Aerodynamic Design of Axial Flow Compressors & Fans
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Lecture 30
Selection of Design Parameters (Contd.)

Hello, and welcome to lecture 30th. We are discussing about the selection of various design parameters.

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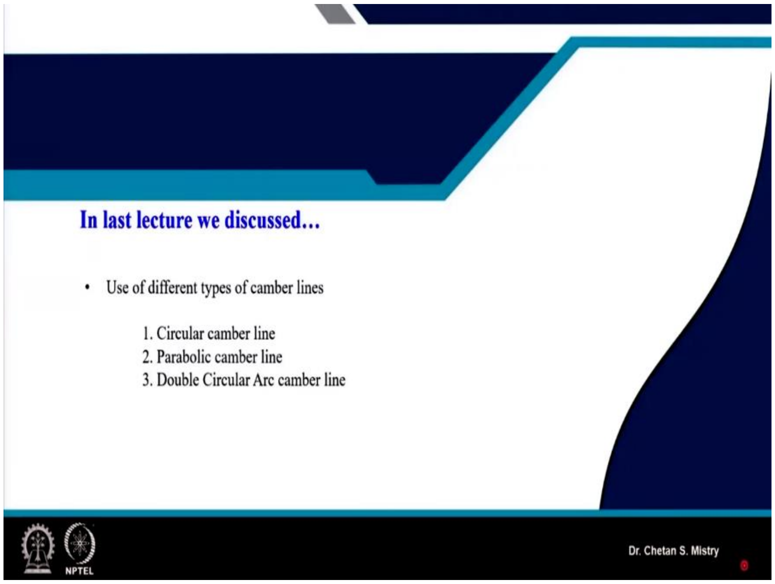

In last lecture, we were discussing about different types of camber lines, which we are using for say our development of airfoil, that's what we will be using for making up our blades. These camber lines, we can say, they are mainly circular arc camber line, parabolic camber line, double circular arc camber line. And if you recall, we have discussed the special kinds of camber lines and their applications.

So, this circular arc camber line, that's what is say conventional kind of camber line that's what we are using for making of initial rotor or stator blades. Later on, as per our requirement, as per our expectation, we will be modifying these camber lines; and, we have shown our own example, what modifications we have done initially with use of circular camber line and later on we have modified with double circular arc camber line.

Then we have seen, say…parabolic arc camber line, that's what is giving the flexibility for making our blade loading to be say, fore-loaded or aft-loaded, we can also go with mid-loaded kind of configuration. There we are having some limitations in sense of location selection and the formation of my shape. When we are talking about double circular arc, that's what is giving us more flexibility in sense at what location we are looking for turning of my blade.

So, we have our β at the junction function, we have our 'x' our percentage chord at the junction point from where we can modify the shape of our camber line. So, when we are modifying the shape of our camber line accordingly, my thickness distribution for airfoil that also will be changing. And we have seen as per our design, we are looking for say having different kinds of degree of reactions.

And even we have discussed earlier, my degree of reaction, that's what is varying all the way from my hub to shroud. And under that configuration, it may be possible that you need to have different kinds of camber lines that need to be used for making of your blade. Now, let us move to the next step.

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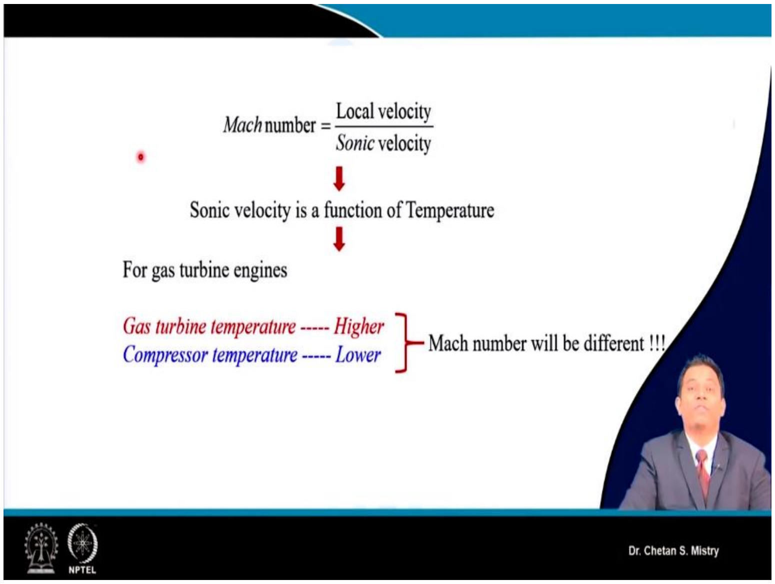

