

Aerodynamic Design of Axial Flow Compressors & Fans
Professor Chetankumar Sureshbhai Mistry
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur
Lecture 46
Design of Low Speed Contra Rotating Fan (Contd)

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Hello, and welcome to Lecture 46. We are discussing design of low speed contra rotating fan.

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Fundamental Design Method

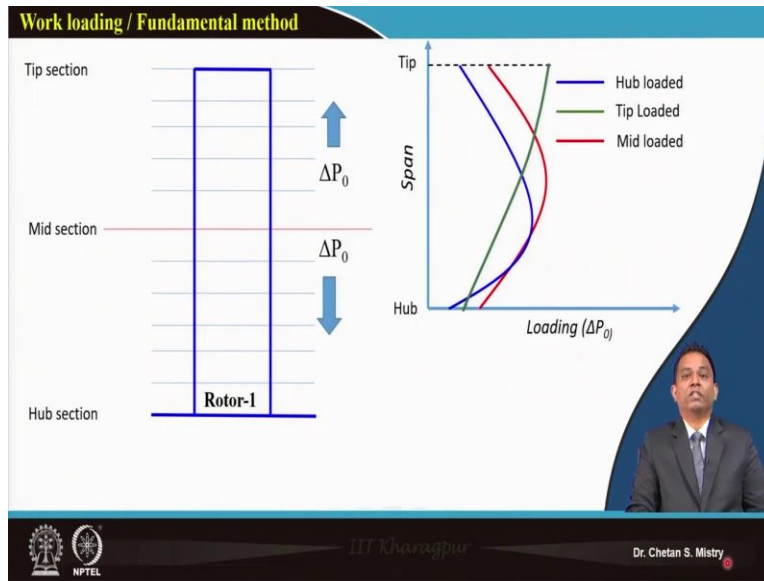
Calculations for Rotor 1 at mid section

Rotor-1	
Solution	6-Mid
r	0.135
r/h	0.667
Peripheral speed U	33.93
Axial velocity C_a	44.40
Total inlet temp T_{t1}	298.88
ΔP_{in}	1200
$\frac{W_1(P_{t1}P_{t2})}{T_{t1}}$	1.01184
ΔT_{in}	1.18520
β_1 (deg)	37.38
w ($C_a \sqrt{\Delta T_1}$)	1191.13
$\tan \beta_2$	-0.0426
β_2 (deg)	-2.44
C_{w2}	35.82
$\tan \alpha_2$	0.8068
α_2 (deg)	38.90
$\Delta \beta$	39.83
Power (kW)	11.21
Pitch σ	0.045
Solidity σ	1.0280
DP	0.52
V_1	55.88
V_2	44.44
DH	0.90
C_u	57.05
m	0.415
Incidence	0.00
Camber angle	67.88
Deviation angle	28.05
Stagger angle	3.45

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In last session, we have discussed about the design calculation for rotor-1 at the mid station, where we have assumed our total pressure rise expected to be 1200 Pa. Then, we have started doing calculation at say mid station as well as we have done our calculation at the hub station.

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Now, say this is what we have expected; here, In this case, we are expecting our total pressure rise at the hub to be say 500 Pa, for what, we have done the calculation at the hub station. Today, we will be moving with say design at the tip station, where we are expecting our total pressure rise to be 1900 Pa. So, let us move with.

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Fundamental Design Method

At Tip

From inlet velocity triangle,

$$\alpha_{1t} = 0^\circ \text{ (Axial Entry)}$$

Hence, $C_{w1t} = 0 \text{ m/s}$

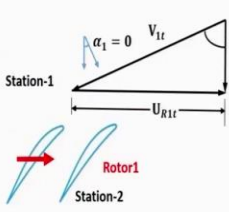
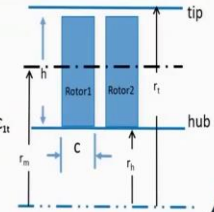
$$U_{R1t} = \frac{\pi N_1 d_t}{60}$$

$$= \frac{\pi \times 2400 \times 0.405}{60}$$

$$\therefore U_{R1t} = 50.89 \text{ m/s}$$

$$\tan \beta_{1t} = \frac{U_{R1t}}{C_a}$$

$$= \frac{50.89}{44.40}$$

$$\beta_{1t} = 48.90^\circ$$



We know

$\alpha_{1t} = 0^\circ$

$d_t = 0.405 \text{ m}$

$N_1 = 2400 \text{ rpm}$

$C_a = 44.40 \text{ m/s}$

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So, this is what we can say our condition. We have our entry to be axial entry. So that is the reason why my α_1 at the tip, that's what is say 0, my C_{w1} at the tip, that is also will be 0. We can say, our peripheral speed, now this is what will be based on my tip diameter, so it is coming 50.89 m/s . Once, we know what is our peripheral speed, and we already know our axial velocity that's what we are assuming to be constant, so that's what will be giving me my relative blade angle at the entry β_1 , as say 48.90 .

At tip,

From inlet velocity triangle,

$$\alpha_{1t} = 0^\circ \text{ (Axial Entry)}$$

$$\text{Hence, } C_{w1t} = 0 \text{ m/s}$$

$$U_{R1t} = \frac{\pi N_1 d_t}{60}$$

$$= \frac{\pi \times 2400 \times 0.405}{60}$$

$$\therefore U_{R1t} = 50.89 \text{ m/s}$$

$$\tan \beta_{1t} = \frac{U_{1t}}{C_a}$$

$$\therefore \tan \beta_{1t} = \frac{50.89}{44.40}$$

$$\therefore \beta_{1t} = 48.90^\circ$$

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Fundamental Design Method

All the parameters for different radial locations can be evaluated similar to mid section from the total pressure rise required at each spanwise location.

The exit Total Pressure at tip

$$P_{02t} = P_{01t} + \Delta P_{0R1t}$$

$$\therefore P_{02t} = 101325 + 1900$$

$$\therefore P_{02t} = 103225 \text{ Pa}$$

And, Pressure Ratio

$$\pi_{1t} = \frac{P_{02t}}{P_{01t}}$$

$$= \frac{103225}{101325}$$

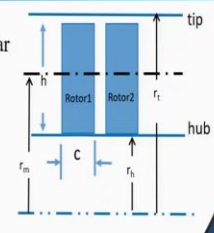
$$= 1.0118$$

Temperature Rise,

$$\Delta T_{0R1t} = \left[\left(\frac{P_{02t} + \Delta P_{0R1t}}{P_{01t}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \times \frac{T_{01t}}{\eta_p}$$

$$\therefore \Delta T_{0R1t} = \left[\left(\frac{101325 + 1900}{101325} \right)^{\frac{1.4-1}{1.4}} - 1 \right] \times \frac{299}{0.85}$$

$$\therefore \Delta T_{0R1t} = 1.87 \text{ K}$$




We know

$P_{01t} = 101325 \text{ Pa}$

$\Delta P_{0R1t} = 1900 \text{ Pa}$

$T_{01t} = 299 \text{ K}$

$\eta_p = 0.8$



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Now, in line to what all we have done, we will be doing our calculation based on our fundamental understanding of thermodynamic and aerodynamic work. So, here at this station, we are expecting our total pressure rise to be say 1900 Pa. And that is the reason if you look at, we are calculating our say pressure at the outlet near the tip, that's what is say it is coming say 103.225 kPa. We are calculating our pressure ratio, that's what is coming 1.011, okay. Now, once we know what is our ΔP_0 , we can do our calculation for say ΔT_0 for rotor-1 at the tip, and that's what is coming 1.87 K.

