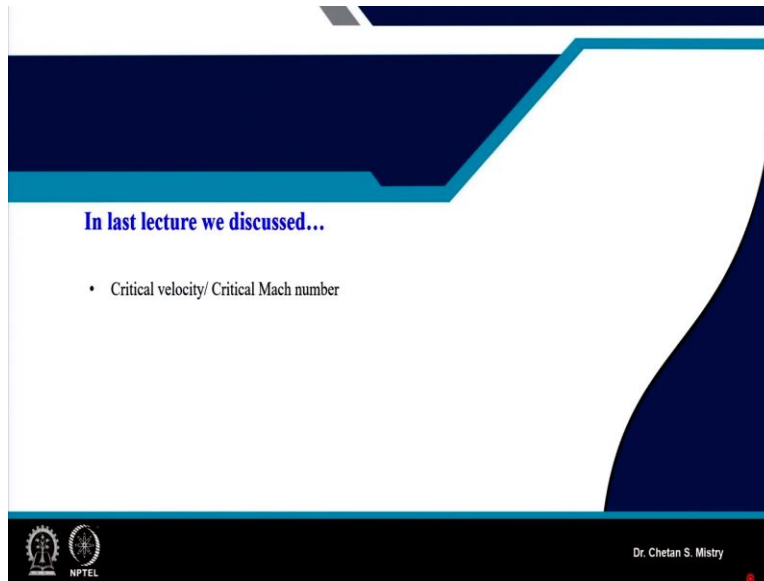


**Aerodynamic Design of Axial Flow Compressors & Fans**  
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**Lecture 31**  
**Selection of Design Parameters (Contd.)**

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Hello, and welcome to Lecture 31. In last lecture, we started discussing about the important parameter, what we have defined as a critical velocity ratio or critical Mach number. We realized, the operating temperature for compressor and operating temperature for...the temperature for the turbine, they both are different.

Say, my compressor is operating at low temperature; turbine, that's what is operating at high temperature. So, based on our conventional definition of Mach number, if we consider, then for compressor, it will always show my flow to be in a transonic range or maybe in supersonic range, though it is working under subsonic condition or high subsonic condition.

Same way, suppose if we consider for the turbine, because in denominator we are having the temperature, and that's the reason why it will always show my flow to be a subsonic flow. And that's what is demanding for special kind of attention. And in order to take care of that, we have introduced a parameter called critical Mach number; and this critical Mach number, that's what will give the idea what will be the type of flow within our flow passage for the turbomachinery.

And that's what we will be using for our further design calculations. Yesterday, we also were discussing about say two different design approaches. First, that's what we say as a sizing problem; and second, we have defined as a rating problem. Sizing in the sense, if we are discussing, we are talking about, say new design of engine, or say maybe new design of the compressor stage for our industrial application.

Under that condition, we may be knowing our input parameters, we know what pressure rise we are expecting, maybe we can do calculation for what will be the rotational speed and all those parameters, that's what are known to us. And based on that, we need to decide with what will be the diameter of my casing, what will be the diameter of my hub, what needs to be the height of my blade, what needs to be the chord of my blade, how my flow angles that will be varying with, we are having different approaches for the design of blade from hub to shroud.

And that is how we are doing our iterations. And finally, we are reaching at say some shape of our stage for rotor and stator. And that's what we have defined as a sizing problem. Now, take a different consideration, suppose say we are already having engine. Now, for that engine, we need to estimate what is a performance.

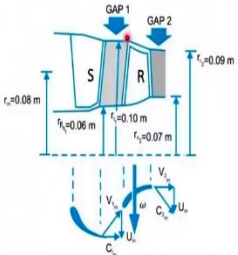
That means you can understand the geometrical parameters, that's what are known to us, say flow angles or blade angles, they are known to us, maybe based on the experimentation or maybe based on actual engine data, we know temperature, we know pressure at different location, even our rotational speeds are also known to us.

If those all things are known to us, we need to verify with say what will be my pressure ratio, what will be my mass flow rate to that engine or to that compressor; and that's what we have defined as a rating problem. So, in line to that, let us try to move ahead, let us take the numerical that's what will give idea how do we use what all we have learned up till now.

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**Tutorial**


Figure shows the meridional flow path of a compressor stage. The rotor speed is 14,000 rpm, and the average specific-heat ratio ( $\gamma$ ) is 1.4. Also, the following conditions apply over the rotor's inlet and exit gaps:



<b>Rotor inlet gap</b>	<b>Rotor exit gap</b>
$T_0 = \text{constant}$	$T_0 = \text{constant}$
$\alpha = 0 \text{ deg}$	Relative flow angle
$\rho = 2.25 \text{ kg/m}^3$	$\beta = 0$
$T_{hub} = 511 \text{ K}$	$\rho_{hub} = 2.4 \text{ kg/m}^3$
	$T_{hub} = 552 \text{ K}$
	$C_{a,hub} = 124 \text{ m/s}$

Using any simple means of integration (such as the trapezoidal rule), and considering only the properties of the hub, mean, and tip sections (for simplicity), calculate the following variables:

- The mass-flow rate through the compressor stage;
- The exit total relative pressure at the tip section.



