Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 39
Design Low Speed Compressor (Contd.)

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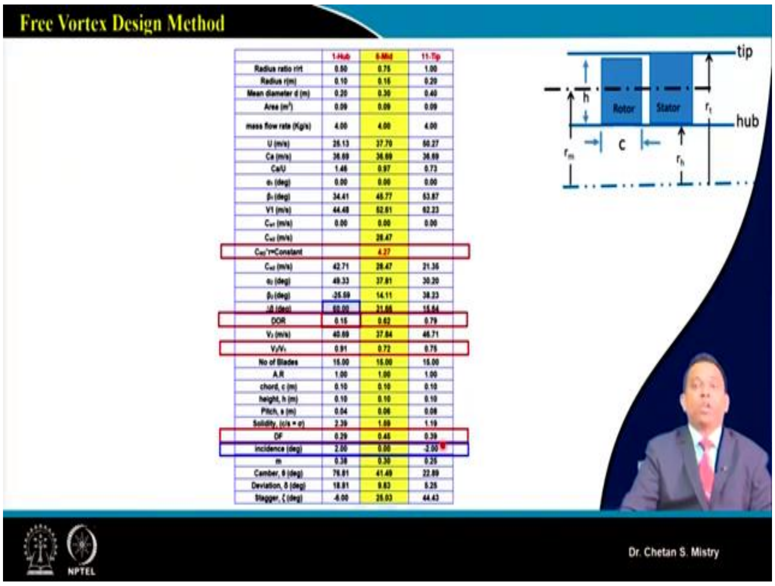

Hello, and welcome to lecture 39. We are discussing Design of Low Speed Axial Flow Compressor. In last lecture, we were discussing about say design of low speed axial flow compressor for given data using free vortex concept. And if you recall, we were discussing about say calculation at the mid station. So, we have done our calculation at the mid station; from mid station calculation, we have done our calculation for $C_w \cdot r$ at mid station, that's what is equal to constant and from that constant value at different radiuses how my C_{w2}, that's what is at the exit of my rotor at hub and tip it is varying.

And, from that variation of C_{w2}, we have done our calculation for different flow angles, we have done our calculation for velocity components, we have done calculation for degree of reaction, diffusion factor, De Haller's factor and different angles say incidence angle, deviation angle, camber angle and stagger angle. And here in this case, if we look at, it says my $\Delta\beta$, as we have discussed at the hub, that's what is coming to be large. And other parameters if he consider, say…degree of reaction near the hub that's what is in the range.

We can say our relative velocity ratio that is also coming in the range, that's what is coming greater than 0.72 and diffusing factor also is in the range of say…maybe say…0.45 at the mid station and the tip, this is what is coming 0.39. And, we have discuss our incidence angle at the hub, that's what we are taking as say +2 and at the tip, that's what we are assuming to be negative or -2.

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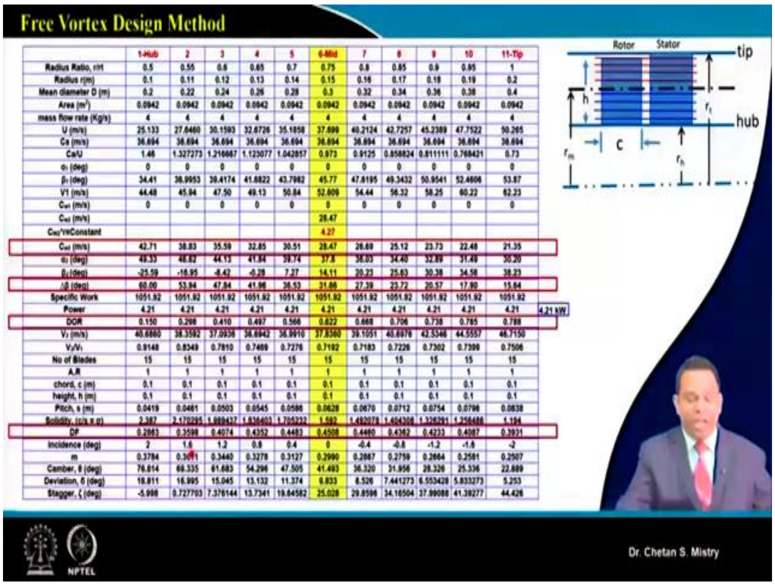

Now, let us see, suppose if you are looking for the whole design or say we are looking for all the stations; so, if we consider, say this is what is our rotor and stator. As we have discussed, this rotor and stator, that's what is we are dividing into say 11 number of stations. So, at mid station we have done our calculation, based on our understanding we have done our calculation at the hub as well as at the tip because those numbers or those radiuses they are known to us.

Now, what we need to do is, this radius, what we have, that's what we need to divide into equal number of stations. Here, we have taken 11 stations, you can take 20 stations, you can take 40 stations, it is up to you; more the number of stations you are taking, more precise will be your design; less your numbers, you will realize that's what is giving you some kind of constraint when you are making those blades.

So, it is preferred that maybe 20 number of station, that's what is most preferred configuration for such low aspect ratio blades. Suppose if we are talking about high aspect ratio blade, it is preferred to go with say 30 number of stations. Now, here in this case, if you look at, we have divided into say 0.01 radius station; so, it is 0.1, 0.11, 0.12, likewise up to 0.2 we are having, okay.

And if we compare the concept what we have done; so, now when you are using yours excel sheet, so, this is what will be my excel sheet program. Now, as I told, you are having so much of flexibility in sense of doing the design. One may say, we will be writing the code for the design, there is nothing wrong in that, but you can understand the amount of time what you are spending in order to realize what parameters they are changing and how it is affecting on other parameters, it will take lot of time for you.

