Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 45
Design of Low Speed Contra Rotating Fan (Contd)

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Hello, and welcome to lecture 45. We are discussing design of low speed contra rotating fan.

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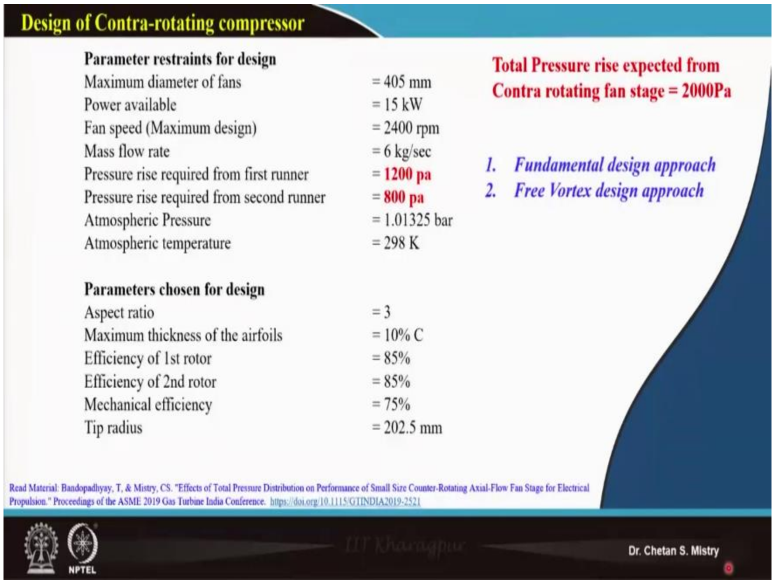

So, this is what configuration we have decided for our design problem. Now, here in this case, we have selected this contra rotating fan that's what is having aspect ratio of 3, that's what is little different from what conventional design what we are expecting. So, if you recall, this is what will give you idea, say, for low speed axial flow compressor, we have taken our aspect ratio of 1. Here, we are considering our aspect ratio of 3.

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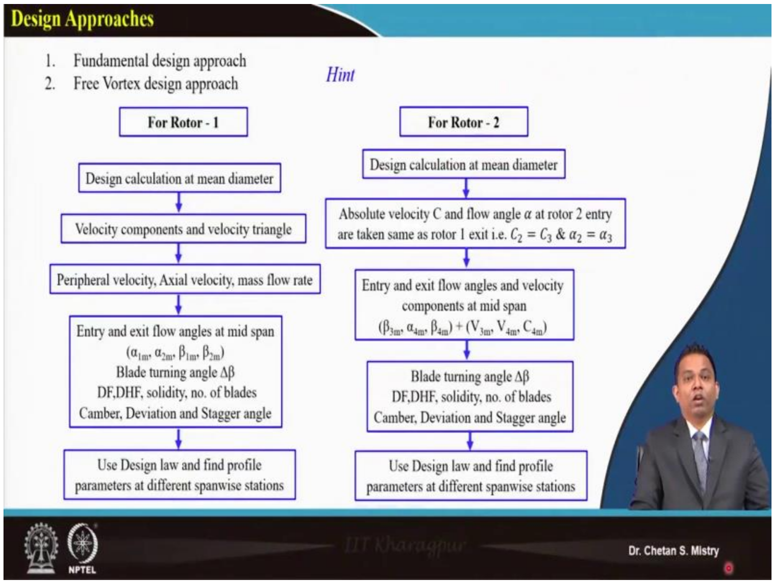

Now, this is what all we have discussed in sense of our hint for solving the problem. So, what we realized? We will be calculating our mean diameter for rotor-1. Then all parameters, that will be calculated at mid station. Based on our design concept or design fundamentals, we will be opting that for say different stations for this blade.

Now for rotor-2, as we have discussed, say for conventional stage, what absolute flow or absolute velocity coming out from the rotor, that's what we are assuming to be going to the stator. So, in line to that here, we are making our assumption, the velocity with which my flow that will be coming out from rotor-1, that's what is say absolute velocity, that's what will be entering inside my rotor-2, okay.

So, we can say our velocity C_2 and velocity C_3, they both are same. In other sense, we can say my absolute flow angle α_2 and α_3, that will be same. Then, based on this assumption, we will be doing our calculation at the mid station. Once, we are having the calculation at the mid station, we can do our calculation at different locations along the span, and we can make our blade ready.

And that is how we will be moving with say design of this contra rotating fan. Now, very first thing, let us take, with say my rotor-1 design. Now if you recall, in order to have this rotor-1 design, we need to have some velocity components that need to be known to me. So, if you recall our general guideline or basic guideline what we have discussed for the design of axial flow compressor, it says maybe mass flow rate or we can say my flow coefficient or say my diameters, if that's what is known to me, I can move with the calculation of say initial parameters.

For this design, if you look at what we know is mass flow rate that's what is known to me, my tip diameter also is known to me and rotational speed for both the rotors for design, we have taken 2400 rpm. So that's what is making our calculation little easy and since of say assumptions. If you recall, we have designed our low speed axial flow compressor where we have assumed our say C_a/U_{tip} to be say some number. So here in this case it is not the case because we know our mass flow rate. So, let us move with the calculation.

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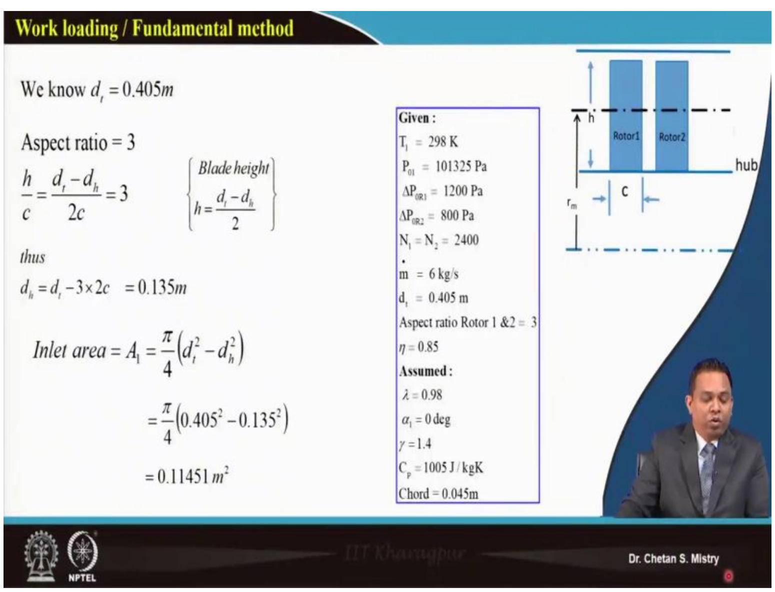

What we know? Say, we know our tip diameter is 0.405 m, okay. And our aspect ratio, that's what is given, it is 3. Even chord for these blades, they are given it is say 45 mm. So, we can say, this is chord, it is say 0.045 m.

$$We\ know\ d_t = 0.405\ m$$

$$Aspect\ ratio = 3$$

If that's what is known to me, say my aspect ratio we can define as say height to chord ratio. You can see, this is my height to chord ratio.

Now this height, we can write down, that is nothing but my tip diameter minus hub diameter divided by say my 2 C, okay. Since I know what is my aspect ratio, we can do calculation for hub diameter. And here, in this case, if you look at my hub diameter, that's what is coming 0.135 m.

$$\frac{h}{c} = \frac{d_t - d_h}{2c} = 3$$

$$\because Blade\ height, h = \frac{d_t - d_h}{2}$$

$$thus, d_h = d_t - 3 \times 2c = 0.135\ m$$

So, you can understand if you are known, not known with the aspect ratio, you can assume your radius ratio. By that way also you can do your calculation, okay.

So initial guess, if it is known to you, better, you go with that part, with the known parameters. If not, then make suitable assumptions, okay, and accordingly, we need to move forward with it. Now once, my hub diameter and tip diameter, they are known to us, we can do the calculation for the inlet area.

You can say it is nothing but $\frac{\pi}{4}(d_t^2 - d_h^2)$, okay and that's what is giving me my area to be 0.114 m^2.

$$Inlet\ area = A_1 = \frac{\pi}{4}(d_t^2 - d_h^2)$$

$$= \frac{\pi}{4}(0.405^2 - 0.135^2)$$

$$= 0.11451\ m^2$$

So, this is what is the area in which my air that's what is going inside. Now, this area, you can understand, we know our basic continuity equation. It says my mass flow rate, we can write down as say $\rho A C_a$.