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So, take a numerical, say...here it says figure shows meridional flow path of compressor stage. The rotor speed is 14,000 rpm and specific heat ratio is 1.4. The following conditions apply over the rotor inlet and outlet gaps. So, information available at rotor inlet gap is my total temperature is constant. My inlet absolute flow angle is 0. Density is  $2.25 \text{ kg/m}^3$ . And static temperature at the hub is 511 K.

$$T_0 = \text{constant}$$

$$\alpha = 0 \text{ deg}$$

$$\rho = 2.25 \frac{\text{kg}}{\text{m}^3}$$

$$T_{hub} = 511 \text{ K}$$

Now, at the rotor exit, so you can understand this is what is the region where this information that's what is given. At rotor exit, it says my  $T_0$  is constant, my relative blade angle  $\beta$  that's what is 0 you can say what we have defined,  $\beta_2$  that's what is equal to 0, density at the hub is  $2.4 \text{ kg/m}^3$ , hub temperature is 552 K and my axial velocity at the hub is 124 m/s.

So, these are the data that's what are given at a rotor entry gap and rotor exit gap. Using any simple means of integration say trapezoidal rule, and considering only the properties of hub, mean, and tip section for simplicity, calculate following variables: the mass flow rate through the compressor stage, and second, exit total relative pressure at the tip section. So, just look at this is what is our

stage and this is what is information that's what is given to us. So, let us try to solve this a numerical.

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**Tutorial**

**Mass flowrate through compressor**

- Continuity equation
- Calculate passage area
- Calculate axial velocity ??
- Is it variable or constant??
- Radial Equilibrium equation

**Rotor inlet gap**  
 $T_0 = \text{constant}$   
 $\alpha = 0 \text{ deg}$   
 $\rho = 2.25 \text{ kg/m}^3$   
 $T_{hub} = 511 \text{ K}$

**Rotor exit gap**  
 $T_0 = \text{constant}$   
 Relative flow angle  $\beta = 0$   
 $\rho_{hub} = 2.4 \text{ kg/m}^3$   
 $T_{hub} = 552 \text{ K}$   
 $C_{a,hub} = 124 \text{ m/s}$

Diagram labels:  $r_m = 0.08 \text{ m}$ ,  $r_{h1} = 0.06 \text{ m}$ ,  $r_{t1} = 0.10 \text{ m}$ ,  $r_{h2} = 0.07 \text{ m}$ ,  $r_{t2} = 0.09 \text{ m}$ ,  $r_t = 0.09 \text{ m}$ ,  $V_{h1}$ ,  $V_{t1}$ ,  $\omega$ ,  $C_{h1}$ ,  $C_{t1}$ ,  $U_{h1}$ ,  $U_{t1}$ .

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Say, this is what is all data that's what is given to us, it says my hub radius is given it is 0.06 meter, my mean radius is 0.08 meter, and we are having our tip radius as 0.09 meter. Here in this case, at the outlet my hub radius is given 0.07 meter. So, in the gap we are having say total temperature as mentioned it is a constant, my alpha is 0, my density is given, temperature is given.

$$r_{h_1} = 0.06 \text{ m}, \quad r_{h_2} = 0.07 \text{ m}$$

$$r_m = 0.08 \text{ m}, \quad r_{t_2} = 0.09 \text{ m}$$

$$r_{t_1} = 0.1 \text{ m}$$

At the outlet, we are having our total temperature, that's what is constant, my blade angle  $\beta$  is 0, my density, temperature and axial velocity they are given to us. What we are asked for is to calculate the mass flow rate.

$$T_0 = \text{constant} \quad T_{hub} = 552 \text{ K}$$

$$\text{Relative flow angle, } \beta = 0 \quad C_{a,hub} = 124 \text{ m/s}$$

$$\rho_{hub} = 2.4 \text{ kg/m}^3$$

Now, the thing is we must have first idea like we can calculate our mass flow rate using continuity equation, okay. When I say continuity equation, we must have say density, area, as well as say our axial velocity.

So, if we consider this density, we say, that's what is the static density. My area, you can say my radius, that's what is given to me. So, that's what will be helping me in calculating my area, and axial velocity. So, here, specifically axial velocity at the hub is given to us. That means we do not know whether our axial velocity is constant or varying.

So, now, if this is what is your question, you can understand, we can use some formulation in order to calculate the variation of this axial velocity, as well as our other properties. So, let us see what will be our strategies. So, as we have discussed our mass flow rate through the compressor, that's what we are looking for. We will be using our continuity equation. We can calculate our passage area. We need to calculate what will be my axial velocity, whether it is variable or constant, we do not know, so for that, we need to go with radial equilibrium equation, okay.

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**Tutorial contd.**

**Solution (a)**

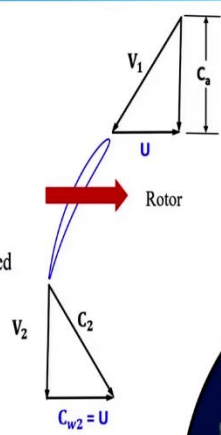
As the relative flow angle at rotor exit is zero,  
 $\beta_2 = 0$  (given)

As observed from the velocity triangle,  
 the relative exit velocity is purely axial  
 $V_w = 0$

The absolute swirl at the rotor exit is equal to rotor speed  
 $C_{w2} = U = \omega r$

As rotational speed is given, angular velocity  
 can be calculated as,  

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 14000}{60} = 1466.07 \text{ rad/s}$$



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So, let us start with. Here, in this case, if you look at the exit of my rotor, my velocity triangle, that's what is given to me. And suppose if I consider this is what is my rotor, we can plot our velocity triangle like this, okay. And at the exit, it says my  $\beta$ , that's what is equal to 0. So, you can say this is what is my  $V_2$ , this will be my  $C_{w2}$ , that's what is equal to peripheral speed  $U$ , okay. And this is what is my absolute velocity.

