200. If you are looking at, it says my De-Haller's factor, that's what is varying from 0.66 to 0.77, okay. Now, if you look at the diffusion factor, that's what is say 0.49 at the hub and 0.46 near say my tip region, okay. And if you are looking at my camber angle, that's what is coming 35° and 18°.

Now, if we look this carefully, when we are expecting our pressure rise from the rotor-2, we need to keep an eye for certain parameters, and here, for this target, we have kept an eye for say our diffusion factor as one of the parameters near the hub and tip region. So that's what we have done here. So, you can see, we have reduced our diffusion factor near the hub region, and we have increased our diffusion factor near the tip region, okay.

And that's what has been done by changing my ΔP_0 at the hub, say that will be reduced to 300 Pa, and this is what has been increased to say 1300 Pa, okay. Now, if you look at my camber angle, accordingly, that's what is coming 17° and 18°. Now, with this all, if you are putting this is what is giving me say my overall distribution.

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Rotor-2											
Solution	1-Hub	2	3	4	5	6-Mid	7	8	9	10	11-Tip
r	0.07	0.08	0.09	0.11	0.12	0.13	0.15	0.16	0.18	0.19	0.20
Peripheral speed U	16.96	20.36	23.75	27.14	30.54	33.93	37.32	40.72	44.11	47.50	50.89
Axial velocity C_a	44.40	44.40	44.40	44.40	44.40	44.40	44.40	44.40	44.40	44.40	44.40
Total inlet temp T_{02}	299.48	299.61	299.75	299.89	300.03	300.17	300.30	300.44	300.58	300.72	300.85
P_{02}	101825	101965	102105	102245	102385	102525	102665	102805	102945	103085	103225
ΔP_{0R2}	300	400	500	600	700	800	900	1000	1100	1200	1300
$\pi_{R2}(P_{03}/P_{04})$	1.0029	1.0039	1.0049	1.0059	1.0068	1.0078	1.0088	1.0097	1.0107	1.0116	1.0126
ΔT_{0R2}	0.30	0.39	0.49	0.59	0.69	0.79	0.88	0.98	1.08	1.17	1.27
C3=C2	53.54	54.67	55.51	56.14	56.65	57.05	57.38	57.66	57.89	58.09	58.26
$\alpha_3=\alpha_2$ (deg)	33.98	35.70	36.88	37.74	38.39	38.90	39.31	39.64	39.92	40.15	40.35
$C_{w3}=C_{w2}$	-29.93	-31.90	-33.31	-34.36	-35.18	-35.82	-36.35	-36.78	-37.15	-37.46	-37.72
β3 (deg)	46.56	49.65	52.11	54.18	55.95	57.52	58.92	60.19	61.35	62.41	63.39
w (C_p* ΔT_0)	297.75	396.50	495.00	593.26	691.27	789.03	886.55	983.83	1080.87	1177.67	1274.23
C_{w4}	-9.98	-9.77	-9.63	-9.53	-9.45	-9.40	-9.35	-9.32	-9.30	-9.28	-9.27
tan β_4	0.61	0.68	0.75	0.83	0.90	0.98	1.05	1.13	1.20	1.28	1.36
β_4 (deg)	31.25	34.16	36.94	39.55	42.01	44.30	46.43	48.42	50.26	51.98	53.57
tan α_4	0.22	0.22	0.22	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
α_4 (deg)	12.67	12.41	12.24	12.11	12.02	11.95	11.90	11.86	11.83	11.81	11.79
Δβ	15.31	15.49	15.18	14.62	13.95	13.22	12.49	11.77	11.08	10.43	9.81
Power (kW)	2.80	3.73	4.66	5.58	6.51	7.43	8.34	9.26	10.17	11.08	11.99
Pitch s	0.025	0.030	0.035	0.040	0.045	0.050	0.055	0.060	0.065	0.070	0.075
Solidity σ	1.80	1.50	1.29	1.13	1.00	0.90	0.82	0.75	0.69	0.64	0.60
DF	0.28	0.32	0.36	0.39	0.41	0.43	0.44	0.46	0.47	0.48	0.48
V_3	64.58	68.58	72.30	75.86	79.31	82.68	86.02	89.31	92.59	95.86	99.12
V_4	51.94	53.66	55.55	57.59	59.75	62.04	64.42	66.90	69.45	72.08	74.77
DH	0.80	0.78	0.77	0.76	0.75	0.75	0.75	0.75	0.75	0.75	0.75
C_4	45.51	45.46	45.43	45.41	45.39	45.38	45.37	45.37	45.36	45.36	45.36
m	0.32	0.31	0.31	0.30	0.30	0.29	0.29	0.29	0.29	0.29	0.28
Incidence	2.00	1.60	1.20	0.80	0.40	0.00	-0.40	-0.80	-1.20	-1.60	-2.00
Camber angle	17.42	18.60	19.13	19.31	19.29	19.18	19.03	18.88	18.75	18.66	18.61
Deviation angle	4.11	4.71	5.15	5.49	5.74	5.96	6.14	6.31	6.47	6.63	6.80
Stagger angle	35.85	38.75	41.35	43.72	45.91	47.93	49.81	51.55	53.17	54.68	56.08