Basically, we are looking for our velocity component, we are looking for axial velocity. So, this axial velocity, we can write down, that is nothing but $\frac{\dot{m}}{\rho A}$. Let us make the assumption, say density we can assume safely for our atmospheric condition as 1.18 kg/m^3. If you are putting mass flow rate that's what is given to us, it is 6 kg/s.

So, 6 divided by this, that's what is giving me my axial velocity to be 44.4 m/s.

$$C_a = \frac{\dot{m}}{\rho A}$$

$$= \frac{6}{1.18 \times 0.11451}$$

$$\therefore C_a = 44.4\ m/s$$

Now, this is what is my axial velocity that's what is known to me. Here in this case, like there is always confusion with total property and static property, say we are considering our static temperature that's what is given, that is 298 K.

So just careful, if this is what is specifically been mentioned as a static temperature, then its better you do calculation for say total property. And that's what we have done here. Say Thermometer, that's what is giving me my atmospheric temperature to be say 298 K, that's what is nothing but my static temperature.

So, we can calculate our total temperature as say $T_1 + \frac{C_a^2}{2C_p}$ because we can assume or we have our entry of the flow that's what is axial. And that is the reason my C_a and C_1, they both are same. So, if you are putting this as a number, it says my total entry temperature that's what is 299 K. So, now we can say we know what is our axial velocity, what is our temperature.

$$Total\ inlet\ temperature$$

$$T_{01} = T_1 + \frac{C_a^2}{2C_p}$$

$$= 298 + \frac{44.4^2}{2 \times 1005}$$

$$\therefore T_{01} = 299\ K$$

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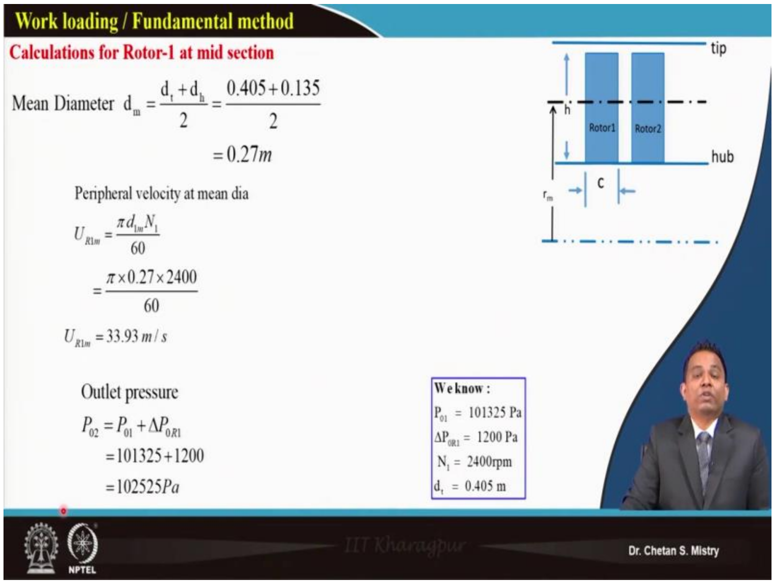

Now, my next parameter for making my velocity triangle, that's what is say my peripheral speed. And as we know, we are doing our calculation at the mid station. That is the reason why mean diameter we can calculate as say $\frac{d_t+d_h}{2}$, that's what is giving me my mid station as say 0.27. This is what is say my 0.27 m, that's what is my diameter at the mid section, okay.

$$Mean\ Diameter\ d_m = \frac{d_t + d_h}{2} = \frac{0.405 + 0.135}{2}$$

$$= 0.27\ m$$

Now, once this is what is known to us, we can do calculation for our say peripheral speed. So, this peripheral speed, we are writing for rotor-1, it is say $\frac{\pi d_{1m} N_1}{60}$. Since my rotational speed for rotor-1 it is giving 2400 rpm, so that is why if you are putting that, it says my peripheral speed it is coming 33.93 m/s.

$$Peripheral\ velocity\ at\ mean\ dia$$

$$U_{R1m} = \frac{\pi d_{1m} N_1}{60}$$

$$= \frac{\pi \times 0.27 \times 2400}{60}$$

$$\therefore U_{R1m} = 33.93\ m/s$$

So, this way you can do your calculation for say C_a as well as U. And as we discussed, maybe if C_a/U_{tip}, that's what is given to you, that will make your life easy, you can go with the calculation straight way. So purposefully, this data that has been given; so that you will get the idea how to do calculation if those parameters are not known and few parameters which are required, that's what is known to you, okay.

Now if we say, my outlet pressure. So, what we have assumed? Say, we are expecting our total pressure by rotor-1, that's what is say 1200 Pa. So, we can say my total pressure at the mid station, that's what is $P_{01} + \Delta P_0$. So, this is what will be my total pressure at the outlet.

$$Outlet\ pressure,$$

$$P_{02} = P_{01} + \Delta P_{0R1}$$

$$= 101325 + 1200$$

$$= 102525\ Pa$$

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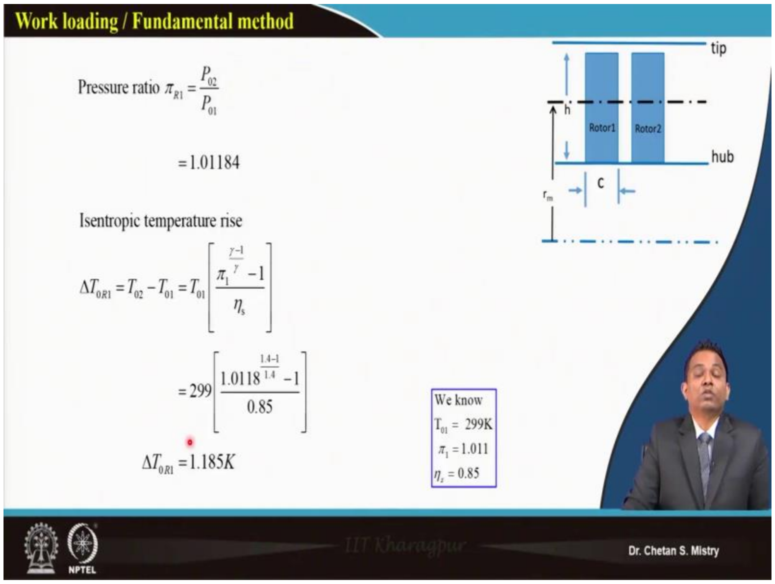

Now, once my total pressure at the outlet, that's what is known to us, we can do our calculation for the pressure ratio for my rotor-1. It says it is P_{02}/P_{01}. And if we are putting that number, it says it is coming 1.011. That's what is my pressure ratio of rotor-1, okay.

$$Pressure\ ratio, \pi_{R1} = \frac{P_{02}}{P_{01}}$$

$$= 1.01184$$

Since my pressure ratio is known to me, we want, we are looking for something, say you can understand, say if we are looking for say…inlet velocity triangle, we are looking for say…three different velocities.

One, it is say peripheral speed, we are looking for absolute velocity or axial velocity and we can calculate what will be our relative velocity. Now, in line to that, suppose say if it is having whirl, you can do calculation for C_{w1}. But since our entry, it is given axial entry, so my C_{w1}, that's what is 0.

But in order to plot my exit velocity triangle, I need to have one of the parameters or the angle, that's what need to be known to me, okay. So, we are moving towards that. What it says? We can say our isentropic temperature rise, that's what we can calculate here by using say T_{01}, what is my pressure ratio and efficiency.

It says efficiency, that's what for two rotors, they are given 85%. So, we can this is what is say 85. That's what say, my ΔT_0 in rotor-1, it is 1.18 K.

$$Isentropic\ temperature\ rise$$

$$\Delta T_{0R1} = T_{02} - T_{01} = T_{01}\left[\frac{\pi_1^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s}\right]$$

$$= 299\left[\frac{1.0118^{\frac{1.4-1}{1.4}} - 1}{0.85}\right]$$

$$\therefore \Delta T_{0R1} = 1.185\ K$$

Be careful! What design we are discussing at this moment, that's what is say subsonic design or low speed design. So, this range of temperature, that will be coming in a lower side.