Now, you know, since by  $\beta_2$ , that's what is given 0 to me, I can say, my  $C_{w2}$ , that's what is equal to my peripheral speed. So, here if you look at, this is what is giving us a hint, how do we start with the calculation of different velocity components. So, very first calculation, that's what we can use by using what data is given to us.

So, it says my  $C_{w2}$ , that's what is equal to  $U$ , that's what is equal to  $\omega r$ . Now, here in this case, we can say my  $\omega$  we can write down as  $\frac{2\pi N}{60}$ . And based on that, if we will be calculating my radial speed, we can say our  $\omega$ , that's what is angular speed, that's what is coming as  $1466.07 \text{ rad/s}$ , okay.

$$C_{w2} = U = \omega r$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 14000}{60} = 1466.07 \text{ rad/s}$$

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**Tutorial contd.**

Using Radial Equilibrium Equation,

$$\frac{1}{2} \frac{dC_a^2}{dr} + \frac{C_w}{r} \frac{d}{dr} (rC_w) = 0$$

Substituting  $C_w = \omega r$

$$\frac{1}{2} \frac{dC_a^2}{dr} + \frac{\omega r}{r} \frac{d}{dr} (r^2 \omega) = 0$$

On simplifying, we will get,

$$C_a \frac{dC_a}{dr} = -2\omega^2 r$$

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Now, we are not having any information about how my axial velocity, that's what is varying. So, in order to take care of that, we know we are having our say radial equilibrium equation. So, if I am writing this radial equilibrium equation, so we will be having our formulation, that's what will be coming like this, okay.

Using Radial Equilibrium Equation,

$$\frac{1}{2} \frac{dC_a^2}{dr} + \frac{C_w}{r} \frac{d}{dr} (rC_w) = 0$$

Now, what we know in place of my  $C_w$ , I can write down that's what is given by  $\omega r$ , okay. So, let me replace this with  $\omega r$ .

Substituting  $C_w = \omega r$

$$\frac{1}{2} \frac{dC_a^2}{dr} + \frac{\omega r}{r} \frac{d}{dr} (r^2 \omega) = 0$$

So, if you are simplifying this equation, it says my axial velocity into  $dC_a$  by  $dr$ , that's what is equal to minus 2 omega square into  $r$ , okay.

On simplifying, we will get,

$$C_a \frac{dC_a}{dr} = -2\omega^2 r$$

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**Tutorial contd.**

$$C_a \frac{dC_a}{dr} = -2\omega^2 r$$

Integrate the radial equilibrium equation from hub to any arbitrary location,

$$\int_{C_{ah}}^{C_a} C_a dC_a = -2\omega^2 \int_{r_h}^r r dr$$

$$C_{ar}^2 - C_{ah}^2 = -2\omega^2 r^2 + 2\omega^2 r_h^2$$


$$C_{ar}^2 = C_{ah}^2 + 2\omega^2 r_h^2 - 2\omega^2 r^2$$

$$C_{ar}^2 = 124^2 + (2 \times 1466.07^2 \times 0.07^2) - 2 \times 1466.07^2 r^2$$

Variation of axial velocity at any location,

$$C_{ar} = \sqrt{36437.7 - 4298312r^2}$$

We know  
 $\omega = 1466.07 \text{ rad/s}$   
 $r_h = 0.07 \text{ m}$   
 $C_{ah} = 124 \text{ m/s}$



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Now, if this is what is our equation, we need to calculate our axial velocity. So, here, in this case, let us consider any arbitrary radius  $r$ . Since, we are having our information available at the hub. So, that is the reason I say, I will be putting my limit from say hub to any radius  $r$ , say this is what

is  $C_a$  into  $dC_a$ , that's what is equal to minus 2 omega square, this limit I am putting that's what is  $r_h$  to  $r$  into  $rdr$ .

$$\int_{C_{ah}}^{C_{ar}} C_a dC_a = -2\omega^2 \int_{r_h}^r r dr$$

If you are doing the integration part, and if you are putting our limits, it says my axial velocity at any radius, that's what is equal to my axial velocity at hub plus this formulation.

$$C_{ar}^2 - C_{ah}^2 = -2\omega^2 r^2 + 2\omega^2 r_h^2$$

$$C_{ar}^2 = C_{ah}^2 + 2\omega^2 r_h^2 - 2\omega^2 r^2$$

Now, the data that's what is available to us, what all we know, we know our  $\omega$ , that's what we have calculated it is  $1466.07 \text{ rad/s}$ . We also know our hub radius at the exit, that's what is  $0.07$  meter, we know our axial velocity at the hub is  $124 \text{ m/s}$ .

So, if I will be putting in this equation, it says my axial velocity at any radial location, that's what will be given by this formula.

$$C_{ar}^2 = 124^2 + (2 \times 1466.07^2 \times 0.07^2) - 2 \times 1466.07^2 r^2$$

*Variation of axial velocity at any location,*

$$C_{ar} = \sqrt{36437.7 - 4298312r^2}$$

So, now, you can understand what all will be the use of our radial equilibrium equation, okay. So, do not forget the formulation of radial equilibrium equation and its application.