So, if you try to look at, this is what will be the distribution of my total pressure. Again, for rotor-2 also, this is what is designer's choice. So here, I have taken 300, maybe you can go with 400, maybe you can go with say 500, nobody will stop. It is your choice. But when you are doing that calculation, you need to do this calculation carefully.

And as we have discussed, we need to check with total pressure rise, average total pressure rises, throughout my span that need to come as per my expectation, okay. So, be careful about doing this calculation. Here in this case, if we look at, this is what is representing my systematic variation of $\Delta\beta$.

You can say, we are having the variation of say diffusion factor, as we discussed, we are having this diffusion factor as one of the keep an eye kind of parameter. If you are looking at the camber angle, this is what is representing my variation of the camber angle. And De-Haller's factor also, if you look at, compared to earlier case, we are modified with, okay.

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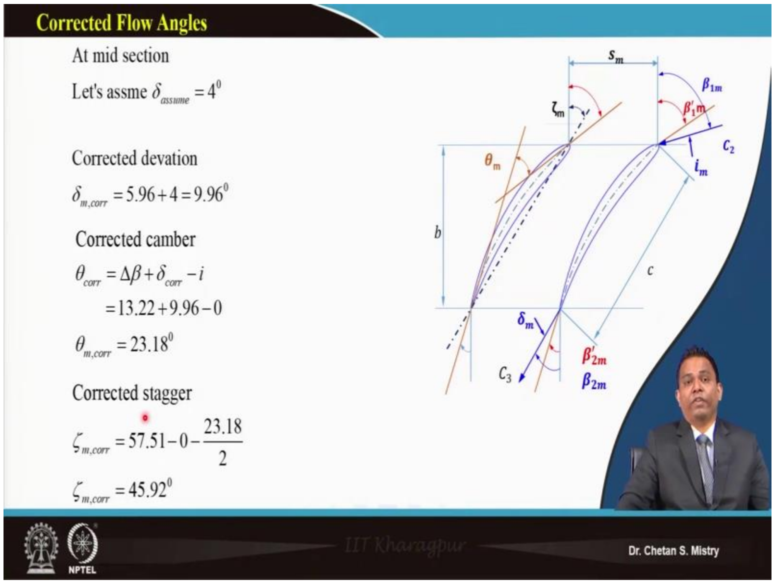

Now, we have discussed for say rotor-1, we have assumed our say deviation angle. And that's what, again say for this rotor-2 also, we can assume our deviation angle to be say assume deviation angle of 4°. If that's what is your case, you will be having say corrected deviation angle. Based on that corrected deviation angle, we need to calculate corrected camber angle, as well as corrected stagger angle. So, we will be having all this calculation for that.

(Refer Slide Time: 25:50)

Rotor-1											
Solution	1-Hub	2.00	3.00	4.00	5.00	6-Mid	7.00	8.00	9.00	10.00	11-Tip
r	0.067	0.081	0.094	0.108	0.121	0.135	0.148	0.162	0.175	0.189	0.203
r/rt	0.33	0.40	0.47	0.53	0.60	0.67	0.73	0.80	0.87	0.93	1.00
Incidence	2.00	1.60	1.20	0.80	0.40	0.00	-0.40	-0.80	-1.20	-1.60	-2.00
Camber angle	17.42	18.60	19.13	19.31	19.29	19.18	19.03	18.88	18.75	18.66	18.61
Deviation angle	4.11	4.71	5.15	5.49	5.74	5.96	6.14	6.31	6.47	6.63	6.80
Stagger angle	35.85	38.75	41.35	43.72	45.91	47.93	49.81	51.55	53.17	54.68	56.08
Deviation (assumed)	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Corrected deviation	8.11	8.71	9.15	9.49	9.74	9.96	10.14	10.31	10.47	10.63	10.80
Corrected camber	21.42	22.60	23.13	23.31	23.29	23.18	23.03	22.88	22.75	22.66	22.61
Corrected Stagger angle	33.85	36.75	39.35	41.72	43.91	45.93	47.81	49.55	51.17	52.68	54.08

And, for this case also, let us say, if you look at, we have assumed this deviation angle to be 4°. Now, this is what is designer's choice. Again, I am telling, like, you know, it is not at 4°, you can go with 3°, you can go with 5°, but that’s what need to make the sense in sense of what performance you are expecting.