The exit total Pressure at tip

$$P_{02t} = P_{01t} + \Delta P_{0R1t}$$

$$\therefore P_{02t} = 101325 + 1900$$

$$\therefore P_{02t} = 103225 \text{ Pa}$$

Pressure Ratio

$$\begin{aligned}\pi_{1t} &= \frac{P_{02t}}{P_{01t}} \\ &= \frac{103225}{101325} \\ &= 1.0118\end{aligned}$$

Temperature Rise,

$$\begin{aligned}\Delta T_{OR1t} &= \left[\left(\frac{P_{01t} + \Delta P_{OR1t}}{P_{01t}} \right)^{\frac{(\gamma-1)}{\gamma}} - 1 \right] \times \frac{T_{01t}}{\eta_p} \\ \therefore \Delta T_{OR1t} &= \left[\left(\frac{101325 + 1900}{101325} \right)^{\frac{1.4-1}{1.4}} - 1 \right] \times \frac{299}{0.85} \\ \therefore \Delta T_{OR1h} &= 1.87 \text{ K}\end{aligned}$$

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Fundamental Design Method

Balancing Aerodynamic and Thermodynamic work

$$C_p \Delta T_{air} = \lambda U_{tip} C_u (\tan \beta_{1t} - \tan \beta_{2t})$$

where $\lambda = 0.98$

$$\therefore 1005 \times 1.87 = 0.98 \times 50.89 \times 44.40 (\tan 48.9^\circ - \tan \beta_{2t})$$

Hence, $\beta_{2t} = 16.53^\circ$

Blade deflection angle

$$\Delta \beta_t = \beta_{1t} - \beta_{2t}$$

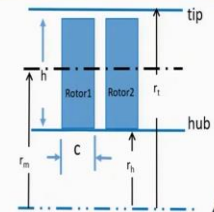
$$\therefore \Delta \beta_t = 48.9^\circ - 16.53^\circ$$

$$\therefore \Delta \beta_t = 32.37^\circ$$

Specific Energy = $C_p \Delta T_{OR1t}$

$$= 1005 \times 1.87$$

$$= 1881.36 \text{ J / kg}$$



We know


$$\Delta T_{air} = 1.987 \text{ K}$$

$$\lambda = 0.98$$


$$U_{tip} = 50.89 \text{ m/s}$$

$$C_u = 44.40 \text{ m/s}$$

$$\beta_{1t} = 48.9^\circ$$



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Now, once this is what is known to us, based on our fundamentals, we can say we are comparing our aerodynamic and thermodynamic work at that station to be same, and based on that we are calculating our β_{2t} , that's what is coming say 16.53. Now once β_{2t} at the tip

is known to us, we can calculate what will be my $\Delta\beta$, and this $\Delta\beta$ at the tip it is coming 32.37° . We can calculate our specific energy for say tip station, okay, that's what is coming 1.88 kJ/kg.

Balancing Aerodynamic and Thermodynamic work,

$$C_p \Delta T_{0R1t} = \lambda U_{R1t} C_a (\tan \beta_{1t} - \tan \beta_{2t})$$

$$\text{where } \lambda = 0.98$$

$$\therefore 1005 \times 1.87 = 0.98 \times 50.89 \times 44.4 (\tan 48.9^\circ - \tan \beta_{2t})$$

$$\therefore \beta_{2t} = 16.53^\circ$$

Blade deflection angle,

$$\Delta\beta_t = \beta_{1t} - \beta_{2t}$$

$$\therefore \Delta\beta_t = 48.9^\circ - (16.53^\circ)$$

$$\therefore \Delta\beta_t = 32.37^\circ$$

$$\text{Specific energy} = C_p \Delta T_{0R1t}$$

$$= 1005 \times 1.87$$

$$= 1881.36 \text{ J/kg}$$

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Fundamental Design Method

Calculation for Flow angles :

$$\tan \beta_{2t} = \frac{U_{R1t} - C_{w2t}}{C_a}$$

$$C_{w2t} = U_{R1t} - C_a \tan \beta_{2t}$$

$$C_{w2t} = 50.89 - 44.40 \tan(16.53)$$

$$\therefore C_{w2t} = 37.72 \text{ m/s}$$

From velocity triangle

$$\tan \alpha_{2t} = \frac{C_{w2t}}{C_{a2t}}$$

$$\tan \alpha_{2t} = \frac{37.72}{44.40}$$

$$\therefore \alpha_{2t} = 40.35^\circ$$

We know

- $\Delta T_{air} = 1.99$
- $\lambda = 0.98$
- $U_t = 50.89 \text{ m/s}$
- $C_a = 44.4 \text{ m/s}$
- $\beta_1 = 48.90^\circ$
- $\beta_2 = 13.69^\circ$

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Now, in order to do the calculation at say tip station, we need to have our C_{w2} at the tip to be known to us. So, what we know from my tan law, it says $\tan \beta_2$ it is nothing but $\frac{U - C_{w2}}{C_a}$. Now, be careful what all we are putting, that's what is at the tip station because we are doing our calculation at the tip station. That's what is giving me my whirl component to be said 37.72, okay.

$$\tan \beta_{2t} = \frac{U_{R1t} - C_{w2t}}{C_a}$$

$$C_{w2t} = U_{R1t} - C_a \tan \beta_{2t}$$

$$C_{w2t} = 50.89 - 44.40 \tan(16.53)$$

$$\therefore C_{w2t} = 37.72 \text{ m/s}$$

So, if you look at here, my velocity triangle, that's what has changed accordingly. So be careful about that part. Once, this is what is known to us, we can do our calculation for α_2 because that's what is my requirement for downstream rotor. So, it says my α_2 , it is coming 40.35°.