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**Tutorial contd.**

Variation of axial velocity at any location,

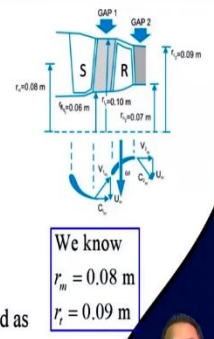
$$C_{ar} = \sqrt{36437.7 - 4298312r^2}$$

The axial velocity at mean and tip can be calculated as,

$$C_{am} = \sqrt{36437.7 - 4298312 \times 0.08^2} = 94.49 \text{ m/s}$$

$$C_{at} = \sqrt{36437.7 - 4298312 \times 0.09^2} = 40.27 \text{ m/s}$$

The mass flow through the compressor annulus can be estimated as

$$m = \rho A C_a = 2\pi\rho \int_{r_h}^{r_t} C_{ar} r dr$$


We know  
 $r_m = 0.08 \text{ m}$   
 $r_t = 0.09 \text{ m}$

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So, here if we go with, say this is what is my variation of axial velocity, that's what we are writing with, okay. Now, we are having different radiuses which are known to me; so, if we consider at hub, my radius is 0.07 meter, at midsection my radius is 0.08 meter, and at my tip, my radius is 0.09 meter. So, if we are putting that in this formula, it says my axial velocity at the midsection is 94.49 m/s, my axial velocity at the tip that's what is 40.27 m/s.

*The axial velocity at mean and tip can be calculated as,*

$$C_{am} = \sqrt{36437.7 - 4298312 \times 0.08^2} = 94.49 \text{ m/s}$$

$$C_{at} = \sqrt{36437.7 - 4298312 \times 0.09^2} = 40.27 \text{ m/s}$$

And, if you look at carefully, just remember the things what it is saying, if you look at the numbers at the hub, we are having maximum axial velocity and at the tip we are getting minimum axial velocity. So, it says, this is what is, you know, it is giving me some different kind of design feeling. You can understand, this is what it says like my design configuration for this blade, that's what is of different kind.

And if you recall, we are having axial velocity, that's what will be varying with different configurations, okay. So, this design may be of such kind, okay. So, this is how we can do our calculation for axial velocity at midsection, our axial velocity at the tip section, okay. Now, as we

have discussed, we are looking for mass flow rate to be calculated. Now, what we know? Mass flow rate that's what we can calculate by using our continuity equation.

And as we have discussed, we can write down that's what is

$$m = \rho AC_a = 2\pi\rho \int_{r_h}^{r_t} C_{ar} r dr$$

Now, here in this case, we are having variation of our axial velocity, okay. So, what is the meaning of that? We need to put our axial velocity variation in terms of radial variation or radius variation. So, we can write down density into area we are writing as  $2\pi r dr$ , okay, that's what we are integrating from hub to tip, okay.

$$m = \rho AC_a = 2\pi\rho \int_{r_h}^{r_t} C_{ar} r dr$$

And we know our axial velocity that is also varying with the radius. So, be careful, straightway, do not try to put this equation as say

$$\frac{\pi}{4} (d_{tip}^2 - d_{hub}^2)$$

Be careful, because here we are not having any information in sense of how my axial velocity that's what is varying. And once we have calculated, we realize, our axial velocity, that's what is varying with the radius. That means, I need to divide all these segments into small - small radius.

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**Tutorial contd.**

The mass flow through the compressor annulus can be estimated as

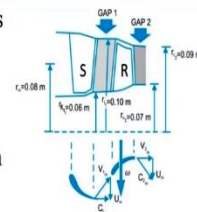
$$m = \rho AC_a = 2\pi\rho \int_{r_h}^{r_t} C_{ar} r dr$$

This can be numerically integrated for radius interval of 0.01 m using trapezoidal rule


$$m = 2\pi\rho \times \frac{\Delta r}{2} \times [C_{ah}r_h + 2C_{am}r_m + C_{at}r_t]$$

Thus

$$m = 2\pi \times 2.4 \times \frac{0.01}{2} \times [124 \times 0.07 + 2 \times 94.49 \times 0.08 + 40.27 \times 0.09]$$

$$m = 2.06 \text{ kg/s}$$


We know  
 $r_m = 0.08 \text{ m}$   
 $r_t = 0.09 \text{ m}$



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So, if we consider this is what is a case, you can write down my mass flow through the compressor annulus, that's what will be given by this formula.

$$m = \rho AC_a = 2\pi\rho \int_{r_h}^{r_t} C_{ar} r dr$$

Now, in order to solve such kind of equation, we need to go with some numerical techniques. And as the hint that's what is given, it says you need to go with using numerical technique, that's what is called trapezoidal rule, okay.

So, if you are writing the trapezoidal rule for that, so, integral part of this  $C_a r dr$ , that's what we can write down in the form of  $\frac{\Delta r}{2} \times [C_{ah}r_h + 2C_{am}r_m + C_{at}r_t]$ .

So, what all you are learning during your graduate studies or maybe for postgraduate studies, remember the concepts what we are using for.

$$m = 2\pi\rho \times \frac{\Delta r}{2} \times [C_{ah}r_h + 2C_{am}r_m + C_{at}r_t]$$

You can understand when we are having the variation of two parameters axial velocity and radius and that too if you are looking for the calculation of these absolute numbers we need to go with this numerical technique, okay. So, if you are considering that as a case, it says I need to select my

$\Delta r$ , suppose if I consider what numbers we are having, it says 0.07, 0.08 and 0.09. So, let me consider  $\Delta r$  to be 0.01. So, I am putting this  $\Delta r = 0.01$ , okay.