Many times, it happens maybe from say hub to tip, you need to have the variation of deviation angle, and maybe from say mid to tip region you will not be having variation of deviation angle. Sometimes it may happen that you need to keep an eye, your deviation angle will be varying all the way from say your hub to tip, I am talking about assumed deviation angle.

So, this is what is a number that’s what is in your hand, you can play with, based on what understanding, or what flow physics you are expecting. And once we are putting this, this is what will be giving me in sense of corrected camber as well as corrected stagger angles, okay.

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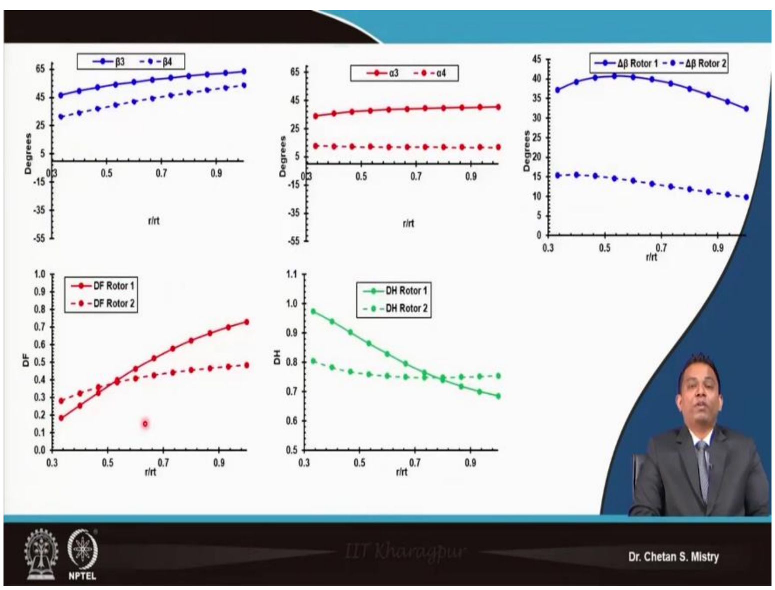

Once, this is what is ready with, we will be doing or we will be checking with how my variation of the flow angle, that’s what is coming. Now, you can understand here our expected pressure rise, it is say 800 Pa. So, my $\Delta\beta$ variation, if you are looking at, that’s what is coming to be on a lower side.

So, just look at, this is what is representing my $\Delta\beta$ variation, okay. And this is what is a variation of my $\Delta\beta$, okay. Now, same way, if you are looking at say my diffusion factor for rotor-1, that's what is having say high load and low load kind of configuration near the hub, my diffusion factor, that's what was coming lower; on say tip side, I am having my diffusion factor to be larger, okay.

But at the same time, you know, you are having say great control of diffusion factor for rotor-2. So, basically how nicely you are doing your design for rotor-1, that's what will be getting reflect on rotor-2, okay. So, when we are doing our design for contra rotating fan, you need to design your rotor-1 nicely, just finalize all parameter using your Excel sheet program, okay.

Once you are confident with, then you go with say design of rotor-2, because they are interrelated. Your small change for rotor-1, that may bring more change in rotor-2. So be careful about these aspects. Here, if you look at, my De-Haller's factor for rotor-1, we are getting very high near the hub region. And compared to that, we are having our De-Haller's factor, that's what is coming to be lower. Now, you must be knowing the reason. We are talking about the relative velocity ratio, okay.

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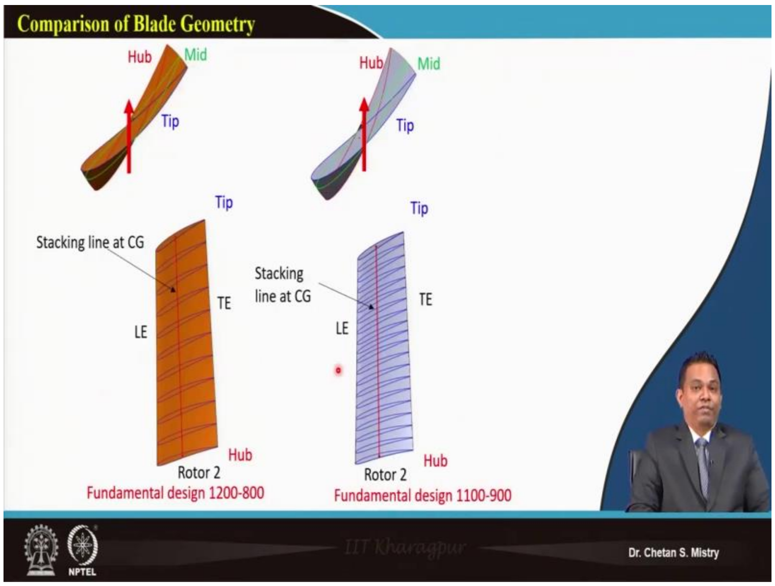