From velocity triangle,

$$\tan \alpha_{2t} = \frac{C_{w2t}}{C_{a2}}$$

$$\therefore \tan \alpha_{2t} = \frac{37.72}{44.4}$$

$$\therefore \alpha_{2t} = 40.35^\circ$$

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Fundamental Design Method

Calculation for relative velocity:

$$V_{1r} = \frac{C_{a1t}}{\cos \beta_{1t}} = \frac{44.4}{\cos(48.9^\circ)} \therefore V_{1r} = 67.54 \text{ m/s}$$

$$V_{2r} = \frac{C_{a2t}}{\cos \beta_{2t}} = \frac{44.4}{\cos(16.53^\circ)} \therefore V_{2r} = 46.31 \text{ m/s}$$

De Haller's Factor

$$\frac{V_{2r}}{V_{1r}} = \frac{46.31}{67.54}$$

$$\therefore DH_t = \frac{V_{2r}}{V_{1r}} = 0.69$$

We know

- $C_{a1t} = C_{a2t} = 44.40 \text{ m/s}$
- $\beta_{1t} = 48.9^\circ$
- $\beta_{2t} = 16.53^\circ$
- $d_t = 0.405 \text{ m}$

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Now, once these angles are known to us, we can do calculation for what all will be my relative velocities. So, my relative velocity at say entry, it is coming 67.54 meter per second, at the outlet it is coming 46.31, and if we are calculating our De-Haller's factor, that's what is coming 0.69. So, you can say, we are having variation of our De-Haller's factor at hub, mid and tip station, okay. When we will see the comparison, that will make more sense.

Calculation for relative velocity:

$$V_{1t} = \frac{C_{a1t}}{\cos \beta_{1t}}$$

$$= \frac{44.4}{\cos(48.9^\circ)}$$

$$\therefore V_{1t} = 67.54 \text{ m/s}$$

$$V_{2t} = \frac{C_{a2t}}{\cos \beta_{2t}}$$

$$= \frac{44.40}{\cos(16.53^\circ)}$$

$$\therefore V_{2t} = 46.31 \text{ m/s}$$

De – Haller's factor

$$\frac{V_{2t}}{V_{1t}} = \frac{46.31}{67.54}$$

$$\therefore DH_t = \frac{V_{2t}}{V_{1t}} = 0.69$$

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Fundamental Design Method

Pitch, $s_t = \frac{\pi d_t}{Z_1}$

We know number of blades, $Z_1 = 19$

$$\therefore s_t = \frac{\pi \times 0.405}{19}$$

$$\therefore s_t = 0.067 \text{ m}$$

Solidity,

$$\sigma_t = \frac{c}{s_t}$$

$$\sigma_t = \frac{0.05}{0.067}$$

$$\therefore \sigma_t = 0.672$$

We know
 $d_t = 0.405 \text{ m}$
 $c = 0.045 \text{ m}$

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Now, we need to calculate what will be our diffusion factor at the tip station. So, we will be calculating our pitch, we will be calculating our solidity. So, pitch, we are calculating based on $\frac{\pi d_t}{Z}$. Here, my number of blades, we are assuming to be 19. So, that's what is giving me my pitch to be 0.067 m. And if you are putting our chord to be 0.045, that's what is giving me say my solidity as 0.67.

$$\text{Pitch}, s_t = \frac{\pi d_t}{Z_1}$$

We have number of blades, $Z_1 = 19$

$$s_t = \frac{\pi d_t}{Z_1}$$

$$\therefore s_t = \frac{\pi \times 0.405}{19}$$

$$\therefore s_t = 0.067 \text{ m}$$

Solidity of rotor at hub station,

$$\sigma_t = \frac{c}{s_t}$$

$$= \frac{0.05}{0.067}$$

$$\therefore \sigma_t = 0.672$$

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Fundamental Design Method

Diffusion factor,

$$(DF)_{\text{tip}} = 1 - \frac{\cos \beta_{1t}}{\cos \beta_{2t}} + \frac{\cos \beta_{1t}}{2 \times \sigma_t} (\tan \beta_{1t} - \tan \beta_{2t})$$

$$= 1 - \frac{\cos(48.9^\circ)}{\cos(16.53^\circ)} + \frac{\cos(48.9^\circ)}{2 \times 0.672} (\tan(48.9^\circ) - \tan(16.53^\circ))$$

$\therefore (DF)_{\text{tip}} = 0.73$

Power required

$$P_{\text{Req}} = \frac{\dot{m} \times C_p \times \Delta T_{0,21}}{\eta_m \times \eta_e}$$

$$= \frac{6 \times 1005 \times 1.87}{0.75 \times 0.85}$$

$$= 17.71 \text{ kW}$$

According to Carter's rule; Slop factor


$$m_t = 0.23 \left(\frac{2a}{c} \right)^2 + 0.1 \frac{(90 - \beta_{2t})}{50}$$

$$= 0.23 (2 \times 0.5)^2 + 0.1 \frac{(90 - (16.53))}{50}$$


$$= 0.38$$

We know

- $\Delta T_{0,21} = 1.87 \text{ K}$
- $c = 0.045 \text{ m}$
- $\sigma_t = 0.672$
- $\beta_{1t} = 48.9^\circ$
- $\beta_{2t} = 16.53^\circ$



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Now, my diffusion factor at the tip, if you are calculating, that's what is coming 0.73, okay. And my power calculation, if we compare at the hub, at mid station, at the tip, this is what

is tip loaded rotor, and that is the reason if you are calculating your power, at that station, this is what is coming 17.71 kW. Now based on Carter's rule, we can do our calculation for say m factor. And this is what is coming 0.38.

Diffusion factor,

$$\begin{aligned}(DF)_{t,rotor} &= 1 - \frac{\cos \beta_{1t}}{\cos \beta_{2t}} + \frac{\cos \beta_{1t}}{2 \times \sigma_t} (\tan \beta_{1t} - \tan \beta_{2t}) \\ &= 1 - \frac{\cos(48.9^\circ)}{\cos(16.53^\circ)} + \frac{\cos(48.9^\circ)}{2 \times 0.672} (\tan(48.9^\circ) - \tan(16.53^\circ))\end{aligned}$$

$$\therefore (DF)_{t,rotor} = 0.73$$

Power required,

$$\begin{aligned}P_{R1,t} &= \frac{\dot{m} \times C_p \times \Delta T_{OR1t}}{\eta_m \times \eta_c} \\ &= \frac{6 \times 1005 \times 1.87}{0.75 \times 0.85} \\ &= 17.71 \text{ kW}\end{aligned}$$

According to Carter's rule; slop factor

$$\begin{aligned}m_t &= 0.23 \left(\frac{2a}{c} \right)^2 + 0.1 \frac{(90 - \beta_{2t})}{50} \\ &= 0.23(2 \times 0.5)^2 + 0.1 \frac{90 - (16.53)}{50} \\ &= 0.38\end{aligned}$$

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Fundamental Design Method

Let's assume Incidence angle 'i' to be -2° at tip.