Now, my axial velocity at the hub is known to me, my radius at the hub is known to me, axial velocity at the midsection that's what is known to me, radius at the midsection that is also known to me, we have our axial velocity at the tip and say radius at the tip, those things are known to us. So, if I will be putting all these numbers, that's what is giving me, my mass flow rate as  $2.06 \text{ kg/s}$ , okay.

$$m = 2\pi \times 2.4 \times \frac{0.01}{2} \times [124 \times 0.07 + 2 \times 94.49 \times 0.08 + 40.27 \times 0.09]$$

$$m = 2.06 \text{ kg/s}$$

So, this is what you can say, based on what data that's what is available to us, just try to recall what all data that's what is available to me. Say, at the exit we are having our  $\beta$ , that's what is given, our axial velocity at some location that's what is given and my density at that location it is given. With these three data, we are able to estimate, you can say, this is what is approximate estimation, you can say roughly, that's what is say  $2.06 \text{ kg/s}$ , that's what is a mass flow rate through this passage, okay.

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**Tutorial contd.**

**Solution (b)**

$r_h = 0.08 \text{ m}$   
 $r_m = 0.06 \text{ m}$   
 $r_t = 0.10 \text{ m}$   
 $r_r = 0.07 \text{ m}$   
 $r_t = 0.09 \text{ m}$

**Rotor inlet gap**  
 $T_0 = \text{constant}$   
 $\alpha = 0 \text{ deg}$   
 $\rho = 2.25 \text{ kg/m}^3$   
 $T_{hub} = 511 \text{ K}$

**Rotor exit gap**  
 $T_0 = \text{constant}$   
 Relative flow angle  
 $\beta = 0$   
 $\rho_{hub} = 2.4 \text{ kg/m}^3$   
 $T_{hub} = 552 \text{ K}$   
 $C_{a,hub} = 124 \text{ m/s}$

Exit Relative total pressure at tip  
 Static pressure at tip ??, Critical velocity??  
 Velocity components and temperature  
 Do we know static pressure at other location at exit?  
 No but, we know static temperature and density at hub!! Any use?? Yes  
 Radial Equilibrium Equation

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Now, let us move with the second configuration, what they are asking for second configuration? We are looking for our total relative pressure at the exit of my rotor, okay. Now, you can understand if I am looking for relative total pressure, that's what is required different parameters known to me. What all parameters are required? We are looking for our static pressure because we know our static pressure and total pressure they are interrelated.

Now, the question is how they are interrelated, they are interrelated with my critical Mach number or critical velocity ratio, okay. Now, we know we need to calculate what will be my static pressure at the tip. So, here in this case, we are not known what will be the exit static pressure at the hub, we do not know what is our exit static pressure at the tip. So, very first thing is we need to calculate what will be our static pressure, okay.

We need to calculate our critical velocity ratio that means we need to have a calculation of the various velocity components at the exit, okay. So, let us see what all strategies we will be using. It says my exit total pressure at the tip that's what we can calculate based on whether we are having the information about the static pressure at the tip, do we know our critical velocity, these two things they are not known to me.

Now, the question is this critical velocity you can calculate based on calculation of your temperature and different velocity components. So, that's what can be done, okay. Means we need to play with this, we are looking for these parameters to be calculated. Next thing is do we know the static pressure at some other location in the exit? At this moment, it says no, but if you look at, we are given with two parameters, that's what is density and temperature.

So, you can say, using our perfect gas equation, we can calculate what will be our static pressure at the hub. Yeah, so once we are having that static pressure at the hub, we need to calculate or we can calculate the other static pressure, how! The question will be coming say how to calculate that part! So, we have a fundamental radial equilibrium equation. So, based on that, we can calculate what will be my static pressure at the tip.

And based on all my velocity components and temperature calculation, we can calculate what is our critical velocity and that is how we need to proceed further, okay. So, whenever this information that's what is given to you, just try to look at what all information are known to me and how do I correlate these things. Many times, the formulas that's what is known to everyone

but the thing is how to apply and where to apply that's what is always a question mark. So, make it this kind of same chart, that's what will give you hint how do we solve the numerical, this will make our method for solution to be very easy, okay.

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**Tutorial contd.**

**Solution (b)**

Let's first target.... Critical velocity

Absolute velocity at rotor exit at hub is given as

$$C_{2h} = \sqrt{C_{2ah}^2 + C_{2wh}^2}$$

$$= \sqrt{124^2 + (1466.07 \times 0.07)^2}$$

$$= 160.96 \text{ m/s}$$

We know

$\rho = 2.4 \text{ kg/m}^3$

$C_{ah} = 124 \text{ m/s}$

$C_{am} = 94.49 \text{ m/s}$

$C_{ar} = 40.27 \text{ m/s}$

$\omega = 1466.07 \text{ rad/s}$

$r_h = 0.07 \text{ m}$

$r_m = 0.08 \text{ m}$

$r_r = 0.09 \text{ m}$

$T_{2h} = 552 \text{ K}$

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So, let us see, very first we are targeting our critical velocity. So, this is what is my velocity triangle, I can say, that's what is at my hub, okay. So, this is what I can say this is what it is at the hub, this is what is at my say inlet and this is what is at my exit. Now, from our velocity triangle, we are looking for critical velocity calculation. That means, we are looking for relative velocity component.