So, conventionally what we know from our basics of gas dynamics, that's what we define our Mach number. This Mach number, we are defining as a local velocity to the sonic velocity. Now, when we say local velocity, that's what is a velocity at particular location, but when we say sonic velocity, that's what is a function of my temperature. This is what is very important.

Suppose, if we are applying this logic, that's what is for say gas turbine engine; if we consider, we are having hot section, we are having cold section; when I say hot section, where we are having higher temperature. So, if we consider say gas turbines, they are on say hot section side, when we are talking about the compressor, few initial stages that's what is on colder side, where we are having this temperature to be lower.

Now, what will happen? The logic, what we say in sense of my Mach number, if I say velocity divided by $\sqrt{\gamma RT}$, then it says for my turbine, since my temperature is higher, my Mach number will be lower. For compressor, if you consider, at the entry maybe we will be having temperature to be low. If our temperature that's what is lower, it says my Mach number is higher.

That means, it says my flow to be transonic or supersonic. For turbine, it says my flow is say subsonic. Now, what we realize? These turbines, if we consider, mainly that's what is acting like a nozzle; so, the flow that's what is coming out from the combustion chamber, that's what will be passing through the nozzles or nozzle vanes, that's what will be accelerating the flow.

And that accelerated flow, that's what will be striking on my rotor. And based on that rotation, we are generating our power. Now, what happens? We are having the acceleration of flow, you can say that's what is giving say choking condition, thermal chocking condition, where my flow is accelerating flow that means my local velocity at that point is higher.

But based on my this definition, it says my flow is subsonic flow. So, that's what is a discrepancy. So, we need to address this issue, when we are talking about say design of our say…axial flow compressors as well as when we are designing our axial flow turbines. How do we address this issue?

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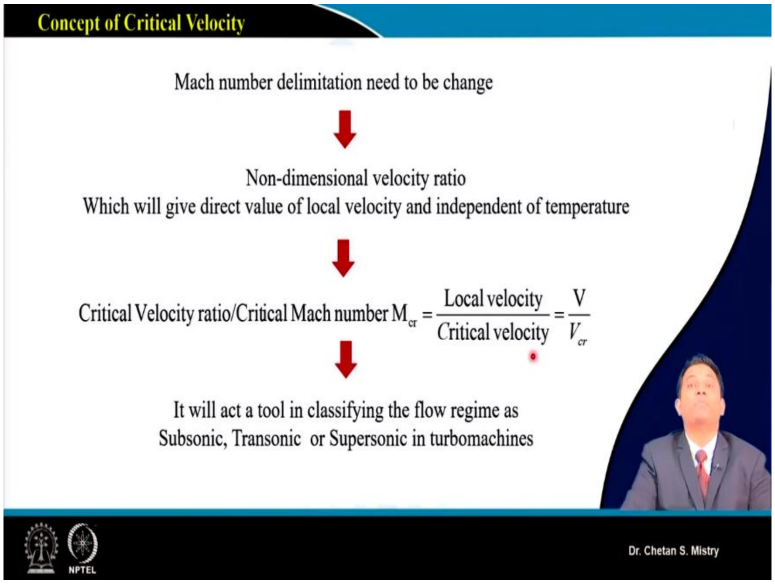

It says, this Mach number, that's what needs to be considered such that it should be different from what conventional definition we are having. And, you know, in order to address this issue, we are introducing new parameter, that's what is called non-dimensional velocity ratio. And if we say non-dimensional velocity ratio, that need to be independent of the temperature, local temperature, okay.

