Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 32
Selection of Design Parameters (Contd.)

Hello, and welcome to Lecture 32. We are discussing about the selection of design parameters. In last lecture, we have solved a numerical that's what is based on calculation of our critical velocity and radial equilibrium. Let us start with say one more numerical.

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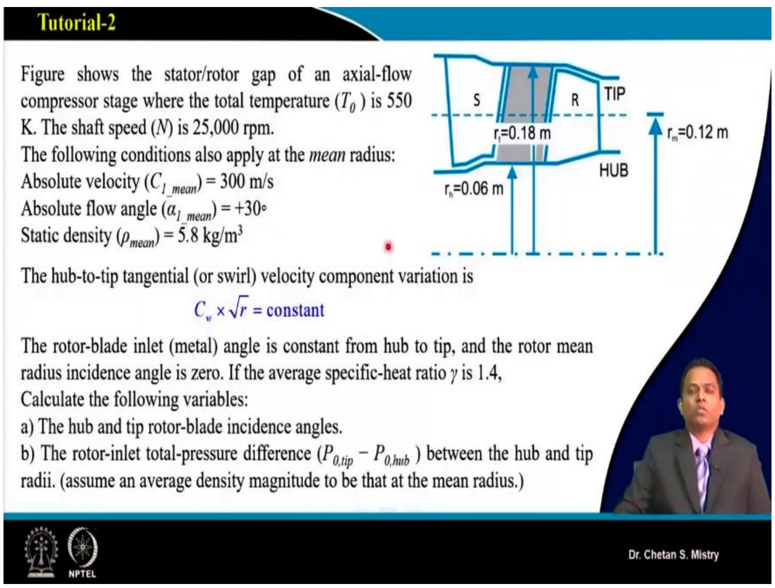

Just look at this, say here, this is what is a figure, that's what is giving us information saying the stator and rotor gap of axial-flow compressor stage, where total temperature is 550 K, rotational speed is 25,000 rpm. Following data, that's what are available at the mean radius now, say absolute velocity at the entry it is 300 m/s, my absolute flow angle at the mid station is 30°, static density at the mid station is 5.8 kg/m^3 and hub to tip tangential or whirl velocity component variation, that's what is given by formula $C_w \times \sqrt{r} = constant$.

The rotor blade inlet or say metal angle is constant from hub to tip and rotor mean radius incidence angle is 0. If the average specific heat ratio is 1.4, calculate following variables; hub and tip rotor blade incidence angles, rotor inlet total pressure difference, that's what is $P_{0,tip} - P_{0,hub}$. At hub and tip radius, assume average density magnitude to be that at the mean radius. So, this numerical that's what is somewhat different from what all we have discussed in the last class.

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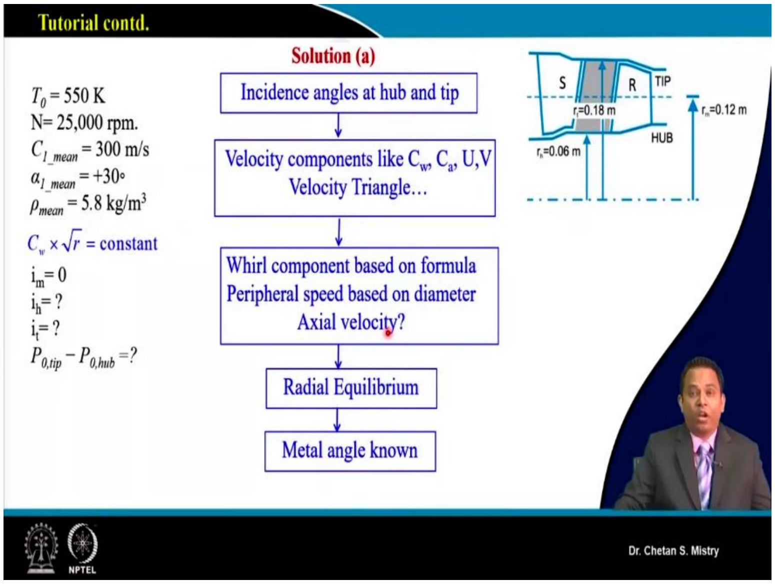

Let us see what all information, that's what is available with us. It says my total temperature is 550 K, rotational speed is given that's what is say 25,000 rpm, my entry absolute velocity is 300 m/s, my absolute flow angle is 30°, density at the mid station it is given and my whirl variation that's what is not say your free-vortex kind of configuration here it is given $C_w \times \sqrt{r} = constant$.

The information that's what is given at midsection, my incidence angle is 0 and we need to calculate what will be the incidence angle at the hub, at the tip. We also need to calculate what will be your total pressure difference. So, you can say this is what is a different kind of numerical where we are looking for other properties, okay. So, let us try to understand how do we solve this kind of numerical.

So, what all we are looking for is we are looking for incidence angle calculation at hub and tip. What we know in order to calculate that say incidence angle, we must know what will be our inlet, blade angle and outlet blade angle. In order to calculate those, we need to have our velocity components and we must know what is our velocity triangle, okay. Now, here in this case, if you look at, we can say, my peripheral speed that's what can be calculated based on available radius.

And my rotational speed is known to me, okay. The whirl component that's what it says it is following $C_w \times \sqrt{r} = constant$. So, that's what will be helping us for calculation of our whirl component, okay. So, this is what all will be helping us in sense of calculating velocity components, but very important velocity component that's what is missing that is axial velocity.

Now, the question is how do we calculate our axial velocity, angles are unknown, few velocity components are known, but at the same time what we know? We have our radial equilibrium equation, that's what will be helping us for calculation of our axial velocity, it may be varying from hub to tip or it may be remains constant, we need to check with; but just understand because this is what is $C_w \times \sqrt{r} = constant$ that definitely says my axial velocity will not remain constant. So, that's what is a hint, we need to calculate what will be my metal blade angle and based on that, we will try to calculate what is our incidence angle. So, this is what is our strategy for say solution 1.