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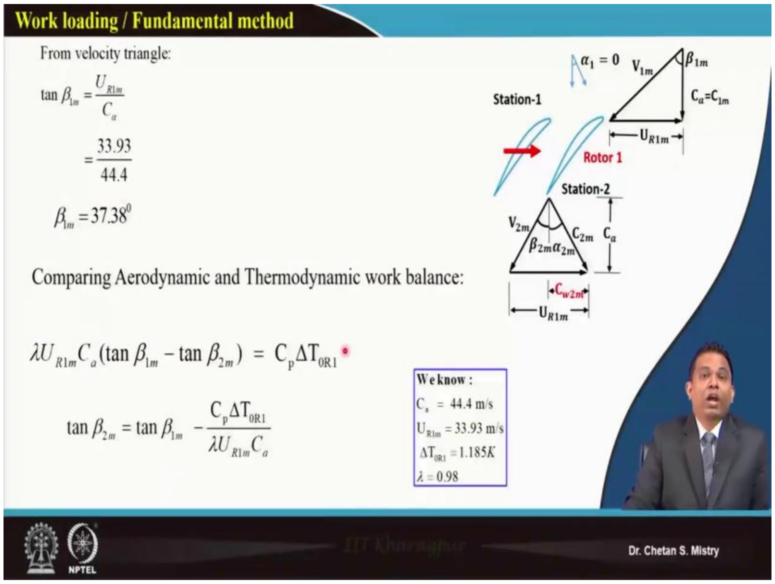

Now, as we have discussed, now suppose if I consider, this is what is say my relative or my velocity triangle at the entry of my rotor-1. Since this is what is say the axial entry, we can write down my $tan\beta_{1m} = \frac{U_{R1m}}{C_a}$. U, we have calculated, it is 33.93. And C_a also, we have calculated, it is 44.4. So, they are the known parameters here, okay.

$$\tan\beta_{1m} = \frac{U_{R1m}}{C_a}$$

$$= \frac{33.93}{44.4}$$

$$\therefore \beta_{1m} = 37.38°$$

If that's what is your case, we can calculate what will be my say air angle or relative flow angle at the entry of my rotor-1. So, this is what is my β_1, okay. Now, we will compare our aerodynamic work and thermodynamic work. You must understand, like we are looking for calculation for say what is the outlet condition.

So, if you are putting that, it says this is what is $\lambda U_{R1m} C_a(\tan\beta_{1m} - tan\,\beta_{2m})$, that is nothing but my aerodynamic work, and this is what is my thermodynamic work, it say $C_p\Delta T_0$. So, this is what we have calculated. And based on that we can calculate what will be my β_2.

Comparing Aerodynamic and Thermodynamic work,

$$\lambda U_{R1m} C_a(\tan\beta_{1m} - tan\,\beta_{2m}) = C_p\Delta T_{0R1}$$

$$\tan\beta_{2m} = \tan\beta_{1m} - \frac{C_p\Delta T_{0R1}}{\lambda U_{R1m} C_a}$$

$$\tan\beta_{2m} = \tan(37.38°) - \frac{1005 \times 1.185}{0.98 \times 33.93 \times 44.4}$$

$$\therefore \beta_{2m} = -2.44°$$

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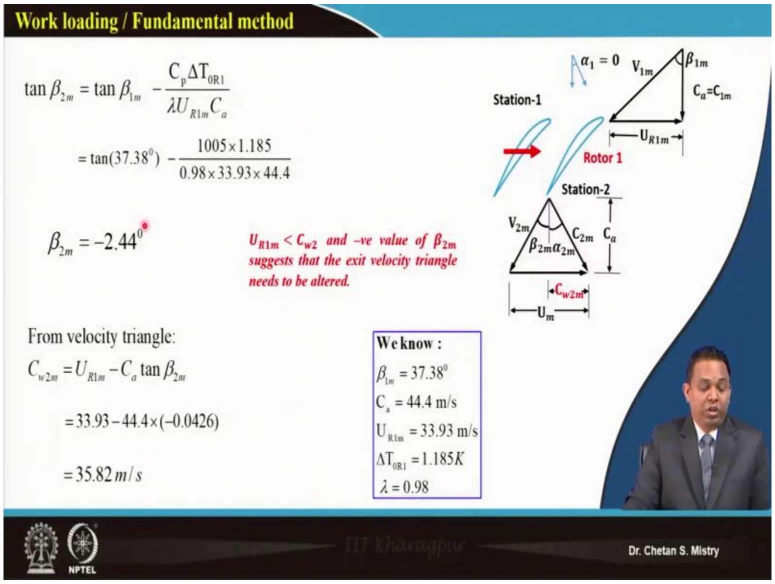

So, here if you look at, my β_2, that's what is coming -2.44, okay. So, this is what we are calculating at the outlet condition. Now, what we know, this is what we can say by using this, we can calculate what will be my C_{w2} because we are looking for all other angles and parameters need to be calculated.

So, my C_{w2}, that's what we can calculate by using $U - C_a \tan\beta_{2m}$, okay. If this is what is your case, my whirl component at the mid station at the outlet, it is coming 35.82 m/s, okay.

$$\textit{From velocity triangle,}$$

$$C_{w2m} = U_{R1m} - C_a \tan\beta_{2m}$$

$$= 33.93 - 44.4 \times (-0.0426)$$

$$= 35.82\ m/s$$

Now, here in this case, what angle we are getting, that need to be very careful about. So, you can see, this is what is the modification of our angle. So, we can say, my velocity triangle, that's what will be changing at the mid station like this. Be careful, okay! So, we need to be very careful when we are doing our calculation. So, this is what is say -2.44, this is what is say this angle. And this is what will be my C_{w2}.

So, if you compare, my C_{w2}, that's what is coming to be larger than that of my peripheral speed U, okay. And if you look at my angle α_2, that's what is with my axial direction we are calculating with, okay. So be careful when you are doing your calculation, make your velocity triangle accordingly.

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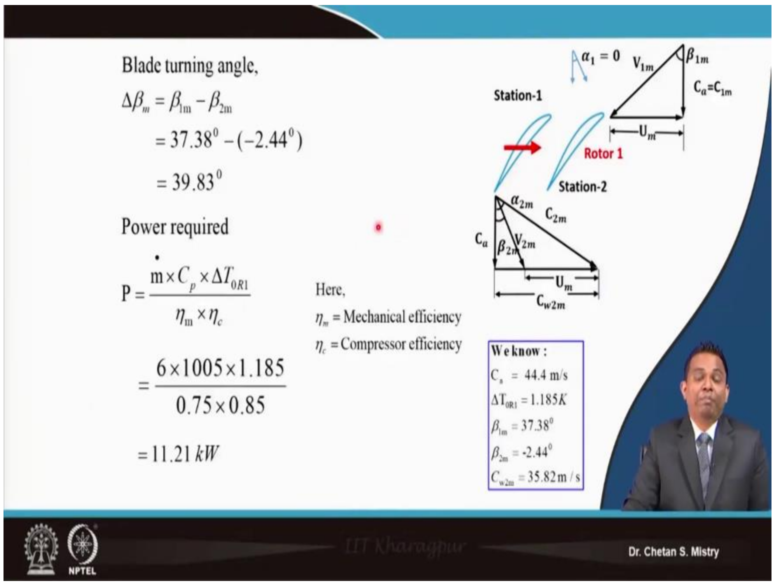

Now, once we have calculated our β_1 and β_2, we can do our calculation for say blade deflection angle, that is nothing but my $\Delta\beta$. So, if we are putting that $\Delta\beta$, it is coming 39.83°, okay.

$$\textit{Blade turning angle,}$$

$$\Delta\beta_m = \beta_{1m} - \beta_{2m}$$

$$= 37.38° - (-2.44°)$$

$$= 39.83°$$

Now, say for this design, it is given my power or motor that's what is available that is having power capacity of 15 kW.

So, in order to check whether this motor is sufficient or not, that also can be verified, when you are doing your calculation using this program, okay. Now here in this case, we can write down my power, that's what is given by $\dot{m}C_p\Delta T_0$ by mechanical efficiency, and this is what is my compressor efficiency.