Here, in this case, when we are having say excel sheet, you change say one of the parameters, suppose say here I am changing my mass flow rate, immediately it will give the reflection what will be the change in all my parameters. Suppose you say, we are having say change in my rotational speed, accordingly of my peripheral speed will be changing and all parameter change that's what will be coming into the picture.

So, it is preferred like as a designer, almost all designers, they are going with such kind of excel sheet design program, okay. So, you just do practice using excel sheet, just learn how to insert the angles, how to do the calculation and as we have discussed, when you are doing your mid-section design using pen and paper, you can verify this station, okay.

Now, here this is what is representing how my say whirl component, that's what is changing from hub to tip, okay. Now, because of this variation, we can say, we will be having $\Delta\beta$ variation that you can say this is what is varying in a systematic way. Remember, if you will be having zigzag kind of variation that means there is something wrong in your design, it should be smooth curve in sense of having this $\Delta\beta$. So be careful and make or keep an eye for these parameters.

Same way, here if you look at, this is what is representing how my degree of reaction that's what is varying from say hub to the tip. Now, here in this case, if you look at, my power requirement, that's what is coming 4.21 kW. You can say, if I will be averaging out throughout my span, it say, this is the amount of work that's what is required. And, as we have discussed earlier, this is what will be helping us in order to decide the capacity of the motor.

Suppose say, we are doing designed for high speed compressor, that's what we will be giving you power maybe in sense of megawatts, or a few 100 kilowatts. So, based on that we need to decide with what motor we can use. So, this is what is one of the good parameters, that's what you can do your calculation with. Same way, here, we have seen, we are having a variation of diffusion factor; we have our variation of camber angle.

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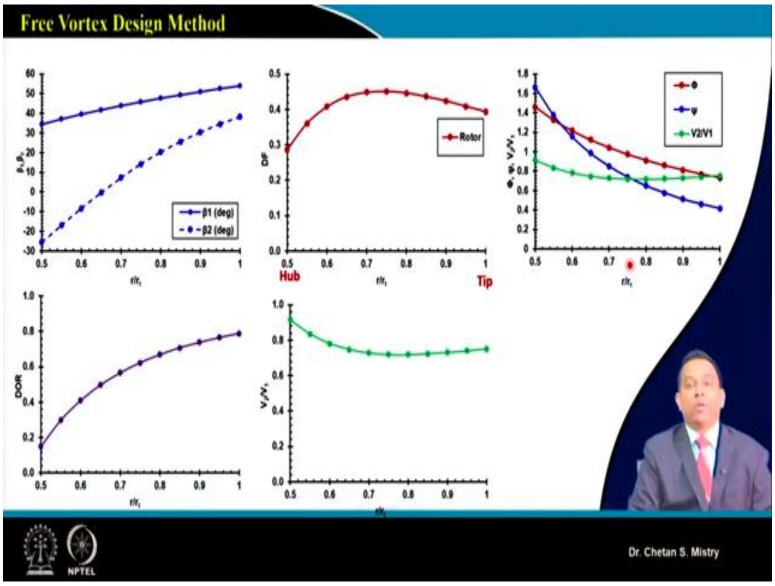

Let us see, let us observe what all are the changes. So here, if you look at, this is what is representing the variation of my β_1. So, β_1, that's what is varying as we have discussed, it is C_a/U, that means with the change of my radius, accordingly, I will be having the variation of β_1. But my β_2, that's what is a function of my C_{w2} and that is the reason, here if you look at, near the hub region, I will be having my $\Delta\beta$ to be large and near the tip region, my $\Delta\beta$ is going to be reduced. This is nothing but that's what is representing what we say in sense of twist of my blade, okay.

So, it says when we are having this, you know free vortex design, we are having highly twisted blade, we will see, okay. Now, this is what is representing the variation of my diffusion factor for the rotor. So, you can say here, my diffusion factor nearly at the mid station, that's what is say the maximum and on both the side, that's what is say lower on say magnitude. But at the same time, if you are looking at the degree of reaction, it says, I am having higher thermodynamic loading, that's what is happening near my tip region.

And if we look at our say De Haller's number or my relative velocity ratio, that's what is coming in the range of what we are looking for. This is what is representing how my flow coefficient, that's what is varying, okay. Along with say my span, we can say, this 0.5 it is representing hub and 1 (one), that's what is representing my tip location. So, we can say this is what is representing my loading.

So, if we look at carefully, say we are having higher loading near the hub region and we are having comparatively low loading near say tip region. So, this is what is representing my ψ parameter. Now, here what we are doing our calculation, we have no other controls, straightway what we say it is, this is what is satisfying our radial equilibrium. That's what will be giving us idea about say axial velocity, that's what is giving us idea about how my swirl component that's what is varying with, okay. Now, once we have done our design for the rotor, my next step, that's what is to design the stator, because we are designing the stage.