Camber angle at tip

$$\theta_t = \frac{(\Delta\beta_t - i_t)}{(1 - \frac{m_t}{\sqrt{\sigma_t}})} = \frac{(32.37 + 2)}{(1 - \frac{0.38}{\sqrt{0.672}})}$$

$$\theta_t = 63.64^\circ$$

Deviation angle at tip

$$\delta_t = \frac{m_t \theta_t}{\sqrt{\sigma_t}} = \frac{0.38 \times 63.64}{\sqrt{0.672}}$$

$$\delta_t = 29.26^\circ$$

Stagger angle at tip

$$\zeta_t = \beta_{tt} - i_t - \frac{\theta_t}{2} = 48.90 + 2 - \frac{63.64}{2}$$

$$\therefore \zeta_t = 19.08^\circ$$

We know :
 $\beta_{tt} = 48.90^\circ$
 $\Delta\beta_t = 32.37^\circ$
 *slop factor $m_t = 0.38$
 $\sigma_t = 0.672$

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Now, say as we have discussed, for say selection of our incidence angle, so we are assuming our incidence angle at the tip to be -2° . What all are the reasons, what we have discussed earlier. Again, in order to take care of your change of incidence, by default, designers, they are adding that angle. So that when the blade that's what will be acting under off design condition, still behave like working in a design condition, okay.

So conventionally, we are assuming this to be say -2° . If you are putting that, it says my camber angle that's what is coming 63.64° , my deviation angle, it is coming 29.26° , and my stagger angle, that's what is coming 19.08° , okay.

Let's assume Incidence angle 'i' to be -2° at tip

Camber angle at tip,

$$\theta_t = \frac{\Delta\beta_t - i_t}{1 - \frac{m_t}{\sqrt{\sigma_t}}} = \frac{32.37 + 2}{1 - \frac{0.38}{\sqrt{0.672}}}$$

$$\therefore \theta_t = 63.64^\circ$$

Deviation angle at tip,

$$\delta_t = \frac{m_t \theta_t}{\sqrt{\sigma_t}} = \frac{0.38 \times 63.64}{\sqrt{0.672}}$$

$$\therefore \delta_t = 29.26^\circ$$

Stagger angle at tip,

$$\zeta_t = \beta_{1t} - i_t - \frac{\theta_t}{2} = 48.9 + 2 - \frac{63.64}{2}$$

$$\therefore \zeta_t = 19.08^\circ$$

Now, we can understand, we have our calculation at mid station, we have calculation at hub station, we have calculation at say tip station. So, all the required parameter for the design, that's what we have done calculation. So, let us see, this is what will be coming in sense of our design sheet.

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Fundamental Design Method

Rotor-1			
Solution	1-Hub	6-Mid	11-Tip
r	0.067	0.135	0.203
r/h	0.333	0.667	1.000
Peripheral speed U	16.96	33.93	50.89
Axial velocity C _a	44.40	44.40	44.40
Total inlet temp T ₀₁	298.98	298.98	298.98
ΔP _{in1}	700	1200	1700
m ₀₁ (P ₀₁ /P _{in1})	1.00891	1.01184	1.01878
ΔT _{in1}	0.86258	1.18320	1.67811
β ₁ (deg)	20.91	37.39	48.90
w (C _a /ΔT ₁)	696.04	1191.13	1684.49
tan β ₁	-0.5609	-0.9428	0.3856
β ₁ (deg)	-28.29	-2.44	21.09
C _w	41.87	35.62	33.77
tan α ₂	0.9429	0.8068	0.7607
α ₂ (deg)	43.32	38.30	37.26
Δβ	50.20	39.83	27.81
Power (kW)	6.55	11.21	15.85
Pitch s	0.022	0.045	0.087
Solidity σ	2.0169	1.0089	0.8729
DP	0.15	0.52	0.87
V ₁	47.53	55.88	67.54
V ₂	50.91	44.44	47.59
DH	1.07	0.80	0.70
C ₁	61.03	57.05	55.79
m	0.468	0.415	0.388
Incidence	2.00	0.00	-2.00
Camber angle	71.94	67.88	54.08
Deviation angle	23.74	28.05	24.26
Stagger angle	-17.06	3.45	23.86

Rotor-1			
Solution	1-Hub	6-Mid	11-Tip
r	0.067	0.135	0.203
r/h	0.333	0.667	1.000
Peripheral speed U	16.96	33.93	50.89
Axial velocity C _a	44.40	44.40	44.40
Total inlet temp T ₀₁	298.98	298.98	298.98
ΔP _{in1}	500	1200	1900
m ₀₁ (P ₀₁ /P _{in1})	1.00493	1.01184	1.01875
ΔT _{in1}	0.49905	1.18320	1.97200
β ₁ (deg)	20.91	37.39	48.90
w (C _a /ΔT ₁)	497.52	1191.13	1881.36
tan β ₁	-0.2919	-0.6426	0.2987
β ₁ (deg)	-16.27	-2.44	16.53
C _w	29.83	35.62	37.72
tan α ₂	0.8740	0.8068	0.8496
α ₂ (deg)	33.88	38.90	40.35
Δβ	37.18	39.83	32.37
Power (kW)	4.88	11.21	17.71
Pitch s	0.022	0.045	0.087
Solidity σ	2.0169	1.0089	0.8729
DP	0.18	0.52	0.72
V ₁	47.53	55.88	67.54
V ₂	46.25	44.44	46.31
DH	0.97	0.80	0.69
C ₁	53.54	57.05	58.28
m	0.443	0.415	0.377
Incidence	2.00	0.00	-2.00
Camber angle	51.12	67.88	63.64
Deviation angle	15.83	28.05	29.28
Stagger angle	-6.65	3.45	19.08

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At hub station, mid station and say tip station. So, if you try to compare, so if you look at my De-Haller's factor, that's what is coming to be 1.07, that's what is at the hub, and 0.7 near the tip region. If you are considering our diffusion factor, it is coming 0.15, and near

tip, that's what is coming 0.67, okay. Now, if you are considering this as your case, it says my camber angle, that's what is coming 17.94, and 54.08.

Let me tell you what exactly is a purpose here. So, you know, this is what is our final design sheet, okay. What we have assumed, we have assumed at hub, our total pressure rise expected, that's what is say 500 in sense, okay. So here if you look at, this is what is giving me 500, 1200 and 1900. Now, in order to showcase what iterations we have done, say...initially we have assumed at hub my total pressure rise as 700, at mid station, we have not done any change, but at the tip, that's what is say 1700 Pa.