Be careful, what we are doing at this moment, say we are assuming our mechanical efficiency to be 75%, which may not be the case when we are talking about actual engine. This may be in the range of 0.85 or 90%. We have discussed this point. So, if you are putting this as a case, it says my power that's what is coming 11.21 kW, okay.

$$Power\ required,$$

$$P = \frac{\dot{m} \times C_p \times \Delta T_0}{\eta_m \times \eta_c}$$

$$= \frac{6 \times 1005 \times 1.185}{0.75 \times 0.85}$$

$$= 11.21\ kW$$

So, this is how we are doing our calculation for power. Remember, this power calculation, that's what is at the mid station, okay.

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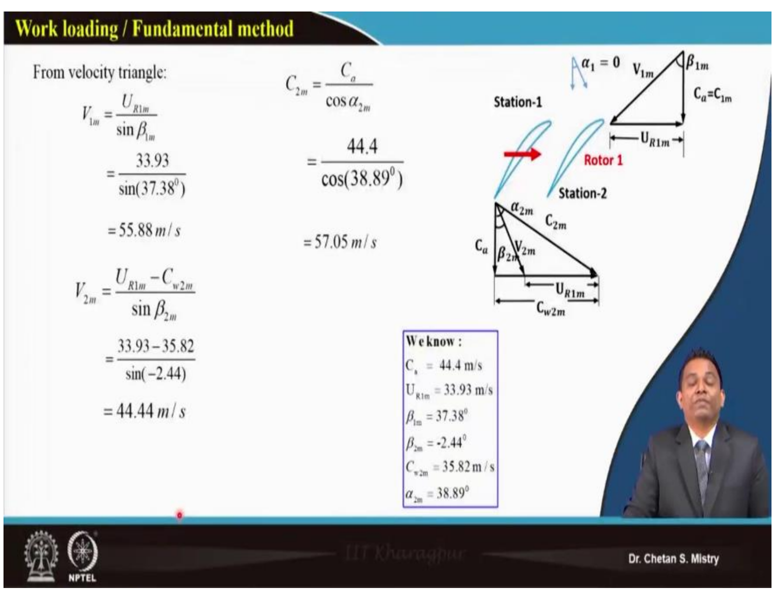

Now, once these all parameters are known to us, we can do calculation for say different velocity components. So, if you are considering say inlet velocity triangle, my relative velocity at the entry of my rotor, that can be calculated based on say sine component. So that's what is given by say $U/\sin\beta$. And that gives me my relative velocity at the entry as 55.85 m/s, okay.

$$From\ velocity\ triangle,$$

$$V_{1m} = \frac{U_{R1m}}{\sin\beta_{1m}}$$

$$= \frac{33.93}{\sin(37.38°)}$$

$$= 55.88\ m/s$$

Same way, based on my outlet velocity triangle, we can do our calculation for relative velocity at the outlet. And this is what is the formulation for that. It says my outlet velocity, it is coming 44.44 m/s, okay.

$$V_{2m} = \frac{U_{R1m} - C_{w2m}}{\sin\beta_{2m}}$$

$$= \frac{33.93 - 35.82}{\sin(-2.44)}$$

$$= 44.44\ m/s$$

Same way, we can do our calculation for absolute velocity because this is what is my requirement for what is going inside my rotor-2, okay. So, this we can write down it is $\frac{C_a}{\cos\alpha_2}$. And that's what is coming 57.05 m/s, okay. So, this is how we are doing calculation for different velocity components.

$$C_{2m} = \frac{C_a}{\cos\alpha_{2m}}$$

$$= \frac{44.4}{\cos(38.89°)}$$

$$= 57.05\ m/s$$

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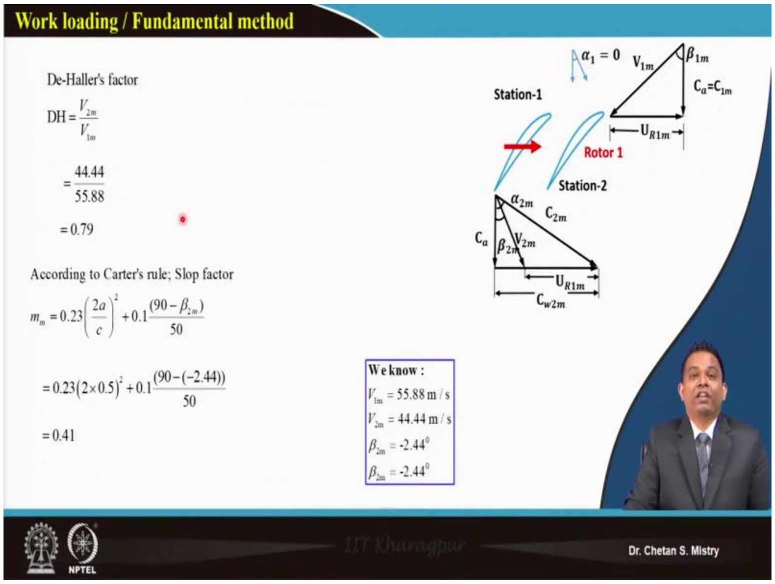

Now, once this is what has been calculated, we will check with the de-Haller's factor. It says, that is nothing but a ratio of relative velocities. So, we can say V_2/V_1, and that's what is coming 0.79.

$$de - Haller's\ fator$$

$$DH = \frac{V_{2m}}{V_{1m}}$$

$$= \frac{44.44}{55.88}$$

$$= 0.79$$

So, this is what is showing our de-Haller's factor. So, one of the parameters for checking, that's what we are considering is say de-Haller's factor, okay.

Remember one thing, this de-Haller's factor, that's what is applicable for the stage, and that too it is a stage, rotor-stator combination, or say stator-rotor combination. But still, we will be checking in order to get the idea, what we are claiming in sense of what we are getting for relative velocity at the entry of the rotor-2, we will see this part.

Now, once this is what has been calculated, our target is to calculate the diffusion factor. And in order to do the calculation for the diffusion factor, we are looking for say parameter that's what is called m factor. So, Carter's rule, that's what it says my slop factor that can be calculated based on what we say my maximum camber, okay. And this is what is giving me my say m factor as say 0.41 at the mid station.

$$According\ to\ Carter's\ rule; Slop\ factor$$

$$m_m = 0.23\left(\frac{2a}{c}\right)^2 + 0.1\frac{(90 - \beta_{2m})}{50}$$

$$= 0.23(2 \times 0.5)^2 + 0.1\frac{90 - (-2.44)}{50}$$

$$= 0.41$$

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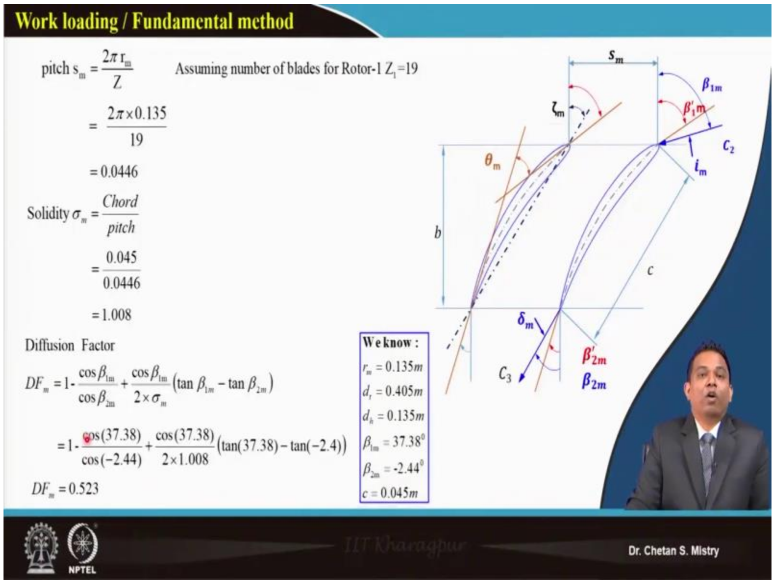

Now, pitch, that's what we are writing as say $2\pi r_m/Z$. Now, the question here it is, the selection of number of blades, okay. Since we are planning to compare our design with the earlier design for say different pressure rise, and that is the reason why we are assuming same number of blades, it says for rotor-1, I am assuming here number of blades to be 19.