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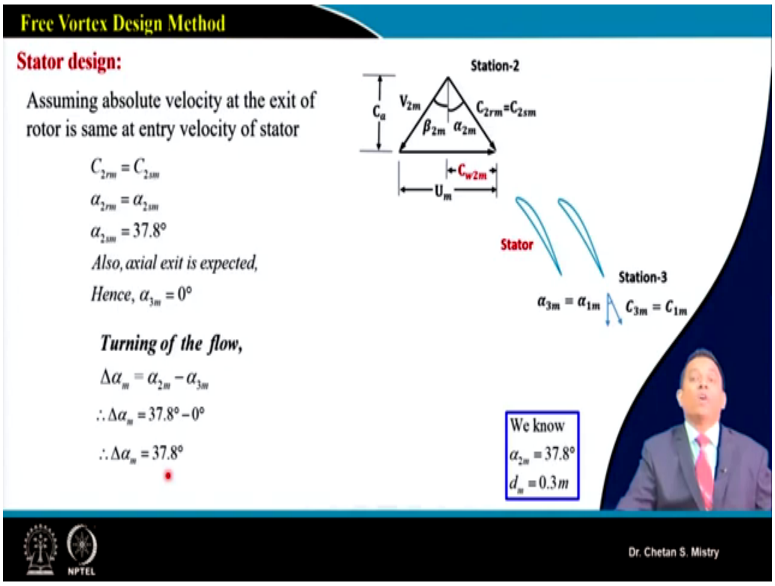

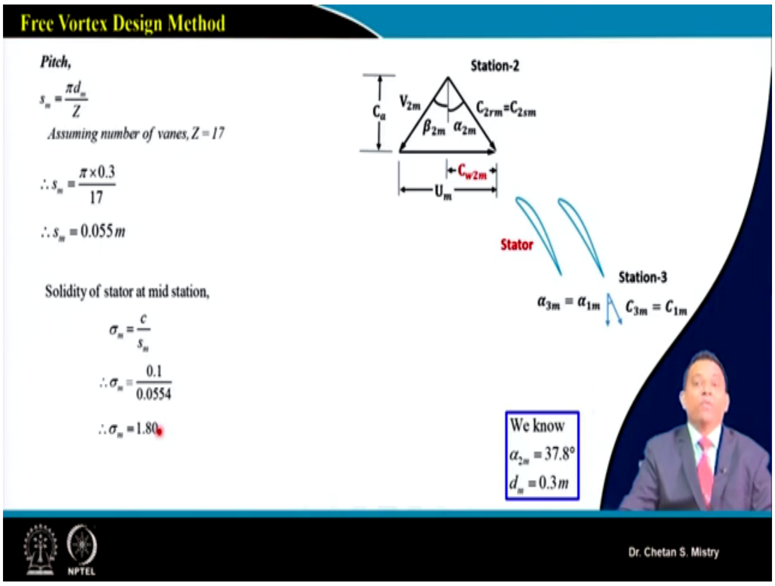

So, in order to do the calculation for say stator, let us see. So, same way what we have done for the rotor, we will be doing our calculation for the stator at the mid station. Here, in this case, what we are assuming, just look at, say, my absolute velocity, that's what is coming out from the rotor, that's what we are assuming same, that's what is entering in my stator. So, you can say, C_2 for rotor and my C_2 for stator, they both are same; this is what is our assumption.

$$C_{2rm} = C_{2sm}$$

$$\alpha_{2rm} = \alpha_{2sm}$$

$$\alpha_{2sm} = 37.8°$$

When I say my velocity, my absolute velocity, that's what is same, we can say my flow angle, or my absolute flow angle, that's what is say α_2, that coming from rotor, and that's what is going to my stator, that will be same. So, we can say α_2 at the entry of my stator, that's what is say 37.8.

Now, for our design configuration, we have considered our exit to be axial one. And when we say we are having our exit to be axial one, we can say α_3, that's what is equal to 0.

$$Also, axial\ exit\ is\ expected,$$

$$Hence, \alpha_{3m} = 0°$$

And, that's what is giving me my flow turning angle. So, it says my flow turning in stator at the mid station, that's what is coming 37.8°, okay. Now, this is what is our calculation for the flow angle; in line to that we are looking for other parameters also. So, we will see, very first we will be calculating with our pitch at the mid station.

$$Turning\ of\ the\ flow,$$

$$\Delta\alpha_m = \alpha_{2m} - \alpha_{3m}$$

$$\therefore \Delta\alpha_m = 37.8° - 0°$$

$$\therefore \Delta\alpha_m = 37.8°$$

Here in this case, our aspect ratio is given 1. Let us take for rotor and stator my chord is 100 mm only. So, both, they are having say aspect ratio of 1. So, if this is what is your case, the pitch we are…we can calculate by using $\pi d/Z$. Here, we need to check with say number of blades for the stator or stator vane. We can say, that's what is we have assumed at this moment to be 17, okay. So, this is what is 17. That's what it says my pitch is coming 0.055, okay.

$$Pitch, s_m = \frac{\pi d_m}{Z}$$

$$Assuming\ number\ of\ vanes, Z = 17$$

$$\therefore s_m = \frac{\pi \times 0.3}{17}$$

$$\therefore s_m = 0.055\ m$$

Now, the solidity, we have defined is in sense of chord to pitch ratio and that solidity for stator it is coming 1.80.

$$Solidity\ of\ stator\ at\ mid\ station,$$

$$\sigma_m = \frac{c}{s_m}$$

$$\therefore \sigma_m = \frac{0.1}{0.0554}$$

$$\therefore \sigma_m = 1.8$$

Now, at mid station, suppose if we know what is our solidity, we can calculate what will be our diffusion factor.