So, what it says? Let me introduce new parameter, that's what we say critical velocity ratio or critical Mach number. So, this critical Mach number, that's what we are defining as

$$Critical\ Velocity\ ratio\ or\ Critical\ Mach\ number,$$
$$M_{cr} = \frac{Local\ Velocity}{Critical\ Velocity} = \frac{V}{V_{cr}}$$

Now, you might have studied in your fundamentals of aerodynamics or in gas dynamics or advanced gas dynamics, there are also, we have discussed about this critical Mach number.

Now, we will see what exactly is the use of this critical Mach number. So, if you are using this kind of velocity ratio, then it is possible that we are able to understand whether my flow is subsonic, it is…is it transonic, or the supersonic in my turbo machines. And that's what is very important.

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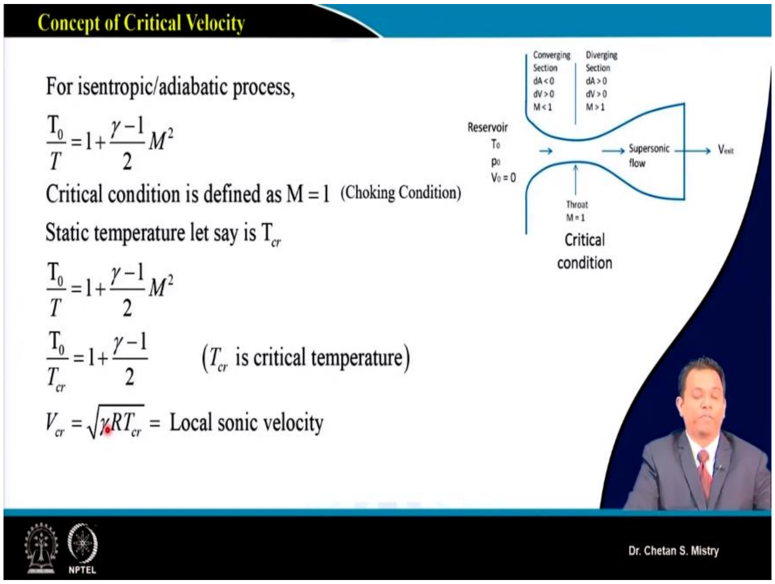

So, let us see, how do we understand that part. So, we are now discussing about the concept of critical velocity ratio. Suppose, if you consider, this is what is my say convergent divergent nozzle. And in convergent divergent nozzle, we have realized my flow acceleration, that's what will be happening from initial subsonic flow to say somewhere or say some location where we will be having maximum mass flow condition, that's what we will be achieving.

And that condition we are defining as a thermal chocking condition. And that thermal chocking condition, that's what is happening at $M = 1$. Later on, we are having diverging passage, that's what will be further increasing our Mach number from say $M = 1$ to say $M = some\ number$, that's what is say my supersonic flow. So, here, our reference, that's what is $M = 1$ condition, that's what we say as a thermal choking condition.

So, in order to define in order to derive the relation for critical velocity ratio, we will be considering $M = 1$ condition as our reference condition. So, here, what we know, if we consider the flow through our axial flow compressor, we are assuming our flow to be say isentropic process. Or maybe we can say it is more or less adiabatic kind of process.

If that's what is your case, you can write down the $\frac{T_0}{T}$, that's what is equal to $1 + \frac{\gamma-1}{2} M^2$. Now, as we have discussed, let us consider $M = 1$ condition. So, here, I am putting $M = 1$, and based on that what temperature we are getting, that temperature I am writing as say T_{cr}. So, here in this equation, if I will be putting my $M = 1$, it says $\frac{T_0}{T_{cr}}$, that's what is equal to $1 + \frac{\gamma-1}{2}$, okay.

So, this T_{cr}, this is nothing but my critical temperature. Now, based on our definition, we can say, my local sonic velocity, that's what will be given by $\sqrt{\gamma R T_{cr}}$. So, you know, this is what we say, as one of our relation.