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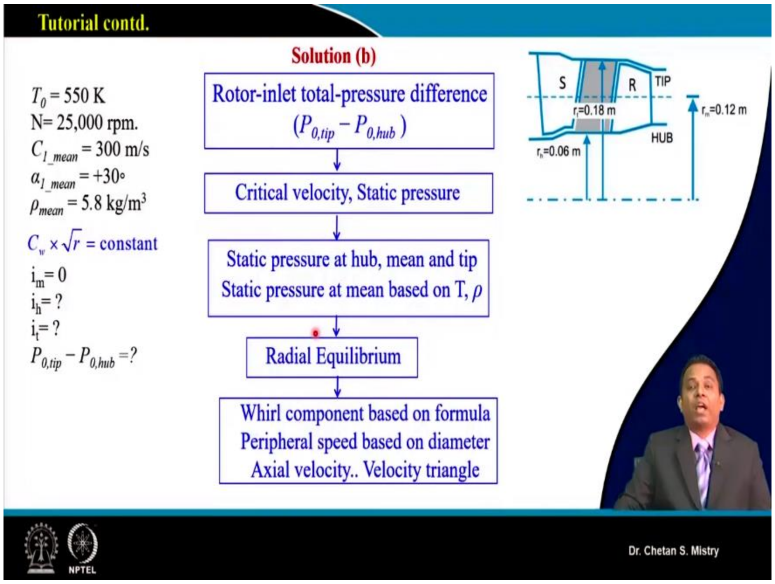

Same way we are looking for calculation of ΔP_0, we can understand if you are looking for say ΔP_0 that is total pressure at the hub, if you are looking for total pressure at the tip, we must know what will be our static pressure. Again, we are having the correlation, that's what is correlating our total pressure and static pressure that's what we say our critical Mach number.

Now, in order to calculate that critical Mach number, we must know what are our velocities and temperature. So, let us see how do we proceed with. It says, like we are looking for rotor inlet total pressure ratio, okay or pressure difference that's what is looking for my critical velocity and static pressure. So, we must know what will be our static pressure at mid station or maybe hub station or at the tip station.

Here, the hint is given at mid station, density is known to us and temperature also is given to us. So, we can say that's what will be helping us for calculating our static pressure at one location. We can use our fundamentals of radial equilibrium equation and based on that we will try to calculate all velocity components and from that we will be calculating our critical velocity ratio and from that we can calculate what is our total pressure difference.

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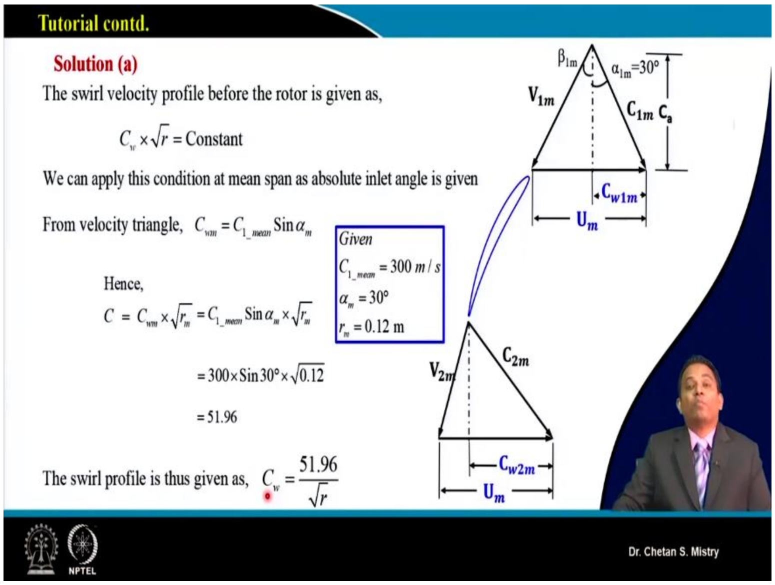

So, let us begin with. Say for first solution, so what all information that's what is given to us, we can plot this kind of velocity triangle. So, what it says? My α angle, that's what is known to us; so, α_1, that's what is given 30°, okay. Now, at mid station my diameter is known, my speed, that's what is known; so, you can calculate what will be my U. So, let us try to understand how do we proceed with.

So, here in this case, from velocity triangle, we can write down our whirl component at the mid station, that's what will be given by $C_{1mean} \sin \alpha_m$, okay, because at mid station I know what is my absolute velocity, I know what is my say absolute flow angle. So, this data, it is known to me, so it says $C_{1\_mean} = 300\ m/s$ and my absolute flow angle is say 30° and at mid radius, I know what is my radius, that's what is 0.12 meter.

So, if I will be putting this, it says, we can calculate what will be my C_w. So, if we are calculating that part it says my $C_{wm} \times \sqrt{r_m}$, that's what is constant. So, basically, we will try to calculate first what is our constant. So, it says 300 into sin 30 into this is what is square root of 0.12, it says my constant as 51.96, okay.

$$Hence, C = C_{wm} \times \sqrt{r_m} = C_{1_{mean}} \sin \alpha_m \times \sqrt{r_m}$$

$$= 300 \times \sin 30° \times \sqrt{0.12}$$

$$= 51.96$$

Now, what is known to me my $C_w \times \sqrt{r} = constant$. So, we can say my C_w that's what is given by

$$C_w = \frac{51.96}{\sqrt{r}}$$

Now, it says if I am looking for what will be my whirl component and hub, I can calculate by putting hub radius, I can calculate at tip by putting tip radius.