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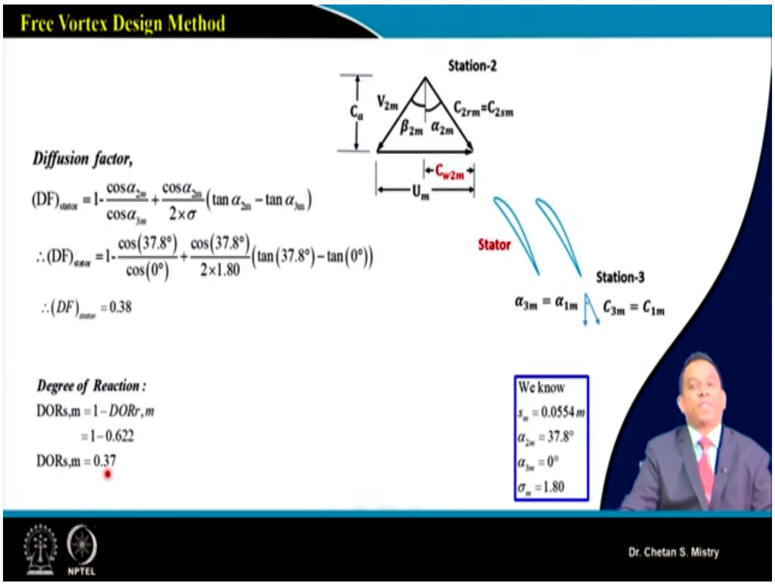

So, this is what is a formulation for the diffusion factor. Remember, when we are discussing the diffusion factor calculation for rotor, we are taking our flow angles as β_1 and β_2. In line to that for stator, we will be considering α_1 and α_2. For our case, this is what is say entry is α_2 and exit is say α_3. So, if you are putting all this number, it says my diffusion factor for the stator, it is coming 0.38, okay.

$$Diffusion\ factor,$$

$$(DF)_{stator} = 1 - \frac{\cos\alpha_{2m}}{\cos\alpha_{3m}} + \frac{\cos\alpha_{2m}}{2\times\sigma}(\tan\alpha_{2m} - \tan\alpha_{3m})$$

$$\therefore (DF)_{stator} = 1 - \frac{\cos(37.8°)}{\cos(0°)} + \frac{\cos(37.8°)}{2\times 1.8}(\tan(37.8°) - \tan(0°))$$

$$\therefore (DF)_{stator} = 0.38$$

Now, next parameter, that's what is of our interest, that's what is say degree of reaction. Now, for stage, we can say maximum possible degree of reaction, that's what is 100%. And, that is the reason why we are considering say degree of reaction for stator, that's what is

$$DOR_{s,m} = 1 - DOR_{r,m}$$

$$= 1 - 0.622$$

$$DOR_{s,m} = 0.37$$

So, we have calculated our degree of reaction for the rotor at mid station is say 0.622, that’s what is giving my degree of reaction for the stator at mid station 0.37, okay.

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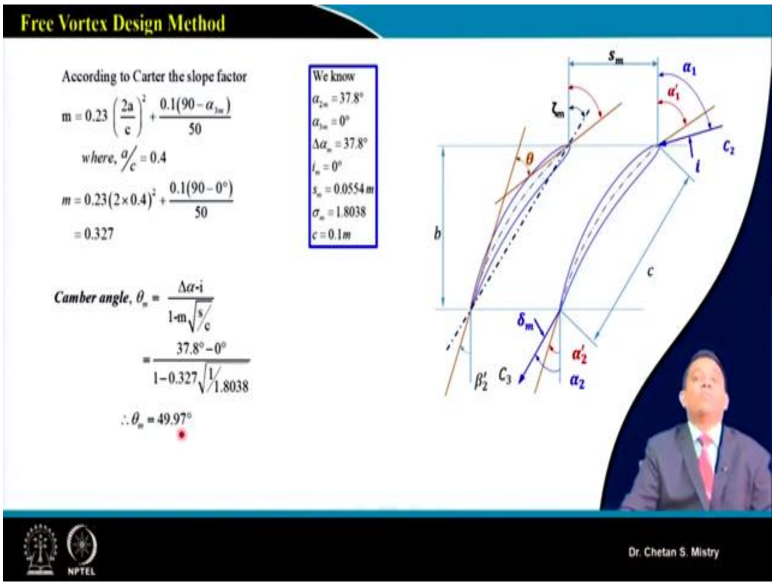

Now, once this parameter they are known to us, we are more interested in our flow angle calculation. And that is the reason we initially start calculation for say ‘m’ factor or Carter factor, okay. So, this Carter's parameter, we can write down. This is a formula for that and for stator also, we are considering our C4 profile. Remember at the initial stage calculation, you can make this assumption. And, as we have discussed, once you are finalized with your design, you will be going for your computational study and that computational study will give insight what is happening based on the detail understanding and your expected requirements, you need to change certain parameters, okay.

$$According\ to\ Carter\ the\ slope\ factor,$$

$$m = 0.23\left(\frac{2a}{c}\right)^2 + \frac{0.1(90 - \alpha_{3m})}{50}$$

$$where, \frac{a}{c} = 0.4$$

$$m = 0.23(2 \times 0.4)^2 + \frac{0.1(90 - 0°)}{50}$$

$$= 0.327$$

But, this is what is our first cut design. So, we are assuming our say C4 airfoil, for that a/c we are considering 0.4. If you are putting this as a number, we can calculate our 'm' parameter is 0.327, okay. Once this is what is known to us, we can do our calculation for the camber angle. It says, this is what is given by $\Delta\alpha$. Remember, we are discussing design for the stator, that's why this is what is $\Delta\alpha$ at the mid station minus incidence, this incident we are considering at the mid station.

$$Camber\ angle, \theta_m = \frac{\Delta\alpha - i}{1 - m\sqrt{\frac{s}{c}}}$$

$$= \frac{37.8° - 0°}{1 - 0.327\sqrt{\frac{1}{1.8038}}}$$

$$= 49.97°$$

Remember, for stator also we are assuming our incidence angle at mid station to be 0, incidence angle at the hub to be +2 and incidence angle at tip, that will be -2. So, if you are assuming this to be 0, this is what it says my camber angle, that's what is coming 49.97° at the mid station, okay.