When we are doing this kind of calculation, just look at what De-Haller's factor we are getting, that's what is say 1.07, that's what is on higher side. But at the same time if you compare my $\Delta\beta$ at the hub, that's what is coming 50.20. That's what is too large, okay. Same way, if at the tip, if you are looking at, that's what is say 27.81. So, you can say the blade what I will be making, that's what will be highly twisted blade, okay.

So, what it indicates? Now, you need to play with your parameters. So, what all parameters we need to play with? So, here if you look at, for this case, for our design, what we have done, at hub we have taken our total pressure rise as say 500 Pa. And that's what is giving me my $\Delta\beta$ as 37.18. You can compare these numbers, okay.

At the same time, near the tip region also, this is what is slightly on higher side but you can say that's what is giving me my angle as say 32.37, $\Delta\beta$. So, by this way, you are having the flexibility to modify these numbers. And basically, you do initial calculation at the mid station with your pen and paper, verify that part, is it coming as a same number, then you make or extend your excel sheet for hub and tip station.

This is what is advisable thing. And that's what will be giving me two extremes. At hub, what need to be my number, at tip, what need to be my number, okay. Now here, if you compare, say...compared to my earlier assumption, diffusion factor that's what is coming 0.18, and here this is what is coming 0.73. It is on higher side, okay. We will discuss about this point, what is the reason why we are expecting this to be on higher side.

Remember one thing, what all numbers that's what is given in open literature, what all people they are discussing, that's what is applicable for stage, rotor and stator combination. That may or may not be straight way applicable for say the design case for contra rotating concept, okay. It is designer's choice, you need to play with the numbers, okay.

Now here in this case, if you are comparing, I am improving my De-Haller's factor in the proper way. At the same time, the camber angle, that also that has reduced from say 51 at the hub to 63 and the tip region, okay. Now, once we have finalized with the two extremes and mid station, what we will be doing? We will be making the number of stations.

Remember one thing, this blade, that's what is having aspect ratio of 3, that means my height of the blade will be taller or it will be larger, compared to aspect ratio one. It means, I will be having blade height to be large. If that's what is your case, it is advisable to go with more number of stations.

(Refer Slide Time: 12:36)

Fundamental Design Method

Solution	Rotor-1										
	1-Hub	2	3	4	5	6-Mid	7	8	9	10	11-Tip
r	0.067	0.061	0.054	0.108	0.121	0.155	0.148	0.162	0.175	0.189	0.203
r/h	0.333	0.460	0.467	0.533	0.600	0.667	0.733	0.800	0.867	0.933	1.000
Peripheral speed U	16.96	20.36	23.75	27.14	30.54	33.93	37.32	40.72	44.11	47.50	50.89
Axial velocity C _z	44.40	44.40	44.40	44.40	44.40	44.40	44.40	44.40	44.40	44.40	44.40
Total inlet temp T ₀₁	298.98	298.98	298.98	298.98	298.98	298.98	298.98	298.98	298.98	298.98	298.98
ΔP_{01}	500	640	780	920	1060	1200	1340	1480	1620	1760	1900
$\pi W(P_{01}/P_{02})$	1.00483	1.00632	1.00770	1.00908	1.01046	1.01184	1.01322	1.01461	1.01599	1.01737	1.01875
ΔT_{01}	0.49555	0.63335	0.77151	0.90955	1.04744	1.18520	1.32283	1.46032	1.59768	1.73491	1.87200
β_1 (deg)	20.91	24.63	28.14	31.44	34.52	37.39	40.05	42.52	44.81	46.93	48.90
w (C _z / ΔT_1)	497.52	636.52	775.37	914.09	1052.68	1191.13	1329.44	1467.62	1605.67	1743.58	1881.36
$\tan \beta_1$	-0.2919	-0.2601	-0.2154	-0.1626	-0.1045	-0.0426	0.0219	0.0886	0.1569	0.2262	0.2967
β_2 (deg)	-16.27	-14.58	-12.15	-9.24	-5.87	-2.44	1.26	5.06	8.91	12.75	16.53
C _w	29.93	31.90	33.31	34.36	35.18	35.82	36.35	36.78	37.15	37.46	37.72
$\tan \alpha_2$	0.8740	0.7186	0.7503	0.7740	0.7923	0.8068	0.8186	0.8284	0.8366	0.8438	0.8496
α_2 (deg)	33.88	35.70	36.88	37.74	38.39	38.90	39.31	39.64	39.92	40.15	40.35
ΔP	37.18	39.21	40.30	40.68	40.40	39.83	38.79	37.48	35.90	34.18	32.37
Power (kW)	4.68	5.99	7.30	8.60	9.91	11.21	12.51	13.81	15.11	16.41	17.71
Pitch α	0.022	0.027	0.031	0.036	0.040	0.045	0.049	0.054	0.058	0.063	0.067
Solidity σ	2.0160	1.6800	1.4400	1.2000	1.1200	1.0000	0.9163	0.8400	0.7754	0.7200	0.6720
DP	0.18	0.26	0.33	0.40	0.46	0.52	0.58	0.62	0.66	0.70	0.73
V ₁	47.53	48.84	50.35	52.04	53.89	55.88	58.00	60.24	62.58	65.02	67.54
V ₂	46.25	45.80	45.42	44.98	44.64	44.44	44.41	44.57	44.94	45.52	46.31
DHP	0.97	0.94	0.90	0.86	0.83	0.80	0.77	0.74	0.73	0.70	0.69
C _z	33.54	54.67	55.51	56.14	56.65	57.05	57.38	57.66	57.89	58.09	58.26
m	0.443	0.439	0.434	0.428	0.422	0.415	0.407	0.400	0.392	0.385	0.377
Incidence	2.00	1.60	1.20	0.80	0.40	0.00	-0.40	-0.80	-1.20	-1.60	-2.00
Camber angle	51.12	56.80	61.27	64.49	66.66	67.88	68.24	67.87	66.89	65.44	63.64
Deviation angle	15.93	19.27	22.18	24.62	26.58	28.05	29.05	29.61	29.79	29.65	29.26
Stagger angle	-6.65	-5.41	-3.89	-1.61	0.79	3.45	6.33	9.39	12.57	15.81	19.08

The slide also includes a schematic diagram of a contra-rotating compressor stage with two rotors (Rotor1 and Rotor2) and a hub. Key parameters shown in the diagram include tip, hub, axial distance C, and radii r₁ and r₂.

Dr. Chetan S. Mistry

With the space as a constraint for this slide, so we have consider only 11 stations, but you can play with say 20 stations, 21 stations, even you can go with 40 stations, nobody will stop you, okay. So here if you look at, this is what is my distribution of ΔP_0 , okay. Now, this ΔP_0 , that's what has not been selected randomly.