Now, once you are doing all this calculation, for 800 Pa, what we are discussing, this is what is a configuration or the blade that's what we are achieving, for rotor-2. Just look at, this is what is my rotor-2. And we are having say hub, mid and tip sections. We are having three sections, this is what is the twist of the blade what we are getting for 11 station. Now, this is what is the case when we are having 1100 and 900 Pa expected pressure rise, okay. If you compare these two, it seems to be a similar kind of thing.

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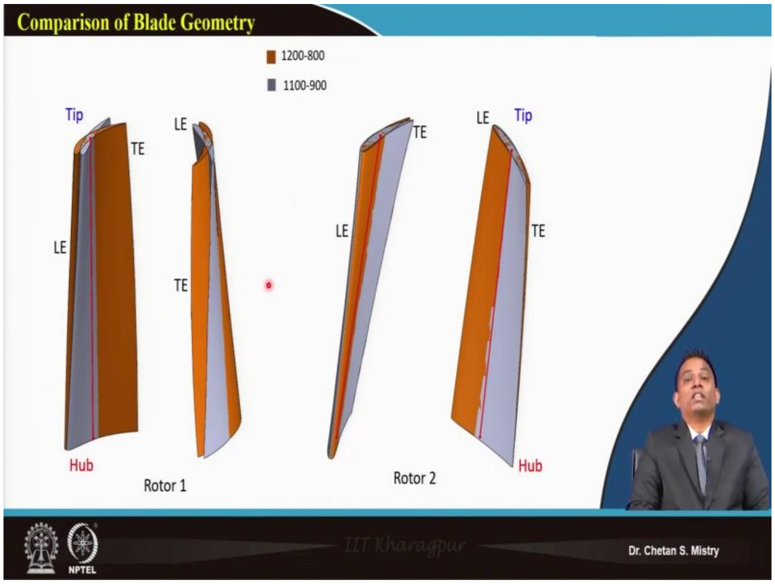

But here if you look at, you will realize the difference for say how my angles or how my blade geometry, that will be look like for two different configurations, okay. Here in this case, we are having combination of 1200 and 800. So, for 1200, what we have realized, we are having our flow angles, that's what is coming to be larger near the tip region because we are designing highly loaded rotor that's what is near the tip region.

Now, when we are comparing for say 800 Pa and 900 Pa, we will be realizing, say when we are having this to be say 900 Pa, our twist, that's what is coming to be slightly on a larger side. Major change we are finding towards the tip region. Near the hub region, not much modification, that's what can be observed. So, like this, you can play with the number of parameters.

Now, here in this case, what all we are discussing is say distribution of my total pressure ratio. The question may come, sir, I am planning to do design for contra rotating configuration in which I will be having my rotor-1 that will be rotating at high speed compared to rotor-2. Yes, same way, same logic, same calculation, you need to do. You make your same Excel sheet program, and you can play with the parameters, you can play with the numbers and get what you are looking for.

Someone will say, I am not interested in say increasing the speed of rotor-1. I am more interested in increasing say the speed of rotor-2. Yes, that is also possible. And if you recall, when I was discussing about the performance map what we have achieved experimentally, it says, when you are increasing the rotational speed of rotor-2, that’s what is improving the performance in sense of pressure rising capacity.

At the same time, we will be having some compromise in overall operating range. So, this is what is designer's choice, what all he will be looking for, okay. And as I told in one of our latest publication where we are having study for say distribution of total pressure ratio between two rotors, and, you know, when we are loading our rotor-1 too high, you will be having more chances for your flow to get separated near the tip region, and that’s what will be making the design more challenging.

So, maybe you need to go with slightly higher loading for rotor-1, not too much loading for rotor-1. And the advantage of having higher loading of rotor-1, we have realized that’s what is giving me say stall free operation, and that’s what is increasing my overall operating range. Now, in next lecture, we will be discussing about the design for the same configuration using free vortex concept. So be with me, thank you for your kind attention.