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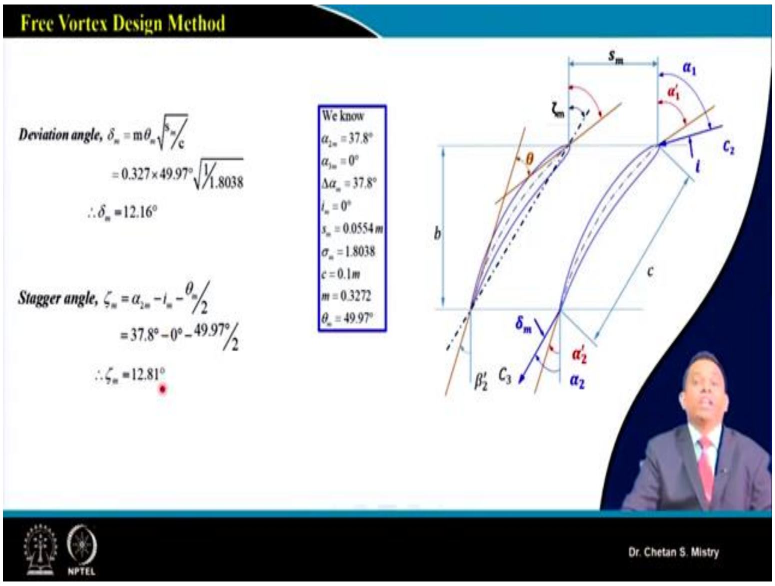

Now, once we have calculated our camber angle, we can do our calculation for the deviation angle. This deviation angle, that's what is a function of my 'm' parameter, my camber angle and solidity. So, based on that we can calculate our deviation angle, that's what is coming 12.16°, okay.

$$Deviation\ angle, \delta_m = m\theta_m\sqrt{\frac{s_m}{c}}$$

$$= 0.327 \times 49.97^\circ\sqrt{\frac{1}{1.8038}}$$

$$\therefore \delta_m = 12.16^\circ$$

Now, in line to that we can do our calculation for blade setting angle or our stagger angle, that's what is a function of my camber angle. It is a function of my α and my incidence angle. So, let us put this here. It says, my stagger angle, that's what is coming 12.81°, okay.

$$Stagger\ angle, \zeta_m = \alpha_{2m} - i_m - {}^{\theta_m}/_2$$

$$= 37.8^\circ - 0^\circ - {}^{49.97^\circ}/_2$$

$$\therefore \zeta_m = 12.81^\circ$$

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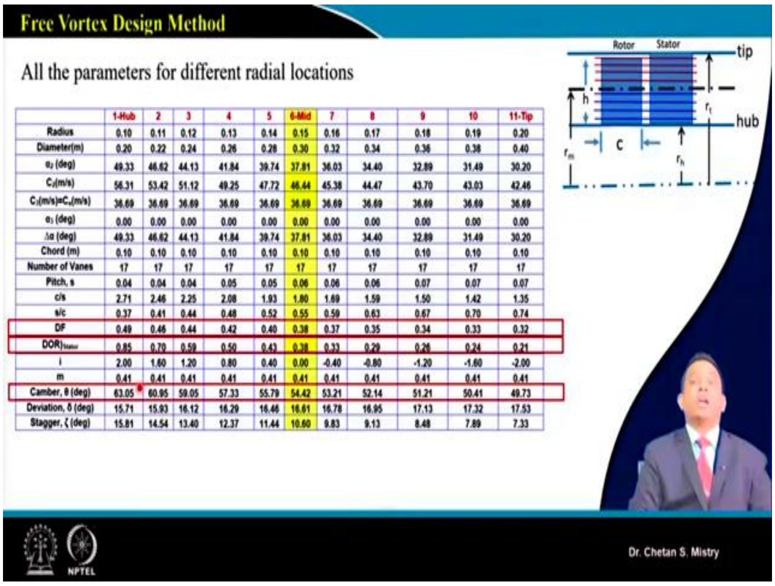

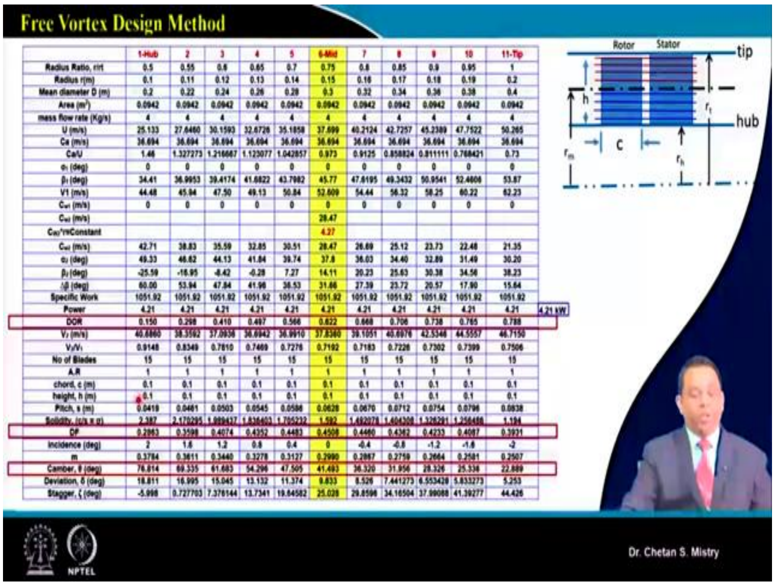

Now, we have done our calculation at the mid station. We already know our 11th station in line to what we have done calculation for the rotor. So, these are our 11 stations. At all 11 stations, we can do our calculation for α_2. We can do our calculation for say C_2, we can do our calculation for C_3, α_3, $\Delta\alpha$, we can take the calculation for diffusion factor, degree of reaction at all the stations.