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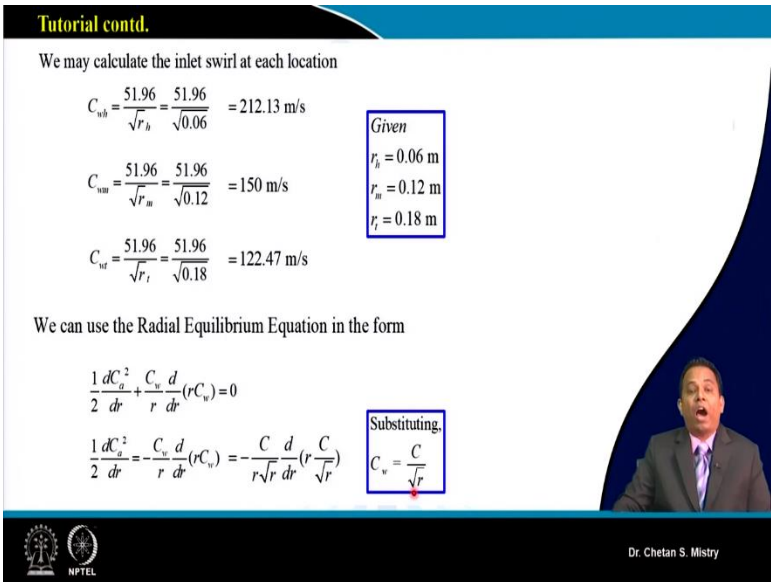

So, let us see. Suppose if you are considering this as our case, at hub, we know our radius as 0.06 meter. At mid station, our radius is 0.12, and at the tip station, my radius is 0.18. So, if we are putting that into formulation, it says my whirl component at the hub is 212.13 m/s. My C_w at the mid station, that's what is 150 m/s. And my C_w at the tip, that's what is 122.47 m/s.

$$C_{wh} = \frac{51.96}{\sqrt{r_h}} = \frac{51.96}{\sqrt{0.06}} = 212.13\ m/s$$

$$C_{wm} = \frac{51.96}{\sqrt{r_m}} = \frac{51.96}{\sqrt{0.12}} = 150\ m/s$$

$$C_{wh} = \frac{51.96}{\sqrt{r_t}} = \frac{51.96}{\sqrt{0.18}} = 122.47\ m/s$$

So, you having higher swirl component or whirl component, that's what is present at the hub and local whirl component, that's what is present at the tip, okay. So, this is what is a calculation for our whirl component. Now, in order to do calculation for our flow angle, we are looking for our axial velocity.

Now, what we know from our formulation of radial equilibrium equation, or say, vertex energy equation, we can write down this is what is a known formula for us. So, in place of my C_w, we can write down, that's what is given by $C_w = \frac{C}{\sqrt{r}}$.

$We\ can\ use\ the\ Radial\ Equilibrium\ Equation\ in\ the\ form$

$$\frac{1}{2}\frac{dC_a^2}{dr} + \frac{C_w}{r}\frac{d}{dr}(rC_w) = 0$$

$$\frac{1}{2}\frac{dC_a^2}{dr} = -\frac{C_w}{r}\frac{d}{dr}(rC_w) = -\frac{C}{r\sqrt{r}}\frac{d}{dr}(r\frac{C}{\sqrt{r}})$$

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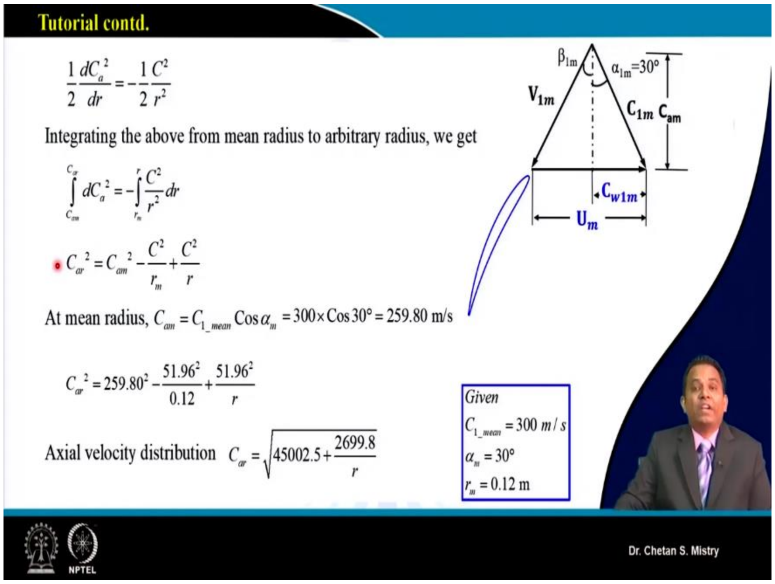

So, if you are putting in this equation and if you will try to put in the formulation form, it says

$$\frac{1}{2}\frac{dC_a^2}{dr} = -\frac{1}{2}\frac{C^2}{r^2}$$

Now here, in this case, we can do the integration. Again, since at mid station my axial velocity, that's what we can calculate. So, let me put this as say mid station to some radius r, here also we are putting at mid station to some radius r.

$$\int_{C_{am}}^{C_{ar}} dC_a^2 = -\int_{r_m}^{r} \frac{C^2}{r^2} dr$$

So, that's what is giving me my axial velocity at any radial location as say

$$C_{ar}^2 = C_{am}^2 - \frac{C^2}{r_m} + \frac{C^2}{r}$$

Now, here in this case, it says, we are not having any information about axial velocity at the mid station. But if you look at from our fundamental understanding the angle alpha, that's what is known to me that is 30°.

Now, we have already calculated our C_{w1} at the mid station. So, based on my formula, I can write down my axial velocity at the mid station, that's what is given by

$$C_{am} = C_{1\_mean} \cos \alpha_m = 300 \times \cos 30° = 259.8\ m/s$$

So, this C_1, that's what is known to me it is 300 m/s, my α is 30°, it says my axial velocity at the mid station is coming to be 259.80 m/s.