Remember one thing, there is no systematic rule you need to apply here. Many times, students they used to put, say you know, they will set this number in such a way that it will give the linear kind of trend or linear variation of total pressure. There is nothing wrong doing that part. But you know, that's what is a mechanical kind of work. We are more towards say aerodynamic design. So, you need to think in an innovative way.

And that is how, you need to keep an eye with the variation of $\Delta\beta$. So, just look at when we are doing this, we are putting these numbers, I will be having particular trend for $\Delta\beta$. At the same time, just keep an eye for De-Haller's factor, okay. Same way, you just keep an eye for what is happening with your camber angle. Just look at, this is what is showing me say particular trend for my camber angle variation, okay.

(Refer Slide Time: 14:00)

Corrected Flow Angles

Corrected deviation--
It can be based on CFD analysis / Experiments or by Experience.

Corrected deviation $\delta_{corr} = \delta + \delta_{assumed}$
Here, $\delta_{assumed}$ = assumed correction value

The modifications need to be made as,
Corrected camber $\theta_{corr} = \Delta\beta + \delta_{corr} - i$

Corrected Stagger angle $\zeta_{corr} = \beta_1 - i - \frac{\theta_{corr}}{2}$

This corrected camber and corrected stagger need to be used for making of blades.....

Dr. Chetan S. Mistry

Now, once whole data sheet that's what is ready with us, then you will say, Sir, we will go with now making of the blade. Now if you recall, I was talking to you what Carter has given in sense of calculation for deviation angle, that's what is not giving the exact features, what we are looking for. So, these days, people, they are comfortable with using the computational tool.

So, based on what all calculation initially we have done in last data sheet, that's what will be the input in sense of my camber and stagger angle, okay. Once, we are putting this

camber angle and stagger angle, we will be observing what is happening near my trailing edge, and that's what will be giving you clue to modify your deviation angle.

So, in one of my paper I have discussed how do we decide with this deviation angle. If you are interested, you can go through, you refer that paper, that will give you idea how we need to check with and why we need to change this angle, okay. So, you know, based on CFD analysis, based on experiments or based on experience, you can modify your deviation angle.

So, once you are modifying your deviation angle, you need to take some numbers. So, this is what is my assumed number. Once we are doing this delta calculation accordingly, my camber angle, I need to correct, because my earlier deviation angle it was different, now I need to put corrected camber angle. So, this is what is my corrected camber angle. Be careful about this.

Same way, when my camber that's what has been changed, I need to do modification for say stagger angle. So, it says my corrected stagger angle, that's what is say $\beta - i$, and $\theta_{corr}/2$, okay. Now, this corrected camber and corrected stagger, these are the angles which we need to use for making of our blade. Be careful about this part, okay!

$$\text{Corrected deviation } \delta_{corr} = \delta + \delta_{assumed}$$

The modifications need to be made as,

$$\text{Corrected camber } \theta_{corr} = \Delta\beta + \delta_{corr} - i$$

$$\text{Corrected Stagger angle } \zeta_{corr} = \beta_1 - i - \frac{\theta_{corr}}{2}$$

It may be possible say like deviation angle, what you are assuming, that may be coming different at different stations. You can say, you will be assuming your deviation angle more near the hub region, you may be reducing your deviation angle to moving towards say tip region. But that need to be in a systematic way, okay. And that's what the computational tool, that's what will be coming into the picture. That's what will give you how do we move forward with.

(Refer Slide Time: 16:45)

Corrected Flow Angles

At mid section
 Let's assume $\delta_{assume} = 4^\circ$

Corrected deviation
 $\delta_{m,corr} = 28.05 + 4 = 32.05^\circ$

Corrected camber
 $\theta_{corr} = \Delta\beta + \delta_{corr} - i$
 $= 39.83 + 32.05 - 0$
 $\theta_{m,corr} = 71.87^\circ$

Corrected stagger
 $\zeta_{m,corr} = 37.38 - 0 - \frac{71.8}{2}$
 $\zeta_{m,corr} = 1.45^\circ$

Dr. Chetan S. Mistry

Now for this case, say we have assumed this to be 4° and if you do it, say at mid station, we have done calculation for corrected deviation, corrected camber and corrected incidence, okay, or corrected our stagger angle, okay. So, you can say my corrected deviation, corrected camber and connected stagger, we have done calculation.

At mid section,

$$\text{Let's assume } \delta_{assume} = 4^\circ$$

Corrected deviation

$$\delta_{m,corr} = 28.05 + 4 = 32.05^\circ$$

Corrected camber,

$$\begin{aligned} \theta_{corr} &= \Delta\beta + \delta_{corr} - i \\ &= 39.83 + 32.05 - 0 \end{aligned}$$

$$\theta_{m,corr} = 71.87^\circ$$

Corrected stagger,

$$\zeta_{m,corr} = 37.38 - 0 - \frac{71.8}{2}$$


$$\zeta_{m,corr} = 1.45^\circ$$

(Refer Slide Time: 17:10)

Corrected Flow Angles

Extending for other radial locations.....

Solution	Rotor-1										
	1-Hub	2.00	3.00	4.00	5.00	6-Mid	7.00	8.00	9.00	10.00	11-Tip
r	0.067	0.081	0.094	0.108	0.121	0.135	0.148	0.162	0.175	0.189	0.203
rit	0.33	0.40	0.47	0.53	0.60	0.67	0.73	0.80	0.87	0.93	1.00
Incidence	2.00	1.60	1.20	0.80	0.40	0.00	-0.40	-0.80	-1.20	-1.60	-2.00
Camber angle	51.12	56.88	61.27	64.49	66.66	67.88	68.24	67.87	66.89	65.44	63.64
Deviation angle	15.93	19.27	22.18	24.62	26.58	28.05	29.05	29.61	29.79	29.65	29.26
Stagger angle	-8.65	-5.41	-3.69	-1.81	0.79	3.45	6.33	9.39	12.57	15.81	19.08
Deviation (assumed)	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Corrected deviation	19.93	23.27	26.18	28.62	30.58	32.05	33.05	33.61	33.79	33.65	33.26
Corrected camber	55.12	60.88	65.27	68.49	70.66	71.88	72.24	71.87	70.89	69.44	67.64
Corrected Stagger angle	-8.65	-7.41	-5.69	-3.81	-1.21	1.45	4.33	7.39	10.57	13.81	17.08



Dr. Chetan S. Mistry

Now, this logic, if you look at, this is what is my assumed deviation angle. As I told, what you need to do is maybe you can plot your velocity contours, you can plot your C_p distribution at particular station, then you just observe how my flow is behaving on my suction surface, on my pressure surface, how it is behaving near the leading edge, how it is behaving near the trailing edge, and based on that, systematically you need to assume this number.