Now, here in this case, you know, this incidence angle as we have discussed, that's what need to be varied in a range. So, just look at, here this is what is say 2° degree, 1.6, 1.2, 0.8, 0.4. So, accordingly you need to make your assumption. Diffusion factor calculation if you look at, it says my diffusion factor at the hub it is coming 0.49 and at tip that's what is coming 0.32, okay.

Same way, when we are doing our calculation for degree of reaction at the hub, that's what is coming 0.85. So, that's what is representing something different, we will see what is the meaning of this number and we can do our calculation for the camber angle. Now, all these parameters what we have calculated for the stator, that's what will be helping us in order to make our blades; both for rotor as well as for the stator, we can say, that's what is helpful for us in order to make our stage. Now, this is if you compare in sense of numbers say my degree of reaction as we have discussed for stator it is coming 0.85 at the hub and here if you look at it is coming 0.15.

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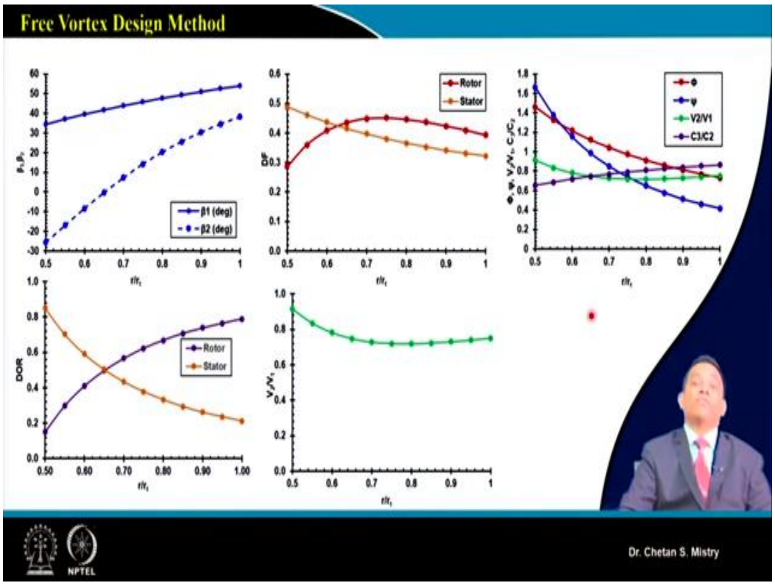

So, let us check with what all we have seen in sense of variation of our angle, okay. Here, in this case, if you look at, this plot that's what is representing how my diffusion factor that's what is varying. And, if you look at, earlier we have done our calculation for rotor and here if you look at, this is what is representing my diffusion factor for the stator. What we can see, is you know, my diffusion factor at the hub that's what is coming to be large; that means it is highly loaded near the hub region, okay.

Same way, if we are comparing our degree of reaction, it says, it reflect the same thing. Here, if you look at this is what is coming to be large. And, this is what is representing what is happening in sense of my say velocity or say absolute velocity ratio. So, when we say, when we are doing our design for the free vortex, that's what is giving us some different kinds of feeling, okay. Now, based on what all angles what we have calculated, we can go with say making of the blade.

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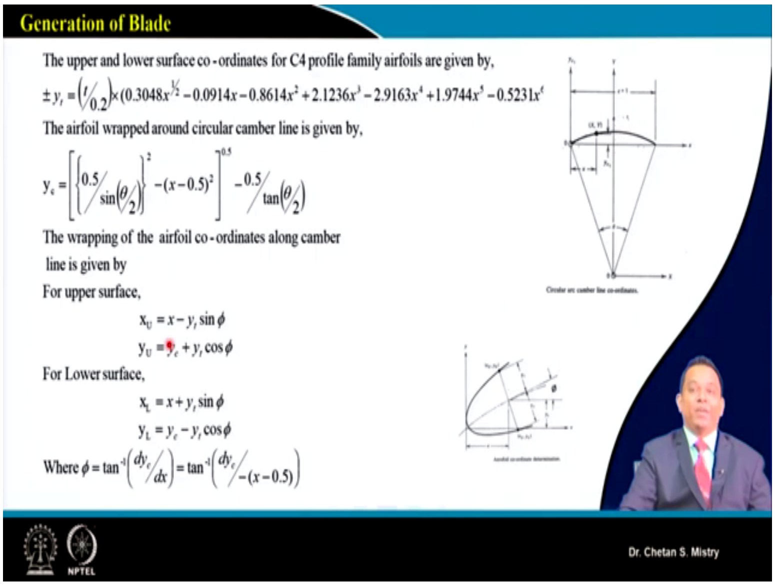

So, in order to understand that part, let us take how do we make our profile or how do we make our blade. So, if you recall, we were discussing about say equation for say thickness distribution for C4 profile. So, this is what is the equation for my C4 profile.