Now, in order to calculate our say relative velocity component, we must know other velocity components. So, let us start with this, it says if you are considering this as my triangle, we can write down my absolute velocity that's what is nothing but it is my axial velocity. And this is what is my whirl velocity component.

Now, be careful! This whirl component what we are calculating, that's what is at the hub. So, specifically I am writing here is hub, so it says at hub we know our axial velocity that's what is  $124 \text{ m/s}$ , we know what is our  $\omega$  and this is what is my 'r'. So, at hub we know our radius as  $0.07$ , okay. So, that's what will be giving me my absolute velocity as  $160.96 \text{ m/s}$ .

$$\begin{aligned}
C_{2h} &= \sqrt{C_{2ah}^2 + C_{2wh}^2} \\
&= \sqrt{124^2 + (1466.07 \times 0.07)^2} \\
&= 160.96 \text{ m/s}
\end{aligned}$$

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**Tutorial contd.**

Now, total temperature at rotor exit (at hub) can be calculated as

$$\begin{aligned}
T_{02h} &= T_{2h} + \frac{C_{2h}^2}{2c_p} \\
&= 552 + \frac{160.96^2}{2 \times 1.005 \times 10^3}
\end{aligned}$$


Hence,  $T_{02h} = 564.9 \text{ K}$

Relative total temperature at tip is given by

$$\begin{aligned}
T_{02r\_tip} &= T_{2r} + \frac{V_{2rel}^2}{2c_p} \\
&= T_{02r} - \frac{C_{2r}^2}{2c_p} + \frac{V_{2rel}^2}{2c_p}
\end{aligned}$$

We know

- $\rho = 2.4 \text{ kg/m}^3$
- $C_{ah} = 124 \text{ m/s}$
- $C_{am} = 94.49 \text{ m/s}$
- $C_{ar} = 40.27 \text{ m/s}$
- $C_{2h} = 160.96 \text{ m/s}$
- $r_h = 0.07 \text{ m}$
- $r_m = 0.08 \text{ m}$
- $r_r = 0.09 \text{ m}$
- $T_{2h} = 552 \text{ K}$



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Now, what we are looking for, we need to have temperature to be known in order to calculate our critical velocity ratio, okay. So, at hub, we can straightway write down say,  $T_{02}$  at the hub, that's what is given by my static pressure plus  $C_2$  square by  $2C_p$ , okay.

$$\begin{aligned}
T_{02rh} &= T_{2h} + \frac{C_{2h}^2}{2C_p} \\
&= 552 + \frac{160.96^2}{2 \times 1.005 \times 10^3}
\end{aligned}$$

$$\text{Hence, } T_{02rh} = 564.9 \text{ K}$$

So, at hub my static pressure it is known to me. We have calculated our absolute velocity and this is what is known to me. So, that's what will be giving me what is my total temperature at the hub, it says this is what is 564.9 K, okay.

Now, what exactly we are looking for, that's what is my relative critical Mach number, and as the reason we must have relative total temperature; so, that's what we can write down here. This is

what is given by this is my static temperature at the tip and this is nothing but this is what is my relative velocity at the exit of my rotor by  $2C_p$ .

$$T_{02r\_tip} = T_{2t} + \frac{V_{2rel}^2}{2C_p}$$

You can understand my static temperature at the tip that's what can be rewritten as

$$T_{02r\_tip} = T_{02t} - \frac{C_{2t}^2}{2C_p} + \frac{V_{2rel}^2}{2C_p}$$

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**Tutorial contd.**

$$T_{02r\_tip} = T_{02t} - \frac{C_{2t}^2}{2c_p} + \frac{V_{2rel}^2}{2c_p}$$

Now,  $V_{2t} = C_{a2t} = 40.24$  m/s (Velocity triangle)

Also,  $C_{2t} = \sqrt{V_{2t}^2 + \omega r_t^2}$

$$= \sqrt{40.24^2 + (1466.07 \times 0.09)^2}$$

We know  
 $\omega = 1466.07$  rad/s  
 $r_t = 0.09$  m

$C_{2t} = 137.94$  m/s \*

Putting these values, we get

$$T_{02r\_tip} = 564.90 - \frac{137.94^2}{2 \times 1.005 \times 10^3} + \frac{40.24^2}{2 \times 1.005 \times 10^3}$$

$$T_{02r\_tip} = 556.2$$
 K

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Now, at tip region, we have calculated our total temperature, what we know, we have our axial velocity, that's what is known to me. So,  $V_2$  at the tip is known to me, okay. So, I can calculate what will be my  $C_{2t}$ , that is nothing but my absolute velocity at the tip. So, what is given to us say, you know, our relative angle  $\beta$ , that's what is equal to 0 at the exit, okay. If that's what is your case, at tip also, I can help this right-angle triangle.

So, we can write down our  $C_2$  at the tip, that's what is given by

$$\text{Now, } V_{2t} = C_{a2t} = 40.24 \text{ m/s (Velocity triangle)}$$

$$\text{Also, } C_{2t} = \sqrt{V_{2t}^2 + \omega r_t^2}$$



$$= \sqrt{40.24^2 + (1466.07 \times 0.09)^2}$$

$$C_{2t} = 137.94 \text{ m/s}$$

Remember here, this is what is my radius at the tip. So, that's what we are putting here and this is giving me my  $C_2$ , it is  $137.94 \text{ m/s}$ , okay. Now, if you are putting these numbers, that's what is giving me my total relative temperature at the tip as  $556.2 \text{ K}$ , okay.