You know better. We are doing our pen paper calculation at this moment. Later on, based on this calculation, we will be making our Excel Sheet program. And in that Excel Sheet program you can play with this number, okay. For say sake of reality, we will be taking same number of blades, say 19 number of blades. It says this is what is giving me pitch as 0.044, okay.

$$pitch, s_m = \frac{2\pi r_m}{Z}$$

$$= 2\pi \times \frac{0.135}{19}$$

$$= 0.0446$$

We can calculate our solidity. It is nothing but chord to pitch ratio. It is coming 1.00.

$$Solidity, \sigma_m = \frac{Chord}{pitch}$$

$$= \frac{0.045}{0.0446}$$

$$= 1.008$$

Now, once my solidity at the mid station is known, we can calculate our diffusion factor based on the formula for say β_1 and β_2. So, if this is what is your case, diffusion factor is coming 0.52 at the mid station. We are presently discussing for say rotor-1.

$$Diffusion\ Factor,$$

$$DF_m = 1 - \frac{\cos\beta_{1m}}{\cos\beta_{2m}} + \frac{\cos\beta_{1m}}{2\times\sigma_m}(\tan\beta_{1m} - \tan\beta_{2m})$$

$$= 1 - \frac{\cos(37.38)}{\cos(-2.44)} + \frac{\cos(37.38)}{2\times 1.008}(\tan(37.38) - \tan(-2.4))$$

$$\therefore DF_m = 0.523$$

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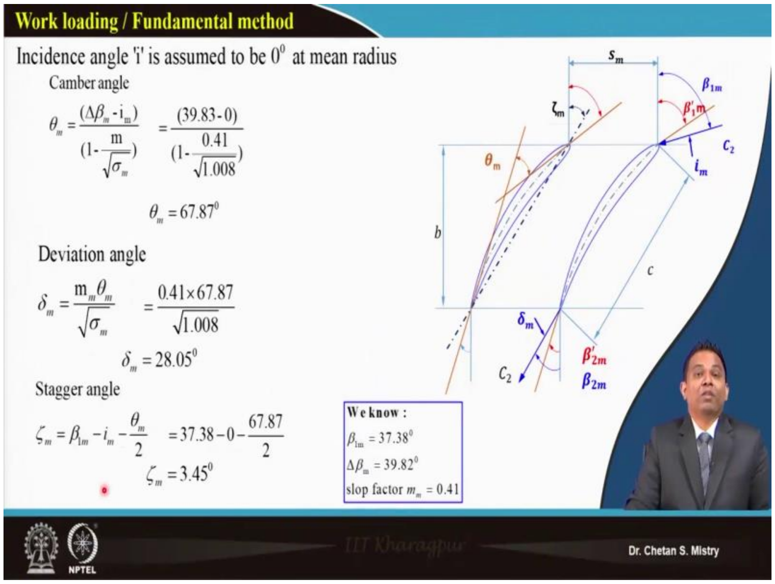

Now, we are more interested in calculating what all will be our angles, say camber angle, my deviation angle and stagger angle. So, let us put, we know based on our equation, this is what is given by $\Delta\beta - i$ divided by this formula. What we realize, at mid station, as we have discussed earlier, we are assuming our incidence angle to be 0.

Same way, at the hub, when we will be doing calculation, we know we are assuming our incidence angle to be positive, and at the tip we are assuming our incidence angle to be negative. So here, we are taking this to be say 0. It says, my camber angle that's what is coming 67.87, okay.

$$Incidence\ angle\ 'i'\ is\ assumed\ tobe\ 0\ at\ mean\ radius$$

$$Camber\ angle,$$

$$\theta_m = \frac{\Delta\beta_m - i_m}{1 - \frac{m}{\sqrt{\sigma_m}}} = \frac{39.83 - 0}{1 - \frac{0.41}{\sqrt{1.008}}}$$

$$\therefore \theta_m = 67.87°$$

Based on this camber angle, we can do calculation what will be my deviation angle. So, this deviation angle is nothing but $\frac{m_m \theta_m}{\sqrt{\sigma_m}}$. If you are doing that calculation, my deviation angle is coming 28.05°, okay.

$$Deviation\ angle,$$

$$\delta_m = \frac{m_m \theta_m}{\sqrt{\sigma_m}} = \frac{0.41 \times 67.87}{\sqrt{1.008}}$$

$$\therefore \delta_m = 28.05°$$

Once these two parameters are known to us, we can do calculation for say our stagger angle. And that stagger angle is coming say 3.45, okay. So, this is what all we are doing calculation at the mid station, let us see.

$$Stagger\ angle,$$

$$\zeta_m = \beta_{1m} - i_m - \frac{\theta_m}{2} = 37.38 - 0 - \frac{67.87}{2}$$

$$\therefore \zeta_m = 3.45°$$

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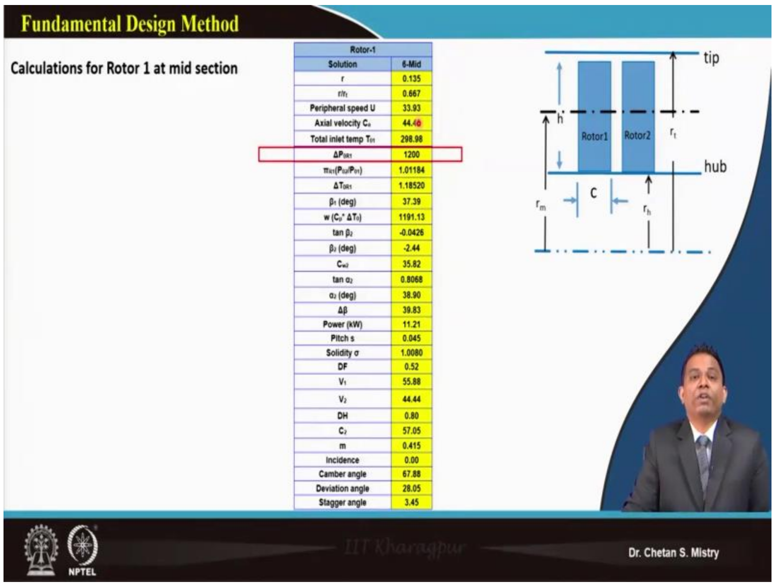

So, we can say this is what is my Excel Sheet program or my parameters, that's what is representing. Here in this case if you are looking at, we are assuming pressure rise by my rotor-1 as say 1200 Pa, okay. This is what is say we are considering as say 1200 Pa. And based on that, these all are the parameters which are calculated.

You can say we are having a diffusion factor 0.52, a De-Haller's factor of 0.80 and our camber angle, that's what is say 67.88. Now, once this station, at…at this particular station, we have done our calculation, we will be targeting what all need to be checked for say my hub station and at my tip station. So, let us move towards that.

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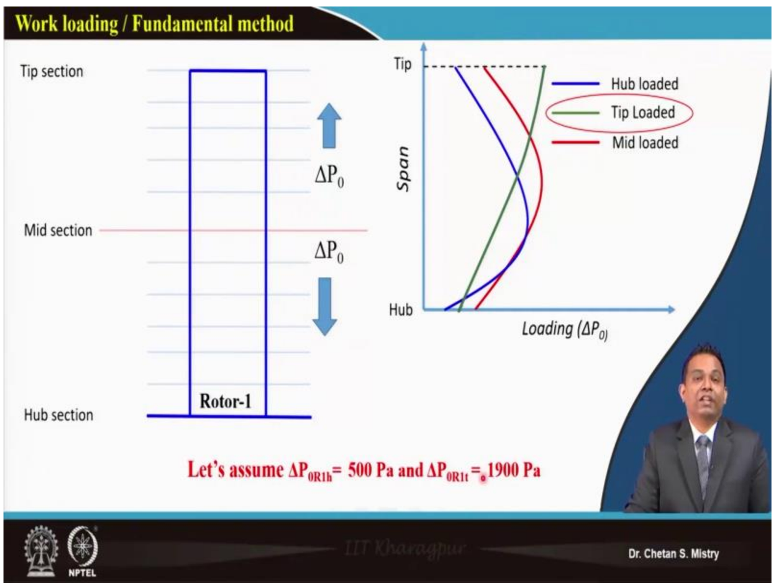

So, what we realize? Say, when we are talking about the fundamental design approach, we know we are having great control in sense of putting the aerodynamic loading to my rotor, okay. We can think of tip loaded rotor, we can think of hub loaded rotor, we can think of mid loaded rotor. So, for present case, we will be considering tip loaded rotor, okay. So, when we say it is a tip loaded rotor, now, I need to assume or I need to select the total pressure that's what is expected or total pressure rise expected at the hub, my total pressure that's what is expected at the tip, and do not forget, we will be doing this total pressure calculation at the outlet as an average total pressure, okay.