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$
$$\frac{T_0}{T_{cr}} = 1 + \frac{\gamma - 1}{2} \ (T_{cr}\ is\ critical\ temperature)$$

$$V_{cr} = \sqrt{\gamma R T_{cr}} = Local\ sonic\ velocity$$

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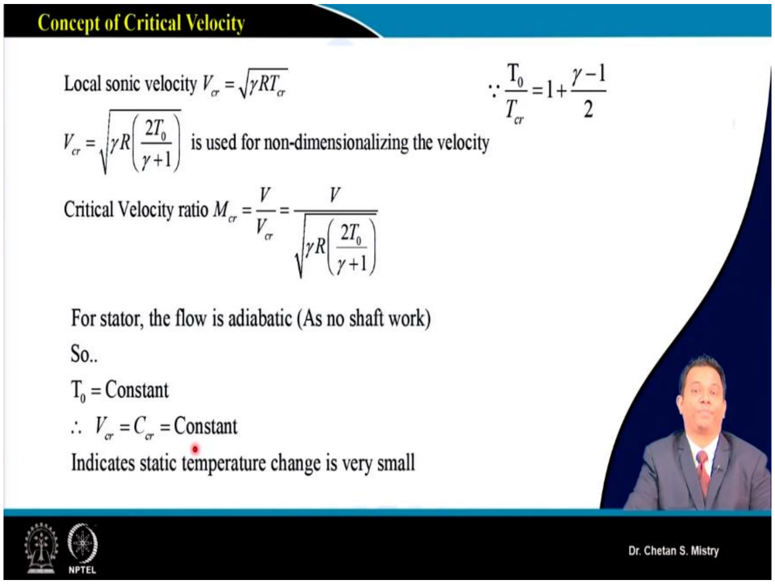

Now, let us move what we are defining in sense of our velocity ratio. So, what it says? If I am writing this, so this T_{cr}, that's what I can write down based on this relation, if I am putting my $M = 1$, that's what is giving me my T_{cr}, that's in the form of T_0. This T_0 is nothing but that's what is my stagnation temperature. So, you know, like, if we will be putting this relation that says my V_{cr}, that's what is given by $\sqrt{\gamma R\left(\frac{2T_0}{\gamma+1}\right)}$.

$$V_{cr} = \sqrt{\gamma R\left(\frac{2T_0}{\gamma+1}\right)}$$

Now, what we have defined? What we have introduced? That's what is my critical velocity ratio, or critical Mach number, so if I am putting that, it says this is what is my velocity divided by say this is what is my critical velocity. So, this is what will be helping us in order to calculate our Mach number at particular location, okay.

$$Critical\ Velocity\ Ratio\ M_{cr} = \frac{V}{V_{cr}} = \frac{V}{\sqrt{\gamma R\left(\frac{2T_0}{\gamma+1}\right)}}$$

So, you can understand, when we are saying local Mach number, that's what is different thing. Now, what Mach number we have defined earlier, that's what is based on my local sonic velocity, that's what is different. And this equation, that's what is different. Now, if we consider, suppose say, we are taking the case of our stator, for stator, we know there is no shaft work. That means if you consider my T_0, that's what will be coming as a constant, okay.

So, if you are putting my T_0, that's what is equal to constant. That's what it says my $V_{cr}\ or\ C_{cr}$, that's what is constant. So, it is indicating my static pressure change is very small, when we are talking for stator. So, we will be having small change of static pressure, that's what is happening in my stator.

$$T_0 = Constant$$
$$\therefore V_{cr} = C_{cr} = Constant$$

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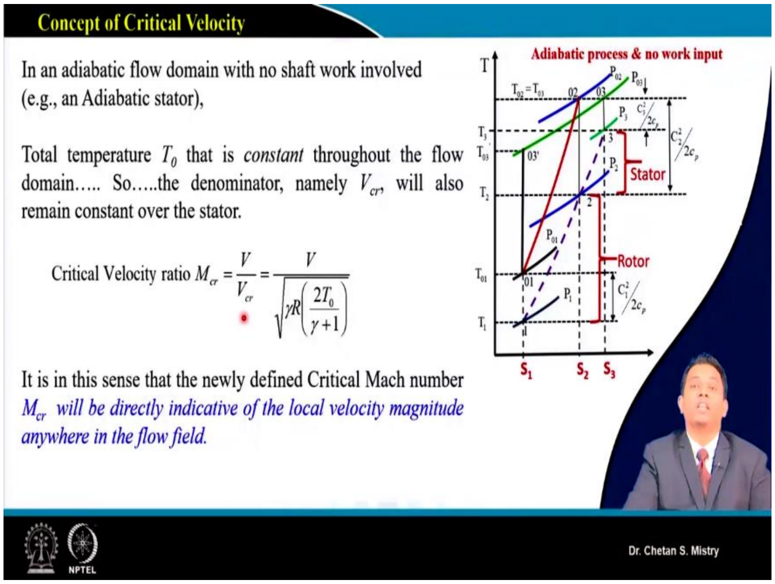