*Putting these values, we get*

$$T_{02r_{tip}} = 564.9 - \frac{137.94^2}{2 \times 1.005 \times 10^3} + \frac{40.24^2}{2 \times 1.005 \times 10^3}$$

$$T_{02r_{tip}} = 556.2 \text{ K}$$

So, be careful at what location which parameter you are calculating. So, make a habit to write down at tip region, what numbers you are putting; at hub, what numbers you are putting, otherwise it will get hotchpotch and maybe it will be difficult to understand. Make a habit of making these velocity triangles; so, that it will make life easy, okay.

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**Tutorial contd.**

Relative critical tip velocity is given by

$$V_{2cr\_tip} = \sqrt{\frac{2\gamma RT_{02r\_tip}}{\gamma + 1}}$$

$$= \sqrt{\frac{2 \times 1.4 \times 287 \times 556.2}{1.4 + 1}}$$

$$V_{2cr\_tip} = 431.26 \text{ m/s}$$


Now we verify the Critical tip Mach No.


$$M_{2t\_cr} = \frac{C_{a2t}}{V_{2cr\_tip}}$$

$$= \frac{40.24}{431.26}$$

$$M_{2t\_cr} = 0.094$$

We know  
 $T_{02r\_tip} = 556.2 \text{ K}$   
 $C_{a2t} = 40.24 \text{ m/s}$





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Now, this is what is known to us. So, we can calculate what will be our relative velocity or relative critical tip velocity, that's what we know that is given by  $2 \gamma RT_{02}$  at the tip, that's what is a relative temperature by  $\gamma + 1$ , that's what we are calculating as say 431.26 m/s, okay.

$$V_{2crt\_tip} = \sqrt{\frac{2\gamma RT_{02r\_tip}}{\gamma + 1}}$$

$$= \sqrt{\frac{2 \times 1.4 \times 287 \times 556.2}{1.4 + 1}}$$

$$V_{2crt\_tip} = 431.26 \text{ m/s}$$

Now, we know what is our say critical Mach number, this critical Mach number we know it is nothing but this is what is say my C or we can say this is what is my  $V_2$ , but since  $\beta$  is given 0, we are writing this as say  $C_{a2}$  divided by  $V_{2crt}$  and this is what is giving my relative Mach number - relative critical Mach number at the tip as 0.094. So, now you can understand in order to do our calculation for static pressure, we are known with say this Mach number.

$$M_{2t\_cr} = \frac{C_{a2t}}{V_{2crt\_tip}}$$

$$= \frac{40.24}{431.26}$$

$$M_{2t\_cr} = 0.094$$

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Tutorial contd.

```
graph TD; A[Exit Relative total pressure at tip] --> B[Static pressure at tip ??, Critical velocity?]; B --> C[Velocity components and temperature]; C --> D[Do we know static pressure at other location at exit?]; D --> E[No but, we know static temperature and density at hub!! Any use?? Yes]; E --> F[Radial Equilibrium Equation];
```

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So, let us move with what all we are looking for, it says, we are looking for exit relative total pressure at the tip. And for that we are looking for say static temperature and critical velocity. And in order to do the calculation of this critical velocity, all these steps what we have followed. Now, we are looking for say static pressure, that's what is our target.

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Tutorial contd.

We know, Radial Equilibrium Equation

$$\frac{dp}{dr} = \frac{\rho C_w^2}{r}$$

Since,  $C_w = \omega r$  (as given)

$$\frac{dp}{dr} = \rho \omega^2 r$$

This can be integrated to give

$$p_2 = \frac{1}{2} \omega^2 r^2 + C$$

Now, we know pressure and temperature at exit near hub

$$p_h = \rho_h R T_h = 2.4 \times 287 \times 552$$
$$p_h = 380217.6 \text{ Pa}$$

We know  
 $T_{2,h} = 552 \text{ K}$   
 $\rho_{2,h} = 2.4 \text{ kg/m}^3$

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So, in order to do that calculation, let us see, how do we proceed with. What we know? My static pressure, that's what is correlated with my whirl velocity component by our radial equilibrium equation. So, we say  $\frac{dp}{dr}$ , that's what is given by

*Radial Equilibrium Equation,*

$$\frac{dp}{dr} = \frac{\rho C_w^2}{r}$$

Now, the  $C_w$ , that's what we are writing as say  $\omega r$ .

*Since,  $C_w = \omega r$  (as given)*

So, if you are putting this in the formulation; so,  $\frac{dp}{dr}$ , that's what will be coming as say

$$\frac{dp}{dr} = \rho \omega^2 r$$

Now, in order to calculate my pressure, we can integrate this equation. So, on integration, that's what is say my pressure is given by

$$p_2 = \frac{1}{2} \omega^2 r^2 + C$$

This is what is say integral constant. And as we have discussed, our static density and static temperature at the hub, that's what is known to us. So, by using our perfect gas equation, say

$$p_h = \rho_h R T_h$$

that's what will give me what is my static pressure at the hub.

$$p_h = 2.4 \times 287 \times 552 = 380217.6 \text{ Pa}$$

So, this is what will be my static pressure at the hub, okay. Be careful, everything we are writing at the hub, okay. Now, this is what is known to me.