So, let us see. Say, let us assume at hub, we are assuming our pressure rise to be 500 Pa. And at the tip, we are considering that to be say 1900 Pa. It is your choice, okay. We will see how these numbers they are coming. Say, initial calculation with iterations, we have assumed these numbers, say 500 Pa and 1900 Pa, okay.

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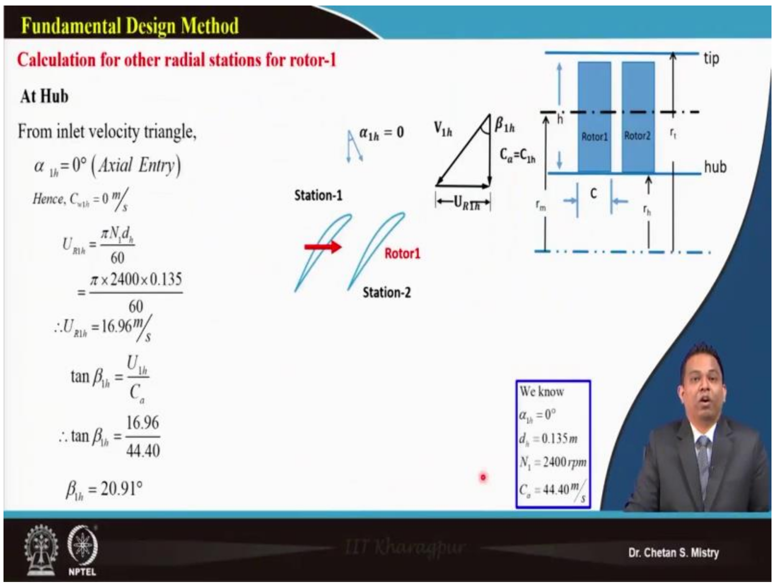

So, if I am putting say 1900 Pa, or say if you are putting at the hub, say 500 Pa, I need to do my calculation at the hub station. Same way, for 1900 Pa also, I need to do my calculation at the tip station, okay. So, let us see what all we are calculating. What we know, my hub, that's what is entry is axial one, so I my α_1 at hub, it is 0. At hub, I can calculate what will be my peripheral speed, okay. So, this peripheral speed, it is coming 19.69 m/s. We can do our calculation for β_1 at the hub. So, you can see here, this is what is coming say 20.91.

At hub,

From inlet velocity triangle,

$$\alpha_{1h} = 0° \ (Axial\ Entry)$$

$$Hence, C_{w1h} = 0\ m/s$$

$$U_{R1h} = \frac{\pi N_1 d_h}{60}$$

$$= \frac{\pi \times 2400 \times 0.135}{60}$$

$$\therefore U_{R1h} = 16.96\, m/s$$

$$\tan \beta_{1h} = \frac{U_{1h}}{C_a}$$

$$\therefore \tan \beta_{1h} = \frac{16.96}{44.40}$$

$$\therefore \beta_{1h} = 20.91°$$

So, in line to what all calculation we have done at the mid station, this calculation need to be done at both hub as well as tip, okay. I can say, you do this calculation but you know it is better to explain; so that, that will build the confidence in you how exactly this calculation need to be done, okay. And after building this confidence, you can go further.

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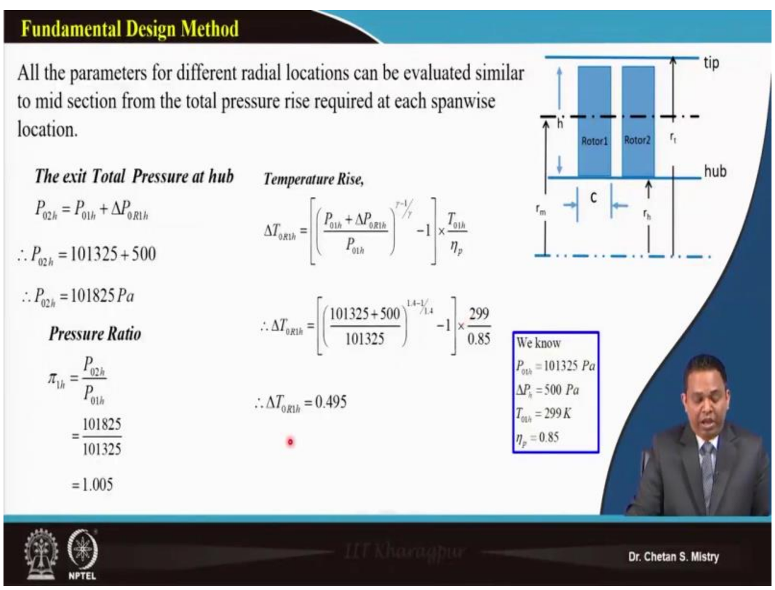

And that is the reason why we are doing all our calculation at the hub as well as at the tip. So, what it says? At the hub, we are expecting our pressure rise to be 500 Pa. So, that’s what will be giving me my pressure ratio and the hub to be 1.005, okay. Be careful, my pressure ratio at all station, that's what is different, okay. We have done earlier calculation that's what is at mid station.

Here in this case, I can do my calculation for say temperature rise at the hub. So, I will be putting P_{01} at the hub and ΔP_0 deltaP$_0$. And this temperature, that's what is we can say, it is 299 K, okay. And that's what is giving me my ΔT_0 as 0.49, okay.

$$The\ exit\ total\ Pressure\ at\ hub$$

$$P_{02h} = P_{01h} + \Delta P_{0R1h}$$

$$\therefore P_{02h} = 101325 + 500$$

$$\therefore P_{02h} = 101825\ Pa$$

$$Pressure\ Ratio$$

$$\pi_{1h} = \frac{P_{02h}}{P_{01h}}$$

$$= \frac{101825}{101325}$$

$$= 1.005$$

$$Temperature\ Rise,$$

$$\Delta T_{0R1h} = \left[\left(\frac{P_{01h} + \Delta P_{0R1h}}{P_{01h}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right] \times \frac{T_{01h}}{\eta_p}$$

$$\therefore \Delta T_{0R1h} = \left[\left(\frac{101325 + 500}{101325}\right)^{\frac{1.4-1}{1.4}} - 1\right] \times \frac{299}{0.85}$$

$$\therefore \Delta T_{0R1h} = 0.495$$

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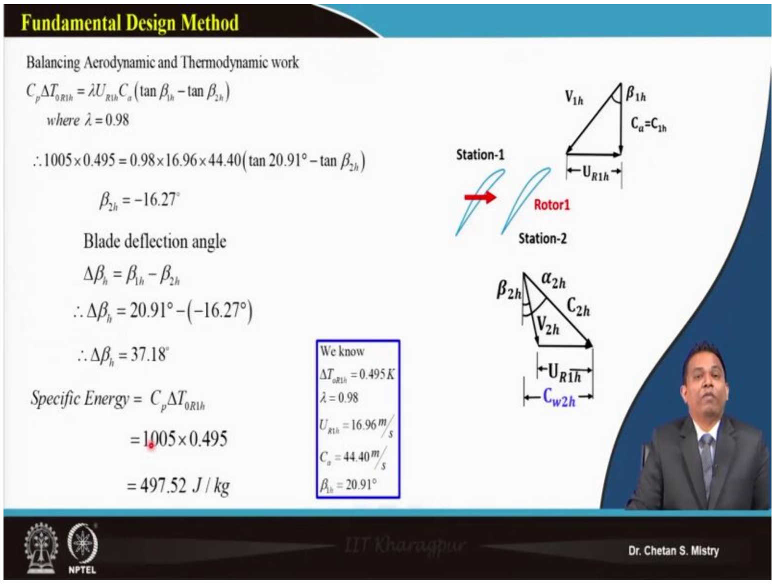

Now, once we have calculated that part, we can do our calculation based on say our comparison, aerodynamic work and thermodynamic work, that's what we are comparing here. And based on this comparison, we can do our calculation for say outlet blade angle, okay, or my air angle, that's what is, or relative flow angle β_2, we are calculating.