Now, let us move ahead. So, this is what we have learned for our T-S diagram. And we have realized, if we consider, we are more interested in our total property, and we have realized we are considering my T_{02} and T_{03}, that's what is representing my stator. We have assumed that process to be adiabatic process, and there is no work input.

And we realize because of presence of friction, my total pressure that's what is going to reduce and that's what is representing P_{02} and P_{03}. Now, this is what will be helpful for us in order to calculate our critical Mach number. Now, at particular location if I know what is my T_0, that's what will be helping us in order to calculating say our critical Mach number. So, this critical Mach number, that is nothing but it is indicative of local velocity magnitude anywhere in the flow field. So, now onwards rather using M, we will be using M_{cr}, okay, for all configuration. So, people those who are working in area of gas turbines, they people, they are always calculating this Mach number based on critical value not based on local temperature value.

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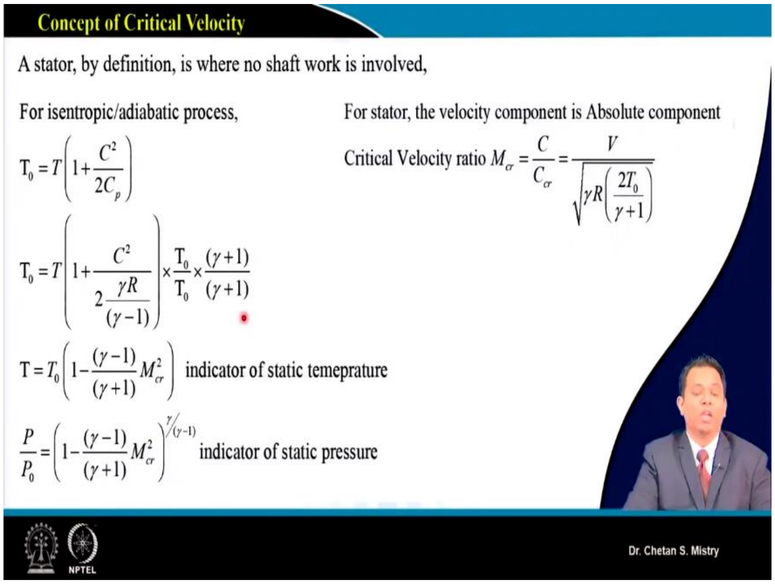

Now, here if we consider, say we are having our stator; so, if we want to calculate that part, we can say for my stator, my process, that's what we are assuming as adiabatic process. So, if I am writing my total temperature, so, based on our total enthalpy definition, we can say $h_0 = h + \frac{C_1^2}{2}$. We can write down in sense of T_0. So, it says

$$T_0 = T\left(1 + \frac{C^2}{2C_p}\right).$$

Now, the C_p, I can replace by $\frac{\gamma R}{\gamma - 1}$. And let me multiply and divide by T_0 and $\gamma + 1$.

$$T_0 = T\left(1 + \frac{C^2}{2\frac{(\gamma R)}{\gamma - 1}}\right) \times \frac{T_0}{T_0} \times \frac{(\gamma + 1)}{(\gamma + 1)}$$

What I know is this is what is my critical Mach number; so, in order to modify my equation, I can say this equation, that's what is been given in this form. It says my T, that's what is given by

$$T = T_0\left(1 - \frac{(\gamma - 1)}{(\gamma + 1)}M_{cr}^2\right) \ (indicator\ of\ static\ temperature)$$

So, now what all temperature we are calculating, all static temperature we are calculating, that's what will be based on my critical Mach number. Same way, local pressure or static pressure, total pressure, these all parameters we will be calculating based on critical Mach number, okay.

$$\frac{P}{P_0} = \left(1 - \frac{(\gamma - 1)}{(\gamma + 1)} M_{cr}^2\right)^{\frac{\gamma}{\gamma - 1}} \ (indicator\ of\ static\ pressure)$$

So, do not get confused, do not use isentropic relation for the calculation of Mach number, okay. Be careful! These relations and those relations for your isotropic flow that's what is different. Now, we are modifying our equation in sense of critical Mach number.