Now, once this is what is known to us, we can write down in the form of this formula, it says my Ca square or my axial velocity at any radius, that's what is given in the form of some radius. So, this is what is the formula, okay.

$$C_{ar}^2 = 259.8^2 - \frac{51.96^2}{0.12} + \frac{51.96^2}{r}$$

So, you can understand, the basic application of radial equilibrium, your vertex energy equation, that's what is always necessary, we need to have some background of mathematical formulation in sense of integration and differentiation, that's what will be helping us in order to get the solution in a proper way. So, this is what is my axial velocity at particular radial location.

$$\mathit{Axial\ velocity\ distribution,}$$

$$C_{ar} = \sqrt{45002.5 + \frac{2699.8}{r}}$$

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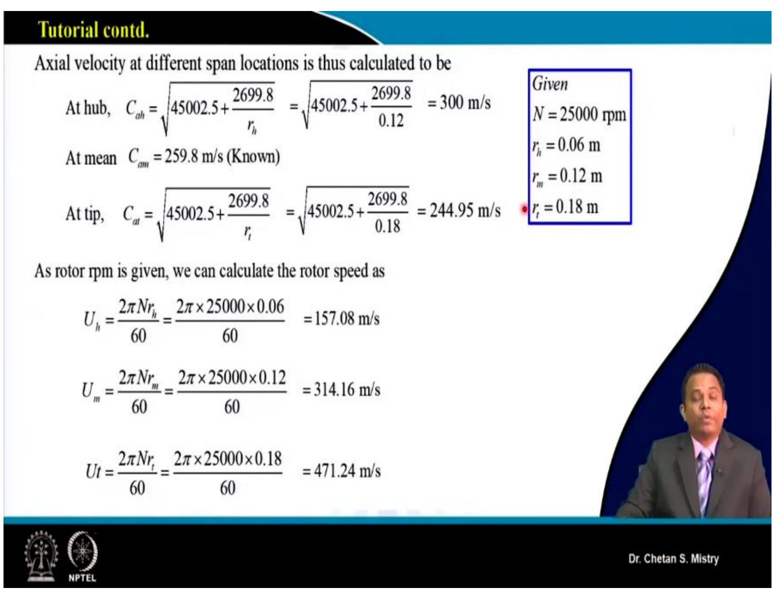

Now, we can say, my radius for hub, my radius for mean and radius at the tip station, they are known to me. It says, at hub it is 0.06 meter, at mid it is 0.12 meter and at the tip it is 0.18 meter. So, we can say this is what we can write down and we can calculate our axial velocity at the hub that's what is coming 300 m/s. At the tip station, this is coming 244.95 m/s, okay.

$Axial\ velocity\ at\ different\ span\ locations\ is\ thus\ calculated\ to\ be$

$$At\ hub, C_{ah} = \sqrt{45002.5 + \frac{2699.8}{r_h}} = \sqrt{45002.5 + \frac{2699.8}{0.12}} = 300\ m/s$$

$$At\ mean, C_{am} = 259.8\ m/s\ \ (known)$$

$$At\ tip, C_{at} = \sqrt{45002.5 + \frac{2699.8}{r_t}} = \sqrt{45002.5 + \frac{2699.8}{0.18}} = 244.95\ m/s$$

So, do not consider or do not assume axial velocity to be constant, be careful! You are given the variation of whirl component and that to it is in a formulation form, that's what is giving you hint my axial velocity is not remaining constant, okay. Now, if this is what is your case, we can calculate what is our peripheral speed, okay. So, in order to calculate that, we know what is our rotational speed, that's what is 25,000, my hub radius is known, that's what will give me my peripheral speed at the hub station, at mid station and at tip station.

So, we can say, at hub it is 157.08 m/s. At mid station, it is 314.16 m/s. And, at the tip station, that's what is say 471.24 m/s. Now, you can say, we are able to calculate different velocity components, this is what will be helping us in sense of calculating different flow angles.

$As\ rotor\ rpm\ is\ given, we\ can\ calculate\ the\ rotor\ speed\ as,$

$$U_h = \frac{2\pi N r_h}{60} = \frac{2\pi \times 25000 \times 0.06}{60} = 157.08\ m/s$$

$$U_m = \frac{2\pi N r_m}{60} = \frac{2\pi \times 25000 \times 0.12}{60} = 314.16\ m/s$$

$$U_t = \frac{2\pi N r_t}{60} = \frac{2\pi \times 25000 \times 0.18}{60} = 471.24\ m/s$$

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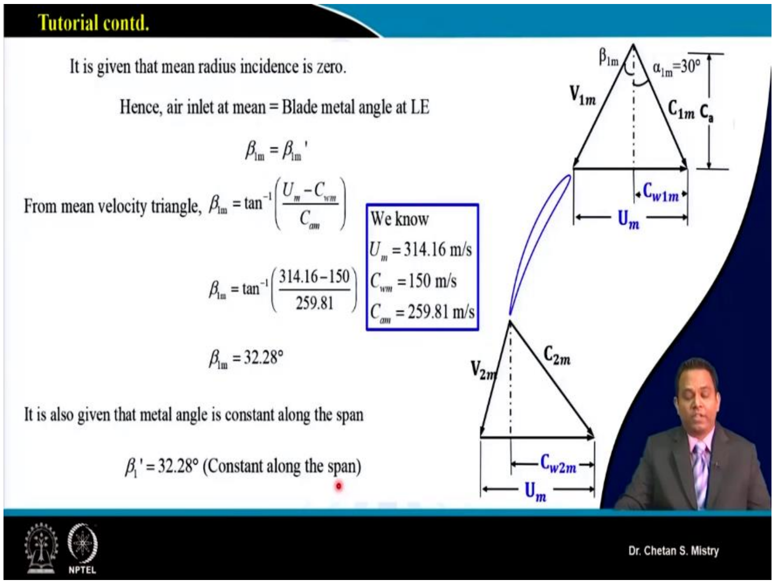

So, let us try to look at this part, what it says at mid radius, my incidence angle, that's what is given to be 0. So, we can say, my β_1 at mid station, that's what is equal to β_1' at mid station. Now, if we are putting our velocity triangle at the mid station, so what we know is my mean peripheral speed it is 314.16 m/s, my C_w is 140 m/s, and my axial velocity is say 259.81.