Now, once this is what is known to us, we can calculate what will be my $\Delta\beta$. Just look at, my $\Delta\beta$ is coming 37.18°.

$$Comparing\ Aerodynamic\ and\ Thermodynamic\ work,$$

$$C_p\Delta T_{0R1h} = \lambda U_{R1h} C_a(\tan\beta_{1h} - tan\ \beta_{2h})$$

$$where\ \lambda = 0.98$$

$$\therefore 1005\times 0.495 = 0.98\times 16.96\times 44.4\ (\tan 20.91° - \tan\beta_{2h})$$

$$\therefore \beta_{2h} = -16.27°$$

$$Blade\ deflectiona\ angle,$$

$$\Delta\beta_h = \beta_{1h} - \beta_{2h}$$

$$\therefore \Delta\beta_h = 20.91° - (-16.27°)$$

$$\therefore \Delta\beta_h = 37.18°$$

Same way, we can do our calculation, as I told, we need to do calculation for the power requirement at that station also. So, this is what is the calculation for the specific energy.

$$Specific\ energy = C_p \Delta T_{0R1h}$$

$$= 1005 \times 0.495$$

$$= 497.52\ J/kg$$

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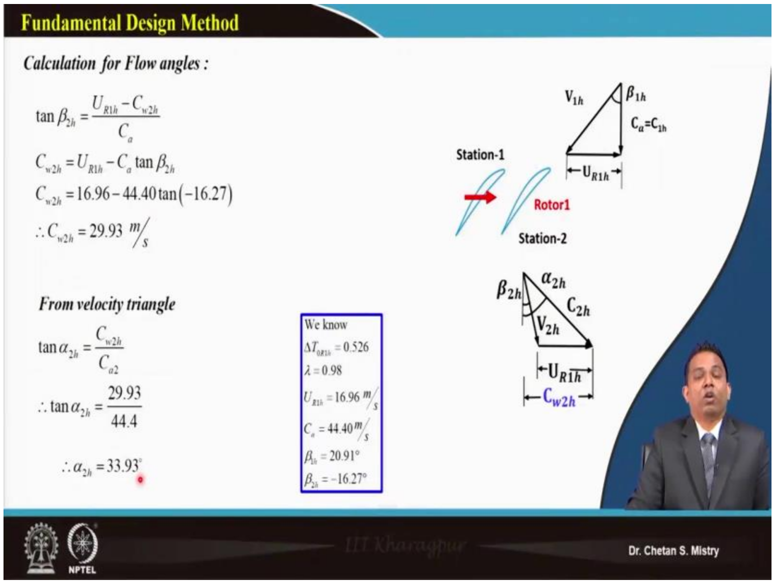

Now, we can do our calculation for β_2, that's what is my outlet angle. We can do our calculation for the whirl component, that's what is coming 29.93 m/s.

$$\tan\beta_{2h} = \frac{U_{R1h} - C_{w2h}}{C_a}$$

$$C_{w2h} = U_{R1h} - C_a \tan\beta_{2h}$$

$$C_{w2h} = 16.96 - 44.40\tan(-16.27)$$

$$\therefore C_{w2h} = 29.93\ m/s$$

And my α_h, that's what is coming or α_{2h}, it is coming 33.93.

$$From\ velocity\ triangle,$$

$$\tan \alpha_{2h} = \frac{C_{w2h}}{C_{a2}}$$

$$\therefore \tan \alpha_{2h} = \frac{29.93}{44.4}$$

$$\therefore \alpha_{2h} = 33.93°$$

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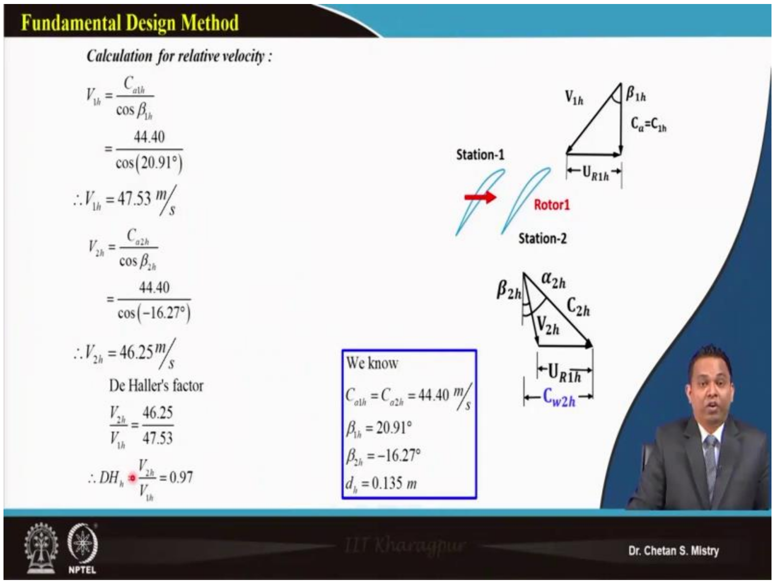

We can do our calculation for relative velocities and relative velocity ratios. So, when we are taking this relative velocity ratios, at the hub it is coming 0.97. Just look it, okay.

$$Calculation\ for\ relative\ velocity:$$

$$V_{1h} = \frac{C_{a1h}}{\cos \beta_{1h}}$$

$$= \frac{44.4}{\cos(20.91°)}$$

$$\therefore V_{1h} = 47.53\ m/s$$

$$V_{2h} = \frac{C_{a2h}}{\cos \beta_{2h}}$$

$$= \frac{44.40}{\cos(-16.27°)}$$

$$\therefore V_{2h} = 46.25\ m/s$$

$$De - Haller'sfactor$$

$$\frac{V_{2h}}{V_{1h}} = \frac{46.25}{47.53}$$

$$\therefore DH_h = \frac{V_{2h}}{V_{1h}} = 0.97$$

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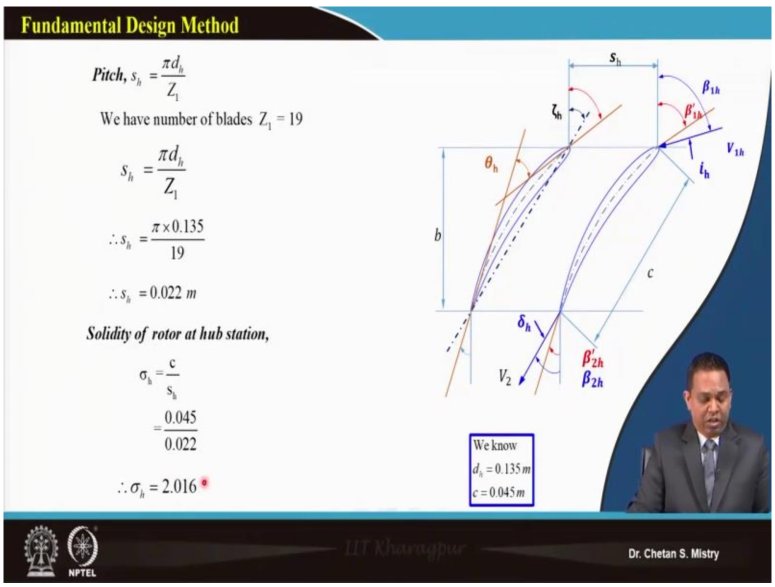

I am sure you are able to do this calculation. So, I am going little fast in sense of showing the calculation. These slides will be with you. Maybe you can verify, you can check also. Here in this case, we can do our calculation for the pitch. Be careful, this is what is for the representation purpose only. You can understand at the hub my radius will be smaller and that is the reason my pitch will be coming to be smaller. At the tip, my radius is higher for same number of blades, my pitch will be coming to be on the higher side. So, if we are putting here, it says my pitch is coming 0.022 m, okay. Now based on this, we can do our calculation for the solidity. At the hub, it is coming 2.016.