So, for this design, we have assumed this to be 4° . And that's what will be giving me my corrected camber and corrected stagger angle, okay. Now, once we are achieving this corrected camber and corrected stagger angle, we can say we have our equation for say C4 profile.

(Refer Slide Time: 18:00)

Generation of Blade

The upper and lower surface co-ordinates for C4 profile family airfoils are given by,

$$\pm y_t = \left(\frac{t}{0.2}\right) \times (0.3048x^{\frac{1}{2}} - 0.0914x - 0.8614x^2 + 2.1236x^3 - 2.9163x^4 + 1.9744x^5 - 0.5231x^6)$$

The airfoil wrapped around circular camber line is given by,

$$y_c = \left[\left\{ \frac{0.5}{\sin\left(\frac{\theta_{corr}}{2}\right)} \right\}^2 - (x-0.5)^2 \right]^{0.5} - \frac{0.5}{\tan\left(\frac{\theta_{corr}}{2}\right)}$$

The wrapping of the airfoil co-ordinates along camber line is given by

For upper surface,

$$x_U = x - y_t \sin \phi$$

$$y_U = y_c + y_t \cos \phi$$

For Lower surface,

$$x_L = x + y_t \sin \phi$$

$$y_L = y_c - y_t \cos \phi$$

Where $\phi = \tan^{-1}\left(\frac{dy_c}{dx}\right) = \tan^{-1}\left(\frac{dy_t}{-(x-0.5)}\right)$

Circular arc camber line co-ordinates.

NPTEL

Dr. Chetan S. Mistry

In that C4 profile, do not forget, what angle we were considering, θ , now that angle need to be corrected angle, okay. Do not miss this part. So purposefully, this slide has been kept intentionally here, okay.

The upper and lower surface co – ordinates for C4 profile family airfoils are given by

$$\pm y_t = \left(\frac{t}{0.2}\right) \times \left(0.3048x^{\frac{1}{2}} - 0.0914x - 0.8614x^2 + 2.1236x^3 - 2.9163x^4 + 1.9744x^5 - 0.5231x^6\right)$$

The airfoil wrapped around circular camber line is given by,

$$y_c = \left[\left\{ \frac{0.5}{\sin\left(\frac{\theta_{corr}}{2}\right)} \right\}^2 - (x - 0.5)^2 \right]^{0.5} - \frac{0.5}{\tan\left(\frac{\theta_{corr}}{2}\right)}$$

The wrapping of the airfoil co – ordinates along camber line is given by

For upper surface,

$$X_U = x - y_t \sin \phi$$

$$Y_U = y_c + y_t \cos \phi$$

For lower surface,

$$X_L = x + y_t \sin \phi$$

$$Y_L = y_c - y_t \cos \phi$$

$$\text{where } \phi = \tan^{-1} \left(\frac{dy_c}{dx} \right) = \tan^{-1} \left(\frac{dy_c}{-(x - 0.5)} \right)$$

(Refer Slide Time: 18:24)

Generation of Blade

Co-ordinates of the CG

$$X_{cg} = 43.5 - 0.0036 \times \theta_{corr}$$

$$Y_{cg} = 0.164 \times \theta_{corr}$$

Upper surface shifting CG to origin,

$$X_{U1} = x_U \times 100 - X_{cg}$$

$$Y_{U1} = y_U \times 100 - Y_{cg}$$

Lower surface shifting CG to origin,

$$X_{L1} = x_L \times 100 - X_{cg}$$

$$Y_{L1} = y_L \times 100 - Y_{cg}$$

The airfoil profiles have been rotated with the **Stagger angle**,

For Upper surface


$$X_U = X_{U1} \times \cos \zeta_{corr} - Y_{U1} \sin \zeta_{corr}$$


$$Y_U = X_{U1} \times \sin \zeta_{corr} + Y_{U1} \cos \zeta_{corr}$$

For lower surface

$$X_L = X_{L1} \times \cos \zeta_{corr} - Y_{L1} \sin \zeta_{corr}$$

$$Y_L = X_{L1} \times \sin \zeta_{corr} + Y_{L1} \cos \zeta_{corr}$$





IIT Kharagpur

Dr. Chetan S. Mistry

Now, same way what stagger angle we have considered for our earlier program, now that input will be corrected stagger angle. So, accordingly, we will be getting our blade geometry, okay. So, for rotor-1, this is what we say, in sense of we are doing calculation for say this corrected camber angle and corrected stagger angle.

Co – ordinates of the CG,

$$X_{cg} = 43.5 - 0.0036 \times \theta_{corr}$$

$$Y_{cg} = 0.164 \times \theta_{corr}$$

Upper surface shifting CG to origin,

$$X_{U1} = X_U \times 100 - X_{cg}$$

$$Y_{U1} = Y_U \times 100 - Y_{cg}$$

Lower surface shifting CG to origin,

$$X_{L1} = X_L \times 100 - X_{cg}$$

$$Y_{L1} = Y_L \times 100 - Y_{cg}$$

The airfoil profiles have been rotated with the stagger angle,

For upper surface,

$$X_U = X_{U1} \times \cos \zeta_{corr} - Y_{U1} \sin \zeta_{corr}$$

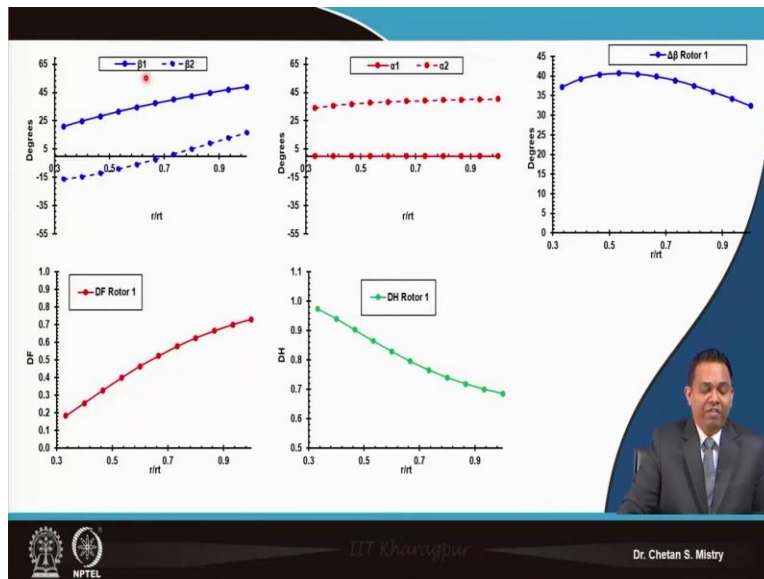
$$Y_U = X_{U1} \times \sin \zeta_{corr} + Y_{U1} \cos \zeta_{corr}$$