So, if you are putting all together, that's what will be giving me this kind of velocity triangle, okay. So, it is always advisable, you make your velocity triangle free hand that's what will help you in sense of calculating different flow angles. So, for this velocity triangle, my β_1, that's what is given by

$$\beta_{1m} = \tan^{-1}\left(\frac{U_m - C_{wm}}{C_{am}}\right)$$

$$\beta_{1m} = \tan^{-1}\left(\frac{314.16 - 150}{259.81}\right)$$

$$\beta_{1m} = 32.28°$$

Now, these numbers are known to us. So, we can calculate our β_{1m} that's what is 32.28°, okay. Here, in this case, my incidence angle, that's what is given 0, so we can write down my β_1', that's what is coming as 32.28°, okay.

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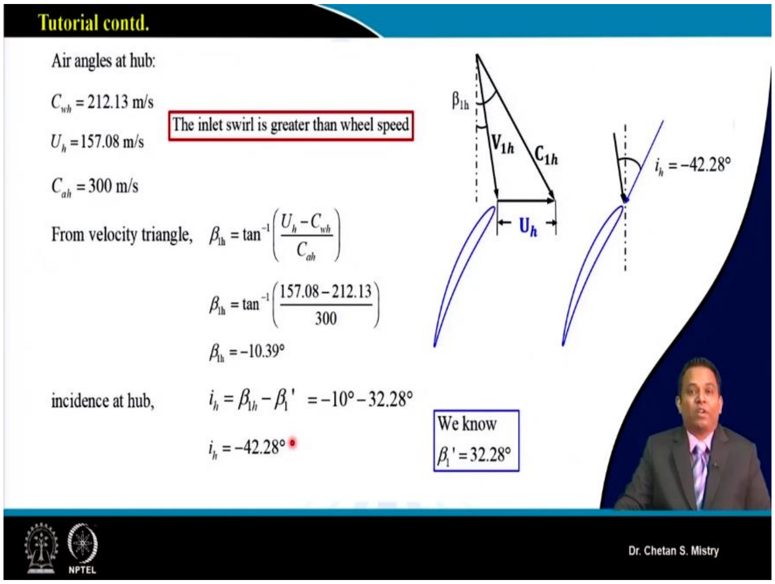

Now, let us try to look at what is happening at the hub. So, we have done our calculation for whirl component it is 212.13 m/s, my peripheral speed at the hub is 157.08 m/s and my axial velocity we have calculated, it is 300 m/s. Be careful here, here if you look at carefully, my whirl component that's what is larger compared to my peripheral speed. So, your velocity triangle that will be changing accordingly.

Do not make any mistake here. So, here if you look at, based on this number, roughly we can say this is what will be my velocity triangle, make a freehand that's what will be helping you in sense of understanding. Same way I can put my equation for β_1 at the hub, it is

$$\beta_{1h} = \tan^{-1}\left(\frac{U_h - C_{wh}}{C_{ah}}\right)$$

$$\beta_{1h} = \tan^{-1}\left(\frac{157.08 - 212.13}{300}\right)$$

$$\beta_{1h} = -10.39°$$

Now, here in this case, my C_w that's what is say it is coming to be higher, that's what will be giving me my β_1 at the hub as $-10.39°$, okay.

What we are asked to calculate for that says, I need to calculate what is my incidence angle at the hub. So, if we are putting that's what is given by

$$i_h = \beta_{1h} - \beta_1'$$

So, this β_1 at hub we have calculated it is $-10°$ and this is what is say it is given it is a constant, that's what is 32.28°.

$$\therefore i_h = -10° - 32.28°$$

$$i_h = -42.28°$$

So, it says my incidence angle, that's what is coming $-42.28°$. Remember, this is what is a numerical this kind of numbers may not come in actual condition, okay. But for understanding you can say this is how we are calculating our incidence angle at the hub, okay.

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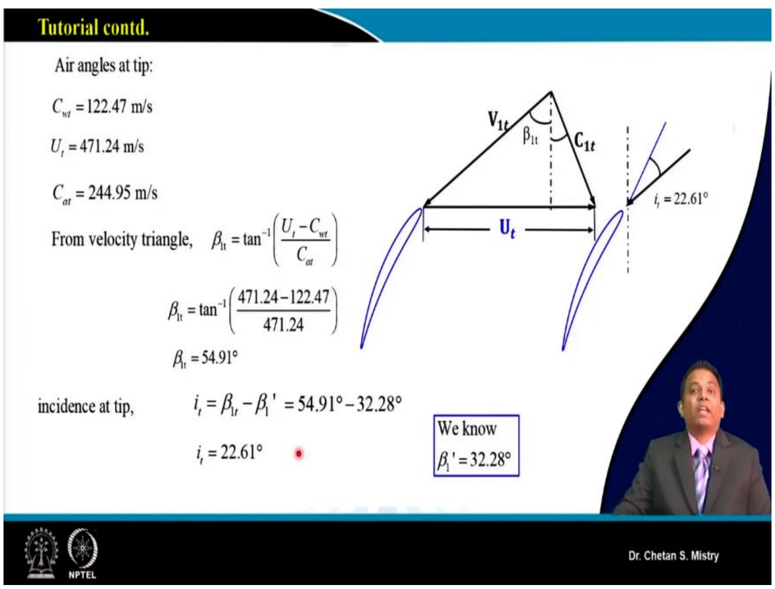

In line to that, we can do our calculation for my tip station. At tip station, we know what is our whirl component, we know what is our peripheral speed, it says my whirl velocity is 122.47 m/s, my peripheral speed is 471.24 m/s and my axial velocity at the tip is 244.25 m/s. So, based on this data, if I am putting my velocity triangle, then this is what will be my velocity triangle, okay.

So, here in this case, we are not having the situation as we have seen for our hub, you can see, here my peripheral speed that's what is larger, and my whirl component that's what is smaller. So you can see here, so this portion that's what is representing my whirl component, okay and this is what is my peripheral speed.