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**Tutorial contd.**

From REE we derived..  $p_2 = \frac{1}{2}\omega^2 r^2 + C$

Then at the hub we can write  $C = p_h - \frac{1}{2}\omega^2 r_h^2$


$$= 380217.6 - \frac{1}{2} \times 1466.07^2 \times 0.07^2$$
$$C = 367580.56 \text{ N/mm}^2$$

The pressure equation thus becomes  $p_2 = \frac{1}{2}\omega^2 r^2 + 367580.56$


The pressure at tip is thus given by

$$p_{2t} = \frac{1}{2} \times 1466.07^2 \times 0.09^2 + 367580.56$$
$$p_{2t} = 388469.79 \text{ Pa}$$

We know  
 $\omega = 1466.07 \text{ rad/s}$   
 $p_h = 380217.6 \text{ Pa}$   
 $r_h = 0.07 \text{ m}$   
 $r_t = 0.09 \text{ m}$



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What we know? We are having our say radial equilibrium equation and from there, we have calculated our static pressure, that's what is given

$$p_2 = \frac{1}{2}\omega^2 r^2 + C$$

Since, our static pressure at the hub is known to us, my radius is known to us, my say angular speed is known to us, we can calculate what will be our constant C.

So, if we are putting this number, this constant, that's what is coming as say 367580.56.

$$C = p_h - \frac{1}{2}\omega^2 r_h^2$$
$$= 380217.6 - \frac{1}{2} \times 1466.07^2 \times 0.07^2$$

$$C = 367580.56 \text{ N/m}^2$$

Be careful, do not forget to write down the unit of this constant. Now, since this is what is a constant known to us, so, we can write down now, our static pressure at the outlet, that's what can be written as

$$p_2 = \frac{1}{2}\omega^2 r^2 + 367580.56$$

Now, this is what is giving us a hint, if I am looking for my static pressure calculation at the tip, I know what is my angular speed, I know what is my radius at the tip. So, we can write down here it is 1466.07 square and my tip radius is 0.09 square, okay.

*The pressure at tip is thus given by,*

$$p_{2t} = \frac{1}{2} \times 1466.07^2 \times 0.09^2 + 367580.56$$

$$p_{2t} = 388469.79 \text{ Pa}$$

If this is what is known, we can calculate our static pressure, that's what is say 388.46 kPa, okay. Now, this is what is giving me our static temperature calculation at the tip, but what exactly we are asked for, we are looking for say our relative total temperature at the tip.

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**Tutorial contd.**

The total relative tip pressure can be obtained by isentropic relationship

$$P_{02rel\_tip} = \frac{P_{2t}}{\left(1 + \frac{\gamma-1}{\gamma+1} \left(\frac{C_{a2t}}{V_{2crit\_tip}}\right)^2\right)^{\frac{\gamma}{\gamma-1}}}$$


$$= \frac{388469.79}{\left(1 + \frac{1.4-1}{1.4+1} \times 0.093^2\right)^{\frac{1.4}{1.4-1}}}$$

$$P_{02rel\_tip} = 390.44 \text{ kPa}$$

We know

$p_{2t} = 388469.79 \text{ Pa}$

$\left(\frac{C_{a2t}}{V_{2crit\_tip}}\right) = 0.093$



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So, let us go with that part. If you are writing, this is what is a formulation what we have already derived in our last lecture. So, straightaway we will put that equation, it says this is what is my  $P_2$ , that's what we have calculated divided by 1 plus gamma minus 1 divided by gamma plus 1, this is nothing but my say critical Mach number square to the power gamma over gamma minus 1. These all numbers they are known to us.

So, we can calculate what will be my total pressure at the tip, okay, and that's what is coming 390.44 kPa.

$$\begin{aligned}
 p_{02rel\_tip} &= \frac{p_{2t}}{\left(1 + \frac{\gamma - 1}{\gamma + 1} \left(\frac{C_{a2t}}{V_{2crt\_tip}}\right)^2\right)^{\frac{\gamma}{\gamma - 1}}} \\
 &= \frac{388469.79}{\left(1 + \frac{1.4 - 1}{1.4 + 1} \times 0.093^2\right)^{\frac{1.4}{1.4 - 1}}} \\
 p_{02rel\_tip} &= 390.44 \text{ kPa}
 \end{aligned}$$

So, this is what is, you know, one way as I discussed, we are looking for say calculating different parameters for the known geometry, it may be possible that you may be given with some information at particular location and you need to check with what all need to be the parameters at that particular location, okay.

So, this is what is we are moving towards, you know, data analysis kind of thing, okay. What exactly say for flying engines, these days, engine companies they are collecting all their data, this data they are in terms of maybe pressure, maybe temperature, say Mach number, all those parameters they are acquiring. And based on that they are assessing the performance of that engine.

And that's what is giving the health of that engine. So, you know, health monitoring, that's what people they are doing by using this data. So, by using that data, maybe you can able to calculate what all need to be my total pressure ratio or my overall pressure ratio, what need to be my temperature; if that temperature is shoot up, that's what is indicating something is wrong, okay. So, that is how all we need to learn with what all we are understanding from our basic learning from fundamental courses and this is what is coming as an applied part. So, thank you very much for your attention! We will be discussing one more numerical in the next session. Thank you.