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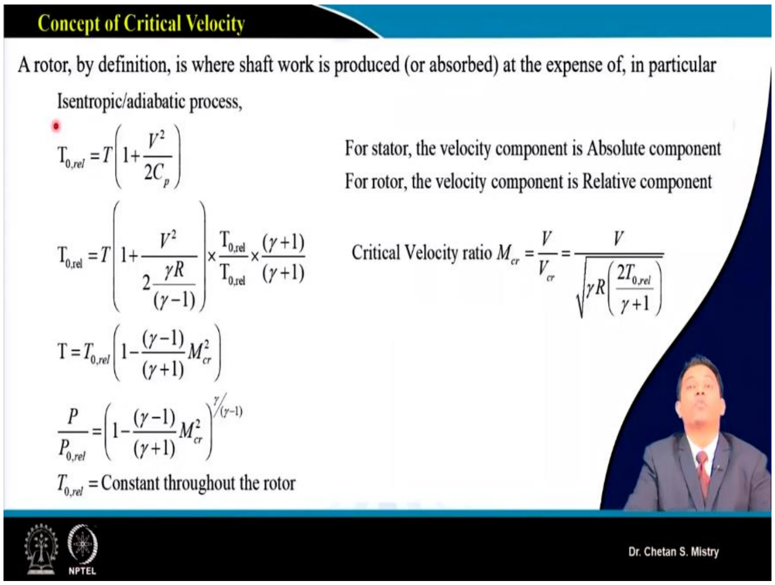

Now, if you are talking about the rotor, what all we know? For say our rotor, that's what is generating the shaft work, if we are talking about say turbine. This is what is say consuming the power, when we are talking about the compressor, so for that part, we can write down say we have discussed in place of using total property, we are using relative property. So, for rotor, we are more interested in relative properties.

So, we can say, that's what is represented by $T_{0,rel}$, okay. And, you know, in place of our absolute velocity, here you will be having relative velocity component, okay. So, here if I will be replacing that, it says my C_p, I can again write down as $\frac{\gamma R}{\gamma - 1}$, we can say, I am multiplying and dividing by $T_{0,rel}$ and $\gamma + 1$.

And if we are putting this equation, that's what is giving me how my static temperature and how my pressure, that's what is varying within the flow passage, okay. So, this is what is very important, it says my $T_{0,rel}$, that's what is remains constant throughout my rotor. Same way, T_0 we have considered, that's what is remains constant for my stator, okay. Now, you know, we are looking for say this kind of calculation at particular location in our rotor and stator or in the stage.

$$T_{0,rel} = T\left(1 + \frac{V^2}{2C_p}\right)$$

$$T_{0,rel} = T\left(1 + \frac{V^2}{2\dfrac{(\gamma R)}{\gamma - 1}}\right) \times \frac{T_{0,rel}}{T_{0,rel}} \times \frac{(\gamma + 1)}{(\gamma + 1)}$$

$$T = T_{0,rel}\left(1 - \frac{(\gamma - 1)}{(\gamma + 1)} M_{cr}^2\right)$$

$$\frac{P}{P_{0,rel}} = \left(1 - \frac{(\gamma - 1)}{(\gamma + 1)} M_{cr}^2\right)^{\frac{\gamma}{\gamma - 1}}$$

Now, the question will come, sir, all of sudden why we are started discussing about this Mach number, okay. So, what design process at this moment we are discussing mainly design process it is been defined in a two way; one, we can say as sizing problem, okay. So, when we say it is a sizing problem that means, you are given with some input parameters that means, you are given with your entry condition, you are given with say rotational speed, you are given with what pressure rise, that's what you are expecting from.