So, if that's what is your case, you can write down my,

$$\tan\beta_{1t} = \frac{(U_t - C_{wt})}{C_{at}}$$

These numbers they are known to us. So, we can do our calculation for, that's what it says, we are getting our incidence angle to be 22.61°.

$$\beta_{1t} = \tan^{-1}\left(\frac{471.24 - 122.47}{471.24}\right)$$

$$\beta_{1t} = 54.91°$$

$$incidence\ at\ tip,$$

$$i_t = \beta_{1t} - \beta_1' = 54.91° - 32.28°$$

$$i_t = 22.61°$$

So, this is what will be giving us idea about what we say in sense of our variation of incidence angle at hub, variation of our incidence angle at the tip, okay.

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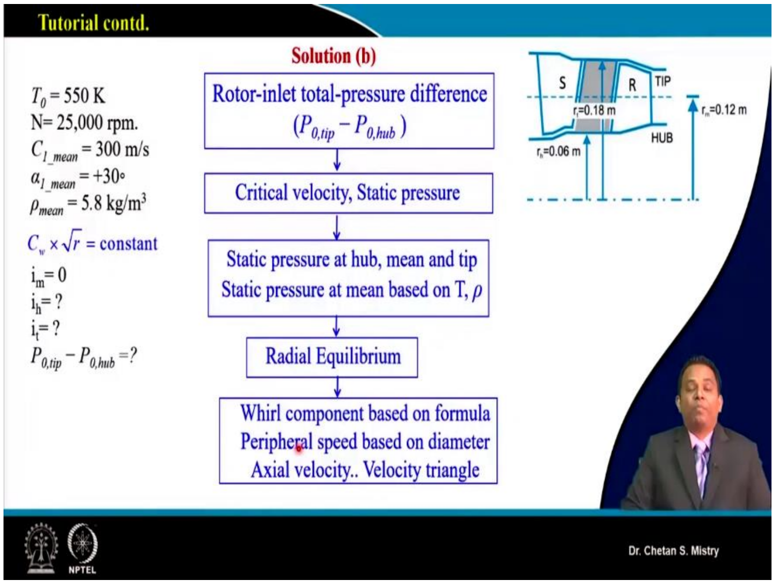

Now, the second part for this numerical as we have discussed, that's where we are looking for variation of ΔP_0 , okay, at hub and what will be the variation of my ΔP_0 or you can say what is my P_0 at the hub and what is my P_0 at the tip, okay. So, here in this case as we have discussed, let us start with the solution for.

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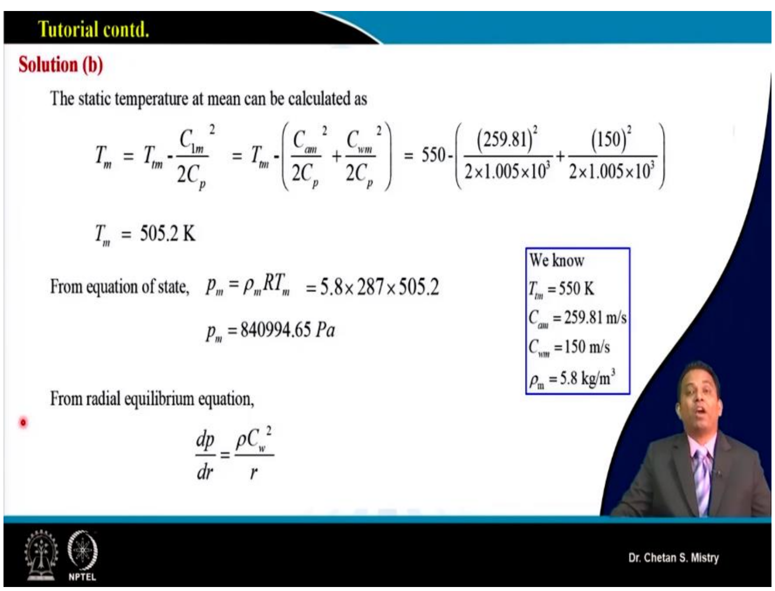

So, very first thing what we are looking for is for calculation of our critical velocity component. Now, in order to calculate a critical velocity component, we must have the temperature value, okay. So, at mid station, I can write down my static temperature that's what is given by

$$T_m = T_{tm} - \frac{C_{1m}^2}{2C_p} = T_{tm} - \left(\frac{C_{am}^2}{2C_p} + \frac{C_{wm}^2}{2C_p}\right)$$

If we are putting that it says my static temperature, that's what is coming as 505.2 K, okay.

$$T_m = 550 - \left(\frac{(259.81)^2}{2 \times 1.005 \times 10^3} + \frac{(150)^2}{2 \times 1.005 \times 10^3}\right)$$

$$T_m = 505.2\ K$$

Now, from our fundamental understanding, we can do our calculation for say my static pressure at the mid station as say

$$p_m = \rho_m R T_m$$

So, we can say, that's what is coming as say 840.99 kPa.

$$p_m = 5.8 \times 287 \times 505.2 = 840994.65\ Pa$$

So, now at mid station, my static pressure, that's what is known to me, okay. So, here in this numerical my temperature, static temperature, that's what is not given straightway. But remember, that's what is related in sense of say my total temperature and absolute velocity, okay.