$$\pm y_t = \left({}^{t}/_{0.2}\right) \times (0.3048\, x^{\frac{1}{2}} - 0.0914\, x - 0.8614\, x^2 + 2.1236\, x^3 - 2.9163\, x^4 + 1.9744 x^5 - 0.5231\, x^6$$

Now, this is what we need to wrap on circular camber line. So, here if you look at, this is what is representing my circular camber line and this is what is the equation for the circular camber line.

$$The\ airfoil\ wrapped\ around\ circular\ camber\ lines\ is\ given\ by,$$

$$y_c = \left[\left\{\frac{0.5}{\sin\left(\frac{\theta}{2}\right)}\right\}^2 - (x-0.5)^2\right]^{0.5} - 0.5/\tan\left(\frac{\theta}{2}\right)$$

As we have discussed, initially, you will be assuming your circular camber line and later on as per your requirement maybe you will go with say other kinds of camber lines what we have already discussed with, okay. And, based on this camber line, now we are looking for what will be my coordinates on upper surface and lower surface.

So, here if you look at, we can calculate what will be our slope factor that is nothing but that's what we are calculating based on our camber line equation. So, here if you look at, this is what is representing my slope factor or say slope that is what is say ϕ. Once we know what is our ϕ, we can calculate what will be my upper coordinates and what will be my lower coordinates.

$$For\ upper\ surface,$$

$$X_U = x - y_t \sin\phi$$

$$Y_U = y_c + y_t \cos\phi$$

$$For\ Lower\ surface,$$

$$X_L = x + y_t \sin\phi$$

$$Y_L = y_c - y_t \cos\phi$$

Let me tell you upper coordinate that means, what are the coordinates for our suction surface and what are the coordinates for our say pressure surface, okay. Now, here in this case we are calculating these parameters. Now, what we are interested in? Say, this is what we can calculate at particular station. We want to make a blade, okay. So, for making up that blade, we are looking for something else, what is our requirement?

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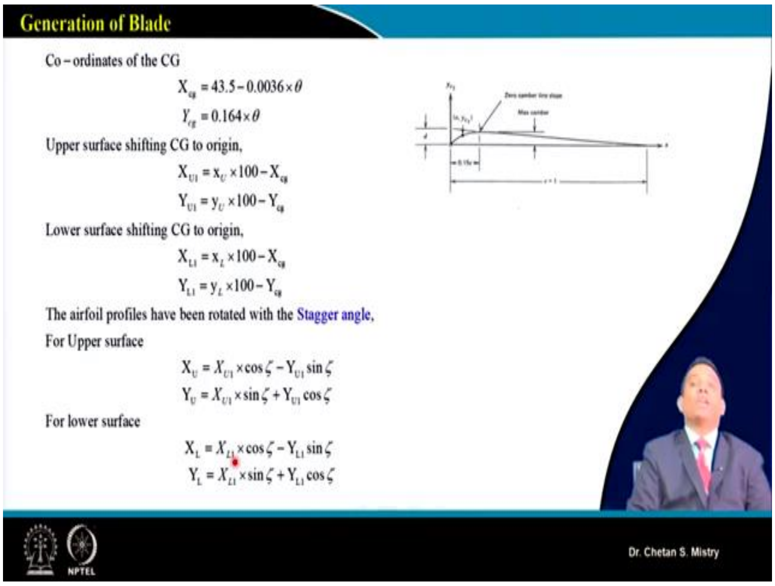

Let us see. So, we want to stack our blade; rotor blade as well as stator blade, about suppose say CG. So, for this profile, we need to have our CG location. For the C4 profile, the CG location, that's what is given in sense of X and Y coordinates, this is what is very important, okay. So, whatever profile you will be selecting, you need to know what is the CG of that particular profile; until and unless you know you cannot do your stacking, okay. So, this is what is my X coordinate and Y coordinate.

$$Coordinates\ of\ the\ CG$$

$$X_{cg} = 43.5 - 0.0036 \times \theta$$

$$Y_{cg} = 0.164 \times \theta$$

Now, once we have this calculation for X coordinates and Y coordinates, I need to shift my coordinates from CG to the origin. So, this is what is a shifting, okay. So, this is what is a shifting for my say CG to the origin. So, you know, like we will be recalculating our upper coordinates or say upper surface or say suction surface X and Y coordinates in line to that we can calculate our pressure surface X and Y coordinates.

So, now, we will be having different kind of arrangement. Now, we are doing our staking about CG, that's what is very important.

$$Upper\ surface\ shifting\ CG\ to\ origin,$$

$$X_{U1} = X_U \times 100 - X_{cg}$$

$$Y_{U1} = Y_U \times 100 - Y_{CG}$$

$$Lower\ surface\ shifting\ CG\ to\ origin,$$

$$X_{L1} = X_L \times 100 - X_{cg}$$

$$Y_{L1} = Y_L \times 100 - Y_{CG}$$

Now, when we are talking about the mechanical aspect, though this course is not belong to that mechanical aspect, but when we are discussing about the making of our blade, it is preferred that the blade, that's what is say rotor blade mainly, that need to be stack about the CG, okay.

Otherwise, it will be having mechanical problem, it will be having some vibrational problem, flutter problem.

So, in order to avoid that kind of situation, mostly for all the rotor, we are staking our blade about say CG, okay. So, we can say, we can calculate what will be my upper coordinate and lower coordinates. Now, once we have done this calculation, we have one more angle, that angle is our setting angle or blade setting angle or we can say that's what is our stagger angle.