And for that, we will be doing our design for rotor and stator with a whole lot of process what all till now what we have discussed with. So, based on what all we are expecting, we are doing our design, that's what is defined as a sizing problem. So, basically, we are sizing our compressor in order to achieve expected pressure rise.

Suppose, we are not getting, we are doing all kinds of modifications in sense of selection of my pressure ratio, in sense of modification of my radius ratio, in sense of modification of my aspect ratio, my radius ratio; so many parameter that's what will be coming into the picture, okay. And once you have done all this calculation, based on your computational study, you will be coming up with verification, it says yes, this is what we are expected in sense of my pressure rise, okay.

So, this is what is one of the configuration. Based on that maybe we will be developing our rotor and stator configuration, we will be doing experimentation. Now, it may possible that you will be having this kind of configuration. Later on, you will be given some tasks, that's what is different task, okay. It says you modify or say resize this compressor.

So, if you recall, when I was discussing about the flow track design, that time we were discussing as per my requirement of smooth shape of flow track, maybe I need to go with the redesign of the stage, maybe many stages or maybe all stages we need to go for redesigning, okay. So, this is what all we are discussing, that's what is defined as sizing problem.

Now, suppose if you consider, say you are a new joinee to the company, or maybe you are already having something with you, suppose the already existing engine is with you, okay. Now, for that engine all specifications, all dimensions, that's what is known to you; you can say, suppose you are having your engine, so you know what is my radius ratio, you know what is my aspect ratio, you know what is my chord; all those parameters are known to you.

Now, you will be asked to check what will be the pressure ratio or what is the pressure ratio for particular stage or you will be asked to check with what will be the pressure ratio of all stages. Now, this is what is a special kind of problem and this is what is defined as a rating problem. So, as I said we are having design problems in a two configuration; one, it is sizing problem, and second, that's what is rating problem.

Now, when I say you need to do your rating problem, the question will come how do we do it, okay. I am having this compressor with me, but what will I do with that? Now, the question need to come with you is: Do I know my flow parameters? When I say my flow parameters, I will say, Do I know my pressure? Do I have details about the temperature? Do I have the details about the velocity? Based on the blades which are available, you can calculate what all are my flow angles, okay.

So, now, you know, this is what is a challenging task. So, when we say this is what is a rating problem there you need to do your calculation based on what all data that's what is available with you. Now, most of the engines when we are looking at those engines, they are having their past data available, okay. So, what all data that may be available with you are maybe pressure; when I say pressure, maybe at the hub, my static pressure at the tip, my static pressure that may be known to me.

By using thermocouples, we can have the temperature value that's what is available with me, okay. Maybe for design configuration, mass flow rate is known to me or it may not be known to me. Rotational speed data that is also available with me. So, now the question is with all this database, I may need to check it what will be my pressure ratio. So, do not you think this is what is a different kind of thought process?

Now, when we are discussing about all these things, that is where our fundamental knowledge that's what is applicable with. Now, what all we have learned in sense of our radial equilibrium, that's what will be coming into the picture. What all learn in sense of my critical Mach number, that's what will be coming into the picture. And based on all these understandings, we will be doing, we will be solving a problem, that's what is called rating problem, okay.

So, it is not only that you will be rating with, it may be possible with this data you are able to diagnose what all are the modifications or the changes that's what has happened in compressor. So, there is a trend that's what is say engine manufacturing company, there is one more form that's what is coming, that's what is called MRO, that's what is Maintenance Repair and Operation; where, people of such kind of knowledge they are of need, they are of demand, okay.

So, with this all these fundamentals what all we have learned, we will try to learn how do we solve this kind of problems. It says the aerodynamic design of axial flow compressors and fans, here we will be solving few numerical based on our requirement for the rating problem. So, in next lecture, we will be solving one of the numerical, that's what will be based on what all data that's what is available to us.

And based on those available data, we will try to predict what is the performance of that machine, okay. So, this is what all we are going to discuss with. Today what all we have discussed is in the sense of introduction of critical Mach number. And this critical Mach number as we have discussed, that's what we will be using for defining local Mach number at particular station in our compressor, this also can be used for defining the local Mach number for the turbines. So, here we are stopping with. Thank you, thank you very much for your kind attention!