So, what all data that's what is given to you just try to locate that, put it on diagram or write down what all data that's what is given to us. And based on that you move ahead with, okay. Now, in order to do our calculation for the variation of my static pressure at the tip, or mean or say maybe hub, we can use our radial equilibrium equation. It says my $\frac{dp}{dr}$, that's what is given by, say

$$\frac{dp}{dr} = \frac{\rho C_w^2}{r}$$

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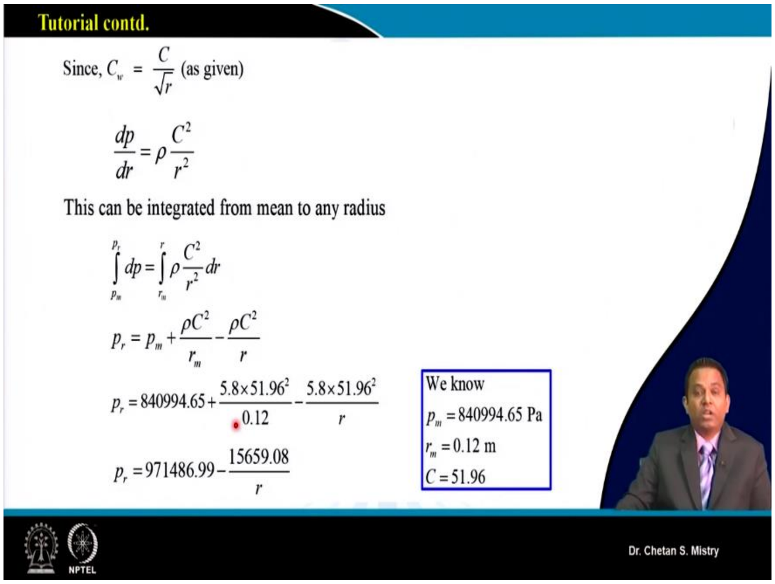

Now, what is given to us? We know what is our whirl component that's what is say $\frac{C}{\sqrt{r}}$. That's what we can write down in this formula.

$$\frac{dp}{dr} = \rho \frac{C^2}{r^2}$$

And if we are integrating that from say, mean station, because at mean station my static pressure is known to me. Be careful, at what location you are known with the pressure or temperature accordingly, you need to decide with your limit, same way for axial velocity.

So, here in this case, at mid station my static pressure is known to me. So, I can calculate my static pressure variation at any station by integrating this form.

$$\int_{p_m}^{p_r} dp = \int_{r_m}^{r} \rho \frac{C^2}{r^2} dr$$

It says my static pressure at any location that's what is given by

$$p_r = p_m + \frac{\rho C^2}{r_m} - \frac{\rho C^2}{r}$$

Now, the main radius that's what is known to us, so if we will be putting all this together, that's what is giving me my static pressure variation with the radius. So, here if you look at, this is what is a formula for the variation of my static pressure at particular radius, okay.

$$p_r = 840994.65 + \frac{5.8 \times 51.96^2}{0.12} - \frac{5.8 \times 51.96^2}{r}$$

$$p_r = 971486.99 - \frac{15659.08}{r}$$

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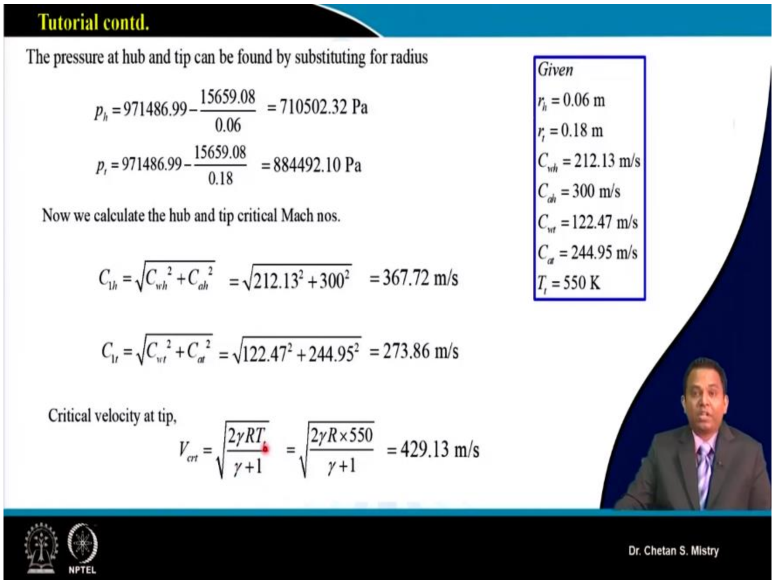

Now, let us put we are looking for our variation of or say we are looking for our static pressure at the hub. So, if you are putting my radius as 0.06, we are getting our static pressure at the hub. If we are putting our radius at the tip as 0.18, we are getting our static pressure at the tip, okay. So, now my static pressure at the hub and tip they are known to us, okay.

$$p_h = 971486.99 - \frac{15659.08}{0.06} = 710502.32\ Pa$$

$$p_t = 971486.99 - \frac{15659.08}{0.18} = 884492.1\ Pa$$

Now, in order to calculate your critical velocity, we must know what all are the velocity components. So, here in this case, if you are looking for say absolute velocity at the hub we can write down that's nothing but C_w^2/C_a^2. And that's what will give me my absolute velocity component as say 367.72 m/s. Same way my absolute velocity at the tip it is 273.86 m/s.

$$C_{1h} = \sqrt{C_{wh}^2 + C_{ah}^2} = \sqrt{212.13^2 + 300^2} = 367.72\ m/s$$

$$C_{1t} = \sqrt{C_{wt}^2 + C_{at}^2} = \sqrt{122.47^2 + 244.95^2} = 273.86\ m/s$$

My critical velocity at the tip we can calculate it is

$$V_{crt} = \sqrt{\frac{2\gamma R T_t}{\gamma + 1}} = \sqrt{\frac{2\gamma R \times 550}{\gamma + 1}} = 429.13\ m/s$$

This temperatures, static temperature at the tip, that's what is known to us. So, we can write down this equation and will get 429.13 m/s.