So, now what all calculation we have done, that's what needs to be staggered, okay. So, again you need to recalculate your say suction surface coordinates and pressure surface coordinates. This is what is little tedious work at this moment and maybe a little complex to understand, but once you will go through in detail you will realize this is what is more interesting. So, if I am incorporating my staggered angle, this is what will be the change in my X and Y coordinates; for upper surface and this is what is say my X and Y coordinate for lower surface.

$$For\ Upper\ surface$$

$$X_U = X_{U1} \times \cos\zeta - Y_{U1}\sin\zeta$$

$$Y_U = X_{U1} \times \sin\zeta + Y_{U1}\cos\zeta$$

$$For\ Lower\ surface$$

$$X_L = X_{L1} \times \cos\zeta - Y_{L1}\sin\zeta$$

$$Y_L = X_{L1} \times \sin\zeta + Y_{L1}\cos\zeta$$

Now, what all we have learned? We have divided our whole span into 11 stations. So, now you can make your program maybe use any code, maybe C, C++ or maybe maltlab code you can develop where you will be having your input parameter as a camber angle and stagger angle; you will be having other parameters like chord and number of stations, you will be putting in an iterative loop and just try to program these particular equations. When you are making that program, as we have discussed, the curve on suction surface and curve on presses surface that need to be smooth curve and when we are looking for this curve to be smooth curve, we need to have more number of points.

So, just decide say more than 500 points for upper surface; same way, more than 500 point for the lower surface, that's what we will be giving you continuous curvature without zigzag, okay.

So, I am sure, you people, you can do this programming for making of this blade, okay. Now, once this is what is ready with you, you can understand, now you are able to do what you are looking for, okay! Finally, we are reaching to the point where we are making our rotor blade as well as our stator blade. We have done whole our design, my design for rotor, we have done design for the stator, okay. And, based on that design, now, we are making our airfoils and we are putting our airfoils in order to make our blade.

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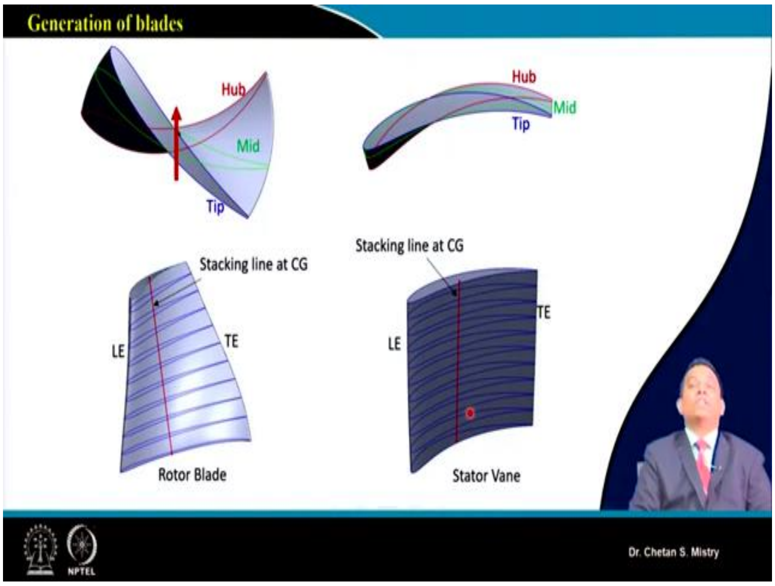

So, let us see. This is what will be our blade. So, here in this case, if you look at, this is what is representing my C4 airfoil at the hub station, this is what is representing at the mid station and this is what is at the tip station, okay. We have total 11 number of say stations. As I told, if you will be having more number of station, you will be having more smoother curve at leading edge as well as trailing edge that is the reason when you are doing your design, you need to be very careful and you can do this design precisely, okay.

So, here you can see, this is what has been stack about the CG point. So, this is what will be making my rotor blade. And, interestingly what we were discussing for our free vortex design, we are having larger curvature, that’s what is happening at the hub, okay. And, at the tip, we will be having our curvature to be lower. So, we can say, this is what is my rotor blade.

Now, based on what all angles we have calculated, what all parameters we have calculated for the stator, you can see, we are having say hub curvature, say mid profile and tip profile and here you can see, this is what is ready with my stator blade and that stator also it is been stack at this moment at CG.

Now, let me tell you, there are different design criteria, different design aspects. Many times, people, they are doing their initial calculation, they are doing their initial design which say stacking about CG for say stator; later on, they realize, you know, this is what is not satisfying what performance they are expecting. Maybe they will be stacking about the trailing edge, maybe they will be staking about the leading edge; this is what is all designer's criteria. And if you recall, for our stator we have shown, where we were having say dihedral, we were having sweep; so, all those kinds of things, that's what you can practice.

So, you know, like the whole purpose for this design course, that's what is to give the idea for initial first cut design and based on this design, you can go with more complexity in order to what all aspects you want to achieve with. And future designs you will see maybe when you are referring the books, when you are referring the research papers or you are attending the conferences, you will realize people they are having different funny kind of shapes for rotor as well as for the stator. And, that's what they are doing with a systematic way, okay. But, for all initial first hand calculation, that's what has been done the way in which we have discussed for first two lectures for our design module, okay, and that too by using free vortex concept.

Now in next lecture, we will be discussing our fundamental method, how do we do our design. And, how that fundamental approach, that's what we will be giving some of the flexibility and how my blades that will be different compared to what all we have designed using this free vortex concept.

So, thank you. Thank you very much! I do prefer that you go through the slides. Do make your excel sheet program, check with all these numbers. And if possible, start making the code in order to make the airfoils as well as to make the blades for both rotor as well as stators. Thank you. Thank you very much!