For lower surface,

$$X_L = X_{L1} \times \cos \zeta_{corr} - Y_{L1} \sin \zeta_{corr}$$

$$Y_L = X_{L1} \times \sin \zeta_{corr} + Y_{L1} \cos \zeta_{corr}$$

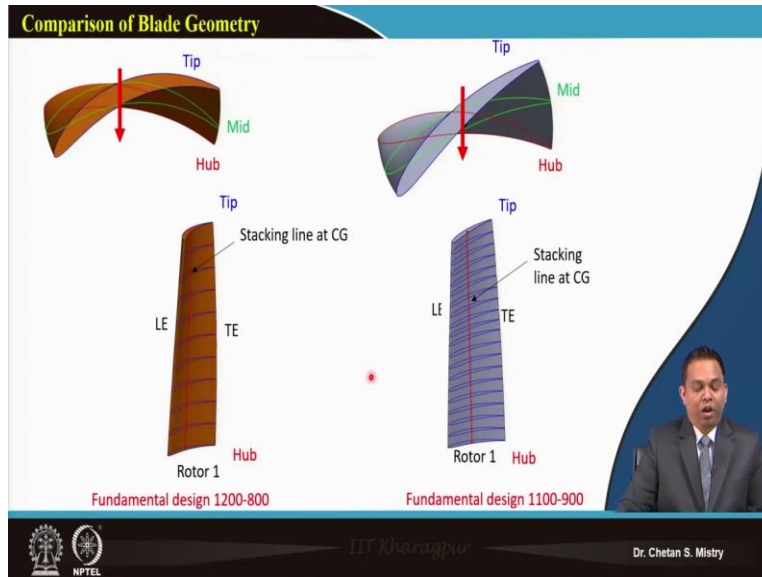
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Let me show you, say how we will be having our variation of β_1 and β_2 . So, you can say, this is what is my $\Delta\beta$ variation. This is what is representing my $\Delta\alpha$ variation, and $\Delta\beta$, that's what is varying in a different way here. My diffusion factor at the hub, it is coming to be

lower, and my diffusion factor at the tip, that's what is larger, that's what is reflecting in sense of our De-Haller's factor also, okay.

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Now, when we are putting all our angle calculation for different 11 stations, so if you look at, this is what is represented by 1200 and 800 Pa distribution. So, this is what is the geometry for my rotor-1, okay. So, here if you look at, say this is what is my hub curvature, my mid station and my tip station, okay.

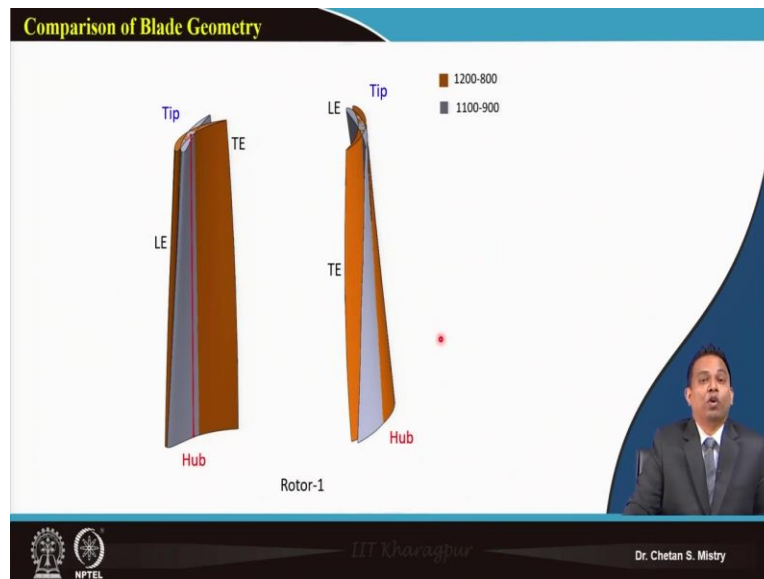
So, same way here, this is what is my hub and tip. Since this is what is a rotating component, we will be taking all our airfoils about CG. Now let me show you here. So, this is what is we have done our initial calculation with, that's what is with 1100 and 900 Pa. So, you can compare the change of the shape for my blade. Just look at, this is what is my rotor-1, this is my rotor-2.

And if you are looking at, how my angles, that's what is changing. Now, be careful, what all we are discussing, that's what is ΔP_0 you are distributing along the span. It may be having some variation. So many times, say initial design, people, they are doing with excel sheet, then they realize they are looking for say particular kind of performance, that's what they may or may not be achieving.

Then at that time, say span wise, they will be doing their calculation for total pressure rise, and based on that comparison, according to the expectation, my ΔP_0 , that's what can be changed. When I am changing ΔP_0 , accordingly I need to make my distribution in a smooth way.

If you look at here, say my variation of β_1 and variation of β_2 , that need to be smooth. It should not be zigzag. Otherwise, it will not permit you to make the blade. And that is the reason as I told, you take more number of stations. So here, we have taken 11 station. You go with say 22 station, 21 station, no one will stop you, okay. And this is what.

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Now, when we are comparing, this is what is a comparison for rotor-1 for two different loadings. So, here if you look at, this golden color what you are observing, that's what is say 1200 and 800 Pa, that means my expected total pressure from rotor-1 is 1200 Pa. And my rotor-2, that's what is we will be calculating immediately. So, here if you look at, this is how my angle variations, that's what is coming into the picture.

So, if you look carefully, from leading edge, okay, so from hub to tip, I will be having the variation of angle. At the same time, towards the trailing edge also, I will be having highly twisted blade for 1200 Pa, okay. So, remember, at all station, my distribution of pressure, that's what is different for both the blades. This is what is for representation purpose, okay.

Now you can understand, if you try to look at, what is happening, here? Near the tip region, I am having my angle, that's what is coming to be larger, okay. Now, that's what, if you go through your aerodynamics, it says when I will be having my blade to be highly cambered blade, or slightly more cambered blade, I may be having chances for my flow to get separated.

So aggressively, if you are doing design, many times, maybe as per your blade shape or as per your computational study, you may need to modify these angles or you may need to reduce your loading. Sometimes you feel, maybe you need to go with the higher loading. So, this is what is all designer's choice, okay. So, I am sure, this is what is giving you idea about the design for rotor-1.

So, here, we are stopping with. In next lecture, we will be discussing about the design of rotor-2, okay. So, be with me, and we will be discussing design of rotor-2 based on fundamental concept, and later on, we will be comparing both the rotors by using two different loadings. Thank you.