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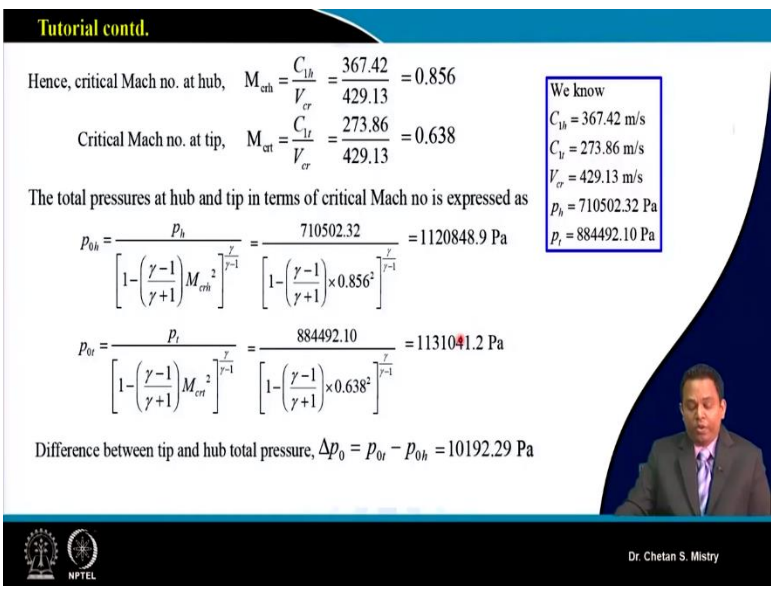

So, now, if you are looking for say calculation of our critical Mach number at the hub, we can write down that by say $\frac{C_1}{V_{cr}}$. Same way for tip, we can write down $\frac{C_1}{V_{cr}}$. So, since my absolute velocities are known to me, so, I can calculate what will be my critical Mach number at the hub and what will be my critical Mach number at the tip and that's what is coming say 0.856 and 0.638.

$Critical\ Mach\ number\ at\ hub,$

$$M_{crh} = \frac{C_{1h}}{V_{cr}} = \frac{367.42}{429.13} = 0.856$$

$Critical\ Mach\ number\ at\ tip,$

$$M_{crt} = \frac{C_{1t}}{V_{cr}} = \frac{273.86}{429.13} = 0.638$$

Now, once this critical Mach number, that's what is known to us, we know there is a relation between say my total pressure and my static pressure in the formula for critical Mach number. So, if you are putting these numbers, it says this is what is

$$p_{0h} = \frac{p_h}{\left[1 - \left(\frac{\gamma - 1}{\gamma + 1}\right) M_{crh}^2\right]^{\frac{\gamma}{\gamma-1}}}$$

Here, at hub my critical Mach number is 0.856. If you are putting that, this is what it says is coming in sense of my Mega pascal pressure, okay.

$$p_{0h} = \frac{710502.32}{\left[1 - \left(\frac{1.4 - 1}{1.4 + 1}\right) \times 0.856^2\right]^{\frac{1.4}{1.4-1}}} = 1120848.9\ Pa$$

Here, at tip also, we can do our calculation.

$$p_{0t} = \frac{p_t}{\left[1 - \left(\frac{\gamma - 1}{\gamma + 1}\right) M_{crt}^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{884492.1}{\left[1 - \left(\frac{1.4 - 1}{1.4 + 1}\right) \times 0.638^2\right]^{\frac{1.4}{1.4-1}}} = 1131041.2\ Pa$$

So, if you are putting all this together, that's what is saying like my ΔP_0, that's what is coming to be say 10.192 kPa, okay.

$$Difference\ between\ tip\ and\ hub\ total\ pressure,$$

$$\Delta p_0 = p_{0t} - p_{0h} = 10192.29\ Pa$$

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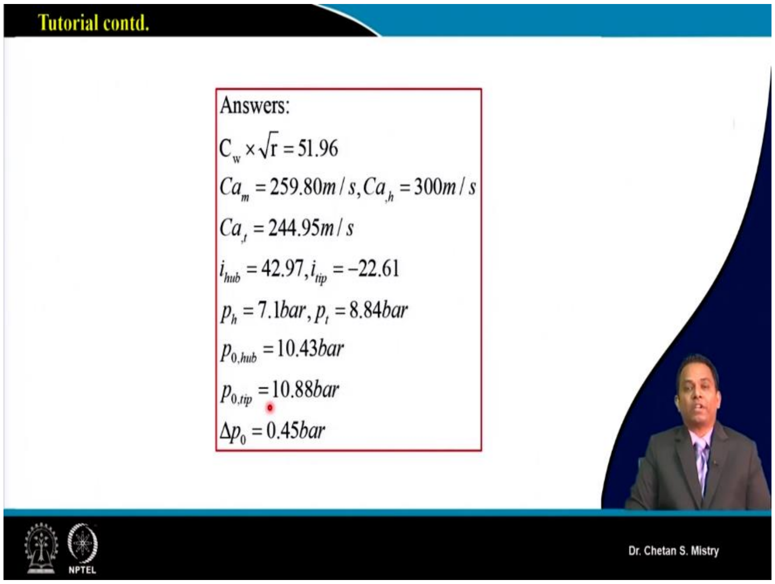

So, let us do the compilation of what all we have learned. Say, we have calculated our say tangential velocity variation, that's what is $C_w \times \sqrt{r} = constant$. That constant we have calculated it is 51.96. We have calculated our axial velocity variation at mid station, hub station and tip station.

Then we have calculated our incidence angle at the hub, we have calculated our incidence angle at the tip, we have calculated static pressure at hub as well as a static pressure at the tip and finally we have calculated our Δp_0.

$$C_w \times \sqrt{r} = 51.96$$

$$C_{am} = 259.80\ m/s$$

$$C_{ah} = 300\ m/s$$

$$C_{at} = 244.95\ m/s$$

$$i_{hub} = 42.97°$$

$$i_{tip} = -22.61°$$

$$p_h = 7.1\ bar$$

$$p_t = 8.84\ bar$$

$$p_{0,hub} = 10.43\ bar$$

$$p_{0,tip} = 10.88\ bar$$

$$\Delta p_0 = 0.45\ bar$$

So, this is how what all we need to use in sense of our understanding for what all formulas we have learned up till now. So, this is what is end of this particular session. Thank you very much for your attention.