$$Pitch, s_h = \frac{\pi d_h}{Z_1}$$

$$We\ have\ number\ of\ blades, Z_1 = 19$$

$$s_h = \frac{\pi d_h}{Z_1}$$

$$\therefore s_h = \frac{\pi \times 0.135}{19}$$

$$\therefore s_h = 0.022\ m$$

$$Solidity\ of\ rotor\ at\ hub\ station,$$

$$\sigma_h = \frac{c}{s_h}$$

$$= \frac{0.045}{0.022}$$

$$\therefore \sigma_h = 2.016$$

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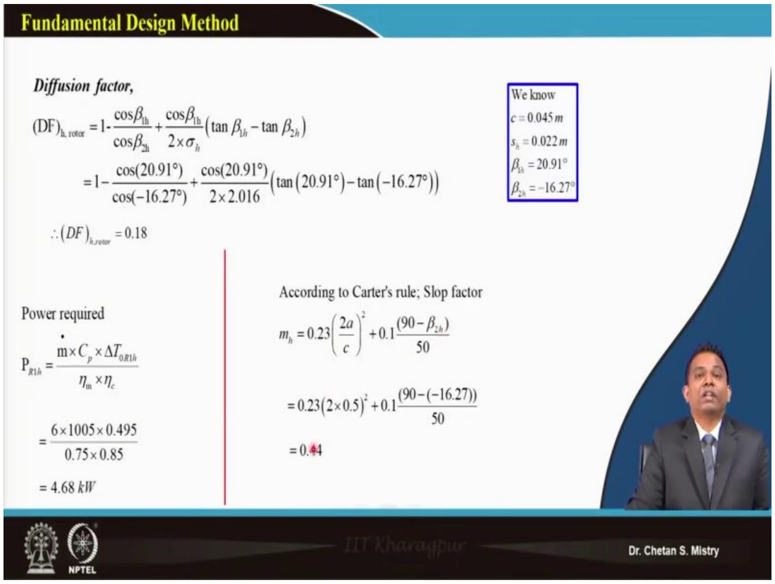

We can also calculate what will be my diffusion factor. So, the diffusion factor, based on β_1 and β_2, it is coming 0.18, okay. So, my power requirement at the hub station, as we have discussed, that's what is a function of ΔT_0 at particular station. And if you are taking mechanical efficiency 75%, and say my compressor efficiency 85%, it says this is what is coming 4.68 kW, okay. We can do our calculation for say m factor. Since here, be careful, we are talking about β_2 at the hub, and that m factor is coming 0.44.

$$Diffusion\ factor,$$

$$(DF)_{h,rotor} = 1 - \frac{\cos\beta_{1h}}{\cos\beta_{2h}} + \frac{\cos\beta_{1h}}{2 \times \sigma_h}(\tan\beta_{1h} - \tan\beta_{2h})$$

$$= 1 - \frac{\cos(20.91°)}{\cos(-16.27°)} + \frac{\cos(20.91°)}{2 \times 2.016}(\tan(20.91°) - \tan(-16.27°))$$

$$\therefore (DF)_{h,rotor} = 0.18$$

$$Power\ required,$$

$$P_{R1,h} = \frac{\dot{m} \times C_p \times \Delta T_{0R1h}}{\eta_m \times \eta_c}$$

$$= \frac{6 \times 1005 \times 0.495}{0.75 \times 0.85}$$

$$= 4.68\ kW$$

$$According\ to\ Carter's\ rule;\ slop\ factor$$

$$m_h = 0.23\left(\frac{2a}{c}\right)^2 + 0.1\frac{(90 - \beta_{2h})}{50}$$

$$= 0.23(2 \times 0.5)^2 + 0.1\frac{90 - (-16.27)}{50}$$

$$= 0.44$$

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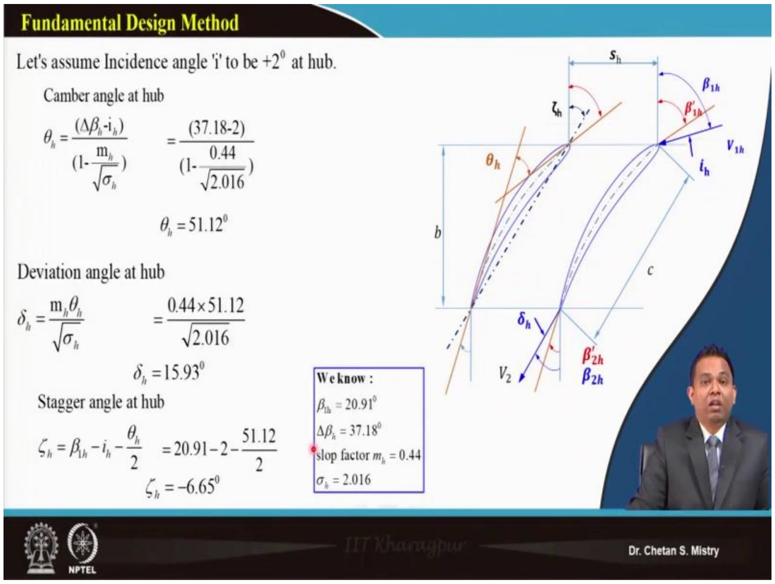

We can do our calculation for say camber angle at the hub, we can do our calculation for the say deviation angle at the hub and we can do our calculation for the say stagger angle at the hub. It says, this is, camber angle is coming 51.12°, my deviation angle, it is coming 15.93, and my stagger angle, that's what is coming -6.65, okay.

$$Let's\ assume\ Incidence\ angle\ 'i'\ tobe + 2°\ at\ hub$$

$$Camber\ angle\ at\ hub,$$

$$\theta_h = \frac{\Delta\beta_h - i_h}{1 - \frac{m_h}{\sqrt{\sigma_h}}} = \frac{37.18 - 2}{1 - \frac{0.44}{\sqrt{2.016}}}$$

$$\therefore \theta_m = 51.12°$$

$$Deviation\ angle\ at\ hub,$$

$$\delta_h = \frac{m_h\theta_h}{\sqrt{\sigma_h}} = \frac{0.44 \times 51.12}{\sqrt{2.016}}$$

$$\therefore \delta_h = 15.93°$$

$Stagger\ angle\ at\ hub,$

$$\zeta_h = \beta_{1h} - i_h - \frac{\theta_h}{2} = 20.91 - 2 - \frac{51.12}{2}$$

$$\therefore \zeta_h = -6.65°$$

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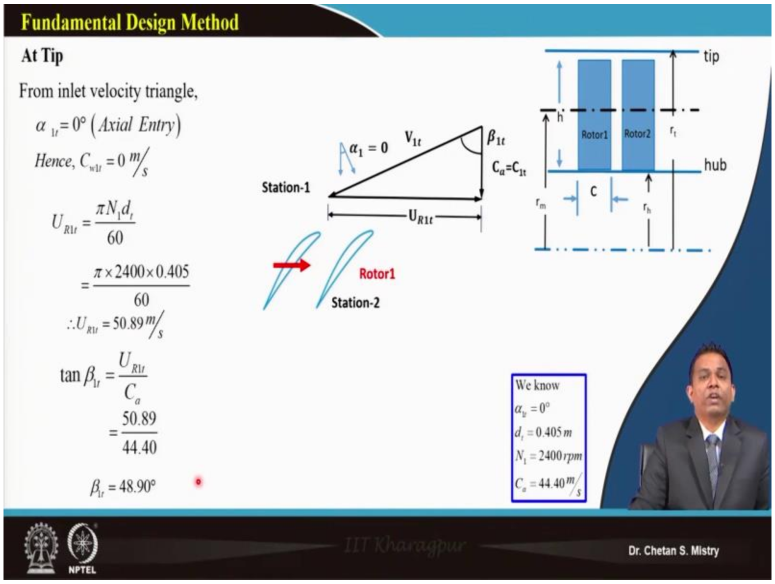

Now, in line to that, as we have discussed, we can do our calculation at the tip station also, okay. So, this is what we will be discussing in next lecture. So, for today, if you realize what we have done, we have done our calculation at the mid station for rotor-1, and based on that calculation we have calculated all parameters which are required for understanding of design.

And we have started calculating other parameters at the hub station. So, in next session, we will be discussing about the calculation at the tip station. Then we will see how we will be setting with these numbers, and why do we need to set these numbers. Then after, we will be starting with the design of say rotor-2. So be with me. Thank you. Thank you very much for your kind attention!